Reilly consider the motion of a ball bouncing on a periodically vibrating table. This nonlinear dynamical system exhibits fixed points, periodic and strange attractors, and period-doubling bifurcations to chaos, similar to the logistic map. Simulations of this system are very interesting, but not straightforward.

Tolga Yalcinkaya and Ying-Cheng Lai, "Chaotic scattering," Computers in Physics **9**, 511–518 (1995). Project 6.28 is based on a draft of this article. The map (6.64) is discussed in more detail in Yun-Tung Lau, John M. Finn, and Edward Ott, "Fractal dimension in nonhyperbolic chaotic scattering," Phys. Rev. Lett. **66**, 978 (1991).

CHAPTER 7

Random Processes

Random processes are introduced in the context of several simple physical systems, including random walks on a lattice, polymers, and diffusion controlled chemical reactions. The generation of random number sequences is also discussed.

7.1 ■ ORDER TO DISORDER

In Chapter 6 we saw several examples of how, under certain conditions, the behavior of a nonlinear deterministic system can appear to be random. In this chapter we will see some examples of how chance can generate statistically predictable outcomes. For example, we know that if we bet often on the outcome of a game for which the probability of winning is less than 50%, eventually we will lose money.

We first discuss an example that illustrates the tendency of systems of many particles to evolve to a well-defined state. Imagine a closed box that is divided into two parts of equal volume (see Figure 7.1). The left half contains a gas of N identical particles and the right half is initially empty. We then make a small hole in the partition between the two halves. What happens? We know that after some time, the average number of particles in each half of the box will become $N/2$, and we say that the system has reached equilibrium.

How can we simulate this process? One way is to give each particle an initial velocity and position and adopt a deterministic model of the motion of the particles. For example, we could assume that each particle moves in a straight line until it hits a wall of the box or another particle and undergoes an elastic collision. We will consider similar deterministic models in Chapter 8. Instead, we first simulate a probabilistic model based on a *random process*.

The basic assumptions of this model are that the motion of the particles is random and the particles do not interact with one another. Hence, the probability per unit time that a particle goes through the hole in the partition is the same for all N particles regardless of the number of particles in either half. We also assume that the size of the hole is such that only one particle can pass through at a time. We first model the motion of a particle passing through the hole by choosing one of the N particles at random and moving it to the other side. For visualization purposes, we will use arrays to specify the position of each particle. We then randomly generate an integer `i` between 0 and $N - 1$ and change the arrays appropriately. A more efficient Monte Carlo algorithm is discussed in Problem 7.2b. The tool we need to simulate this random process is a random number generator.

It is counterintuitive that we can use a deterministic computer to generate sequences of random numbers. In Section 7.9 we discuss some of the methods for computing a set of numbers that appear statistically random but are in fact generated by a deterministic