Quaternions to Euler angles. Convert the quaternion to a rotation matrix and then convert the matrix to Euler angles. This conversion is easy if these two conversion algorithms have already been programmed.

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CHAPTER 18 Seeing in Special and General Relativity

We compute how objects appear at relativistic speeds and in the vicinity of a large spherically symmetric mass.

18.1 ■ SPECIAL RELATIVITY

How do objects appear at relativistic speeds? The Lorentz–Fitzgerald length contraction in the direction of motion is not the only effect that needs to be considered when determining the apparent shape of an object. A single observer forms an image of an object by collecting light emitted from the entire object. When an observer *sees* the object, the observer does not see its current position nor its true shape but sees each part of the object where it was when the light was emitted. This position is known as the *retarded position*. Because of the finite speed of light, we must calculate when and where along the object's trajectory each light ray originated to determine the image formed on the observer's retina.

The relative velocity of an object with respect to a single observer defines a direction, which we take to be the direction of the x-axis in the observer's frame of reference S. Let S' be the rest frame of the object and v be the velocity of S with respect to S' such that the origins coincide at $t = t' = 0$. The Lorentz transformation connecting S and S' is

$$x' = \frac{1}{\gamma}(x - vt) \tag{18.1a}$$

$$y' = y \tag{18.1b}$$

$$z' = z \tag{18.1c}$$

$$t' = \gamma(t - vx/c^2), \tag{18.1d}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - \beta^2}$ and $\beta = v/c$. In the rest frame of the object, the spatial separation between two points on the object is

$$d' = \sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2}. \tag{18.2}$$

In the rest frame of the observer, the separation is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}, \tag{18.3}$$