Differences Between Discrete and Continuous Distributions

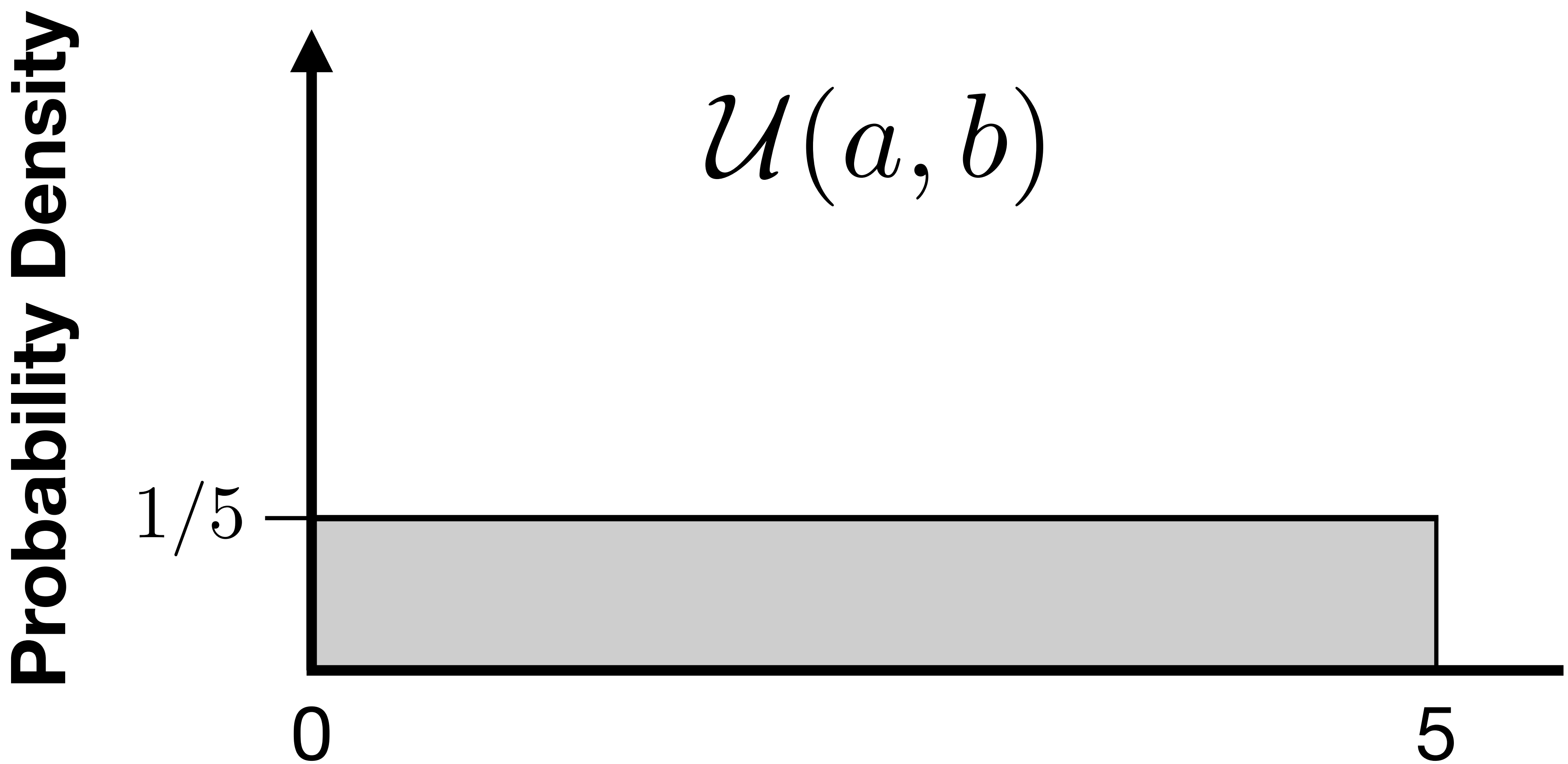
- Probability masses in discrete distributions sum to 1, while probability densities in continuous distributions integrate to 1.

- Probability densities can be greater than 1 (yet they can still integrate to 1).

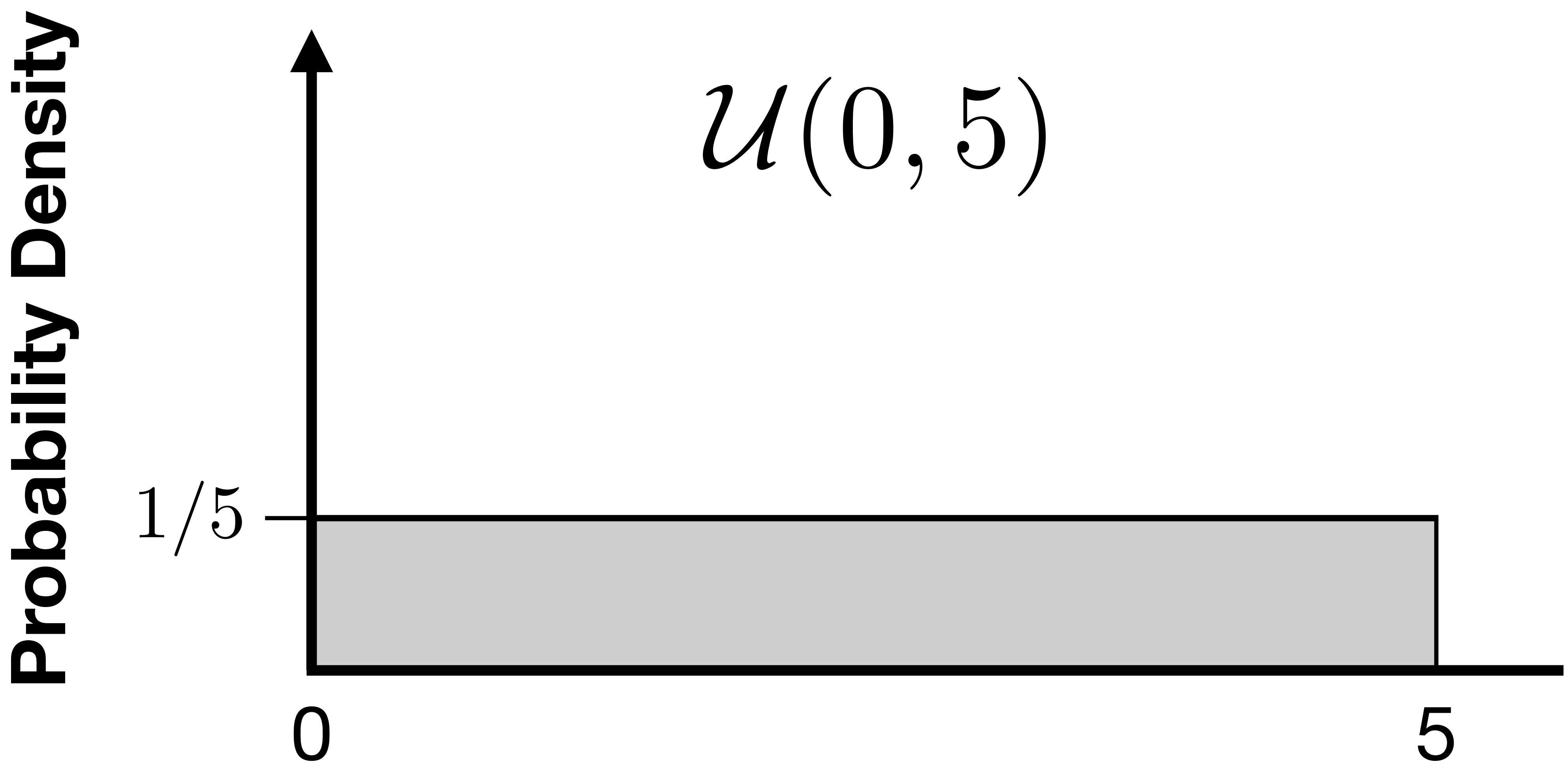
- The probability mass of any precise value (e.g., 1.000…) in a continuous distribution is always infinitely small.

