

# Computational Phonology Workshop

## Introduction & Tutorial

Thomas Graf

Stony Brook University  
[mail@thomasgraf.net](mailto:mail@thomasgraf.net)  
<http://thomasgraf.net>

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# Outline

- 1** The Subregular Enterprise
- 2** (Tier-Based) Strictly Local Phonotactics
- 3** Subregular Mappings for Phonology
- 4** (Tier-Based) Strictly Local Syntax

# Computational View of Language

In formal language theory, string sets are classified according to their formal complexity.

regular < context-free < mildly context-sensitive < ...

Phonology

Morphology

Syntax

This allows predictions for

- ▶ typology (e.g. no center embedding in phonology)
- ▶ learning (required input, what must be in UG)
- ▶ cognitive architecture (type of memory, structural inference rules)

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Kaplan and Kay (1994)

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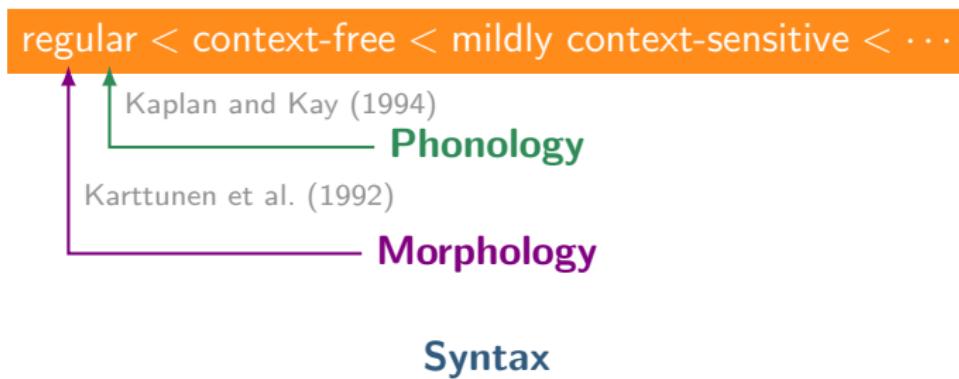
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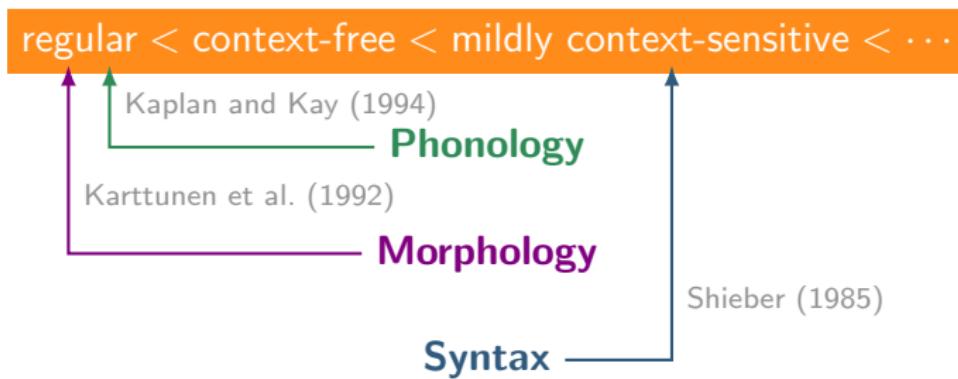


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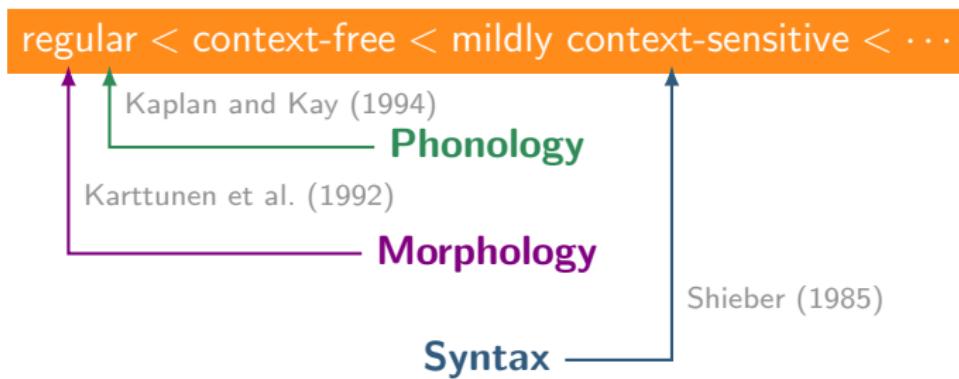


