1 Abstract

2 Introduction

Short paragraph defining homogenization and how it can be used to development a better understanding of rock mechanics. Statement about gap in the literature addressed in this article, i.e., not yet applied to HF.

Discussion of the natural fractured rocks as a multiscale material.

Brief discussion of literature describing multiscale (upscaling) modelling methods. Concurrent vs hierarchical multiscale analysis. See my review article on my website and [Gracie and Belytschko(2011)Gracie, and Belytschko]. Justify that a hierarchical approach is preferable here, do to long simulated time; concurrent approach is to restrictive in terms of time step size. Also justify/explain selection of homogenization instead of an alternative method of upscaling.

Discussion of literature around modeling using homogenization.

- Continuum to continuum and DEM to Continuum. rigid versus deformable particals
- Analogy to debonding of particle -¿ note lack of contact.
- What are the common characteristics of the models.
- What are the unique characteristics of the models.

Overview of DEM literature with a focus on HF

Outline of the article

3 Distinct Element Method

4 Homogenization Approach

4.1 Stress Homogenization

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A} \int_{\Omega} \boldsymbol{\sigma} dA \tag{1}$$

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A} \left[\int_{\Omega_r} \boldsymbol{\sigma} dA_r + \int_{\Omega_f} \boldsymbol{\sigma} dA_f \right] \tag{2}$$

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A} \left[\sum_{i=1}^{N_b} \boldsymbol{\sigma}^i A_b^i + \sum_{i=1}^{N_d} \boldsymbol{\sigma}^i A_d^i \right] \tag{3}$$

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A} \left[\sum_{i=1}^{N_b} \sum_{j=1}^{N_z} \boldsymbol{\sigma}_z^{ij} A_z^{ij} + \sum_{i=1}^{N_d} p_d^i \mathbf{I} A_d^i \right] \tag{4}$$

4.2 Strain Homogenization

$$\boldsymbol{\epsilon} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right] \tag{5}$$

$$\langle\boldsymbol{\epsilon}\rangle^{\Omega} = \frac{1}{A}\int_{\Omega}\boldsymbol{\epsilon} dA \tag{6}$$

$$\langle\boldsymbol{\epsilon}\rangle^{\Omega} = \frac{1}{A}\left[\int_{\Omega_r}\boldsymbol{\epsilon} dA_r + \int_{\Omega_f}\boldsymbol{\epsilon} dA_f\right] \tag{7}$$

$$\langle\boldsymbol{\epsilon}\rangle^{\Omega} = \frac{1}{2A}\left[\int_{\Omega_r}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right] dA_r + \int_{\Omega_f}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right] dA_f\right] \tag{8}$$

$$\langle\boldsymbol{\epsilon}\rangle^{\Omega} = \frac{1}{2A}\left[\oint_{\Gamma_r}\left[\mathbf{u}\otimes\mathbf{n} + \mathbf{n}\otimes\mathbf{u}\right] d\Gamma_r + \oint_{\Gamma_f}\left[\mathbf{u}\otimes\mathbf{n} + \mathbf{n}\otimes\mathbf{u}\right] d\Gamma_f\right] \tag{9}$$

$$\langle\boldsymbol{\epsilon}\rangle^{\Omega} = \frac{1}{2A}\left[\sum_{i=1}^{N_{rb}}\left[\mathbf{u}^i_{rb}\otimes\mathbf{n}^i_{rb} + \mathbf{n}^i_{rb}\otimes\mathbf{u}^i_{rb}\right] L^i_{rb} + \sum_{i=1}^{N_{db}}\left[\mathbf{u}^i_{db}\otimes\mathbf{n}^i_{db} + \mathbf{n}^i_{db}\otimes\mathbf{u}^i_{db}\right] L^i_{db}\right] \tag{10}$$

5 Damage Plasticty Model For Quasi-Brittle Materials

Damage plasticity model based on Strain can be decomposed into elastic and plastic components:

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^{el} + \boldsymbol{\epsilon}^{pl} \tag{11}$$