Continuous Uniform Distribution

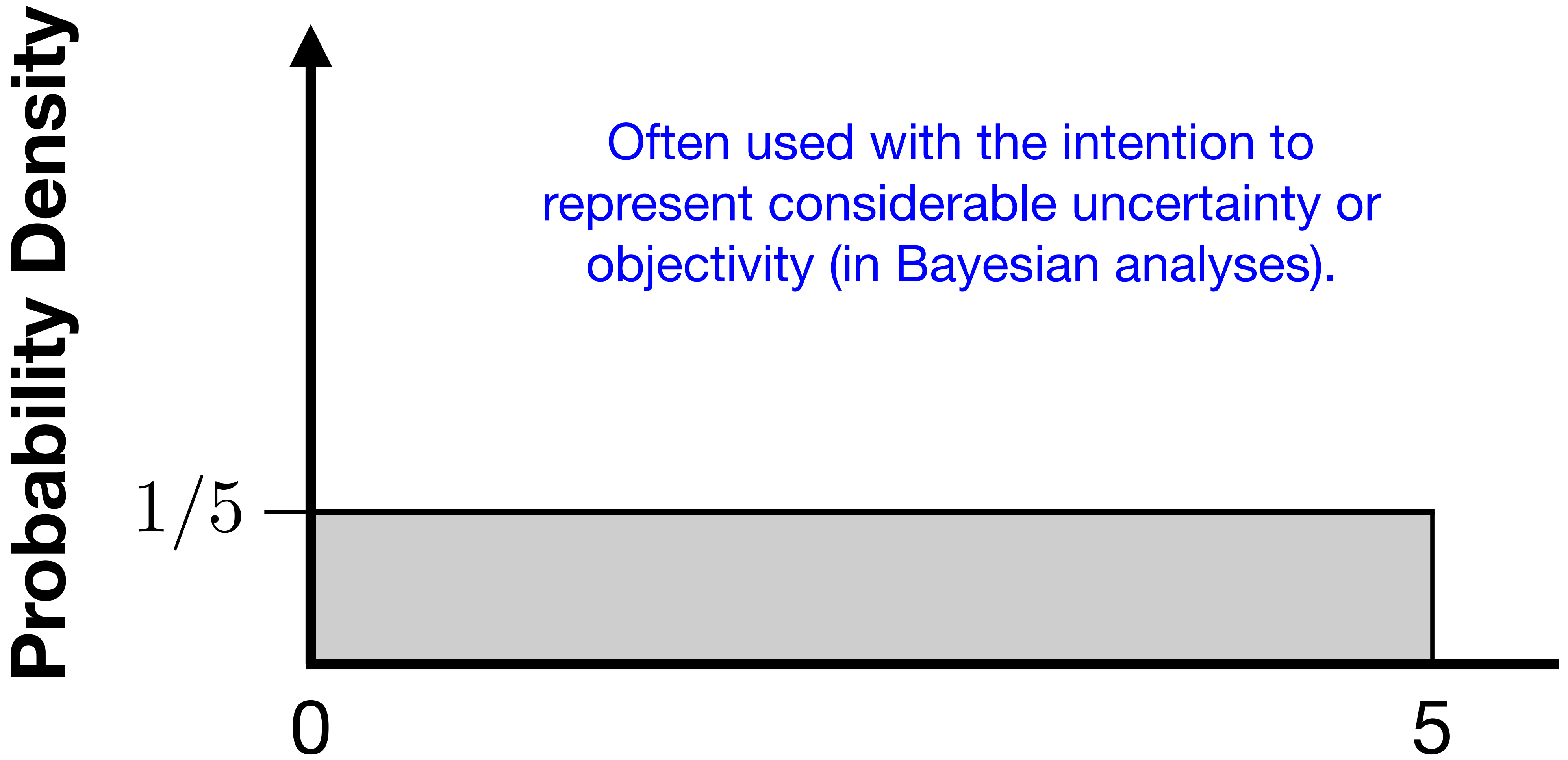
Continuous Uniform Distribution



Continuous Uniform Distribution



Continuous Uniform Distribution



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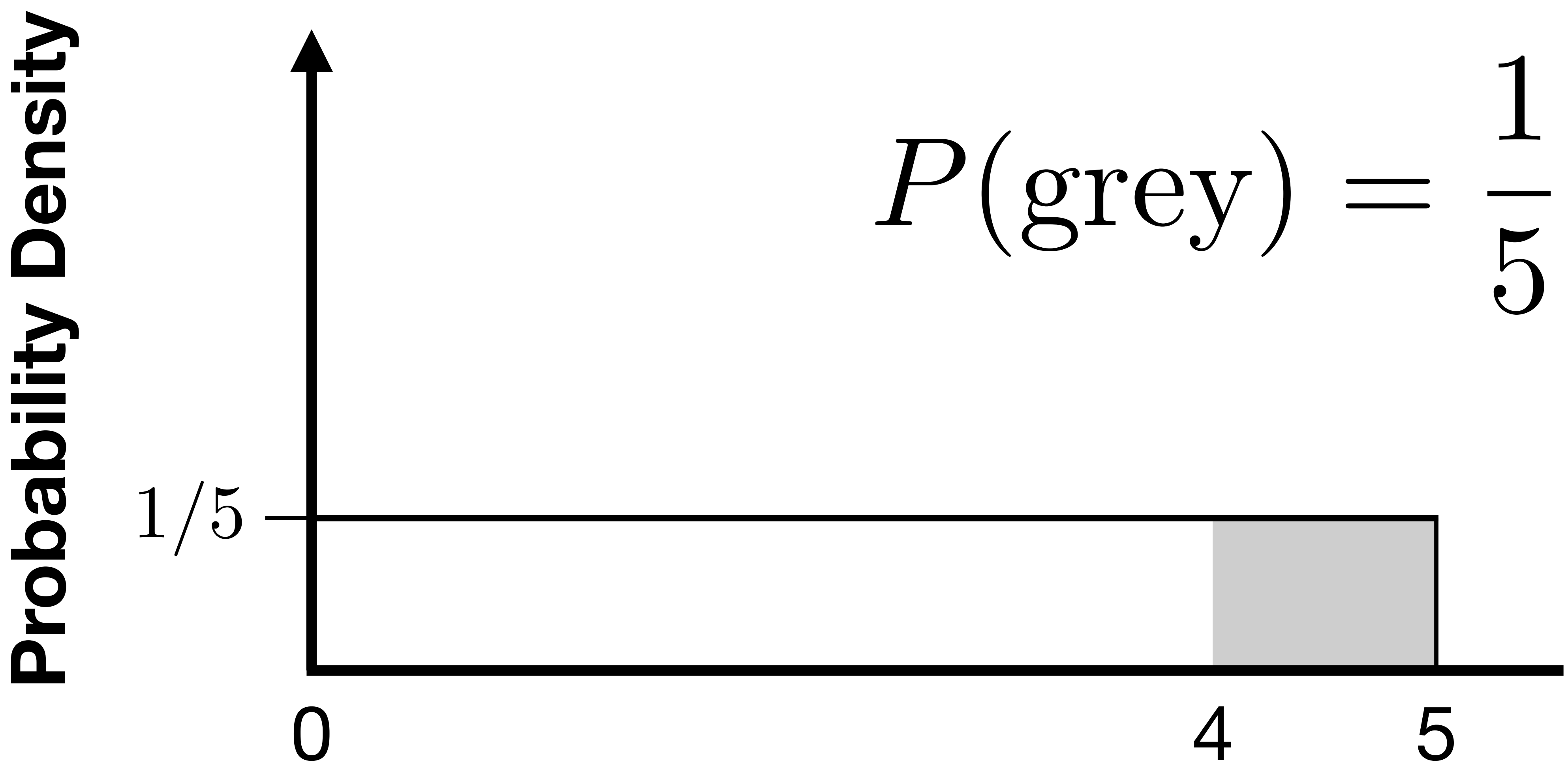
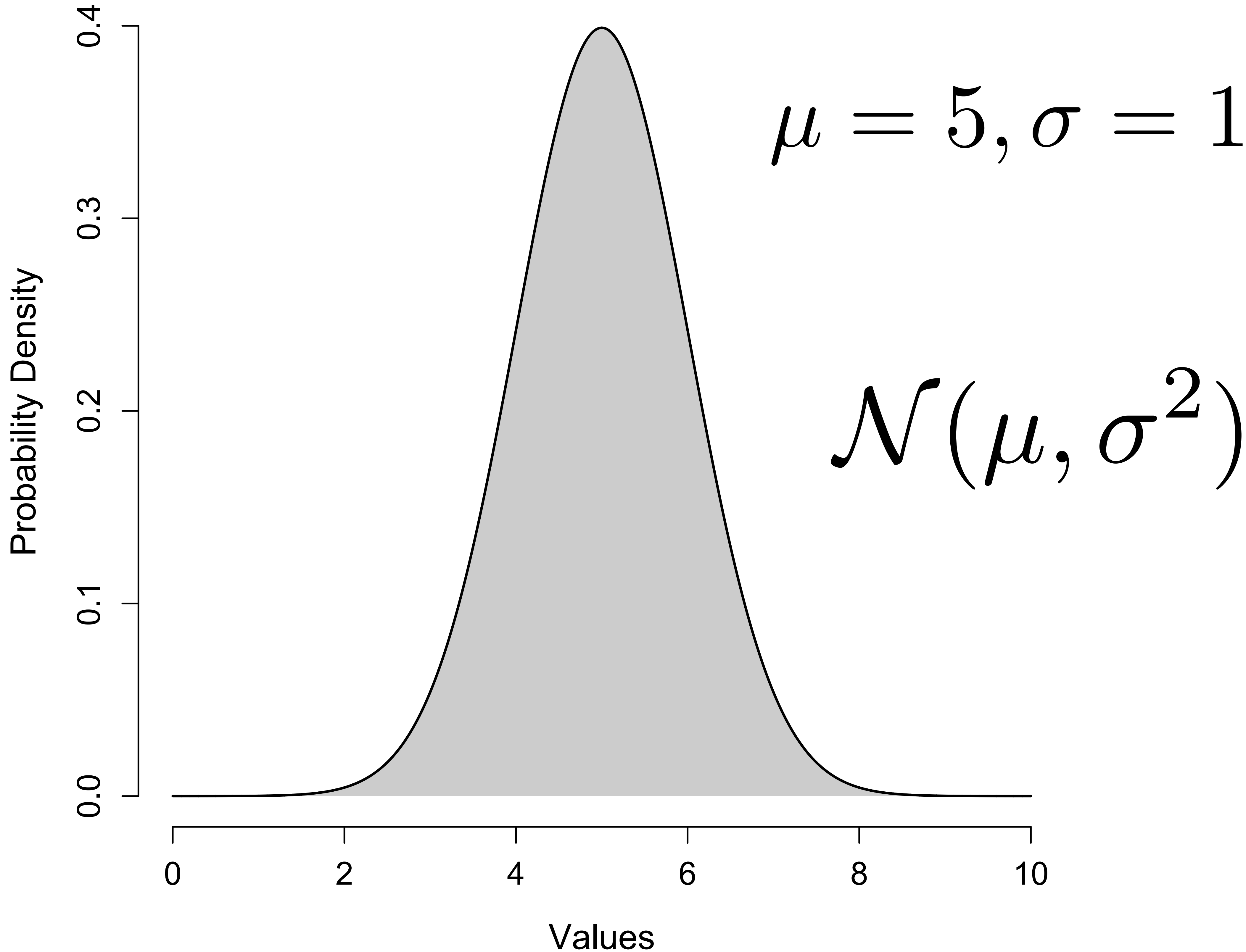