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# Too Many Patterns are Regular

## ► Problem

- ▶ All phonological and morphological patterns are regular.
- ▶ But not all regular patterns occur in phonology.
- ▶ Regularity is **too loose an upper bound**.

### Examples of Regular yet Unattested Patterns

- ▶ First-last consonant harmony
- ▶ Every word with a plosive contains an open syllable.
- ▶ Words with at least 3 suffixes must have exactly 5 prefixes.

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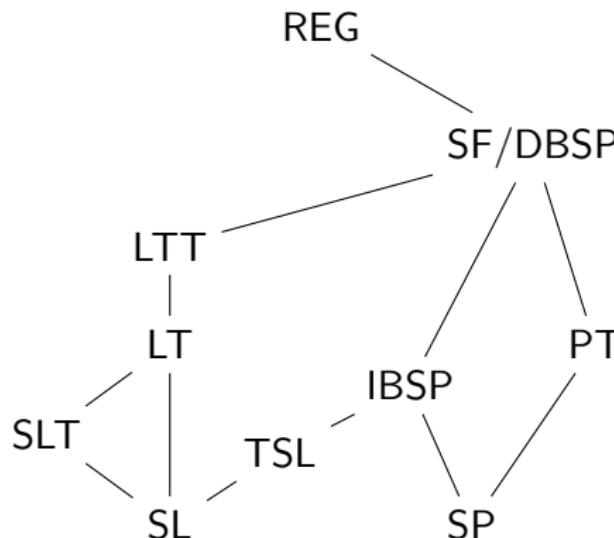
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# Subregular Languages

Often forgotten: hierarchy of **subregular languages**

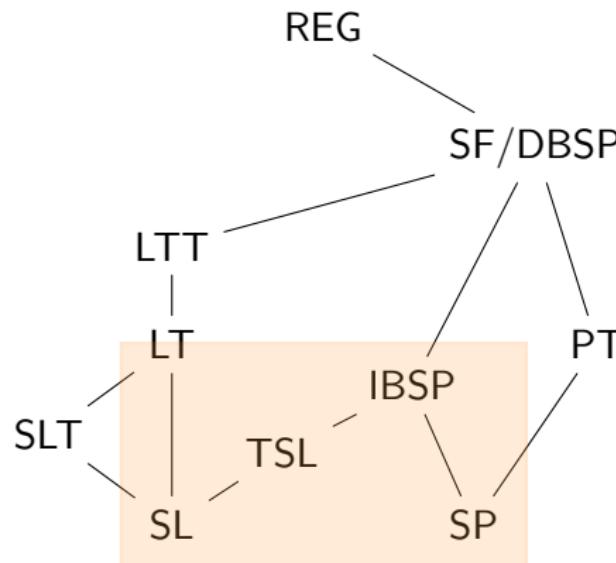
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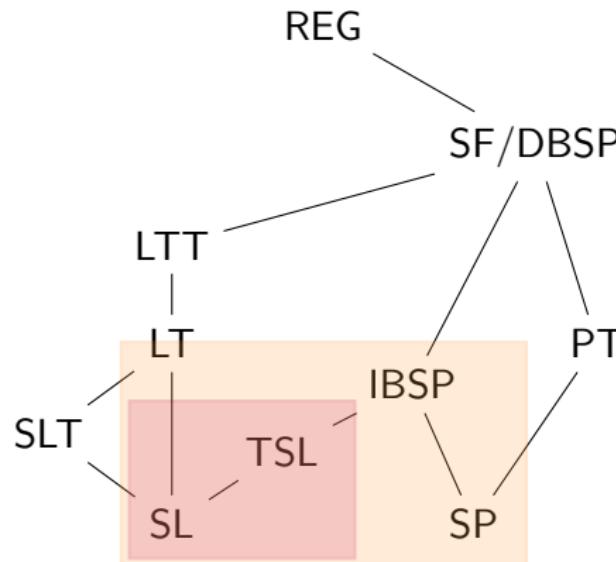
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# SL: Strictly Local

