```
      control.setValue("epsilon", 0.0000001); // desired precision
      control.setValue("xleft", 0.01);        // guess for xleft
      control.setValue("xright", 0.99);       // guess for xright
   }

   public void calculate() {
      // desired precision
      double epsilon = control.getDouble("epsilon");
      r = control.getDouble("r");
      period = control.getInt("period");
      xleft = control.getDouble("xleft");
      xright = control.getDouble("xright");
      gleft = map(xleft, r, period)-xleft;
      gright = map(xright, r, period)-xright;
      if(gleft*gright<0) {
         while(Math.abs(xleft-xright)>epsilon) {
            bisection();
         }
         double x = 0.5*(xleft+xright);
         control.println("search for period "+period+);
         control.println(0+"\t"+x); // result
         for(int i = 1;i<=2*period+1;i++) {
            x = map(x, r, 1);
            control.println(i+"\t"+x);
         }
      } else {
         control.println("range does not enclose a root");
      }
   }

   public void bisection() {
      // midpoint between xleft and xright
      double xmid = 0.5*(xleft+xright);
      double gmid = map(xmid, r, period)-xmid;
      if(gmid*gleft>0) {
         xleft = xmid;  // change xleft
         gleft = gmid;
      } else {
         xright = xmid; // change xright
         gright = gmid;
      }
   }

   double map(double x, double r, double period) {
      if(period>1) {
         double y = map(x, r, period-1);
         return 4*r*y*(1-y);
      } else {
         return 4*r*x*(1-x);
      }
   }

   public static void main(String[] args) {
      CalculationControl.createApp(new RecursiveFixedPointApp());
   }
}
```


Problem 6.13 Unstable periodic trajectories for the logistic map

(a) Test `RecursiveFixedPointApp` for values of r for which the logistic map has a stable period with $p = 1$ and $p = 2$. Set the desired precision ϵ equal to 10^{-7}. Initially use $x_{\rm left} = 0.01$ and $x_{\rm right} = 0.99$. Calculate the stable attractor analytically and compare the results of your program with the analytical results.

(b) Set $r = 0.95$ and find the periodic trajectories for $p = 1, 2, 5, 6, 7, 12, 13,$ and 19.

(c) Modify `RecursiveFixedPointApp` so that n_b, the number of bisections needed to obtain the unstable trajectory, is listed. Choose three of the cases considered in part (b) and compute n_b for the precision $\epsilon = 0.01, 0.001, 0.0001,$ and 0.00001. Determine the functional dependence of n_b on ϵ. ■

Now that we know how to find the values of the unstable periodic trajectories, we discuss an algorithm for stabilizing this period. Suppose that we wish to stabilize the unstable trajectory of period p for a choice of $r = r_0$. The idea is to make small adjustments of $r = r_0 + \Delta r$ at each iteration so that the difference between the actual trajectory and the target periodic trajectory is small. If the actual trajectory is x_n and we wish the trajectory to be at $x(i)$, we make the next iterate x_{n+1} equal to $x(i+1)$ by expanding the difference $x_{n+1} - x(i+1)$ in a Taylor series and setting the difference to zero to first order. We have $x_{n+1} - x(i+1) = f(x_n, r) - f(x(i), r_0)$. If we expand $f(x_n, r)$ about $(x(i), r_0)$, we have to first order

$$x_{n+1} - x(i+1) = \frac{\partial f(x,r)}{\partial x}[x_n - x(i)] + \frac{\partial f(x,r)}{\partial r}\Delta r = 0. \tag{6.29}$$

The partial derivatives in (6.29) are evaluated at $x = x(i)$ and $r = r_0$. The result is

$$4r_0[1 - 2x(i)][x_n - x(i)] + 4x(i)[1 - x(i)]\Delta r = 0, \tag{6.30}$$

and the solution of (6.30) for Δr can be written as

$$\Delta r = -r_0\frac{[1 - 2x(i)][x_n - x(i)]}{x(i)[1 - x(i)]}. \tag{6.31}$$

The procedure is to iterate the logistic map at $r = r_0$ until x_n is sufficiently close to an $x(i)$. The nature of chaotic systems is that the trajectory is guaranteed to eventually come close to the desired unstable trajectory. Then we use (6.31) to change the value of r so that the next iteration is closer to $x(i+1)$. We summarize the algorithm for controlling chaos as follows:

1. Find the unstable periodic trajectory $x(1), x(2), \ldots, x(p)$ for the desired value of r_0.
2. Iterate the map with $r = r_0$ until x_n is within ϵ of $x(i)$. Then use (6.31) to determine r.
3. Turn off the control by setting $r = r_0$.