Normal Distribution



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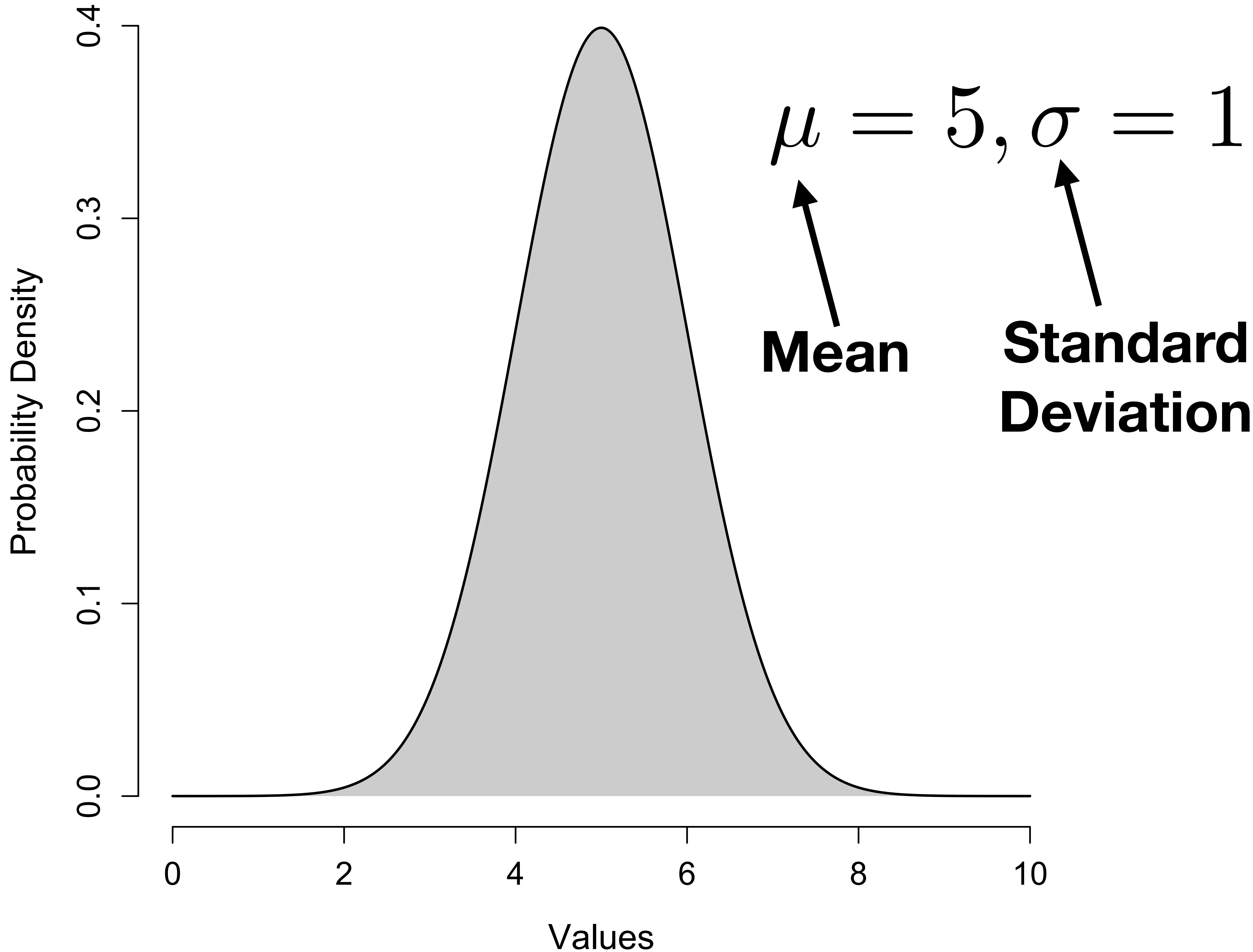
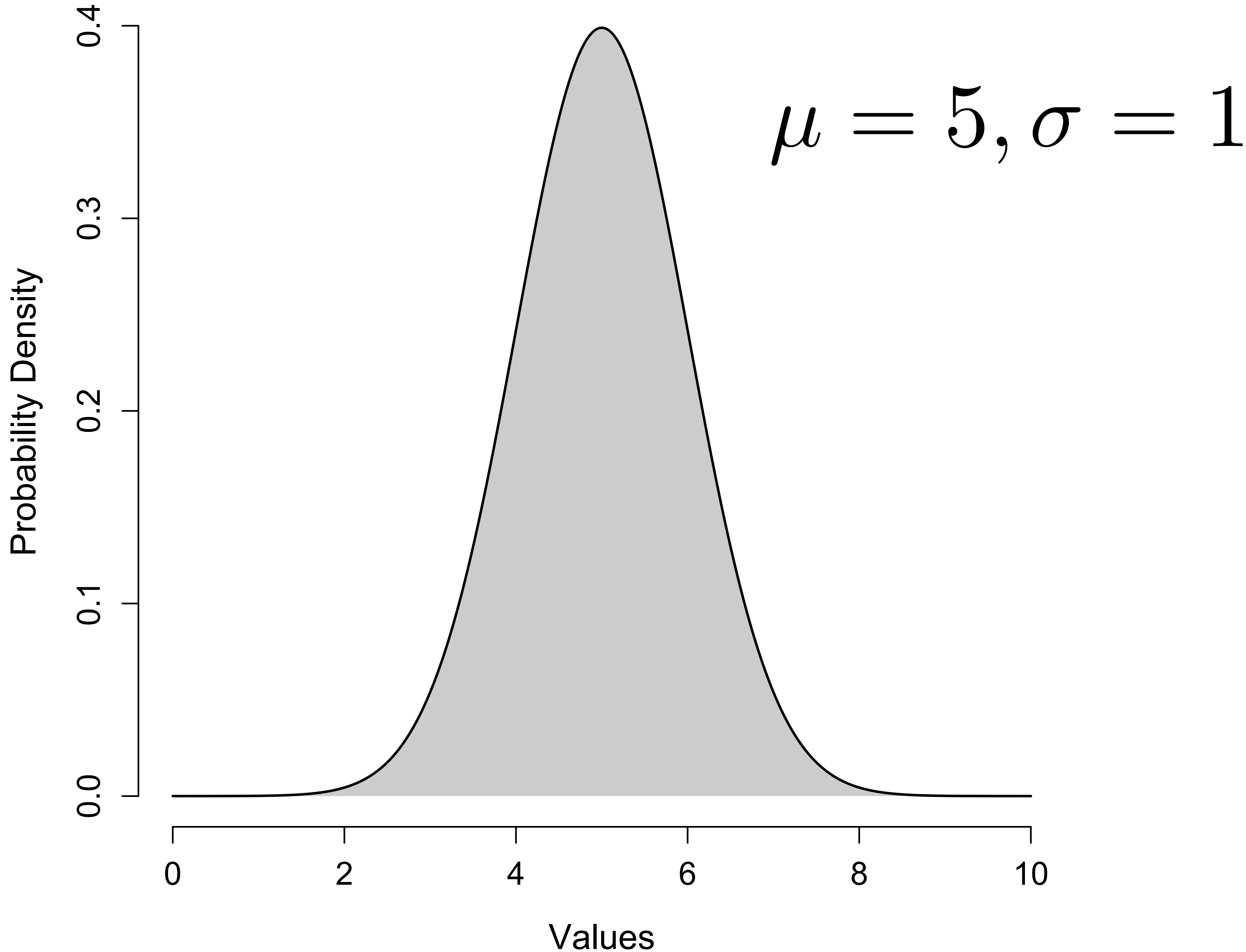


Normal Distribution



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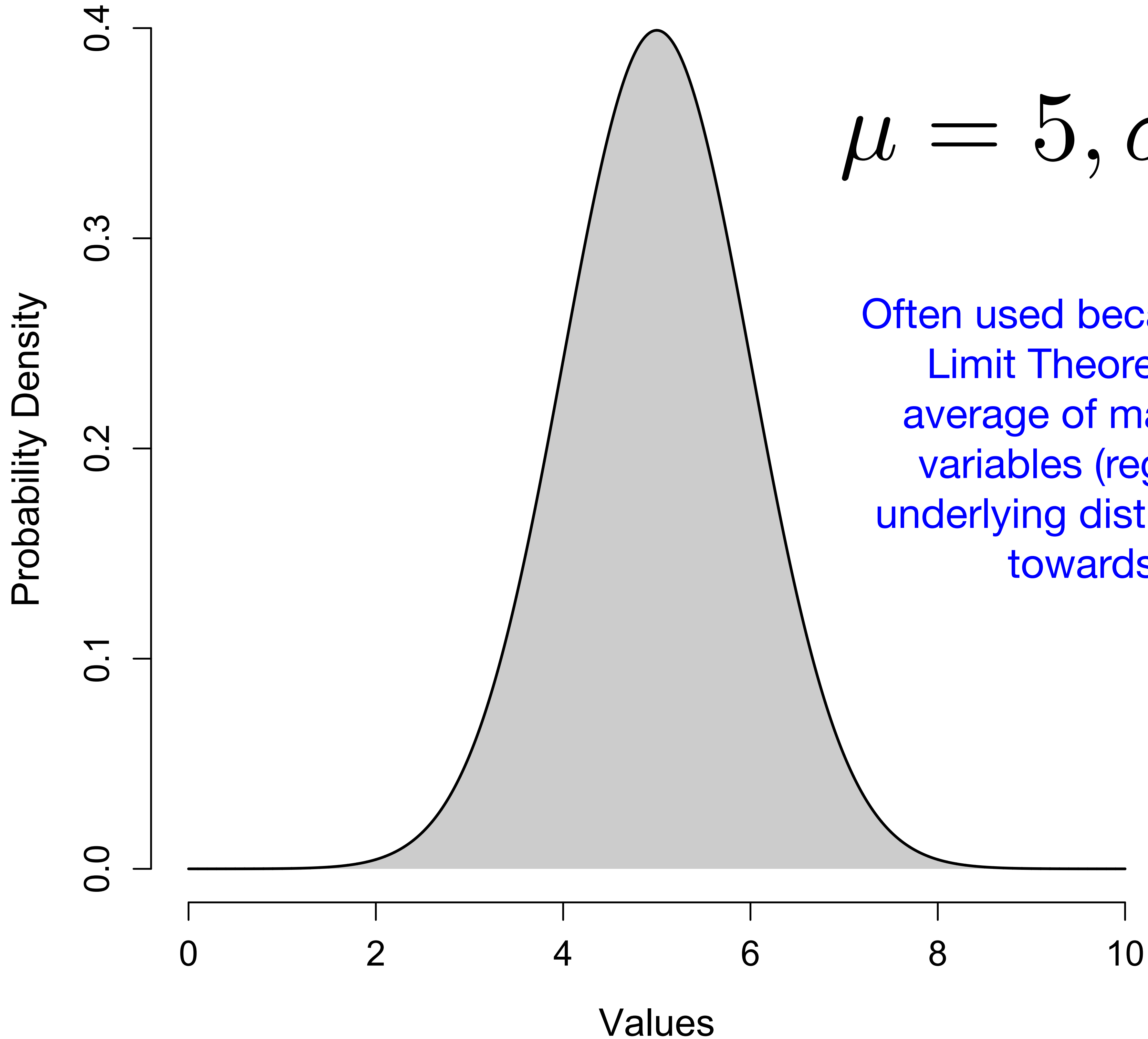


$$\mu = 5, \sigma = 1$$

Often used because of the Central Limit Theorem - the sum or average of many independent variables (regardless of their underlying distributions) will tend towards a Normal.