- ▶ SL formalizes **local dependencies**.
- ▶ SL grammars are collections of markedness constraints that are
  - ▶ hard/non-violable,
  - ▶ locally bounded.

## Strictly Local Grammars & Languages

$SL_n$  grammar finite set of forbidden  $n$ -grams

$SL_n$  language all strings except those with forbidden  $n$ -grams

## Example: SL Constraints

Process	Constraint	Forbidden $n$ -grams
Word-final devoicing	*[+voice]×	z×
Intervocalic voicing	*V[-voice]V	asa, asi, ..., isa, isi, ..., afa, afi, ..., ifa, ifi, ...
CV template	* $\times$ V	$\times$ a, $\times$ i, ...
	*CC	pp, pb, ..., bp, bb, ...
	*VV	aa, ai, ..., ia, ii, ...
	*C×	p×

# SL is Too Weak

- ▶ SL grammars only handle locally bounded dependencies.
- ▶ But some processes in phonology are unbounded.

Samala Sibilant Harmony (Heinz 2015:16)

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We can make Samala SL-like if we **create new locality domains**.

## Tier-Based Strictly Local Grammars & Languages

$TSL_n$  grammar finite set of forbidden  $n$ -grams + tier alphabet

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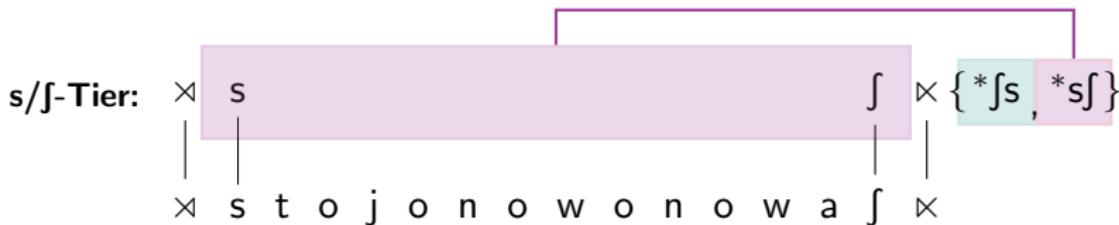
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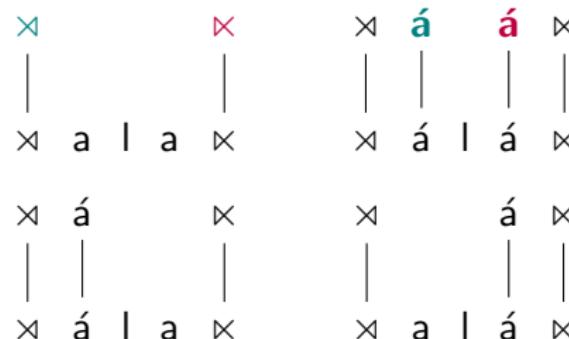


# Example: Stress Assignment

Culminativity every word has exactly one primary stress

**Tier** contains segments with primary stress

**n-grams**  $\textcolor{teal}{\times}\textcolor{red}{\times}$  and  $\textcolor{teal}{\acute{s}}\textcolor{red}{s}$



# What TSL Cannot do

- ▶ **First-Last Harmony**

because first/last is not a segmental property

- ▶ **Plosive implies open syllable**

cannot enforce Boolean conditionals

- ▶ **Mix local and non-local dependencies**

e.g. sibilant harmony does not apply to sibilants followed by a nasal

## Attested Patterns Beyond TSL?

- ▶ **Tone Plateauing** (but IBSP; Graf 2016)

