```
   // new lattice
   public void newLattice(int L, double p, LatticeFrame lattice) {
      lattice.resizeLattice(L, L);
      for(int i = 0;i<L;i++) {
         for(int j = 0;j<L;j++) {
            if(Math.random()<p) {
               lattice.setValue(i, j, 1);
            }
         }
      }
   }

   public void block(LatticeFrame lattice, LatticeFrame blockedLattice,
         int Lb) {
      blockedLattice.resizeLattice(Lb, Lb);
      for(int ib = 0;ib<Lb;ib++) {
         for(int jb = 0;jb<Lb;jb++) {
            int leftCellsProduct = lattice.getValue(2*ib, 2*jb)*
                 lattice.getValue(2*ib, 2*jb+1);
            int rightCellsProduct = lattice.getValue(2*ib+1, 2*jb)*
                 lattice.getValue(2*ib+1, 2*jb+1);
            if(leftCellsProduct==1||rightCellsProduct==1) {
               // vertical spanning rule
               blockedLattice.setValue(ib, jb, 1);
            }
         }
      }
   }

   public void setLatticeColors(LatticeFrame lattice) {
      lattice.setIndexedColor(0, Color.WHITE);
      lattice.setIndexedColor(1, Color.BLUE);
   }

   public static void main(String[] args) {
      CalculationControl.createApp(new RGApp());
   }
}
```

Problem 12.9 Visual renormalization group

Use RGApp with $L = 64$ to estimate the value of the percolation threshold. For example, confirm that for small p, such as $p = 0.4$, the renormalized lattice almost always renormalizes to a nonspanning cluster. What happens for $p = 0.8$? How can you use the properties of the renormalized lattices to estimate p_c? ■

Although a visual implementation of the renormalization group allows us to estimate p_c, it does not allow us to estimate the critical exponents. In the following, we present a renormalization group method that allows us to obtain p_c and the critical exponent ν associated with the connectedness length. This analysis follows closely the method presented by Reynolds et al.

We adopt the same procedure as before; that is, we replace the b^d sites within a cell of linear dimension b by a single site that represents whether or not the original lattice sites span the cell. The second step is to determine the parameters that specify the new

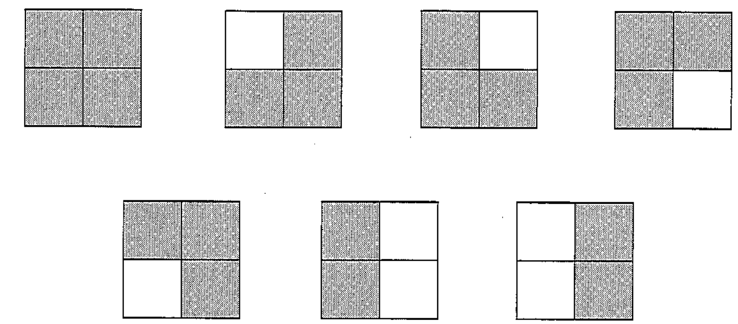

Figure 12.14 The seven (vertically) spanning configurations on a $b = 2$ cell.

renormalized configuration. We make the simple approximation that each cell is independent of all the other cells and is characterized only by the probability p' that the cell is occupied. The relation between p and p' reflects the fact that the basic physics of percolation is connectedness, because we define a cell to be occupied only if it contains a set of sites that span the cell. If the sites are occupied with probability p, then the cells are occupied with probability p', where p' is given by a *renormalization transformation* or a *recursion relation* of the form

$$p' = R(p). \tag{12.17}$$

The quantity $R(p)$ is the total probability that the sites form a spanning path.

An example will make the formal relation (12.17) more clear. In Figure 12.14, we show the seven vertically spanning site configurations for a $b = 2$ cell. The probability p' that the renormalized site is occupied is given by the sum of the probabilities of all the spanning configurations:

$$p' = R(p) = p^4 + 4p^3(1-p) + 2p^2(1-p)^2. \tag{12.18}$$

(Note that $q = 1 - p$ is the probability that a site is empty.) In general, the probability p' of the occupied renormalized sites is different than the occupation probability p of the original sites. For example, suppose that we begin with $p = p_0 = 0.5$. After a single renormalization transformation, the value of p' from (12.18) is $p_1 = p' = R(p_0 = 0.5) = 0.44$. If we perform a second renormalization transformation, we have $p_2 = R(p_1) = 0.35$. It is easy to see that further transformations drive the system to the fixed point $p = 0$. Similarly, if we begin with $p = p_0 = 0.7$, we find that successive transformations drive the system to the fixed point $p = 1$. This behavior is qualitatively similar to what we observed in the visual renormalization group.

To find the nontrivial fixed point associated with the critical threshold p_c, we need to find the special value of p such that

$$p^* = R(p^*). \tag{12.19}$$

For the recursion relation (12.18), we find that the solution of the fourth-degree equation for p^* yields the two trivial fixed points, $p^* = 0$ and $p^* = 1$, and the nontrivial fixed point $p^* = 0.61804$ which we associate with p_c. This calculated value of p^* for $b = 2$ should be compared with $p_c \approx 0.5927$.