Variance

Variance is the expectation of the squared deviation of a random variable from its mean.

$$\mathrm{Var}(X) = \sigma^2(X) = \mathrm{E}[(X - \mu)^2]$$

Variance (Standard Deviation)

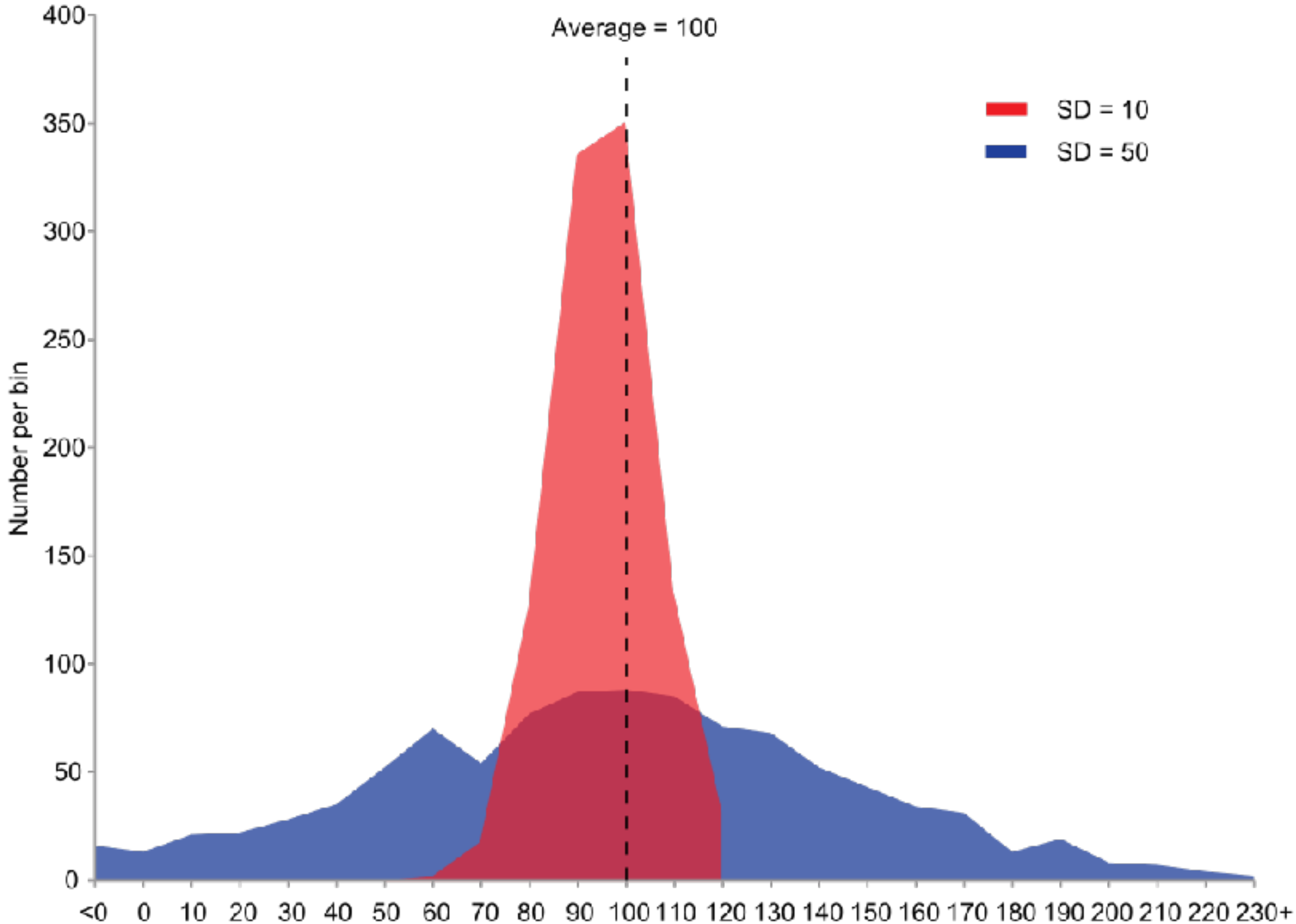
Variance is the expectation of the squared deviation of a random variable from its mean.

$$\mathrm{Var}(X) = \sigma^2(X) = \mathrm{E}[(X - \mu)^2]$$

The standard deviation of a random variable is the square root of its variance.

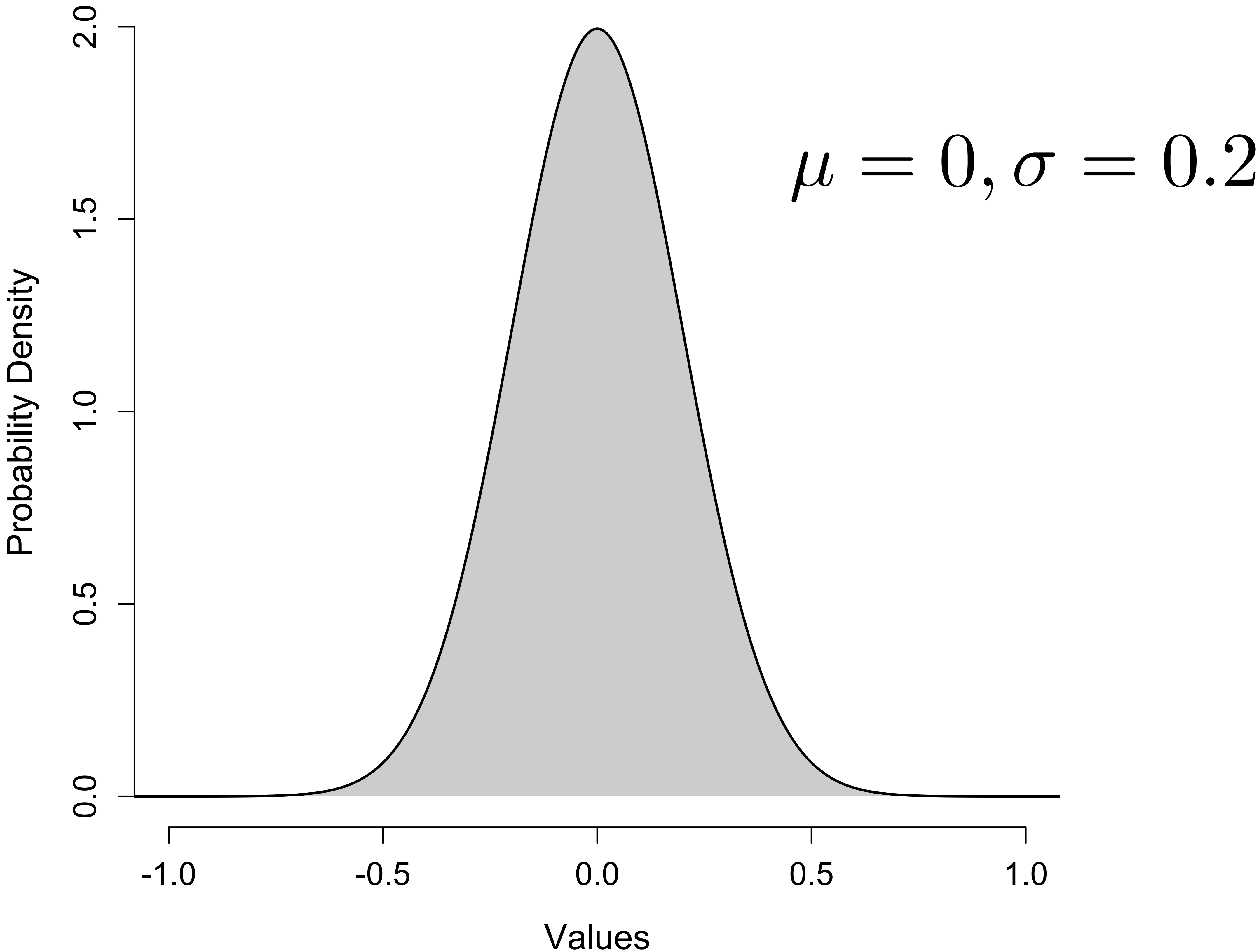
$$\mathrm{sd}(X) = \sigma(X) = \sqrt{\mathrm{E}[(X - \mu)^2]}$$

Standard Deviation / Variance

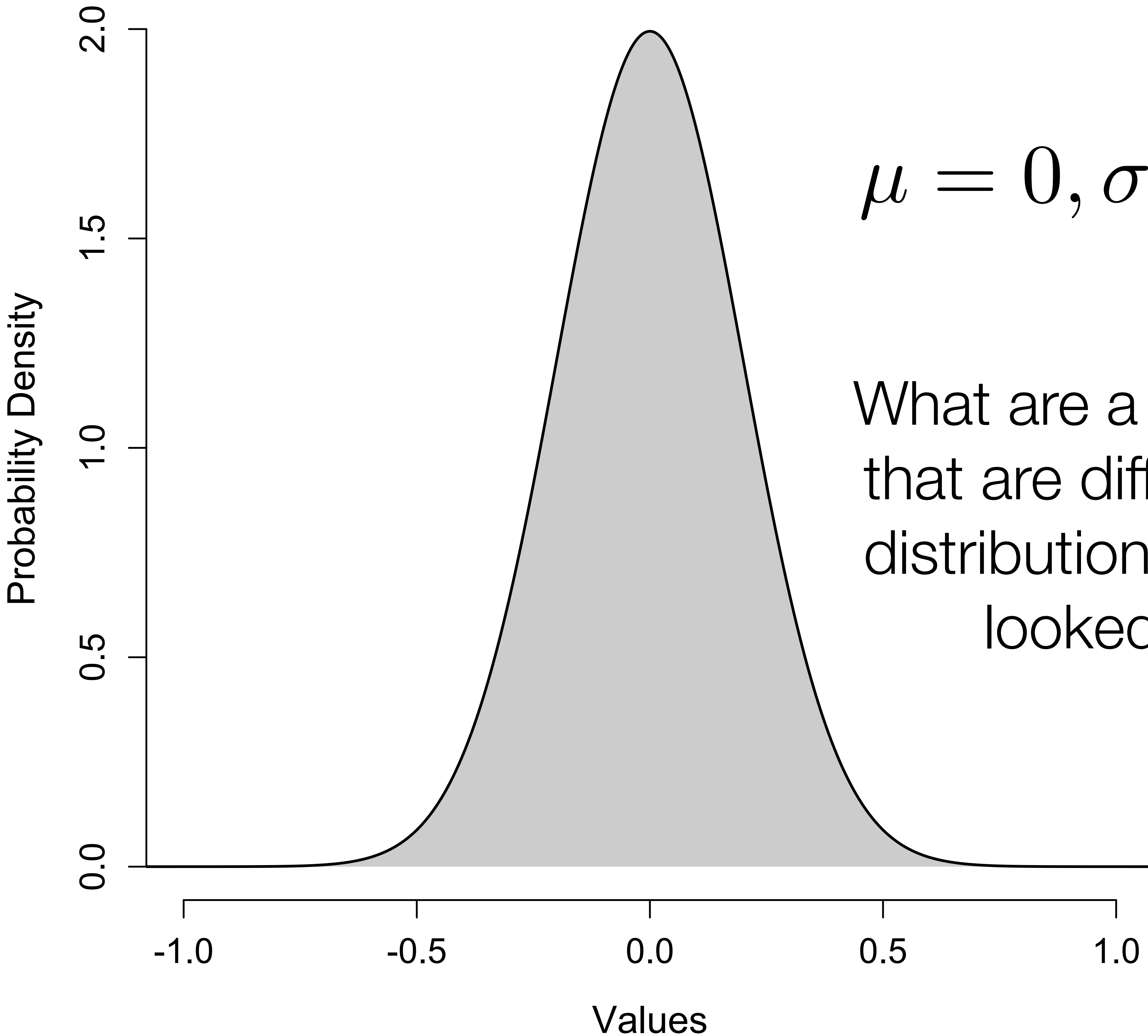


https://en.wikipedia.org/wiki/Standard_deviation#/media/File:Comparison_standard_deviations.svg