Taking the time derivative gives the decomposition of the strain rate, $\dot{\boldsymbol{\epsilon}}$:

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^{el} + \dot{\boldsymbol{\epsilon}}^{pl} \tag{12}$$

The constitutive stress-strain relationship including a scalar damage parameter, $\mathbf{D}$ can be written as follows:

$$\boldsymbol{\sigma} = (1-\mathbf{D})\mathbf{E} : \boldsymbol{\epsilon}^{\boldsymbol{el}} \tag{13}$$

For simplicity, the damaged elastic stiffness is described as the reduced stiffness due to the damage:

$$\mathbf{E}^{\mathbf{d}} = (1-\mathbf{D})\mathbf{E} \tag{14}$$

Substituting 11 and 14 into 13 results in the following:

$$\boldsymbol{\sigma} = \mathbf{E}^{\mathbf{d}} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{pl}) \tag{15}$$

Using the "usual notions of CDM" (find reference), the effective stress. $\bar{\boldsymbol{\sigma}}$, can be defined as:

$$\bar{\boldsymbol{\sigma}} = \mathbf{E} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{pl}) \tag{16}$$

Such that the cauchy stress tensor can be realted to the effective stress tensor as follows:

$$\boldsymbol{\sigma} = (1 - \mathbf{D})\bar{\boldsymbol{\sigma}} \tag{17}$$

The nature of the damage evolution is assumed to be a function of the effective stress and the equivalent plastic strain, $\bar{\boldsymbol{\epsilon}}^{\boldsymbol{pl}}$:

$$\mathbf{D} = \mathbf{D}(\bar{\boldsymbol{\sigma}}, \bar{\boldsymbol{\epsilon}}^{\boldsymbol{pl}}) \tag{18}$$

In this formulation, the brittle nature of rock neccessitates seperate characterization of tensile and compressive damage. In the case where a rock sample fails completely in tension, (i.e. the tensile stiffness becomes effectively 0), the compressive strength remains intacts to a fairly high degree such that two seperate damage variables for tensile damage and compressive damage. As such, the equivalent plastice strain is also considered seperately for tension and compression and is represented as follows:

$$\bar{\boldsymbol{\epsilon}}^{\boldsymbol{pl}} = \begin{bmatrix} \bar{\boldsymbol{\epsilon}}_{\boldsymbol{t}}^{\boldsymbol{pl}} \\ \bar{\boldsymbol{\epsilon}}_{\boldsymbol{c}}^{\boldsymbol{pl}} \end{bmatrix} \tag{19}$$

5.1 Governing equations of the coarse-scale (continuum) model

Consider the dynamic equilibrium of a naturally fractured rock mass Ω. Let Γ denote the boundary of Ω and let Γ be divided into mutually exclusive sets Γ_u and Γ_t. The body contains a set of natural fractures denoted by Γ_{cr} and is subjected to a body force $\mathbf{g}$. Let material points in the the undeformed and the deformed configuration be denoted by $\mathbf{X}$ and $\mathbf{x}$, respectively. Let $\mathbf{u}(\mathbf{X}, t) = \mathbf{x}(\mathbf{X}, t) - \mathbf{X}$ denote the displacement of material point $\mathbf{x}$ at time t. Equilibirum of Ω is governed by

$$\rho_s \ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{g},\ \forall \mathbf{x} \in \Omega, t \geq 0, \tag{20}$$

in which $\ddot{\mathbf{u}} = \ddot{\mathbf{u}}(\mathbf{x}, t)$ denotes the second partial derivative of the displacement field and ρ_s is the density of the rock mass.

5.2 FEM Forumulation

6 Constitutive Relationship Parameter Estimation

6.1 Levenburg-Marquardt Algorithm

6.2 Parameter Initial and Boundary Estimates

7 Material Testing Example

8 Results

9 Conclusions

References

[Gracie and Belytschko(2011)Gracie, and Belytschko] Gracie, R.; Belytschko, T. *International Journal for Numerical Methods in Engineering* **2011**, *86*, 575–597.