1 Abstract

2 Introduction

Short paragraph defining homogenization and how it can be used to development a better understanding of rock mechanics. Statement about gap in the literature addressed in this article, i.e., not yet applied to HF.

Discussion of the natural fractured rocks as a multiscale material.

Brief discussion of literature describing multiscale (upscaling) modelling methods. Concurrent vs hierarchical multiscale analysis. See my review article on my website and [Gracie and Belytschko(2011)Gracie, and Belytschko]. Justify that a hierarchical approach is preferable here, do to long simulated time; concurrent approach is to restrictive in terms of time step size. Also justify/explain selection of homogenization instead of an alternative method of upscaling.

Discussion of literature around modeling using homogenization.

- Continuum to continuum and DEM to Continuum. rigid versus deformable particals
- Analogy to debonding of particle -¿ note lack of contact.
- What are the common characteristics of the models.
- What are the unique characteristics of the models.

Overview of DEM literature with a focus on HF

Outline of the article

3 Formulation

3.1 Governing equations of the coarse-scale (continuum) model

Consider the dynamic equilibrium of a naturally fractured rock mass Ω. Let Γ denote the boundary of Ω and let Γ be divided into mutually exclusive sets Γ_u and Γ_t. The body contains a set of natural fractures denoted by Γ_{cr} and is subjected to a body force $\mathbf{g}$. Let material points in the the undeformed and the deformed configuration be denoted by $\mathbf{X}$ and $\mathbf{x}$, respectively. Let $\mathbf{u}(\mathbf{X}, t) = \mathbf{x}(\mathbf{X}, t) - \mathbf{X}$ denote the displacement of material point $\mathbf{x}$ at time t. Equilibirum of Ω is governed by

$$\rho_s \ddot{\mathbf{u}} = \nabla \cdot \boldsymbol{\sigma} + \mathbf{g}, \; \forall \mathbf{x} \in \Omega, t \geq 0, \tag{1}$$

in which $\ddot{\mathbf{u}} = \ddot{\mathbf{u}}(\mathbf{x}, t)$ denotes the second partial derivative of the displacement field and ρ_s is the density of the rock mass.

3.2 Governing equations of the fine-scale (discrete) model

4 Homogenization Approach

4.1 Stress Homogenization

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A} \int_{\Omega} \boldsymbol{\sigma} dA \tag{2}$$

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A} \left[\int_{\Omega_r} \boldsymbol{\sigma} dA_r + \int_{\Omega_f} \boldsymbol{\sigma} dA_f \right] \tag{3}$$

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A_r} \sum_{i=1}^{N_b} \boldsymbol{\sigma}^i A_b^i + \frac{1}{A_f} \sum_{i=1}^{N_d} \boldsymbol{\sigma}^i A_d^i \tag{4}$$

$$\langle \boldsymbol{\sigma} \rangle^{\Omega} = \frac{1}{A_r} \sum_{i=1}^{N_b} \sum_{j=1}^{N_z} \boldsymbol{\sigma}_z^{ij} A_z^{ij} + \frac{1}{A_f} \sum_{i=1}^{N_d} p_d^i \mathbf{I} A_d^i \tag{5}$$

4.2 Strain Homogenization

$$\boldsymbol{\epsilon} = \frac{1}{2} \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] \tag{6}$$

$$\langle \boldsymbol{\epsilon} \rangle^{\Omega} = \frac{1}{A} \int_{\Omega} \boldsymbol{\epsilon} dA \tag{7}$$

$$\langle \boldsymbol{\epsilon} \rangle^{\Omega} = \frac{1}{A} \left[\int_{\Omega_r} \boldsymbol{\epsilon} dA_r + \int_{\Omega_f} \boldsymbol{\epsilon} dA_f \right] \tag{8}$$

$$\langle \boldsymbol{\epsilon} \rangle^{\Omega} = \frac{1}{2A} \left[\int_{\Omega_r} \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] dA_r + \int_{\Omega_f} \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] dA_f \right] \tag{9}$$

$$\langle \boldsymbol{\epsilon} \rangle^{\Omega} = \frac{1}{2A} \left[\oint_{\Gamma_r} \left[\mathbf{u} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u} \right] d\Gamma_r + \oint_{\Gamma_f} \left[\mathbf{u} \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{u} \right] d\Gamma_f \right] \tag{10}$$

$$\langle \boldsymbol{\epsilon} \rangle^{\Omega} = \frac{1}{2A} \left[\sum_{i=1}^{N_{rb}} \left[\mathbf{u}_{rb}^i \otimes \mathbf{n}_{rb}^i + \mathbf{n}_{rb}^i \otimes \mathbf{u}_{rb}^i \right] L_{rb}^i + \sum_{i=1}^{N_{db}} \left[\mathbf{u}_{db}^i \otimes \mathbf{n}_{db}^i + \mathbf{n}_{db}^i \otimes \mathbf{u}_{db}^i \right] L_{db}^i \right] \tag{11}$$

References

[Gracie and Belytschko(2011)Gracie, and Belytschko] Gracie, R.; Belytschko, T. *International Journal for Numerical Methods in Engineering* **2011**, *86*, 575–597.