Normal Distribution



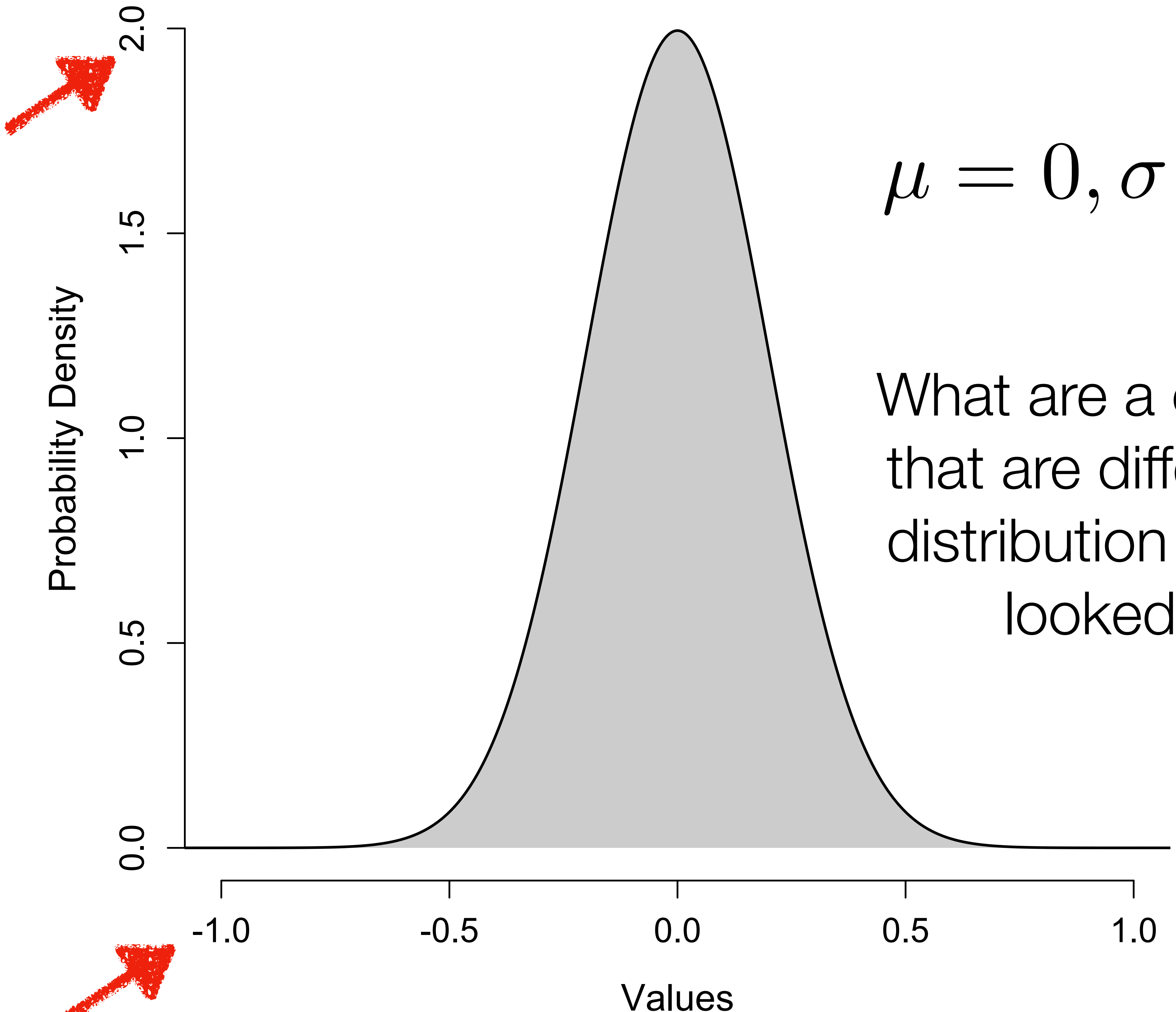
Normal Distribution



$$\mu = 0, \sigma = 0.2$$

What are a couple of things that are different about this distribution than any we've looked at before?

Normal Distribution



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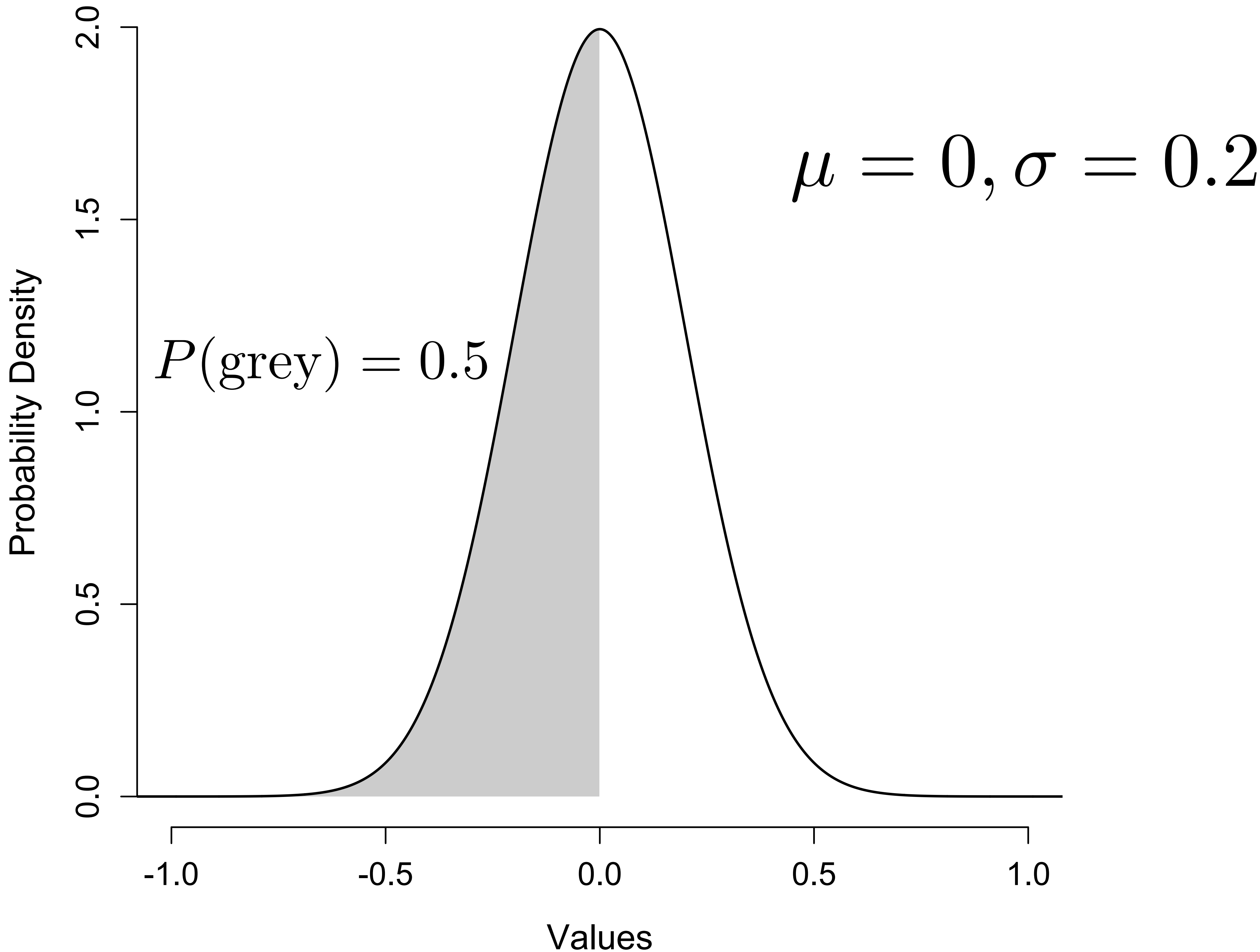
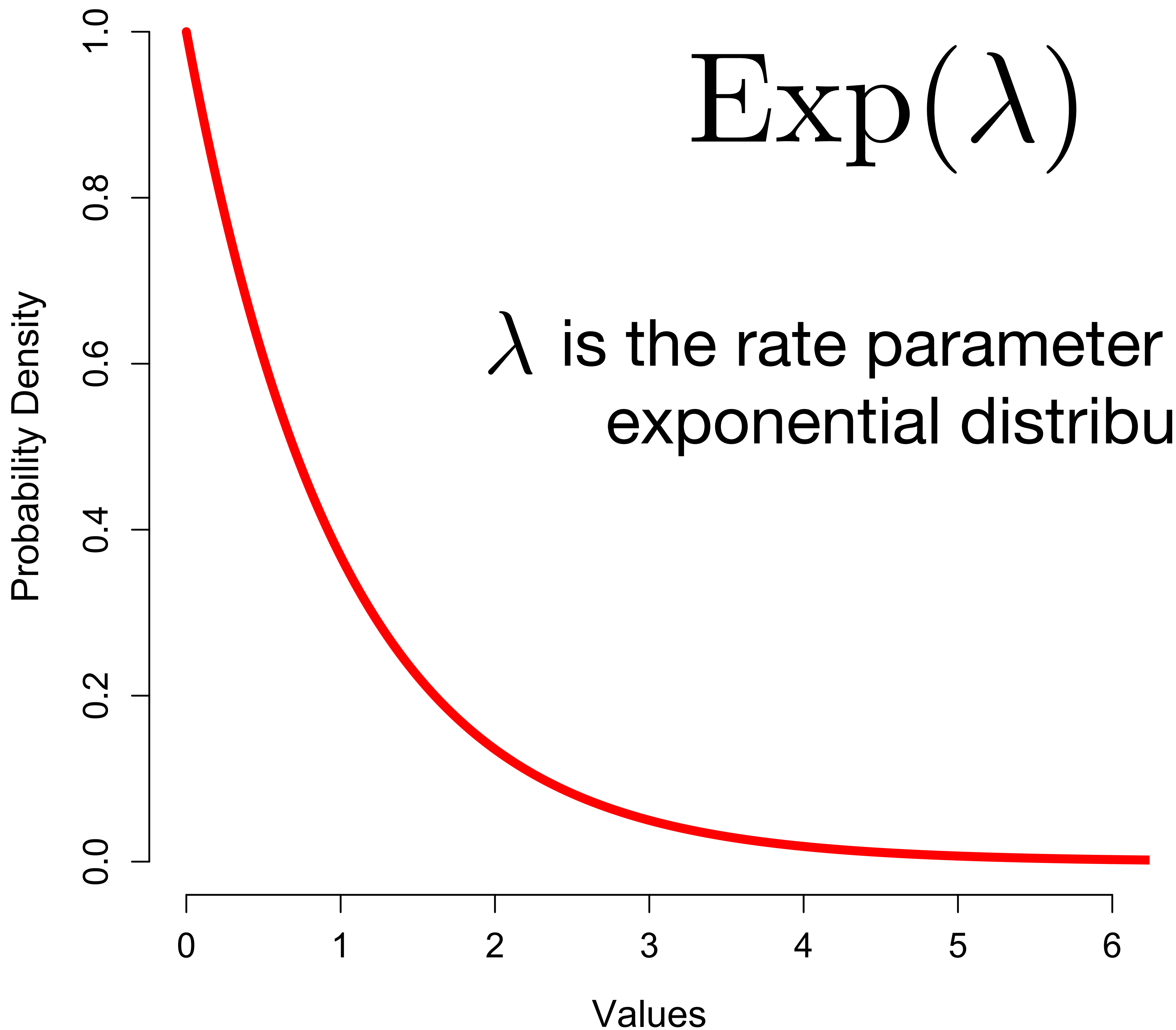