- ▶ **Non-Final Rhol** [Baek's talk]

- ▶ **Multiple Harmony** [Aksënova's talk]

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# Complexity of Phonology

- ▶ All local phonological constraints are SL.
- ▶ All segmental long-distance constraints are TSL.
- ▶ Suprsegmental constraints (tone, stress) may go beyond TSL.  
(Graf 2010a,b; Jardine 2015)

# Cognitive Implications

- ▶ SL and TSL languages are **learnable** from positive data.  
(Heinz et al. 2012; Jardine and Heinz 2016)
  - ▶ UG: specifies upper bound on size of  $n$ -grams
  - ▶ memorize which sequences have not been seen so far
  - ▶ induce tier (more complex)
  - ▶ learning input can be relatively small
- ▶ What cognitive resources are required?
  - ▶ Only memorization of the last  $n$  segments of a specific type
  - ▶ For most processes  $n \leq 3$ , and for all  $n \leq 7$
  - ▶ Fits **within bounds of human working memory**

# Interim Summary: Phonotactics

- ▶ Natural languages have **TSL phonotactics**.
- ▶ gives tighter bound on typology
- ▶ solves poverty of stimulus by greatly simplifying learning
- ▶ reduces cognitive resource requirements

Next

- ▶ phonological mappings
- ▶ SL & TSL syntax

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# Phonological Mappings

- ▶ So far we have only considered phonotactics.
- ▶ But mappings from underlying representations to surface forms can be studied, too.
- ▶ Regular mappings are enough.  
(Kaplan and Kay 1994)
- ▶ What about **subregular mappings?**

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	ISL Output	OSL Output
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<b>Input string:</b> CVCV р CVCV		

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  - output string: output strictly local (**OSL**)
- ▶ Mapping never considers both input and output.

## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCVCV	CVVCVCVC
<b>Input string:</b> CVCVCVCV	CVCCCCCC	C

# Strictly Local Mappings

## Strictly Local (SL; Chandlee 2014)

- ▶ Move through string from left to right.
- ▶ Rewrite  $x$  as  $y$  based on previous  $n$  symbols in...
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## Example

**SL<sub>3</sub>-mapping:**  $V \rightarrow C \mid VC_$   
**Input string:** CVCVCVCV

**ISL Output**  
CVCVCVCV  
CVCCCCCC

**OSL Output**  
CVVCVCVC  
C

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## Example

**SL<sub>3</sub>-mapping:**  $V \rightarrow C \mid VC_$   
**Input string:** CVCVCVCV

**ISL Output**  
CVCVCVCV  
CVCCCCCC

**OSL Output**  
CV CVCVCV  
CV

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## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCVCV	CV <b>C</b> VCVCV
<b>Input string:</b> CVCVCVCV	CVCCCCCC	CV

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**SL<sub>3</sub>-mapping:**  $V \rightarrow C \mid VC_$   
**Input string:** CVCVCVCV

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CVCVCVCV  
CVCCCCCC

**OSL Output**  
CV**CVCVCV**  
**CV**

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## Example

**SL<sub>3</sub>-mapping:**  $V \rightarrow C \mid VC_$   
**Input string:** CVCVCVCV

**ISL Output**  
CVCVCVCV  
CVCCCCCC

**OSL Output**  
CV**C**VCVCV  
**CVC**

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## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCVCV	CVC <span style="color:red">V</span> CVCV
<b>Input string:</b> CVCVCVCV	CVCCCCCC	CVC

