CHAPTER

4 Oscillatory Systems

We explore the behavior of oscillatory systems, including the simple harmonic oscillator, a simple pendulum, and electrical circuits, and introduce the concept of phase space.

4.1 ■ SIMPLE HARMONIC MOTION

There are many physical systems that undergo regular, repeating motion. Motion that repeats itself at definite intervals, for example, the motion of the earth about the sun, is said to be *periodic*. If an object undergoes periodic motion between two limits over the same path, we call the motion *oscillatory*. Examples of oscillatory motion that are familiar to us from our everyday experience include a plucked guitar string and the pendulum in a grandfather clock. Less obvious examples are microscopic phenomena such as the oscillations of the atoms in crystalline solids.

To illustrate the important concepts associated with oscillatory phenomena, consider a block of mass m connected to the free end of a spring. The block slides on a frictionless, horizontal surface (see Figure 4.1). We specify the position of the block by x and take $x = 0$ to be the equilibrium position of the block, that is, the position when the spring is relaxed. If the block is moved from $x = 0$ and then released, the block oscillates along a horizontal line. If the spring is not compressed or stretched too far from $x = 0$, the force on the block at position x is proportional to x:

$$F = -kx. \tag{4.1}$$

The force constant k is a measure of the stiffness of the spring. The negative sign in (4.1) implies that the force acts to restore the block to its equilibrium position. Newton's equation of motion for the block can be written as

$$\frac{d^2x}{dt^2} = -\omega_0^2 x, \tag{4.2}$$

where the angular frequency ω_0 is defined by

$$\omega_0^2 = \frac{k}{m}. \tag{4.3}$$

The dynamical behavior described by (4.2) is called *simple harmonic motion* and can be solved analytically in terms of sine and cosine functions. Because the form of the solution will help us introduce some of the terminology needed to discuss oscillatory motion, we

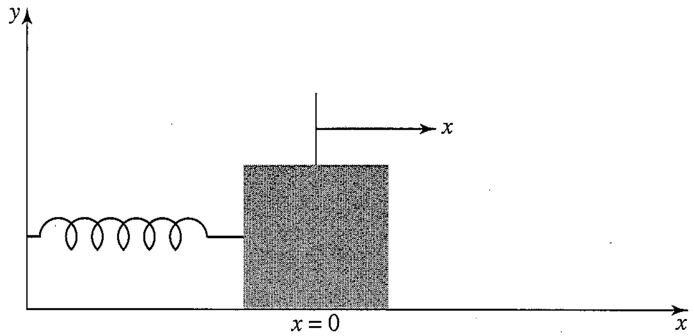

Figure 4.1 A one-dimensional harmonic oscillator. The block slides horizontally on the frictionless surface.

include the solution here. One form of the solution is

$$x(t) = A\cos(\omega_0 t + \delta), \tag{4.4}$$

where A and δ are constants and the argument of the cosine is in radians. It is straightforward to check by substitution that (4.4) is a solution of (4.2). The constants A and δ are called the amplitude and the phase, respectively, and are determined by the initial conditions for x and the velocity $v = dx/dt$.

Because the cosine is a periodic function with period 2π, we know that $x(t)$ in (4.4) is also periodic. We define the period T as the smallest time for which the motion repeats itself, that is,

$$x(t + T) = x(t). \tag{4.5}$$

Because $\omega_0 T$ corresponds to one cycle, we have

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{k/m}}. \tag{4.6}$$

The frequency ν of the motion is the number of cycles per second and is given by $\nu = 1/T$. Note that T depends on the ratio k/m and not on A and δ. Hence, the period of simple harmonic motion is independent of the amplitude of the motion.

Although the position and velocity of the oscillator are continuously changing, the total energy E remains constant and is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2. \tag{4.7}$$

The two terms in (4.7) are the kinetic and potential energies, respectively.

Problem 4.1 Energy conservation

(a) Use the Euler `ODESolver` to solve the dynamical equations for a simple harmonic oscillator by extending `AbstractSimulation` and implementing the `doStep` method. (See Section 4.2 for an example of such a program for the pendulum.) Have your program plot $\Delta E_n = E_n - E_0$, where E_0 is the initial energy and E_n is the total