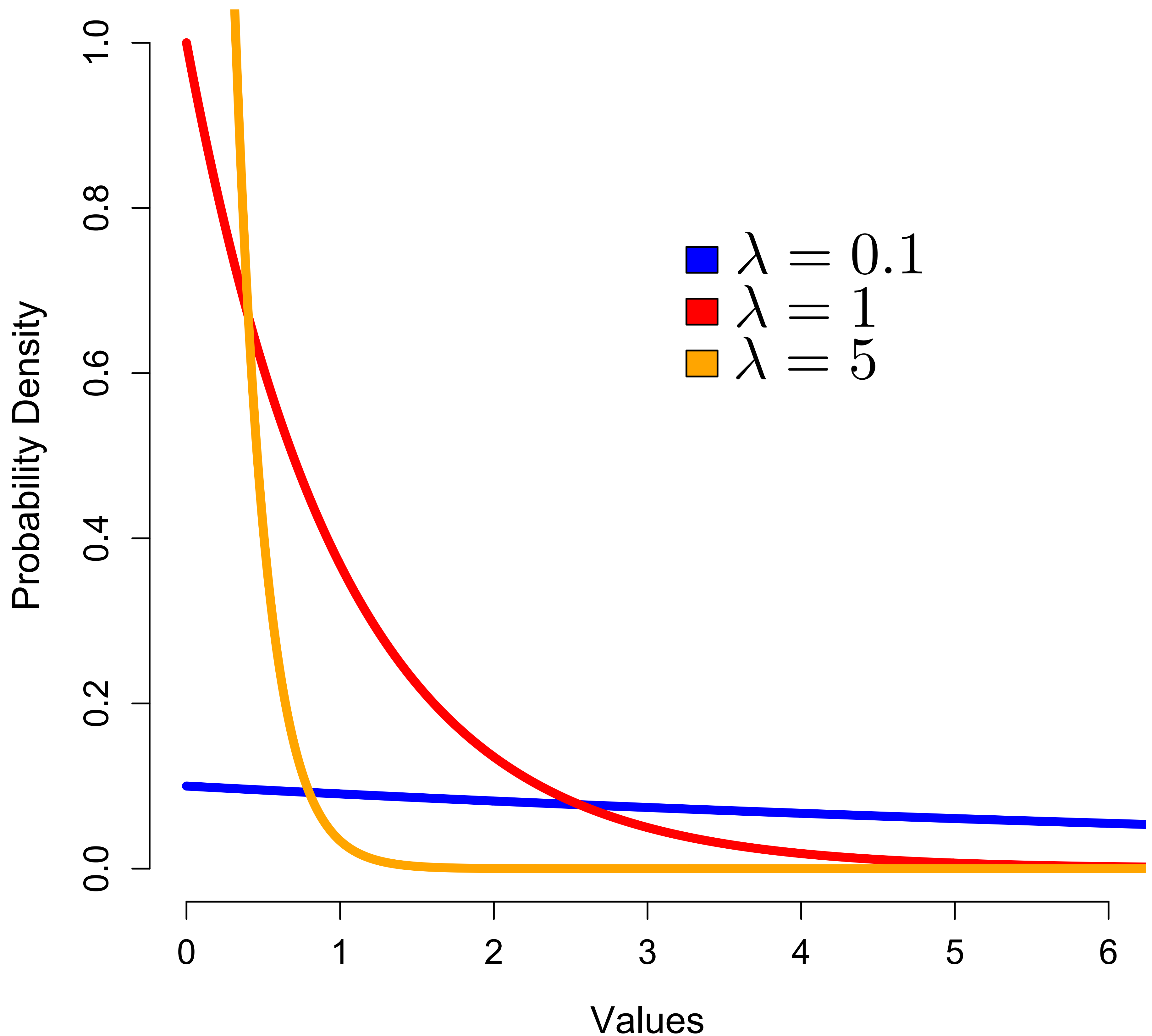
Exponential Distribution



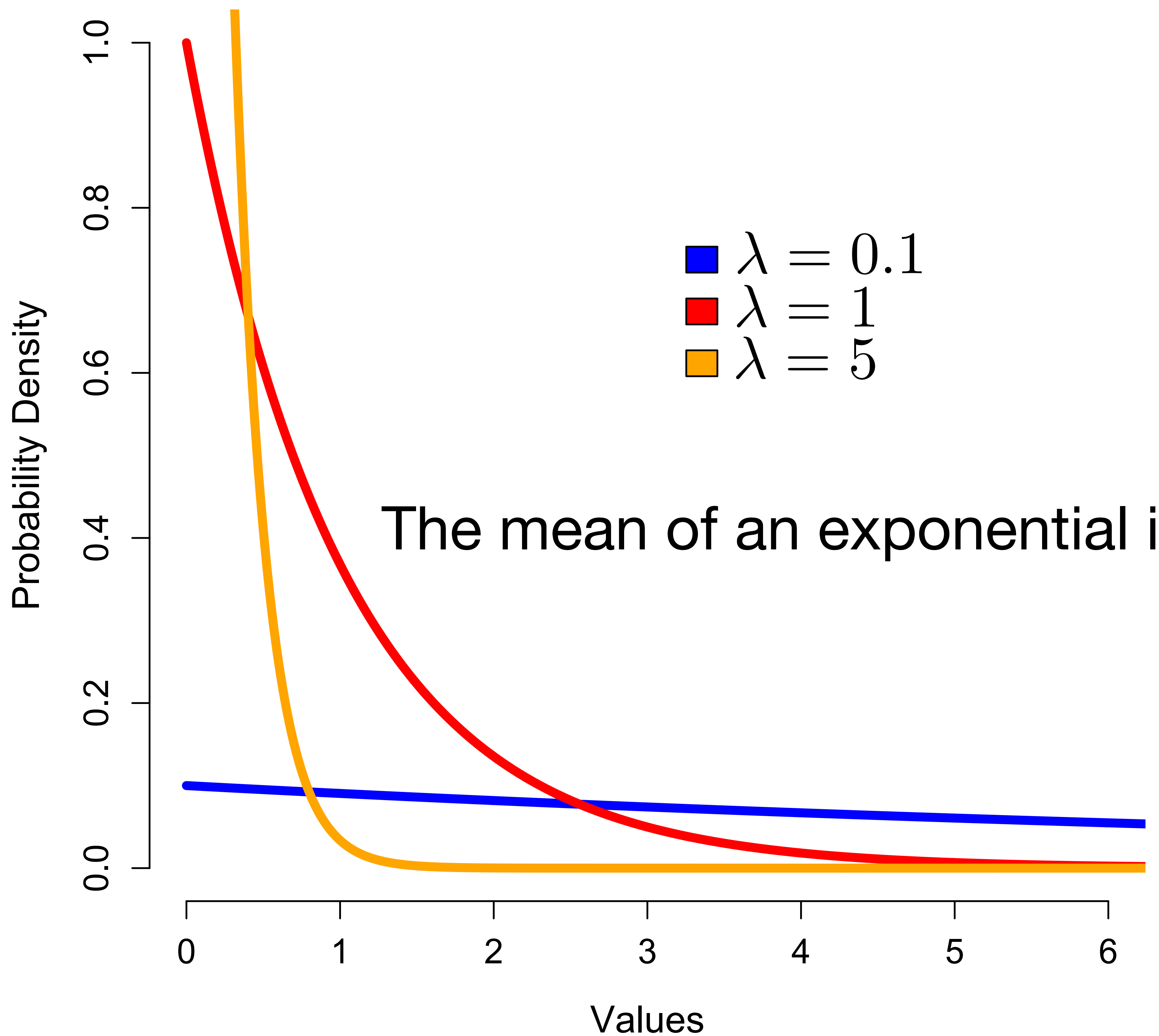
$$\mathrm{Exp}(\lambda)$$

λ is the rate parameter of the exponential distribution.

