The target application, `Planet2App`, extends `AbstractSimulation` in the usual way. Because it is almost identical to Listing 5.2, it is not shown here. The complete program is available in the ch05 package.

Problem 5.10 Planetary perturbations
Use `Planet2App` with the initial conditions given in the program. For illustrative purposes, we have adopted the numerial values $m_1/M = 10^{-3}$ and $m_2/M = 4 \times 10^{-2}$, and hence `GM1` $= (m_2/M)GM = 0.04$`GM` and `GM2` $= (m_1/M)GM = 0.001$`GM`. What would be the shape of the orbits and the periods of the two planets if they did not mutually interact? What is the qualitative effect of their mutual interaction? Describe the shape of the two orbits. Why is one planet affected more by their mutual interaction than the other? Are the angular momentum and the total energy of planet one conserved? Are the total energy and total angular momentum of the two planets conserved? A related but more time consuming problem is given in Project 5.18. ■

Problem 5.11 Double stars
Another interesting dynamical system consists of one planet orbiting about two fixed stars of equal mass. In this case there are no closed orbits, but the orbits can be classified as either stable or unstable. Stable orbits may be open loops that encircle both stars, figure eights, or orbits that encircle only one star. Unstable orbits will eventually collide with one of the stars. Modify `Planet2` to simulate the double-star system, with the first star located at $(-1, 0)$ and the second star of equal mass located at $(1, 0)$. Place the planet at $(0.1, 1)$ and systematically vary the x and y components of the velocity to obtain different types of orbits. Then try other initial positions. ■

5.10 ■ TWO-BODY SCATTERING

Much of our understanding of the structure of matter comes from scattering experiments. In this section we explore one of the more difficult concepts in the theory of scattering, the *differential cross section*.

A typical scattering experiment involves a beam with many incident particles all with the same kinetic energy. The coordinate system is shown in Figure 5.7. The incident particles come from the left with an initial velocity $\mathbf{v}$ in the $+x$ direction. We take the center of the beam and the center of the target to be on the x-axis. The *impact parameter* b is the perpendicular distance from the initial trajectory to a parallel line through the center of the target (see Figure 5.7). We assume that the width of the beam is larger than the size of the target. The target contains many scattering centers, but for calculational purposes, we may consider scattering off only one particle if the target is sufficiently thin.

When an incident particle comes close to the target, it is deflected. In a typical experiment, the scattered particles are counted in a detector that is far from the target. The final velocity of the scattered particles is $\mathbf{v}'$, and the angle between $\mathbf{v}$ and $\mathbf{v}'$ is the scattering angle θ.

Let us assume that the scattering is elastic and that the target is much more massive than the beam particles so that the target can be considered to be fixed. (The latter condition can be relaxed by using center of mass coordinates.) We also assume that no incident particle is scattered more than once. These considerations imply that the initial speed and final speed of the incident particles are equal. The functional dependence of θ on b depends on the force on the beam particles due to the target. In a typical experiment, the number of particles in

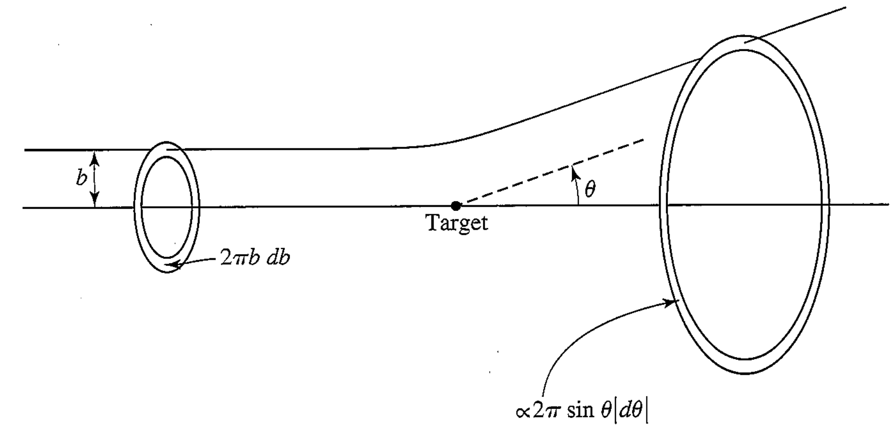

Figure 5.7 The coordinate system used to define the differential scattering cross section. Particles passing through the beam area $2\pi b\, db$ are scattered into the solid angle $d\Omega$.

an angular region between θ and $\theta + d\theta$ is detected for many values of θ. These detectors measure the number of particles scattered into the solid angle $d\Omega = \sin\theta\, d\theta\, d\phi$ centered about θ. The *differential cross section* $\sigma(\theta)$ is defined by the relation

$$\frac{dN}{N} = n\sigma(\theta)\, d\Omega, \tag{5.27}$$

where dN is the number of particles scattered into the solid angle $d\Omega$ centered about θ and the azimuthal angle ϕ, N is the total number of particles in the beam, and n is the target density defined as the number of targets per unit area.

The interpretation of (5.27) is that the fraction of particles scattered into the solid angle $d\Omega$ is proportional to $d\Omega$ and the density of the target. From (5.27) we see that $\sigma(\theta)$ can be interpreted as the effective area of a target particle for the scattering of an incident particle into the element of solid angle $d\Omega$. Particles that are not scattered are ignored. Another way of thinking about $\sigma(\theta)$ is that it is the ratio of the area $b\, db\, d\phi$ to the solid angle $d\Omega = \sin\theta\, d\theta\, d\phi$, where $b\, db\, d\phi$ is the infinitesimal cross-sectional area of the beam that scatters into the solid angle defined by θ to $\theta + d\theta$ and ϕ to $\phi + d\phi$. The alternative notation for the differential cross section, $d\sigma/d\Omega$, comes from this interpretation.

To do an analytical calculation of $\sigma(\theta)$, we write

$$\sigma(\theta) = \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|. \tag{5.28}$$

We see from (5.28) that the analytical calculation of $\sigma(\theta)$ involves b as a function of θ or, more precisely, how b changes to give scattering through an infinitesimally larger angle $\theta + d\theta$.

In a scattering experiment, particles enter from the left (see Figure 5.7) with random values of the impact parameter b and azimuthal angle ϕ, and the number of particles scattered into the various detectors is measured. In our simulation, we know the value of b, and we can integrate Newton's equations of motion to find the angle at which the incident particle is scattered. Hence, in contrast to the analytical calculation, a simulation naturally yields θ as a function of b.