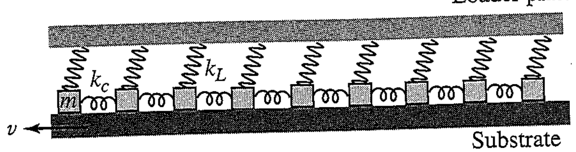

Figure 14.9 Schematic of the Burridge–Knopoff model. Blocks with mass m are attached to their nearest neighbors by springs with spring constant k_c. They are also attached to a fixed loader plate with spring constant k_L. The substrate moves with speed v to the left.

Write a program to implement (14.15) using periodic boundary conditions. Choose $c = 1$, $D = 2^{-15}$, and the initial condition

$$n(x, t = 0) = 1.0 + 0.4\cos(2\pi x), \tag{14.16}$$

where x denotes the position of a lattice site. Boghosian and Levermore used $2^{16} = 65{,}536$ lattice sites so that $\Delta x = 2^{-16}$. The bias is given by $\alpha = c\Delta x/2D = 0.25$ and the time step is $\Delta t = (\Delta x)^2/2D = 2^{-18}$. Average your results for 128 lattice sites and plot the average density as a function of x for different values of t up to $t = 1$. Do you see any evidence of a shock wave (a sharp discontinuity in $n(x)$)? ■

Project 14.26 Spring-block model of earthquakes

The first simulations of earthquakes were done by Burridge and Knopoff in 1967. Their model represents the motion of one side of a lateral fault that is driven by a slow shear deformation and subject to a nonlinear, velocity-dependent friction force. The model consists of a one-dimensional array of blocks on a substrate (see Figure 14.9). Each block is connected to its nearest neighbors by springs with spring constant k_c, which represent the linear elastic response of the system to compressional deformations. Each block is also connected by a spring with spring constant k_L to a fixed loader plate.

The system is loaded by moving the substrate at a constant speed v to the left. Eventually, the force on a block exceeds the static friction threshold F_0 and the block slips. As the block moves, the springs connecting it to its neighbors change length, thus changing the forces acting on them. The neighboring blocks begin to accelerate if the force is sufficient, that is, if the force due to the springs is greater than the static friction force.

The equation of motion of the Burridge-Knopoff model can be written as

$$m\ddot{x}_j = k_c(x_{j+1} - 2x_j + x_{j-1}) - k_L x_j - F(v + \dot{x}_j), \tag{14.17}$$

where x_j is the displacement of the jth block. The force between the blocks is given by $k_c(x_{j+1} - 2x_j + x_{j-1})$, the force from the loader plate is $-k_L x_j$, and F represents the friction force due to the substrate. Periodic boundary conditions are not used.

As usual, it is convenient to introduce dimensionless variables, which we take to be $u_j = (k_L/F_0)x_j$, $\omega_L^2 = k_L/m$, and $\tau = \omega_L t$. We rewrite (14.17) as

$$\ddot{u}_j = \ell^2(u_{j+1} - 2u_j + u_{j-1}) - u_j - \phi(2\alpha v + 2\alpha\dot{u}_j), \tag{14.18}$$

where $\phi(w) = F(w)/F_0$, the stiffness parameter $\ell = \sqrt{k_c/k_L}$, $v = vk_L/\omega_L F_0$, and $2\alpha = \omega_L F_0/k_L v$; the dot now denotes differentiation with respect to τ. The equation of motion (14.18) can be solved using the Euler–Richardson algorithm with $\Delta\tau = 10^{-3}$.

The velocity of a block is set to zero if at any time the speed of the block relative to the substrate is less than a parameter v_0, its speed is decreasing, and the force due to the springs is less than F_0. Otherwise, the friction force is given by

$$\phi(w) = \frac{1-\sigma}{1 + \dfrac{w}{1-\sigma}} \quad (w > 0), \tag{14.19}$$

where the parameter σ represents the drop of the friction force at the onset of the slip. If a block is stuck, the calculation of the static friction force is a bit more involved. If the total force on a block due to the springs is to the right, then the static friction force is set equal and opposite to the total spring force up to a maximum value of F_0. However, if the total spring force is to the left, the static friction is chosen so that the acceleration of the block is zero. Typical values of the parameters are $F_0 = 1$, $\ell = 10$, $\sigma = 0.01$, $\alpha = 2.5$, and $v_0 = 10^{-5}$.

Initially we set $\dot{u}_j = 0$ for all j and assign small random displacements to all the blocks. The blocks will then move according to (14.18). For simplicity we set the substrate velocity $v = 0$, and when all the blocks become stuck, we move all the blocks to the left by an equal amount such that the total force due to the springs on one block equals unity (F_0). This procedure will then cause one block to move or slip. As this block moves, other neighboring blocks may move leading to an earthquake. Eventually, all the blocks will again become stuck. The main quantities of interest are $P(s)$, the distribution of the number of blocks that have moved during an earthquake, and $P(M)$, the distribution of the net displacement of the blocks during an earthquake, where

$$M = \sum_i \Delta u_i. \tag{14.20}$$

The sum over i in (14.20) is over the blocks involved in an earthquake and Δu_i is the net displacement of the blocks during the earthquake. Do $P(s)$ and $P(M)$ exhibit scaling consistent with Gutenberg–Richter?

The movement of the blocks represents the slip of the two surfaces of a fault past one another during an earthquake. The stick-slip behavior of this model is similar to that of a real earthquake fault. Other interesting questions are posed in the references (see Klein et al., Ferguson et al., and Mori and Kawamura). ■

REFERENCES AND SUGGESTIONS FOR FURTHER READING

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