Exponential Distribution

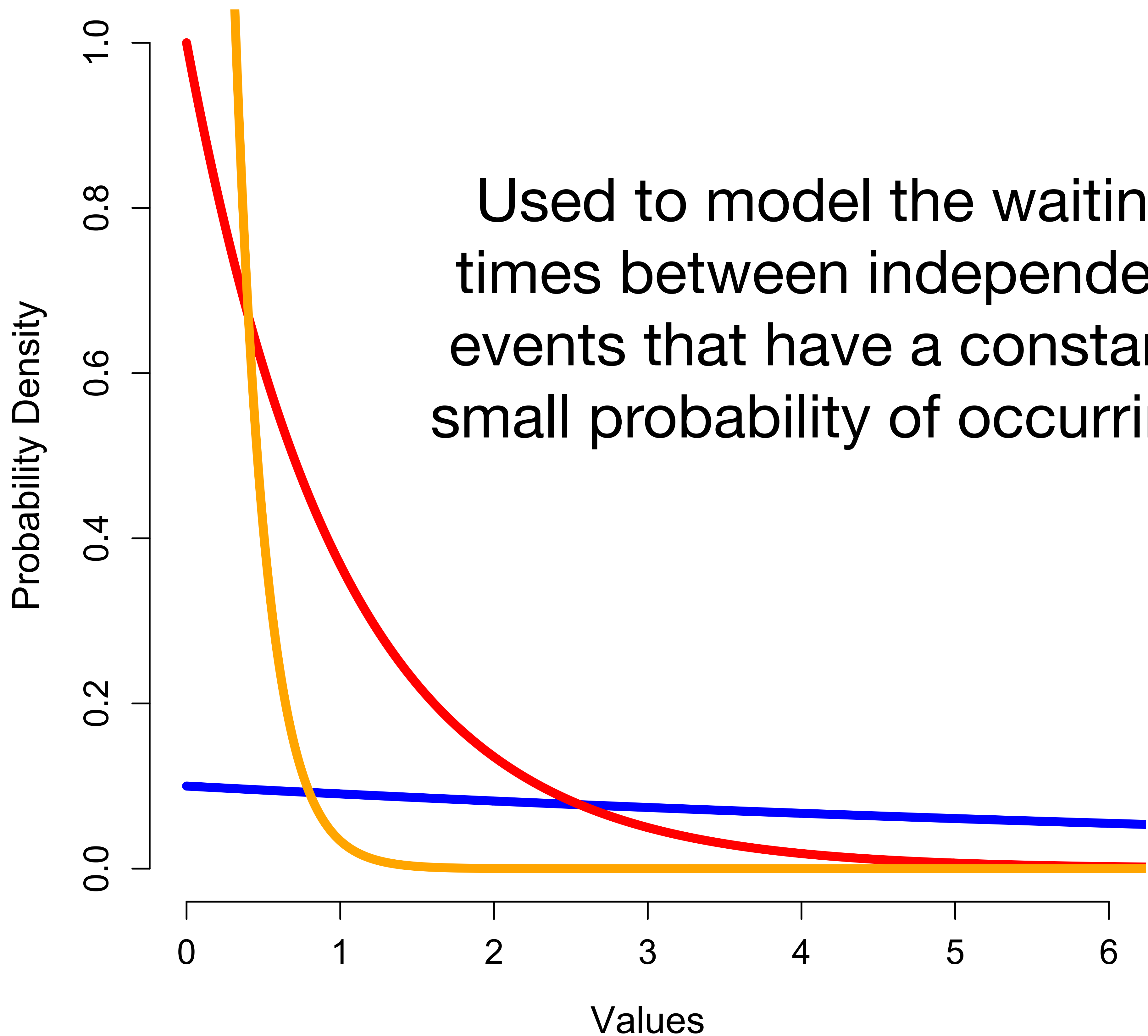


Exponential Distribution



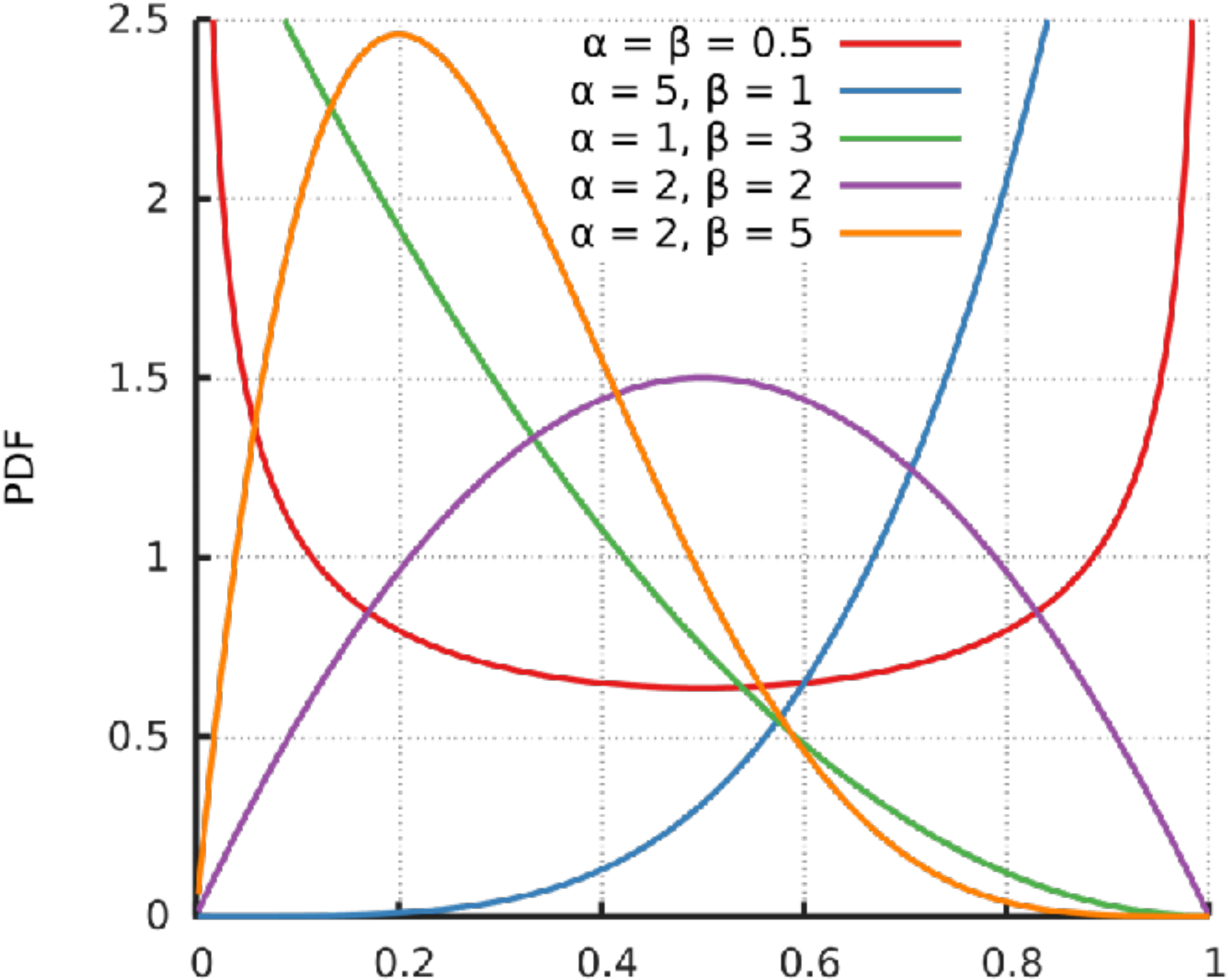
The mean of an exponential is $\frac{1}{\lambda}$

Exponential Distribution



Used to model the waiting times between independent events that have a constant, small probability of occurring.

Beta Distribution

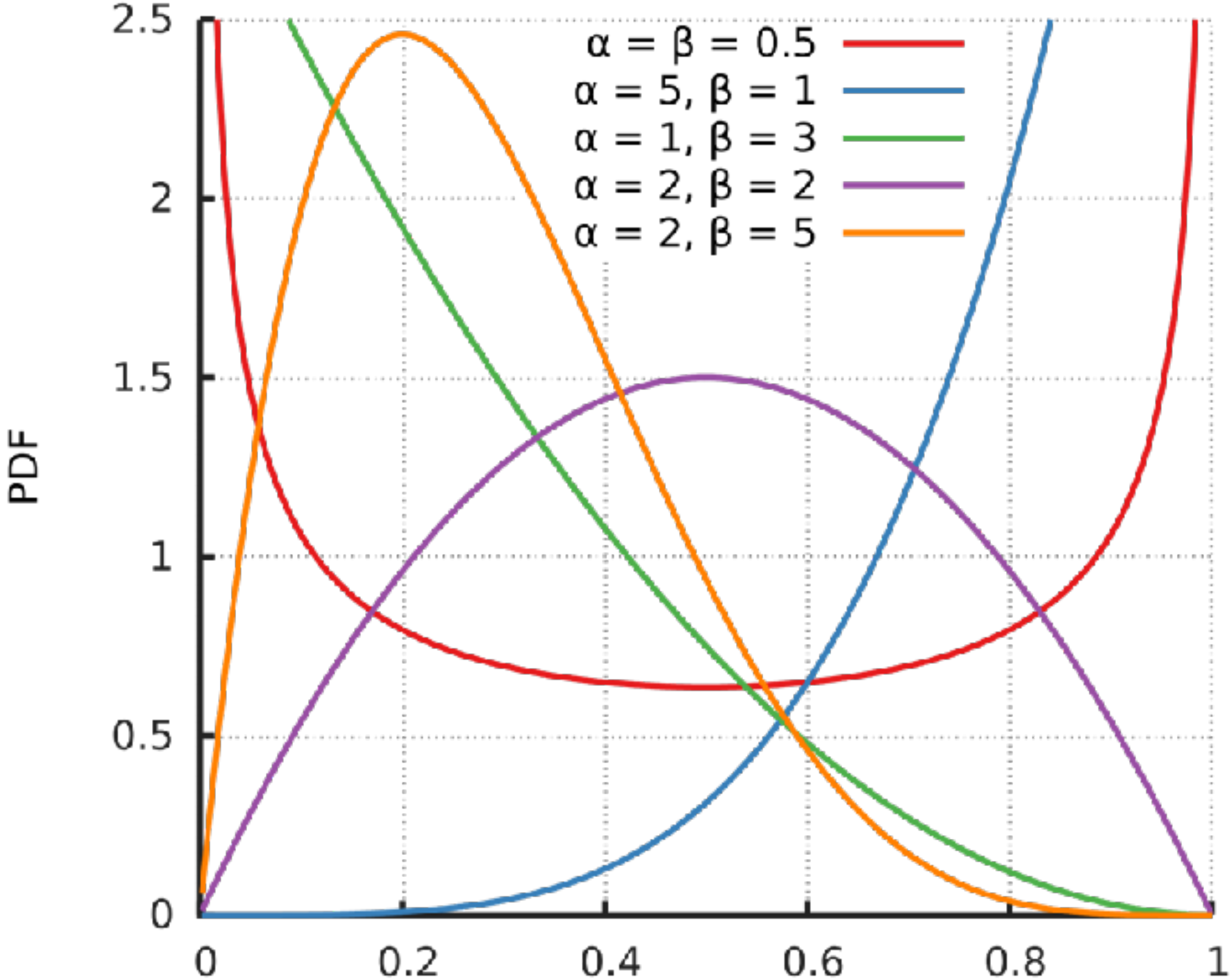


Two Shape Parameters
α and β

What is the effect of changing these parameter values?

What is the **support** of this distribution?

