```
        control.setAdjustableValue("steps per display", 100);
    }

    public static void main(String[] args) {
        SimulationControl.createApp(new LatticeGasApp());
    }
}
```

An important application of lattice gas models is to simulate the flow in and around various geometries. In Problem 14.20 we will see that the fluid velocity field develops vortices, wakes, and other fluid structures near obstacles. Method `initialize` in class `LatticeGas` places an obstacle in the middle of the lattice and provides initial values for each site. Large lattices are required to obtain quantitative results, because it is necessary to average the velocity over many sites. The parameter `density` is the average number of particles divided by the maximum possible. The pressure can be varied by changing the `flowSpeed` parameter.

Problem 14.20 Flow past a barrier

(a) Convince yourself that you understand the collision rules and their implementation in `LatticeGas`. Then download the class `FastLatticeGas` from the ch14 directory. This latter class uses all 32 bits of an `int` variable and runs about twice as fast. The tradeoff is that the code is more difficult to debug and understand. Use the parameters in Listing 14.14. Describe the flow once a steady-state velocity field begins to appear. Do you see a wake appearing behind the obstacle? Are there vortices?

(b) Repeat part (a) with different size obstacles. Are there any systematic trends? (One limitation of the present program is that it naively redraws a circle to represent each barrier site. This redrawing requires a significant amount of computer resources and limits the size of the obstacles that we can consider.)

(c) Reduce the pressure by reducing the flow speed. Are there any noticeable changes in behavior from parts (a) and (b)? Reduce the pressure still further and describe any changes in the fluid flow. ■

Problem 14.21 Approach to equilibrium

(a) Consider the approach of a lattice gas to equilibrium. Modify `LatticeGas` so that the initial configuration has zero net momentum, the particles are localized in a $b \times b$ region, and there are no barrier sites. Choose $L = 30$ and $b = 4$ and place six particles at every site in the localized region. The other sites in the lattice are initially empty. Describe what happens to the particles as a function of time. Approximately how many time steps does it take for the system to come to equilibrium? Do the particles appear to be at random positions with random velocities? What is your visual algorithm for determining when equilibrium has been reached?

(b) Repeat part (a) for $b = 2$, 6, 8, and 10. Estimate the equilibration time in each case. What is the qualitative dependence of the equilibration time on b? How does the equilibration time depend on the number density ρ?

(c) Repeat part (a) with $b = 4$, but with $L = 10$, 20, and 40. Estimate the equilibration time in each case. How does the equilibration time depend on ρ? ■

Problem 14.22 Fluid flow in porous media

(a) Modify class `LatticeGas` so that instead of a rectangular barrier, the barrier sites are placed at random in the system. We define the porosity ϕ as the fraction of sites without a barrier. The interesting quantity to measure is the permeability k, which is a measure of the fluid conductivity. We can compute the permeability using the relation

$$k \propto \frac{\phi \sum_i \langle v_{i,x} \rangle}{\sum_j \langle \Delta p_{j,x} \rangle}, \tag{14.8}$$

where the sum in the numerator is over the horizontal velocity of all particles in the pore space (the sites at which there are no barriers), and the sum in the denominator is over the injected momentum at all sites used to maintain the flow. The brackets refer to averages over time. Compute the permeability as a function of the porosity ϕ and display your results on a log-log plot. You should average over at least 10 configurations of random barrier sites for each value of the porosity. What value of ϕ corresponds to the percolation threshold, defined by $k = 0$? See Rothman and Zaleski for a discussion of the comparison of this type of simulation with results for real rocks.

*(b) Vary the size of the lattice and use the finite size scaling procedure discussed in Section 12.4 to estimate the critical exponent μ defined by the dependence of the permeability on the porosity, that is, $k \sim (\phi - \phi_c)^\mu$. Assume that you know the value of the percolation exponent ν defined by the critical behavior of the connectedness length $\xi \sim |p - p_c|^{-\nu}$ (see Table 12.1). ■

The principal virtues of lattice gas models are their use of simultaneous updating, which makes them very fast on parallel computers, and their use of integer or boolean arithmetic and bit manipulation, which is faster than floating point arithmetic. Their major limitation is that it is necessary to average over many sites to obtain quantitative results. It is not yet clear whether lattice gas models are more efficient than standard simulations of the Navier–Stokes equation. The greatest promise for lattice gas models may not be with simple single component fluids, but with multicomponent fluids such as binary fluids and fluids containing bubbles (see the book by Rothman and Zaleski). A related technique that might hold greater promise is the lattice Boltzmann method (see the references).

14.7 ■ OVERVIEW AND PROJECTS

The models we have discussed in this chapter have been presented as algorithms rather than in terms of differential equations and are a reflection of the way that technology affects the way we think. Can you discuss the models in this chapter without thinking about their implementation on a computer? Can you imagine understanding these models without the use of computer graphics?

We have given only a brief introduction to cellular automata and other models that are relevant to the rapidly developing study of complex systems. There are many more models and applications that we have not discussed, ranging from aging, the immune system, economic cycles, and pedestrian movements to name just a few.