We use the speed of light to convert distance to light travel time ($r_{\text{ret}} = c\tau$), substitute for δ, and obtain

$$c\tau = \sqrt{(x - v\tau)^2 + y^2}. \tag{18.7}$$

If we square both sides and solve for τ, we find

$$\tau = \frac{-x\beta + \sqrt{x^2\beta^2 + (1-\beta^2)(x^2+y^2)}}{c(1-\beta^2)}, \tag{18.8}$$

where we have chosen the positive square root to make the travel time τ positive.

We can incorporate the time delay (18.8) into a program to obtain the image seen by a stationary observer. We subclass `ContractedRing` and add methods to compute and draw the retarded positions of the points on the ring. Retarded points are stored in an array and the retarded positions are computed by solving (18.8). The class `ObservedRing` is shown in Listing 18.2.

Listing 18.2 The `ObservedRing` class models the appearance of a ring traveling in the x direction at relativistic speeds.

```
package org.opensourcephysics.sip.ch18;
import java.awt.*;
import java.awt.geom.*;
import org.opensourcephysics.display.*;

public class ObservedRing extends ContractedRing {
   Point2D[] retardPts;

   public ObservedRing(double x0, double y0, double vx, int numPts) {
      super(x0, y0, vx, numPts); // x would change to numberOfPoints
      retardPts = new Point2D[numPts];
      for(int i = 0;i<numPts;i++) {
         retardPts[i] = new Point2D.Double();
      }
   }

   void setRetardedPts() {
      double oneOverGammaSquared = (1-vx*vx);
      for(int i = 0, n = labPoints.length;i<n;i++) {
         double x = labPoints[i].getX();
         double y = labPoints[i].getY();
         double tau = (-vx*x+Math.sqrt(x*x*vx*vx+
            oneOverGammaSquared*(x*x+y*y)))/oneOverGammaSquared;
         retardPts[i].setLocation(x-vx*tau, y);
      }
   }

   void drawObservedShape(DrawingPanel panel, Graphics2D g2) {
      setRetardedPts();
      // converts from view to pixel coordinates
      AffineTransform at = panel.getPixelTransform();
      at.transform(retardPts, 0, pixPoints, 0, retardPts.length);
      g2.setColor(Color.BLACK);
      for(int i = 1, n = retardPts.length;i<n;i++) {
```



```
         g2.draw(new Line2D.Double(pixPoints[i-1], pixPoints[i]));
      }
   }

   public void draw(DrawingPanel panel, Graphics g) {
      Graphics2D g2 = (Graphics2D) g;
      drawShape(panel, g2);
      drawObservedShape(panel, g2);
   }
}
```

Exercise 18.2 Relativistic ring

Write a test program that instantiates and displays the apparent shape of a rapidly moving ring. Explain the sharp convex point when the front edge of the ring touches (reaches) the observer. Does the ring ever appear to be concave? Why? ■

Exercise 18.3 Relativistic ruler

Modify the `ObservedRing` class to display a long narrow rectangle. Can an observer see the Lorentz–Fitzgerald contraction if this "ruler" is moving along the x-axis? Click-drag within the display to measure the apparent length of the ruler at various positions. Explain the meaning of the term *observer* in relativity. Some authors (see Taylor and Wheeler) prefer the term *bookkeeper*. Why might this term be better? ■

What an observer sees is quite different from what is given by the Lorentz contraction. What makes Einstein's special theory of relativity profound is not the appearance, but rather that length really does contract and time really does slow down.

Exercise 18.4 Relativistic square

Write a target class that instantiates and displays the apparent shape of a moving square whose trajectory is $(x_0 - vt, b)$ past a stationary observer at $(0, 0)$. Is the apparent shape still a square? Explain why the observer can see the square's hidden side. This effect is known as *Terrell rotation*. ■

We can rotate the shape seen in the simulation around the x-axis to visualize the apparent shape of a three-dimensional sphere approaching an observer head-on. This case was treated analytically by Suffern, but most other two- and three-dimensional objects cannot be treated analytically and are best visualized using the help of a computer. A complete and accurate visualization must take into account additional physics such as the Doppler effect, aberration, and angular changes in the intensity distribution of the emitted light (see Weisskopf).

18.2 ■ GENERAL RELATIVITY

The idea that space is curved was first tested by Gauss who measured the interior angles of a large triangle by placing lanterns on three mountain tops. Although Gauss obtained the Euclidian (flat-space) result of 180°, measurements of stellar positions during the 1919