# Strictly Local Mappings

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## Example

**SL<sub>3</sub>-mapping:**  $V \rightarrow C \mid VC_$   
**Input string:** CVCVCVCV

**ISL Output**  
CVCVCVCV  
CVCCCCCC

**OSL Output**  
CVC~~V~~CVCV  
C~~V~~C

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## Example

**SL<sub>3</sub>-mapping:**  $V \rightarrow C \mid VC_$   
**Input string:** CVCVCV

**ISL Output**  
CVCVCV  
CVCCCC

**OSL Output**  
CVCVCVCV  
CVCC

# Strictly Local Mappings

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## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCV	CVCV <b>C</b> CV
<b>Input string:</b> CVCVCV	CVCCCC	CVCC

# Strictly Local Mappings

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## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCV	CVCVCV
<b>Input string:</b> CVCVCV	CVCCCC	CVCC

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<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCV	CVCVCV
<b>Input string:</b> CVCVCV	CVCCCC	CVCCC

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<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCV	CVCVCV
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## Example

**SL<sub>3</sub>-mapping:**  $V \rightarrow C \mid VC_$   
**Input string:** CVCVCVCV

**ISL Output**  
CVCVCVCV  
CVCCCCCC

**OSL Output**  
CVCVC**V**CV  
CVC**CC**

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	<b>ISL Output</b>	<b>OSL Output</b>
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<b>Input string:</b> CVCVCV	CVCCCC	CVCCV

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	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCV	CVCVCV
<b>Input string:</b> CVCVCV	CVCCCC	CVCCV

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<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCV	CVCVCV
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## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCV	CVCVCV
<b>Input string:</b> CVCVCV	CVCCCC	CVCCC

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## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCVCV	CVCVCVC <span style="color:red">V</span>
<b>Input string:</b> CVCVCVCV	CVCCCCCC	CVCCCVC

# Strictly Local Mappings

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## Example

	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCVCV	CVCVCVCV
<b>Input string:</b> CVCVCVCV	CVCCCCCC	CVCCCVC

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	<b>ISL Output</b>	<b>OSL Output</b>
<b>SL<sub>3</sub>-mapping:</b> $V \rightarrow C \mid VC_$	CVCVCVCV	CVCVCVCV
<b>Input string:</b> CVCVCVCV	CVCCCCCC	CVCCC <b>VCC</b>

# How Many Processes are SL?

- ▶ All locally bounded processes  
assimilation, dissimilation, vowel harmony, even metathesis
- ▶ **But:** mappings where local processes interact may not be SL  
multiple metathesis **[Takahasi's talk]**
- ▶ Some processes are not locally bounded and thus not SL.  
sibilant harmony, tone plateauing, sour grapes harmony

# A Note on TSL

Every  $TSL_n$  grammar can be decomposed into

- 1 an **ISL<sub>1</sub>** function (the tier projection), and
- 2 an  $SL_n$  grammar.

## An Interesting Puzzle

- ▶ What happens if we use an  $ISL_k$  function for tier projection?
- ▶ Addressed in **Aniello De Santo**'s talk

# A Note on TSL

Every  $TSL_n$  grammar can be decomposed into

- 1 an **ISL<sub>1</sub>** function (the tier projection), and
- 2 an  $SL_n$  grammar.

## An Interesting Puzzle

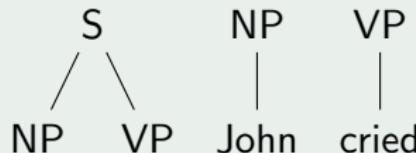
- ▶ What happens if we use an  $ISL_k$  function for tier projection?
- ▶ Addressed in **Aniello De Santo**'s talk

# (Tier-Based) Strictly Local Syntax

- ▶ SL tree grammars are common in computational linguistics:  
**context-free grammars**
- ▶ By adding tier projection, we get TSL tree grammars.

