```
        frame.setVisible(true);
        frame.setDefaultCloseOperation(javax.swing.JFrame.EXIT_ON_CLOSE);
    }
}
```

To emphasize the weaker regions of the diffraction pattern, the program plots the logarithm of the intensity. First the intensity is normalized to its peak value; then the logarithm is taken and all values less than a cutoff are truncated. The resulting range is mapped linearly to (0, 255) to set the gray scale.

Problem 9.36 Two-dimensional apertures
Modify `FraunhoferApp` to show the diffraction pattern from a double slit and compare the computed diffraction pattern to the analytic result. ■

The `Fraunhofer2DApp` program computes the Fraunhofer diffraction pattern for a circular aperture using a two-dimensional FFT. This program is used in Problem 9.37 but is not listed because it is too long and because it is similar to Listing 9.11.

Problem 9.37 Two-dimensional apertures

(a) Compile and run the `Fraunhofer2DApp` program. Compute the diffraction pattern using aperture radii of 4λ and 0.25λ in a mask with dimension $a\lambda$. How does the radius influence the diffraction pattern? How does the rectangular grid influence the pattern? How can the effect of the rectangular grid be reduced?
(b) Because a typical computer monitor displays only 256 gray scale values, the class `Fraunhofer2DApp` uses a logarithmic scale to enhance the visibility of the fringes. Add code to display a linear plot of intensity as a function of radius using a slice through the center of the pattern.
(c) Compute the diffraction pattern for an annular ring with inner radius 1.8λ and outer radius 2.2λ. Why is there a bright spot at the center of the diffraction pattern? What effect does the finite width of the annular ring produce?
(d) Compute the diffraction pattern for a rectangular aperture with width 2λ and height 6λ. Describe the effect of the asymmetry of the slit. ■

Problem 9.38 Diffraction patterns due to multiple apertures

(a) Compute the diffraction pattern due to a 5×5 array of rectangular slits. Each slit has a width of 0.5 and a height of 0.25 and is offset by (1,1) from neighboring slits using units such that $\lambda = 1$. The aperture array is centered within a mask ten units on a side. What is the effect of the asymmetry of the slit? What happens if the number of slits is decreased or increased?
(b) Compute the diffraction pattern due to 25 randomly placed rectangular slits. Each slit should have a width of 0.5 and a height of 0.25 and be placed within a region ten units on a side. Do not be concerned if rectangles overlap.
(c) Compare the results from (a) and (b). What effect does the random placement have on the pattern? ■

9.10 ■ FRESNEL DIFFRACTION

Fourier analysis can be used to compute the Fresnel diffraction pattern by decomposing a wave incident on an aperture into a sum of plane waves and then propagating each plane wave from the aperture mask to the screen. A plane wave with wavenumber (q_x, q_y, q_z) propagating in a homogenous environment can be written as

$$\mathcal{U} = \mathcal{U}_0 e^{i(q_x x + q_y y + q_z z)}, \tag{9.67}$$

where $\mathcal{U}_0$ is the amplitude of the field at the origin, and (q_x, q_y, q_z) is a vector of length $2\pi/\lambda$ in the direction of propagation. If we place a viewing screen perpendicular to the direction of the incoming light at a point z_0 along the z-axis, then the field on the screen is

$$\mathcal{U} = \mathcal{U}_0 e^{iq_z z_0} = \mathcal{U}_0 e^{iz_0(q^2 - q_x^2 - q_y^2)^{1/2}}, \tag{9.68}$$

where we have used the fact that $q_x^2 + q_y^2 + q_z^2 = q^2$.

We now place an aperture mask at the origin $z = 0$ in the xy-plane and illuminate it from the left by a plane wave. Because the aperture truncates the incident plane wave, we obtain a more complicated field $\mathcal{U}_0(q_x, q_y)$ that contains both q_x and q_y spatial components:

$$\mathcal{U}_0(q_x, q_y) = \iint_{\text{aperture}} e^{i(q_x x + q_y y)}\, dx\, dy. \tag{9.69}$$

The field that propagates from the origin contains the Fourier components of the aperture mask. In other words, because we have truncated the wave, we have a field composed of a mixture of plane waves with wavenumbers (q_x, q_y, q_z). Each field component is multiplied by the $e^{iq_z z_0}$ phase factor in (9.68) as it propagates toward the viewing screen at z_0. The following steps summarize the algorithm:

1. Compute the Fourier transformation of the aperture (9.69) to obtain the field's components in the plane of the aperture.
2. Multiply each component by the propagation phase factor $e^{iz_0(q^2 - q_x^2 - q_y^2)^{1/2}}$.
3. Compute the inverse transformation to obtain the amplitude.
4. Compute the magnitude squared of the amplitude to obtain the intensity.

The Fresnel diffraction pattern algorithm is implemented in Listing 9.12. Note that the field includes evanescent waves if $q^2 - q_x^2 - q_y^2 < 0$.

Listing 9.12 The `FresnelApp` program computes the Fresnel diffraction pattern from a circular aperture.

```
package org.opensourcephysics.sip.ch09;
import org.opensourcephysics.frames.RasterFrame;
import org.opensourcephysics.numerics.FFT2D;

public class FresnelApp {
   final static double PI2 = Math.PI*2;
   final static double PI4 = PI2*PI2;

   public static void main(String[] args) {
```