Beta Distribution

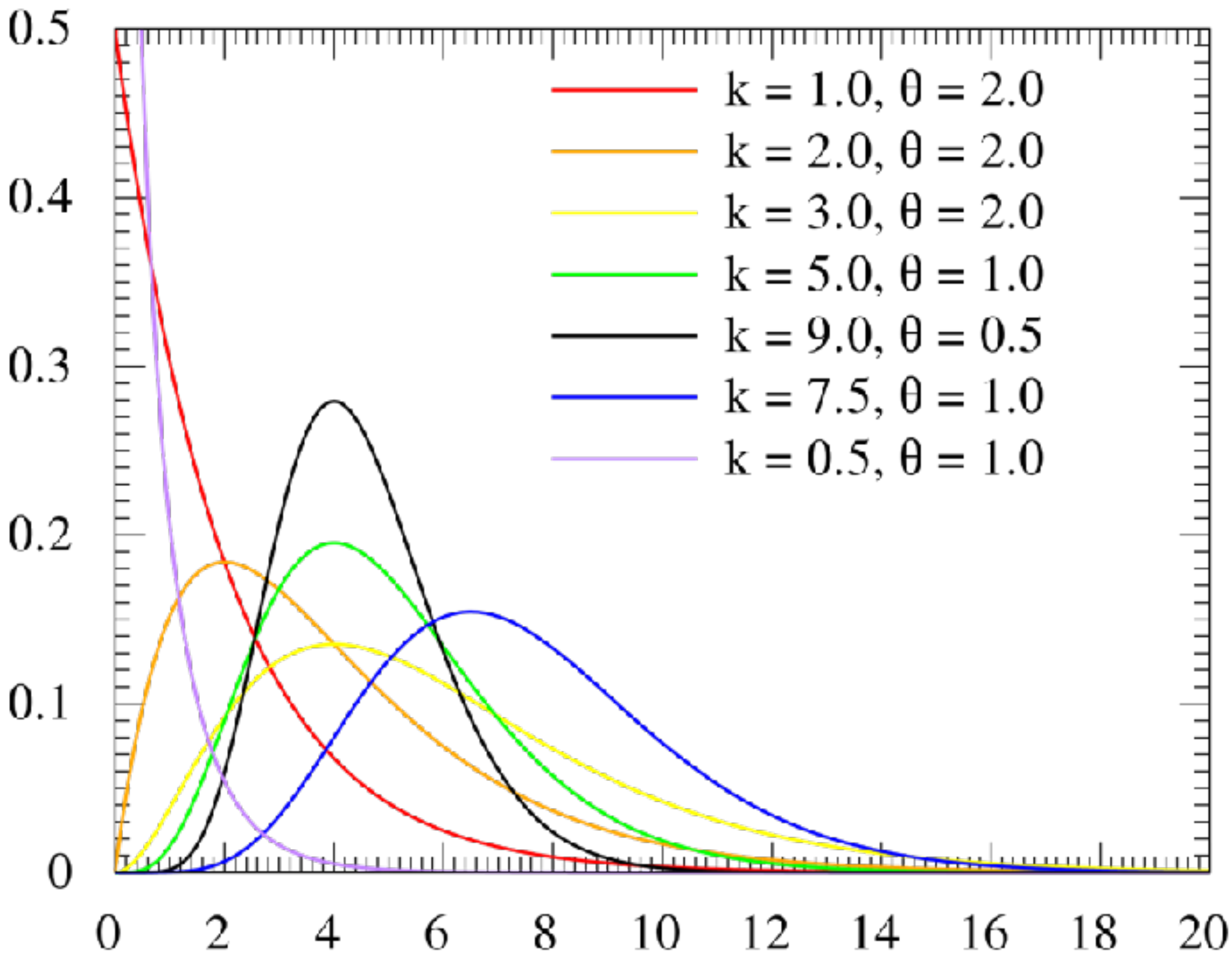


Support

$$x \in (0, 1)$$

Often used as a flexible distribution to model values between 0 and 1.

Gamma Distribution



Two Shape Parameters

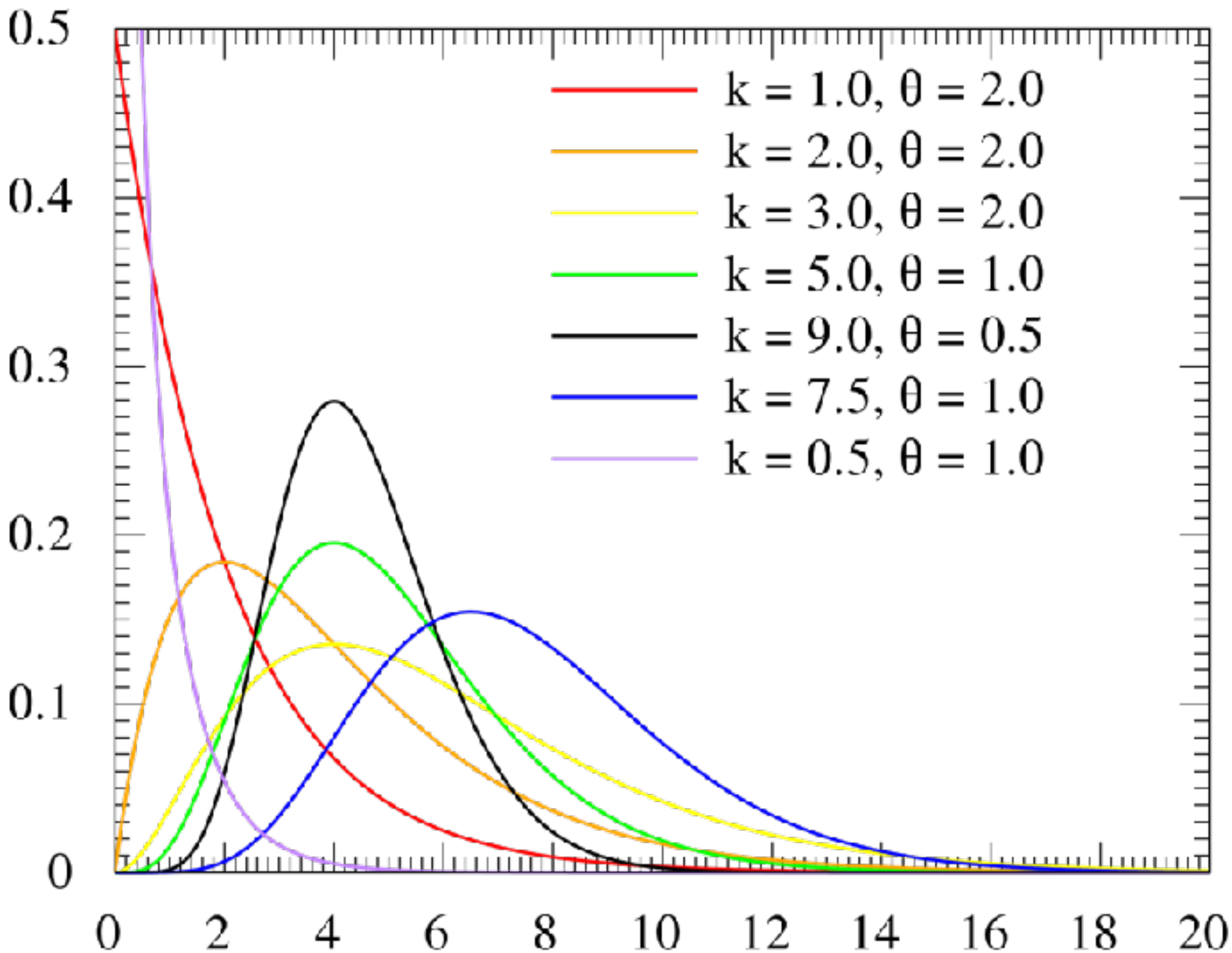
Shape and Scale

k and θ

Shape and Rate

α and β

Gamma Distribution



$$X \sim \Gamma(k, \theta)$$

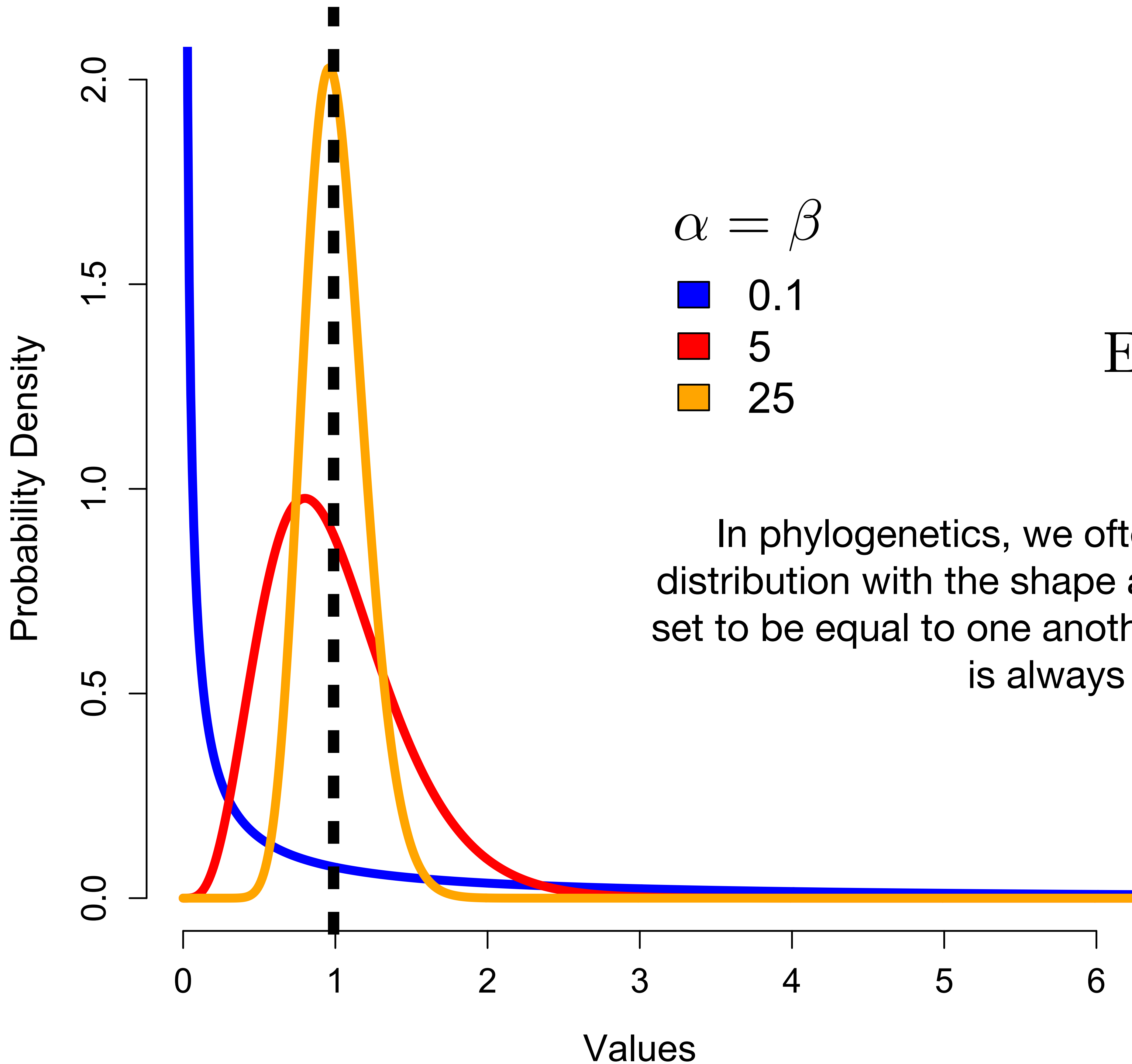
$$X \sim \Gamma(\alpha, \beta)$$

$$\mathrm{E}[X] = k\theta = \frac{\alpha}{\beta}$$

Support

$$x \in (0, \infty)$$

Gamma Distribution



$$\mathrm{E}[X] = k\theta = \frac{\alpha}{\beta}$$

In phylogenetics, we often use a gamma distribution with the shape and rate parameters set to be equal to one another, so that the mean is always 1.