```
      int ypivot = drawingPanel.yToPix(0);
      int xpix = drawingPanel.xToPix(Math.sin(state[0]));
      int ypix = drawingPanel.yToPix(-Math.cos(state[0]));
      g.setColor(Color.black);
      g.drawLine(xpivot, ypivot, xpix, ypix);          // the string
      g.setColor(color);
      g.fillOval(xpix-pixRadius, ypix-pixRadius, 2*pixRadius,
         2*pixRadius); // bob
   }
}
```

Note that Pendulum implements the draw method as required by the Drawable interface.

The target class, PendulumApp, is shown in Listing 4.2. The angle θ is plotted as a function of time, and an animation of the motion is drawn.

Listing 4.2 Visualization of the motion of a pendulum.

```
package org.opensourcephysics.sip.ch04;
import org.opensourcephysics.controls.*;
import org.opensourcephysics.frames.*;

public class PendulumApp extends AbstractSimulation {
   PlotFrame plotFrame = new PlotFrame("Time", "Theta",
                         "Theta versus time");
   Pendulum pendulum = new Pendulum();
   DisplayFrame displayFrame = new DisplayFrame("Pendulum");

   public PendulumApp() {
      displayFrame.addDrawable(pendulum);
      displayFrame.setPreferredMinMax(-1.2, 1.2, -1.2, 1.2);
   }

   public void initialize() {
      double dt = control.getDouble("dt");
      double theta = control.getDouble("initial theta");
      double thetaDot = control.getDouble("initial dtheta/dt");
      pendulum.setState(theta, thetaDot);
      pendulum.setStepSize(dt);
   }

   public void doStep() {
      // angle vs time data added
      plotFrame.append(0, pendulum.state[2], pendulum.state[0]);
      pendulum.step(); // advances the state by one time step
   }

   public void reset() {
      pendulum.state[2] = 0; // set time = 0
      control.setValue("initial theta", 0.2);
      control.setValue("initial dtheta/dt", 0);
      control.setValue("dt", 0.1);
   }

   // creates a simulation control structure using this class
   public static void main(String[] args) {
      SimulationControl.createApp(new PendulumApp());
   }
}
```

Problem 4.5 Oscillations of a pendulum

(a) Make the necessary changes so that the analytical solution for small angles is also plotted.

(b) Test the program at sufficiently small amplitudes so that $\sin\theta \approx \theta$. Choose $\omega_0 = \sqrt{g/L} = 3$ and the initial conditions $\theta(t=0) = 0.2$ and $d\theta(t=0)/dt = 0$. Determine the period numerically and compare your result to the expected analytical result for small amplitudes. Explain your method for determining the period. Estimate the error due to the small angle approximation for these initial conditions.

(c) Consider larger amplitudes and make plots of $\theta(t)$ and $d\theta(t)/dt$ versus t for the initial conditions $\theta(t=0) = 0.1, 0.2, 0.4, 0.8$, and 1.0 with $d\theta(t=0)/dt = 0$. Choose Δt so that the numerical algorithm generates a stable solution; that is, monitor the total energy and ensure that it does not drift from its initial value. Describe the qualitative behavior of θ and $d\theta/dt$. What is the period T and the amplitude $\theta_{\max}$ in each case? Plot T versus $\theta_{\max}$ and discuss the qualitative dependence of the period on the amplitude. How do your results for T compare in the linear and nonlinear cases; for example, which period is larger? Explain the relative values of T in terms of the relative magnitudes of the restoring force in the two cases. ■

4.3 ■ DAMPED HARMONIC OSCILLATOR

We know from experience that most oscillatory motion in nature gradually decreases until the displacement becomes zero; such motion is said to be *damped* and the system is said to be *dissipative* rather than conservative. As an example of a damped harmonic oscillator, consider the motion of the block in Figure 4.1 when a horizontal drag force is included. For small velocities, it is a reasonable approximation to assume that the drag force is proportional to the first power of the velocity. In this case the equation of motion can be written as

$$\frac{d^2x}{dt^2} = -\omega_0^2 x - \gamma\frac{dx}{dt}. \tag{4.16}$$

The *damping coefficient* γ is a measure of the magnitude of the drag term. Note that the drag force in (4.16) opposes the motion. We simulate the behavior of the damped linear oscillator in Problem 4.6.

Problem 4.6 Damped linear oscillator

(a) Incorporate the effects of damping into your harmonic oscillator simulation and plot the time dependence of the position and the velocity. Describe the qualitative behavior of $x(t)$ and $v(t)$ for $\omega_0 = 3$ and $\gamma = 0.5$ with $x(t=0) = 1$, $v(t=0) = 0$.

(b) The period of the motion is the time between successive maxima of $x(t)$. Compute the period and corresponding angular frequency and compare their values to the undamped case. Is the period longer or shorter? Make additional runs for $\gamma = 1, 2$, and 3. Does the period increase or decrease with greater damping? Why?

(c) The amplitude is the maximum value of x during one cycle. Compute the *relaxation time* τ, the time it takes for the amplitude of an oscillation to decrease by $1/e \approx 0.37$