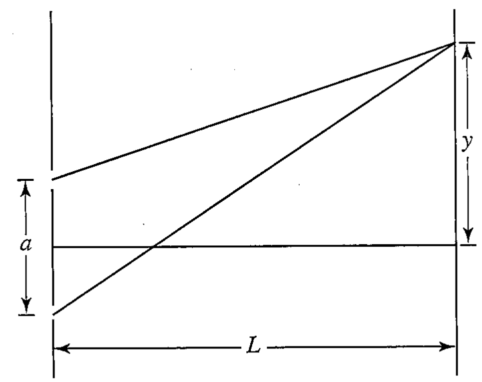

Figure 9.4 Young's double slit experiment. The figure defines the quantities a, L, and y used in Problem 9.31.

The classic example of interference is Young's double slit experiment (see Figure 9.4). Imagine two narrow parallel slits separated by a distance a and illuminated by a light source that emits light of only one frequency (monochromatic light). If the light source is placed on the line bisecting the two slits and the slit opening is very narrow, the two slits become coherent light sources with equal phases. We first assume that the slits act as point sources, for example, pinholes. A screen that displays the intensity of the light from the two sources is placed a distance L away. What do we see on the screen?

In the following problems, we discuss writing programs to determine the intensity of light that is observed on a screen due to a variety of geometries. The wavelength of the light sources, the positions of the sources $\mathbf{r}_i$, and the observation points on the screen need to be specified. Your program should instantiate the necessary point sources and compute a plot showing the intensity on the obervation screen located to the right of the sources by summing the phasors. Although we suggest that you use Listing 9.10 as a guide, it is unrealistic to compute a two-dimensional grid that covers the entire region from the source to the screen. It is more efficient to plot the intensity on the screen, thereby reducing the computation to a single loop over the screen coordinate y.

Problem 9.31 Point source interference

(a) Derive an analytic expression for E from two and three point sources if the screen is far from the sources.

(b) Compute and plot the intensity of light on a screen due to two small sources (a source that emits spherical waves). Compute the phasors using (9.61) and find the intensity by taking the magnitude of $\mathcal{E}$. Let a be the distance between the sources and y be the vertical position along the screen as measured from the central maximum. Set $L = 200\,\text{mm}$, $a = 0.1\,\text{mm}$, the wavelength of light $\lambda = 5000\,\text{Å}$ ($1\,\text{Å} = 10^{-7}\,\text{mm}$), and consider $-5.0\,\text{mm} \leq y \leq 5.0\,\text{mm}$ (see Figure 9.4). Describe the interference pattern you observe. Identify the locations of the intensity maxima and plot the intensity of the maxima as a function of y. Compare your result to the analytic expression for a two slit diffraction pattern.

(c) Repeat part (b) for $L = 0.5\,\text{mm}$, and $1.0\,\text{mm} \leq y \leq 1.0\,\text{mm}$. Note that in this case L is not much greater than a, and hence we cannot ignore the r dependence of $|\mathbf{r} - \mathbf{r}_i|^{-1}$ in (9.61). ∎

Problem 9.32 Diffraction grating

High resolution optical spectroscopy is done with multiple slits. In its simplest form, a diffraction grating consists of N parallel slits. Compute the intensity of light for $N = 3$, 4, 5, and 10 slits with $\lambda = 5000\,\text{Å}$, slit separation $a = 0.01\,\text{mm}$, screen distance $L = 200\,\text{mm}$, and $-15\,\text{mm} \leq y \leq 15\,\text{mm}$. How do the intensity of the peaks and their separation vary with N? ∎

In our analysis of the double slit and the diffraction grating, we assumed that each slit was a pinhole that emits spherical waves. In practice, real slits are much wider than the wavelength of visible light. In Problem 9.33 we consider the pattern of light produced when a plane wave is incident on an aperture such as a single slit. To do so, we use Huygens's principle and replace the slit by many coherent sources of spherical waves. This equivalence is not exact, but is applicable when the aperture width is large compared to the wavelength.

Problem 9.33 Single slit diffraction

(a) Compute the time averaged intensity of light diffracted from a single slit of width $0.02\,\text{mm}$ by replacing the slit by $N = 20$ point sources spaced $0.001\,\text{mm}$ apart. Choose $\lambda = 5000\,\text{Å}$, $L = 200\,\text{mm}$, and consider $-30\,\text{mm} \leq y \leq 30\,\text{mm}$. What is the width of the central peak? How does the width of the central peak compare to the width of the slit? Do your results change if N is increased?

(b) Determine the position of the first minimum of the diffraction pattern as a function of the wavelength, slit width, and distance to the screen.

(c) Compute the intensity pattern for $L = 1\,\text{mm}$ and $50\,\text{mm}$. Is the far field condition satisfied in this case? How do the patterns differ? ∎

Problem 9.34 A more realistic double slit simulation

Reconsider the intensity distribution for double slit interference using slits of finite width. Modify your program to simulate two "thick" slits by replacing each slit by 20 point sources spaced $0.001\,\text{mm}$ apart. The centers of the thick slits are $a = 0.1\,\text{mm}$ apart. How does the intensity pattern change? ∎

***Problem 9.35 Diffraction pattern from a rectangular aperture**

We can use a similar approach to determine the diffraction pattern due to a two-dimensional thin opaque mask with an aperture of finite width and height near the center. The simplest approach is to divide the aperture into little squares and to consider each square as a source of spherical waves. Similarly, we can divide the viewing screen or photographic plate into small regions or cells and calculate the time averaged intensity at the center of each cell. The calculations are straightforward, but time consuming because of the necessity of evaluating the cosine function many times. The less straightforward part of the problem is deciding how to plot the different values of the calculated intensity on the screen. One way is to plot