Computational Physics Homework Assignment #1

March 11, 2019; Due April 01, 2019

Reading/Pre-Lab Assignment

1. Read lecture notes and references; Recall your previous learned; Get familiar with computational resources, some useful software, and various operating system commands.
2. Study the sample programs either in the textbook or lecture notes, and write your own computer programs for coffee-cooling, falling objects, oscillatory motion, numerical integrations, etc., use any programming language as you prefer.
3. **Complete** one Lab Assignments before you leave the laboratory).

Laboratory Assignments (select 4 or 5) (Total Points: 200; Optional 50), on March 18

1. (Problem 2.1) **Coffee cooling program** (Points: 20, 15, 10, 10, 5; 25).
   (a) **Determine Cooling Constant** *r*: based on Newton's law of cooling, find approximated values of *r* for black coffee and coffee with cream that describe the experimental results shown in the following Table (is *r* a constant from the following Table?). What is your implicit criterion for determining the best value of *r*? Give two ways of determining the best value of *r* and argue which is better and why it is better. Because time is measured in minutes, the unit of the cooling constant *r* is min^{-1}.
   (b) Use the value of *r* found in part (a) and make a graph showing the dependence of temperature on time. Plot the data given in the Table on the same graph and compare them with your results. You may use any graphic package to make the plot.
   (c) Does the time step Δt have any physical significance? Make sure your choice of Δt is sufficiently small so that it does not affect your results. You should estimate the error of your results.
   (d) The initial difference in temperature between the black coffee and its surroundings is approximately 64^oC. How long does it take for the coffee to cool so that the difference is $64/2 = 32^oC$? How long does it take the difference to become 64/4 and 64/8? Try to understand your results in simple terms without first using a computer.
   (e) Refer to the Table, discuss if Newton's law of cooling is applicable to the cooling of a cup of coffee, and what modifications can one make?
   (f) (Optional, 25 points) Try to visualize the change of coffee temperature as your program is running. You can use color to represent the temperature, say, red means high temperature. (**Show it to TA or Lecturer**)

Table The temperature of coffee in a glass. The temperature was recorded with an estimated accuracy of 0.1^oC. The air temperature was 17^oC. The second column corresponds to black coffee and the third column corresponds to coffee with heavy cream.

time (min)	T^oC **(black)**	T^oC **(cream)**	**time (min)**	T^oC **(black)**	T^oC **(cream)**
0	82.3	68.8	24	51.2	45.9
2	78.5	64.8	26	49.9	44.8
4	74.3	62.1	28	48.6	43.7
6	70.7	59.9	30	47.2	42.6

8	67.6	57.7	32	46.1	41.7
10	65.0	55.9	34	45.0	40.8
12	62.5	53.9	36	43.9	39.9
14	60.1	52.3	38	43.0	39.3
16	58.1	50.8	40	41.9	38.6
18	56.1	49.5	42	41.0	37.7
20	54.3	48.1	44	40.1	37.0
22	52.8	46.8	46	39.5	36.4

2. (Problem 2.2) **Is it faster to add cream first or later?** (Points: 20).
Now we come to a practical question: suppose that the initial temperature of the coffee is 95^oC, but that the coffee can be sipped comfortable only when its temperature is below 70^oC. Assume that the addition of cream cools the coffee by $\Delta T = 8^oC$. If you are in a hurry and want to wait the shortest possible time, should the cream be added first and the coffee be allowed to cool to 70^oC, or should you wait until the coffee has cooled to 78^oC before adding the cream? Use your program and the two values of r you extrapolated from the Table to "simulate" the two cases. Does your general conclusion depend on using a different value of r when the cream is added? Does your answer depend on the value of ΔT?

3. **Comparison of algorithm.** (Points: 15, 5, 15, 15; <u>25).</u>
   (a) Use your programs to determine the time dependence of the velocity and position of a freely falling body near the earth's surface. Assume the values $y = 30m$ and $v = 0$ at $t = 0s$. The coordinate system is shown in the slide 8 of lecture-3 note. Compare your output to the exact results given by $v = v_0 - gt$, $y(t) = y_0 + v_0t - gt^2/2$. To reach the accuracy of 10^{-5}, what is a suitable value of Δt for the Euler, the Euler-Cromer, and the Euler-Richardson algorithms? (Make a table of comparison.)
   (b) Use any graphic tool to plot y and v of falling object as functions of time.
   (c) Apply your program to a simple harmonic oscillator for which $F = -kx$, taking units such that $k = 1$ and $m = 1$. Assuming $x(t = 0) = 1.0$ and $v(t = 0) = 0.0$, determine $x(t)$ by using the three algorithms and compare them with the exact result (I hope you remember how to get it) at $t = n\pi/4$, n = 1, 2, · · · , 32. What happens if you run for longer time, say $n = 201$, or 501? Try different values of Δt such that the accuracy as compared to the exact solution at $t = \pi$ is of 10^{-5}.
   (d) From your simulation results, is the Euler-Cromer algorithm better than the Euler algorithm? Think of a simple modification of either algorithm that yields exact results for the case of a freely falling body without air resistance.
   (e) (<u>Optional 25 points</u>) Write a subroutine to observe and visualize the position of the falling object at equal time intervals. Is the distance that the object has travelled over equal time intervals equal?

