of many clusters (not counting the spanning cluster), and the central limit theorem implies that $P(R)$ converges to a Gaussian for large systems (except at $p = p_c$). Confirm these statements and find the shape of $P(R)$ for $p = p_c$ and $p_a = 0.25$. Variations of the Cont–Bouchaud model can be found in the references. The application of methods of statistical physics and simulations to economics and finance is now an active area of research and is commonly known as *econophysics*. ■

Project 12.16 The connectedness length

(a) Modify class `Clusters` so that the connectedness length ξ defined in (12.9) is computed. One way to do so is to introduce four additional arrays, `xAccum`, `yAccum`, `xSquaredAccum`, and `ySquaredAccum`, with the data stored at indices corresponding to the root sites. We visit each occupied site in the lattice and determine its root site. For example, if the site x, y is occupied and its root is `root`, we set `xAccum[root] += x`, `xSquaredAccum[root] += x*x`, `yAccum[root] += y`, and `ySquaredAccum[root] += y*y`. Then R_s^2 for an individual cluster is given by

$$R_s^2 = \texttt{xSquaredAccum[root]}/s + \texttt{ySquaredAccum[root]}/s - (\texttt{xAccum[root]}/s)^2 - (\texttt{xAccum/s[root]})^2, \tag{12.40}$$

where s is the number of sites in the cluster, which is given by `-parent[root]`.

(b) What is the qualitative behavior of $\xi(p)$ as a function of p for different size lattices? Is $\xi(p)$ a monotonically increasing or decreasing function of p for $p < p_c$ and $p > p_c$? Remember that ξ does not include the spanning cluster. ■

Project 12.17 Spanning clusters and periodic boundary conditions

For simplicity, we have used open boundary conditions, partly for historical reasons and partly because a spanning cluster is easier to visualize for open boundary conditions. An alternative is to use periodic boundary conditions and define a spanning cluster as one that wraps all the way around the lattice (see Figure 12.16).

A method for detecting cluster wrapping has been proposed by Machta et al. In addition to the `parent` array introduced on page 464, we define two integer arrays that give the net displacement in the x and y direction of each site to its parent site. When we traverse a site's cluster tree, we sum these displacements to find the total displacement to the root site. When an added site neighbors two (or more) sites that belong to the same cluster, we compare the total displacements to the root site for these two sites. If these displacements differ by an amount that does not equal the minimum displacement between these two sites, then cluster wrapping has occurred (see Figure 12.17).

Modify the Newman–Ziff algorithm so that periodic boundary conditions are used to define the clusters and the existence of a spanning cluster. Use your program to estimate p_c and n_s and show that periodic boundary conditions give better results for the percolation threshold p_c and the cluster size distribution n_s for the same size lattice. ■

Project 12.18 Conductivity in a random resistor network

(a) An important critical exponent for percolation is the conductivity exponent t defined by

$$\sigma \sim (p - p_c)^t, \tag{12.41}$$

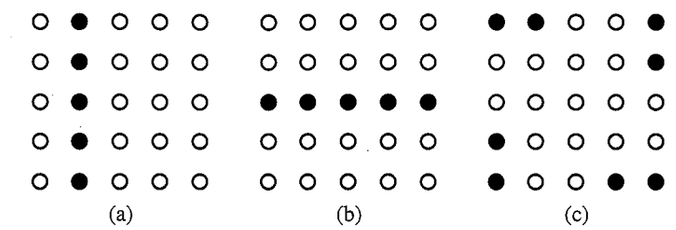

Figure 12.16 (a) Example of a cluster that wraps vertically. (b) Example of a cluster that wraps vertically and horizontally. (c) Example of a single cluster that does not wrap. Periodic boundary conditions are used in each case.

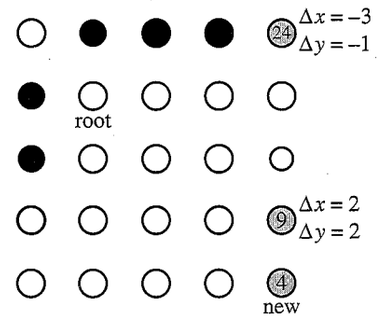

Figure 12.17 Example of cluster wrapping for periodic boundary conditions. When site 4 is occupied, it is a neighbor of sites 9 and 24 which belong to a single cluster. We compare the horizontal and vertical displacements of neighbors 9 and 24 to their root. If the difference between these displacements is not equal to the minimum displacement between them ($\Delta x_{\text{min}} = 0$, $\Delta y_{\text{min}} = 2$), then wrapping has occurred, as is the case here.

where σ is the conductance (or inverse resistance) per unit length in two dimensions. Consider bond percolation on a square lattice where each occupied bond between two neighboring sites is a resistor of unit resistance. Unoccupied bonds have infinite resistance. Because the total current into any node must equal zero by Kirchhoff's law, the voltage at any site (node) is equal to the average of the voltages of all nearest neighbor sites connected by resistors (occupied bonds). Because this relation for the voltage is the same as the algorithm for solving Laplace's equation on a lattice, the voltage at each site can be computed using the relaxation method discussed in Chapter 10. To compute the conductivity for a given $L \times L$ resistor network, we fix the voltage $V = 0$ at sites for which $x = 0$ and fix $V = 1$ at sites for which $x = L + 1$. In the y direction we use periodic boundary conditions. We then compute the voltage at all sites using the relaxation method. The current through each resistor connected to a site at $x = 0$ is $I = \Delta V/R = (V - 0)/1 = V$. The conductivity is the sum of the currents through all the resistors connected to $x = 0$ divided by L. In a similar way, the conductivity can be computed from the resistors attached to the $x = L + 1$ boundary. Write a program to implement the relaxation method for the conductivity of a random resistor network on a square lattice. An indirect, but easier way of computing the conductivity, is considered in Problem 13.8.