```
PlotFrame plotFrame = new PlotFrame("iterations", "x",
                                    "graphical solution");
double r;     // control parameter
int iterate; // iterate of f(x)
double x, y;
double x0, y0;

public GraphicalSolutionApp() {
   plotFrame.setPreferredMinMax(0, 1, 0, 1);
   plotFrame.setConnected(true);
   plotFrame.setXPointsLinked(true);
   // second argument indicates no marker
   plotFrame.setMarkerShape(0, 0);
   plotFrame.setMarkerShape(2, 0);
}

public void reset() {
   control.setValue("r", 0.89);
   control.setValue("x", 0.2);
   control.setAdjustableValue("iterate", 1);
}

public void initialize() {
   r = control.getDouble("r");
   x = control.getDouble("x");
   iterate = control.getInt("iterate");
   x0 = x;
   y0 = 0;
   clear();
}

public void startRunning() {
   if(iterate!=control.getInt("iterate")) {
      iterate = control.getInt("iterate");
      clear();
   }
   r = control.getDouble("r");
}

public void doStep() {
   y = f(x, r, iterate);
   plotFrame.append(1, x0, y0);
   plotFrame.append(1, x0, y);
   plotFrame.append(1, y, y);
   x = x0 = y0 = y;
   control.setValue("x", x);
}

void drawFunction() {
   int nplot = 200; // # of points at which function computed
   double delta = 1.0/nplot;
   double x = 0;
   double y = 0;
   for(int i = 0;i<=nplot;i++) {
      y = f(x, r, iterate);
      plotFrame.append(0, x, y);
```



```
      x += delta;
   }
}

void drawLine() { // draws line y = x
   for(double x = 0;x<1;x += 0.001) {
      plotFrame.append(2, x, x);
   }
}

public double f(double x, double r, int iterate) {
   if(iterate>1) {
      double y = f(x, r, iterate-1);
      return 4*r*y*(1-y);
   } else {
      return 4*r*x*(1-x);
   }
}

public void clear() {
   plotFrame.clearData();
   drawFunction();
   drawLine();
   plotFrame.repaint();
}

public static void main(String[] args) {
   SimulationControl control = SimulationControl.createApp(
                                  new GraphicalSolutionApp());
   control.addButton("clear", "Clear", "Clears the trajectory.");
}
}
```

Problem 6.4 Qualitative properties of the fixed points

(a) Use `GraphicalSolutionApp` to show graphically that there is a single stable fixed point of $f(x)$ for $r < 3/4$. It would be instructive to modify the program so that the value of the slope $df/dx|_{x=x_n}$ is shown as you step each iteration. At what value of r does the absolute value of this slope exceed unity? Let b_1 denote the value of r at which the fixed point of $f(x)$ bifurcates and becomes unstable. Verify that $b_1 = 0.75$.

(b) Describe the trajectory of $f(x)$ for $r = 0.785$. Is the fixed point given by $x = 1 - 1/4r$ stable or unstable? What is the nature of the trajectory if $x_0 = 1 - 1/4r$? What is the period of $f(x)$ for all other choices of x_0? What are the values of the two-point attractor?

(c) The function $f(x)$ is symmetrical about $x = 1/2$ where $f(x)$ is a maximum. What are the qualitative features of the second iterate $f^{(2)}(x)$ for $r = 0.785$? Is $f^{(2)}(x)$ symmetrical about $x = 1/2$? For what value of x does $f^{(2)}(x)$ have a minimum? Iterate $x_{n+1} = f^{(2)}(x_n)$ for $r = 0.785$ and find its two fixed points x_1^* and x_2^*. (Try $x_0 = 0.1$ and $x_0 = 0.3$.) Are the fixed points of $f^{(2)}(x)$ stable or unstable for this value of r? How do these values of x_1^* and x_2^* compare with the values of the two-point attractor of $f(x)$? Verify that the slopes of $f^{(2)}(x)$ at x_1^* and x_2^* are equal.