4. **Trajectory of a shot.** (Points: 10, 10, 15, 15). (Select Problem 4 or 5)
   (a) Use (or modify) your program(s) to compute the two-dimensional trajectory of a ball moving in air and plot y as function of x. Neglect air resistance first so that you can compare your computed results with the exact results. Assume that a ball is thrown from ground level at an angle θ_0 above the horizontal with an initial velocity $v_0 = 22m/s$. Vary

θ_0 (at least 5 θ_0 values) and show that the maximum range occurs at $\theta_0 = \theta_{max} = 45^o$. What is R_{max}, the maximum range of the ball, divided by v_0^2/g at corresponding angles?

(b) Suppose that a ball is thrown from a height h at an angle θ_0 above the horizontal with the same initial speed as in part (a). Again neglect air resistance, do you expect θ_{max} to be larger or smaller than 45^o? What is θ_{max} for $h = 2.0m$? By what percent is the range R changed if θ is varied by 3.0% from θ_{max}?

(c) Consider the effects of air resistance on the range and optimum angle of the ball in part (b). Assume $F_d(v) = k_2v^2$. What is the unit of k_2? Compute the optimum angle for $h = 3.3m$, $v_0 = 30m/s$, and $C = k_2/m = 0.1$, and compare your answer to the value found in part (b). Is R more or less sensitive to changes in θ_0 from θ_{max} than in part (b)? Determine the optimum launch angle and the corresponding range for the more realistic value of C = 0.002.

(d) Consider the motion of two identical objects that both start from a height $h = 13m$. One object is dropped vertically from rest and the other is thrown with a horizontal velocity $v_x = 30m/s$. Answer the following two questions by intuition and then by simulation results.

   i. Which object reaches the ground first?

   ii. Assume that there is air resistance and that the drag force is proportional to v^2 with $C = k_2/m = 0.03$. Which object reaches the ground first now? What if the drag force is proportional to v with $C = k_1/m = 0.055$?

5. **Motion of a linear oscillator** (Points: 10, 10, 15, 15). (Select Problem 4 or 5)

(a) Set $\omega_0 = 3$, $\gamma = 0.4$, $\omega = 2$ and the amplitude of the external force $A_0 = 3.0$ for all runs unless otherwise stated. This corresponds to an underdamped oscillator in the absence of an external force.

Plot $x(t)$ versus t for $A_0 = 0$ and $A_0 = 3.0$.

The initial conditions are: $x(t = 0) = 1$, $v(t = 0) = 0$.

Q1: How does the qualitative behavior of $x(t)$ differ when $A_0 = 3.0$ from the non-perturbed ($A_0 = 0$) case?

Q2: What is the period and angular frequency of $x(t)$ after several oscillations have occurred? (*Just estimate it by counting.*)

(b) Repeat (a) for another initial condition $x(t = 0) = 0.5$, $v(t = 0) = 1.0$.

Q3: Does $x(t)$ approach a limiting behavior independently of the initial conditions?

(Note: Try to identify a *transient* part of $x(t)$ that depends on the initial conditions and decays in time, and a *steady state* part that dominates at longer times and is independent of the initial conditions.)

(c) Modify your program so that E_n, the total energy per unit mass, is computed at each time $t_n = t_0+n\Delta t$. Plot the difference $\Delta E_n = E_n - E_0$ as a function of time for part (a).

Q4: Is E_n a conserved quantity within the algorithm accuracy?

Q5: Try the 4th order Runge-Kutta algorithm for **Q4**.

(d) Compute $x(t)$ for several combinations of ω_0 and ω (3 sets at least).

Q6: What is the period and angular frequency of the steady state in each case?

Q7: What parameters determine the frequency of the steady behavior? Assume that T is proportional to ω^α and estimate the exponent α from a log-log plot of T versus ω.

6. **Numerical Integration** (Points: 20). Using the asymptotic error formulae for the Trapezoid

and the Simpson's rules, estimate the number of subdivisions n to evaluate the following integrals to the given accuracy ε.

$$I_1 = \int_2^4 dx \log(x), \quad \varepsilon = 10^{-7}$$

$$I_2 = \int_0^1 dx e^{-x^2}, \quad \varepsilon = 10^{-9}$$

$$I_3 = \int_{1/2}^{5/2} \frac{dx}{1+x^2}, \quad \varepsilon = 10^{-11}$$