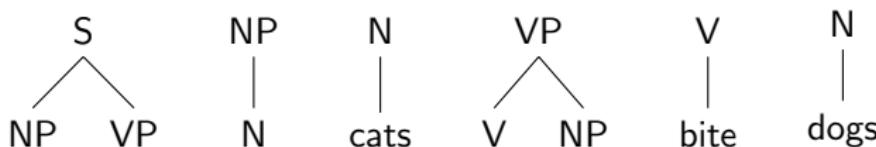
## Example: CFGs as $SL_2$ Tree Grammars

S → NP VP  
NP → John  
VP → cried

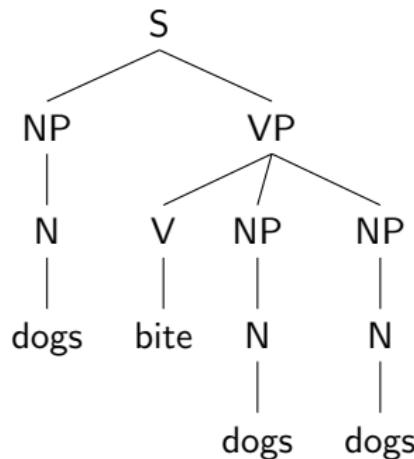


# Example: An Illicit Tree

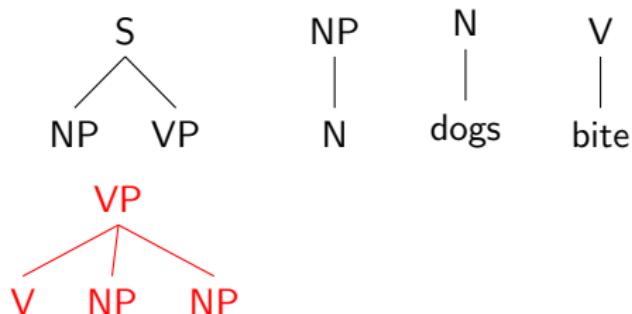
## SL<sub>2</sub> Tree Grammar



## Example Tree

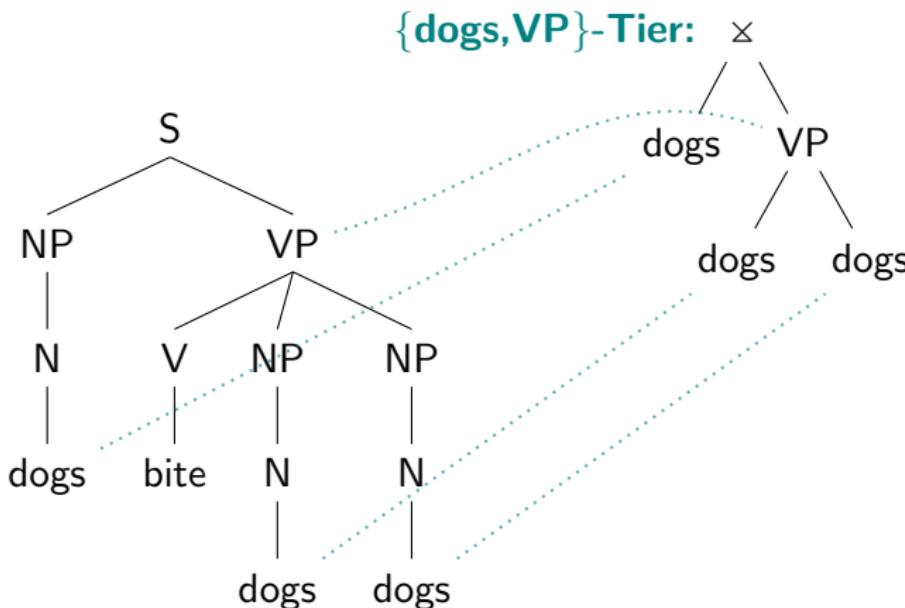


## Tree Bigrams of Example Tree



# Tier Projection for Trees

Just as for strings, we can project tiers for trees.  
(Graf and Heinz 2016)



# Towards TSL-Syntax

While TSL-Syntax is still young, it holds promise:

- ▶ movement dependencies are TSL (Graf and Heinz 2016)
- ▶ Mandarin negation in **Hongchen Wu's talk**
- ▶ scope ambiguities in **Lei Liu's blitz talk**

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