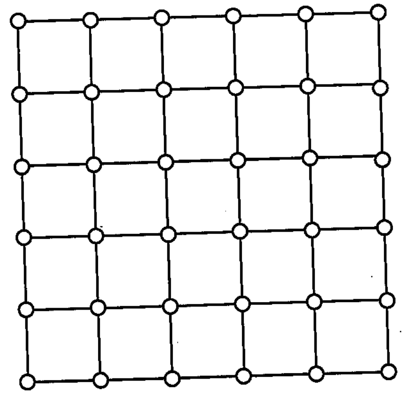

Figure 7.2 An example of a 6 × 6 square lattice. Note that each site or node has four nearest neighbors.

Problem 7.7 More random walks in one dimension

(a) Suppose that the probability of a step to the right is $p = 0.7$. Compute $\langle x \rangle$ and Δx^2 for $N = 4, 8, 16$, and 32. What is the interpretation of $\langle x \rangle$ in this case? What is the qualitative dependence of Δx^2 on N?

(b) An interesting property of random walks is the mean number $\langle D_N \rangle$ of *distinct* lattice sites visited during the course of an N-step walk. Do a Monte Carlo simulation of $\langle D_N \rangle$ and determine its N dependence. ■

We can consider either a large number of successive walks as in Problem 7.7 or a large number of noninteracting walkers moving at the same time as in Problem 7.8.

Problem 7.8 A random walk in two dimensions

(a) Consider a collection of walkers initially at the origin of a square lattice (see Figure 7.2). At each unit of time, each of the walkers moves at random with equal probability in one of the four possible directions. Create a drawable class, `Walker2D`, which contains the positions of M walkers moving in two dimensions and draws their location, and modify `WalkerApp`. Unlike `WalkerApp`, this new class need not specify the maximum number of steps. Instead, the number of walkers should be specified.

(b) Run your application with the number of walkers $M \geq 1000$ and allow the walkers to take at least 500 steps. If each walker represents a bee, what is the qualitative nature of the shape of the swarm of bees? Describe the qualitative nature of the surface of the swarm as a function of the number of steps N. Is the surface jagged or smooth?

(c) Compute the quantities $\langle x \rangle$, $\langle y \rangle$, Δx^2, and Δy^2 as a function of N. The average is over the M walkers. Also compute the mean square displacement R^2 given by

$$R^2 = \langle x^2 \rangle - \langle x \rangle^2 + \langle y^2 \rangle - \langle y \rangle^2 = \Delta x^2 + \Delta y^2. \tag{7.12}$$

What is the dependence of each quantity on N? (As before, we will frequently write R^2 instead of R_N^2.)

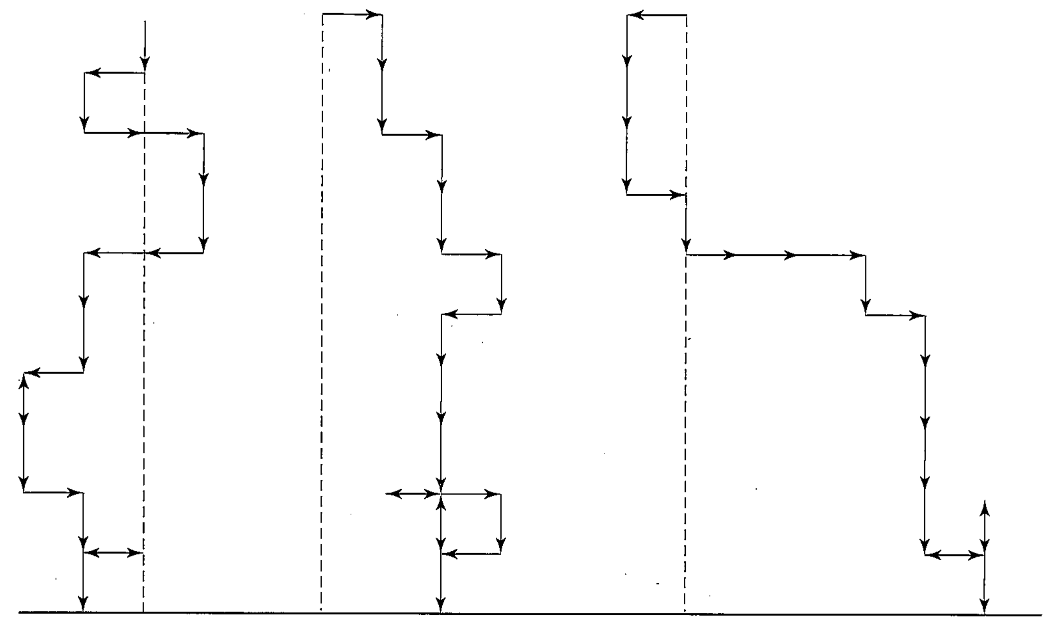

Figure 7.3 Examples of the random path of a raindrop to the ground. The step probabilities are given in Problem 7.9. The walker starts at $x = 0$, $y = h$.

(d) Estimate R^2 for $N = 8, 16, 32$, and 64 by averaging over a large number of walkers for each value of N. Assume that $R = \sqrt{R^2}$ has the asymptotic N dependence:

$$R \sim N^{\nu} \quad (N \gg 1), \tag{7.13}$$

and estimate the exponent ν from a log-log plot of R^2 versus N. We will see in Chapter 13 that the exponent $1/\nu$ is related to how a random walk fills space. If $\nu \approx 1/2$, estimate the magnitude of the self-diffusion coefficient D from the relation $R^2 = 4DN$.

(e) Do a Monte Carlo simulation of R^2 on a triangular lattice (see Figure 8.5) and estimate ν. Can you conclude that the exponent ν is independent of the symmetry of the lattice? Does D depend on the symmetry of the lattice? If so, give a qualitative explanation for this dependence.

*(f) Enumerate all the random walks on a square lattice for $N = 4$ and obtain exact results for $\langle x \rangle$, $\langle y \rangle$, and R^2. Assume that all four directions are equally probable. Verify your program by comparing the Monte Carlo and exact enumeration results. ■

Problem 7.9 The fall of a rain drop

Consider a random walk that starts at a site a distance $y = h$ above a horizontal line (see Figure 7.3). If the probability of a step down is greater than the probability of a step up, we expect that the walker will eventually reach a site on the horizontal line. This walk is a simple model of the fall of a rain drop in the presence of a random swirling breeze. Do a Monte Carlo simulation to determine the mean time τ for the walker to reach any site on the