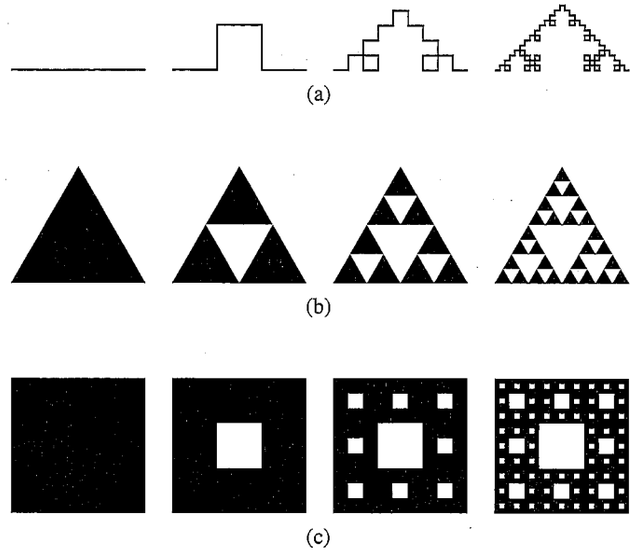

Figure 13.7 (a) The first few iterations of the quadric Koch curve. (b) The first few iterations of the Sierpiński gasket. (c) The first few iterations of the Sierpiński carpet.

(b) Regular fractals can be generated from a pattern that is used in a self-replicating manner. Write a program to generate the quadric Koch curve shown in Figure 13.7a. What is its fractal dimension?

(c) What is the fractal dimension of the Sierpiński gasket shown in Figure 13.7b? Write a program that generates the next several iterations.

(d) What is the fractal dimension of the Sierpiński carpet shown in Figure 13.7c? How does the fractal dimension of the Sierpiński carpet compare to the fractal dimension of a percolation cluster? Are the two fractals visually similar? ■

13.3 ■ KINETIC GROWTH PROCESSES

Many systems in nature exhibit fractal geometry. Fractals have been used to describe the irregular shapes of such varied objects as coastlines, clouds, coral reefs, and the human lung. Why are fractal structures so common? How do fractal structures form? In this section we discuss several growth models that generate structures that show a remarkable similarity to forms observed in nature. The first two models are already familiar to us and exemplify the flexibility and utility of kinetic growth models.

Epidemic model. In the context of the spread of disease, we usually want to know the conditions for an epidemic. A simple lattice model of the spread of a disease can be formulated as follows. Suppose that an occupied site corresponds to an infected person. Initially there is a single infected person and the four nearest neighbor sites (on the square lattice) correspond to susceptible people. At the next time step, we visit the four susceptible sites

and occupy (infect) each site with probability p. If a susceptible site is not occupied, we say that the site is immune and we do not test it again. We then find the new susceptible sites and continue until either the disease is controlled or reaches the boundary of the lattice. Convince yourself that this growth model of a disease generates a cluster of infected sites that is identical to a percolation cluster at probability p. The only difference is that we have introduced a discrete time step into the model. Some of the properties of this model are explored in Problem 13.5.

Problem 13.5 A simple epidemic model

(a) Explain why the simple epidemic model discussed in the text generates the same clusters as in the percolation model. What is the minimum value of p necessary for an epidemic to occur? Recall that in one time step, all susceptible sites are visited simultaneously and infected with probability p. Determine how n, the number of infected sites, depends on the time t (the number of time steps) for various values of p. A straightforward way to proceed is to modify class `SingleCluster` so that all susceptible sites are visited and occupied with probability p before new susceptible sites are found. In Chapter 14 we will learn that this model is an example of a cellular automaton.

(b) What are some ways that you could modify the model to make it more realistic? For example, the infected sites might recover after a certain time. ■

Eden model. An even simpler example of a growth model was proposed by Eden in 1958 to simulate the growth of tumors or a bacterial colony. Although we will find that the resultant mass distribution is not a fractal, the description of the Eden model illustrates the general nature of the fractal growth models we will discuss.

Choose a seed site at the center of the lattice for simplicity. The unoccupied nearest neighbors of the occupied sites are the perimeter or *growth* sites. In the simplest version of the model, a growth site is chosen at random and occupied. The newly occupied site is removed from the list of growth sites and the new growth sites are added to the list. This process is repeated many times until a large cluster of occupied sites is formed. The difference between this model and the simple epidemic model is that all tested sites are occupied. In other words, no growth sites ever become "immune." Some of the properties of Eden clusters are investigated in Problem 13.6.

Problem 13.6 The Eden model

(a) Modify class `SingleCluster` so that clusters are generated on a square lattice according to the Eden model. A straightforward procedure is to occupy perimeter sites with probability $p = 1$. The simulation should be stopped when the cluster just reaches the edge of the lattice. What would happen if we were to occupy perimeter sites indefinitely? Follow the procedure of Problem 13.3 and determine the number of occupied sites $M(r)$ within a distance r of the seed site. Assume that $M(r) \sim r^D$ for sufficiently large r and estimate D from the slope of a log-log plot of M versus r. A typical log-log plot is shown in Figure 13.8 for $L = 61$. Can you conclude from your data that Eden clusters are compact?