1 Abstract

2 Introduction

Short paragraph defining homogenization and how it can be used to development a better understanding of rock mechanics. Statement about gap in the literature addressed in this article, i.e., not yet applied to HF.

Discussion of the natural fractured rocks as a multiscale material.

Brief discussion of literature describing multiscale (upscaling) modelling methods. Concurrent vs hierarchical multiscale analysis. See my review article on my website and [Gracie and Belytschko(2011)Gracie, and Belytschko]. Justify that a hierarchical approach is preferable here, do to long simulated time; concurrent approach is to restrictive in terms of time step size. Also justify/explain selection of homogenization instead of an alternative method of upscaling.

Discussion of literature around modeling using homogenization. Continuum to continuum and DEM to Continuum. What are the common characteristics of the models. What are the unique characteristics of the models.

Overview of DEM literature with a focus on HF

Outline of the article

3 Formulation

3.1 Governing equations of the coarse-scale (continuum) model

Consider the dynamic equilibrium of a naturally fractured rock mass Ω. Let Γ denote the boundary of Ω and let Γ be divided into mutually exclusive sets Γ_u and Γ_t. The body contains a set of natural fractures denoted by Γ_{cr} and is subjected to a body force $\mathbf{g}$. Let material points in the the undeformed and the deformed configuration be denoted by $\mathbf{X}$ and $\mathbf{x}$, respectively. Let $\mathbf{u}\left(\mathbf{X},t\right) = \mathbf{x}\left(\mathbf{X},t\right) - \mathbf{X}$ denote the displacement of material point $\mathbf{x}$ at time t. Equilibirum of Ω is governed by

$$\rho_s \ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{g}, \ \forall \mathbf{x} \in \Omega, t \geq 0, \tag{1}$$

in which $\ddot{\mathbf{u}} = \ddot{\mathbf{u}}\left(\mathbf{x},t\right)$ denotes the second partial derivative of the displacement field and ρ_s is the density of the rock mass.

3.2 Governing equations of the fine-scale (discrete) model

3.3 Homogenization of an HF fine-scale model

3.4 Strain Homogenization

$$\langle \boldsymbol{\sigma} \rangle_\Omega = \frac{1}{|A|} \int_\Omega \boldsymbol{\sigma} dA \tag{2}$$

References

[Gracie and Belytschko(2011)Gracie, and Belytschko] Gracie, R.; Belytschko, T. *International Journal for Numerical Methods in Engineering* **2011**, *86*, 575–597.