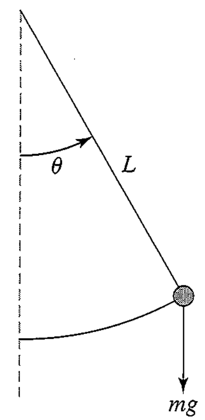

Figure 4.2 Force diagram for a simple pendulum. The angle θ is measured from the vertical direction and is positive if the mass is to the right of the vertical and negative if it is to the left.

In the absence of friction, two forces act on the bob: the force mg vertically downward and the force of the rod which is directed inward to the center if $|\theta| < \pi/2$. Note that the effect of the rigid rod is to constrain the motion of the bob along the arc. From Figure 4.2, we can see that the component of mg along the arc is $mg \sin\theta$ in the direction of decreasing θ. Hence, the equation of motion can be written as

$$mL\frac{d^2\theta}{dt^2} = -mg\sin\theta, \tag{4.10}$$

or

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta. \tag{4.11}$$

Equation (4.11) is an example of a nonlinear equation because $\sin\theta$ rather than θ appears. Most nonlinear equations do not have analytical solutions in terms of well-known functions, and (4.11) is no exception. However, if the amplitude of the pendulum oscillations is sufficiently small, then $\sin\theta \approx \theta$, and (4.11) reduces to

$$\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta \quad (\theta \ll 1). \tag{4.12}$$

Remember that θ is measured in radians.

Part of the fun of studying physics comes from realizing that equations that appear in different contexts are often similar. An example can be seen by comparing (4.2) and (4.12). If we associate x with θ, we see that the two equations are identical in form, and we can immediately conclude that for $\theta \ll 1$, the period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{(small amplitude oscillations)}. \tag{4.13}$$

One way to understand the motion of a pendulum with large oscillations is to solve (4.11) numerically. Because we know that the numerical solutions must be consistent with conservation of energy, we derive the form of the total energy here. The potential energy

can be found from the following considerations. If the rod is deflected by the angle θ, then the bob is raised by the distance $h = L - L\cos\theta$ (see Figure 4.2). Hence, the potential energy of the bob in the gravitational field of the earth is

$$U = mgh = mgL(1 - \cos\theta), \tag{4.14}$$

where the zero of the potential energy corresponds to $\theta = 0$. Because the kinetic energy of the pendulum is $\frac{1}{2}mv^2 = \frac{1}{2}mL^2(d\theta/dt)^2$, the total energy E of the pendulum is

$$E = \frac{1}{2}mL^2\left(\frac{d\theta}{dt}\right)^2 + mgL(1 - \cos\theta). \tag{4.15}$$

We use two classes to simulate and visualize the motion of a pendulum problem, `Pendulum` and `PendulumApp`. The `Pendulum` class implements the `Drawable` and `ODE` interfaces and solves the dynamical equations using the Euler–Richardson algorithm.

Listing 4.1 A `Drawable` class that models the simple pendulum.

```
package org.opensourcephysics.sip.ch04;
import java.awt.*;
import org.opensourcephysics.display.*;
import org.opensourcephysics.numerics.*;

public class Pendulum implements Drawable, ODE {
   double omega0Squared = 3;                      // g/L
   double[] state = new double[] {0, 0, 0}; // {theta, dtheta/dt, t}
   Color color = Color.RED;
   int pixRadius = 6;
   EulerRichardson odeSolver = new EulerRichardson(this);

   public void setStepSize(double dt) {
      odeSolver.setStepSize(dt);
   }

   public void step() {
      odeSolver.step(); // execute one Euler-Richardson step
   }

   public void setState(double theta, double thetaDot) {
      state[0] = theta;
      state[1] = thetaDot; // time rate of change of theta
   }

   public double[] getState() {
      return state;
   }

   public void getRate(double[] state, double[] rate) {
      rate[0] = state[1];                    // rate of change of angle
      // rate of change of dtheta/dt
      rate[1] = -omega0Squared*Math.sin(state[0]);
      rate[2] = 1; // rate of change of time dt/dt = 1
   }

   public void draw(DrawingPanel drawingPanel, Graphics g) {
      int xpivot = drawingPanel.xToPix(0);
```