where we will assume that ω is a constant. The rate for each of the three position variables is the corresponding velocity. Typical parameter values for a 149 gram baseball are $C_D = 6 \times 10^{-3}$ and $C_M = 4 \times 10^{-4}$. See the book by Adair for a more complete discussion.

Problem 3.15 Curveballs

(a) Create a class that implements (3.26). Assume that the initial ball is released at $z = 1.8$ m above and $x = 18$ m from home plate. Set the initial angle above the horizontal and the initial speed using the constructor.

(b) Write a program that plots the vertical and horizontal deflection of the baseball as it travels toward home plate. First set the drag and Magnus forces to zero and test your program using analytical results for a 40 m/s fastball. What initial angle is required for the pitch to pass over home plate at a height of 1.5 m?

(c) Add the drag force with $C_D = 6 \times 10^{-3}$. What initial angle is required for this pitch to be a strike assuming that the other initial conditions are unchanged? Plot the vertical deflection with and without drag for comparison.

(d) Add topspin to the pitch using a typical spin of $\omega_y = 200$ rad/s and $C_M = 4 \times 10^{-4}$. How much does topspin change the height of the ball as it passes over the plate? What about backspin?

(e) How much does a 35 m/s curveball deflect if it is pitched with an initial spin of 200 rad/s? ■

Problem 3.16 Visualizing baseball trajectories in three dimensions
Add a 3D visualization of the baseball's trajectory to Problem 3.15 using `ElementTrail` to display the path of the ball. The following code fragment shows how a trail is created and used.

```
ElementTrail trail = new ElementTrail();
trail.setMaximumPoints(500);
trail.getStyle().setLineColor(java.awt.Color.RED);
// frame3D is an OSP3DFrame
frame3D.addElement(trail);
// points are added to a trail to show a trajectory
trail.addPoint(x,y,z);  // adds a point to the trace
```
■

Coupled three-dimensional equations of motion occur in electrodynamics when a charged particle travels through electric and magnetic fields. The equation of motion can be written in vector form as

$$m\dot{\mathbf{v}} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}), \tag{3.27}$$

where m is the mass of the particle, q is the charge, and $\mathbf{E}$ and $\mathbf{B}$ represent the electric and magnetic fields, respectively. For the special case of a constant magnetic field, the trajectory of a charged particle is a spiral along the field lines with a cyclotron orbit whose period of revolution is $2\pi m/qB$. The addition of an electric field changes this motion dramatically.

The rates for the velocity components of a charged particle using units such that $m = q = 1$ are

$$\frac{dv_x}{dt} = E_x + v_y B_z - v_z B_y \tag{3.28a}$$

$$\frac{dv_y}{dt} = E_y + v_z B_x - v_x B_z \tag{3.28b}$$

$$\frac{dv_z}{dt} = E_z + v_x B_y - v_y B_x. \tag{3.28c}$$

The rate for each of the three position variables is again the corresponding velocity.

Problem 3.17 Motion in electric and magnetic fields

(a) Write a program to simulate the two-dimensional motion of a charged particle in a constant electric and magnetic field with the magnetic field in the z direction and the electric field in the y direction. Assume that the initial velocity is in the xy-plane.

(b) Why does the trajectory in part (a) remain in the xy-plane?

(c) In what direction does the charged particle drift if there is an electric field in the x direction and a magnetic field in the z direction if it starts at rest from the origin? What type of curve does the charged particle follow?

(d) Create a three-dimensional simulation of the trajectory of a particle in constant electric and magnetic fields. Verify that a charged particle undergoes spiral motion in a constant magnetic field and zero electric field. Predict the trajectory if an electric field is added and compare the results of the simulation to your prediction. Consider electric fields that are parallel to and perpendicular to the magnetic field. ■

Although the trajectory of a charged particle in constant electric and magnetic fields can be solved analytically, the trajectories in the presence of dipole fields cannot. A magnetic dipole with dipole moment $\mathbf{p} = |p|\hat{\mathbf{p}}$ produces the following magnetic field:

$$\mathbf{B} = \frac{\mu_0 m}{4\pi\epsilon_0 r^3}[3\hat{\mathbf{p}} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \hat{\mathbf{p}}]. \tag{3.29}$$

(The distinction between the symbol p for the dipole moment and p for momentum should be clear from the context.)

***Problem 3.18 Motion in a magnetic dipole field**
Model the Earth's Van Allen radiation belt using the following formula for the dipole field:

$$\mathbf{B} = B_0\left(\frac{R_E}{R}\right)^3[3\hat{\mathbf{p}} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \hat{\mathbf{p}}], \tag{3.30}$$

where R_E is the radius of the Earth, and the magnetic field at the equator is $B_0 = 3.5 \times 10^{-5}$ tesla. Note that a 1 MeV electron at 2 Earth radii travels in very tight spirals with a cyclotron period that is much smaller than the travel time between the north and south poles. Better visual results can be obtained by raising the electron energies by a factor of ~ 1000. Use classical dynamics, but include the relativistic dependence of the mass on the particle speed. ■