digital computers are often used to optimize the design of circuits for special applications. The RCApp program is not shown here because it is similar to PendulumApp, but this program is available in the Chapter 4 package. The RCApp program simulates an RC circuit with an alternating current (AC) voltage source of the form $V_s(t) = \cos \omega t$ and plots the time dependence of the charge on the capacitor. You are asked to modify this program in Problem 4.12.

Problem 4.12 Simple filter circuits

(a) Modify the RCApp program to simulate the voltages in an RC filter. Your program should plot the voltage across the resistor V_R and the voltage across the source V_s in addition to the voltage across the capacitor V_C. Run this program with $R = 1000\,\Omega$ and $C = 1.0\,\mu$F (10^{-6} farads). Find the steady state amplitude of the voltage drops across the resistor and across the capacitor as a function of the angular frequency ω of the source voltage $V_s = \cos \omega t$. Consider the frequencies $f = 10$, 50, 100, 160, 200, 500, 1000, 5000, and 10000 Hz. (Remember that $\omega = 2\pi f$.) Choose Δt to be no more than 0.0001 s for $f = 10$ Hz. What is a reasonable value of Δt for $f = 10000$ Hz?

(b) The output voltage depends on where the digital oscilloscope is connected. What is the output voltage of the oscilloscope in Figure 4.6a? Plot the ratio of the amplitude of the output voltage to the amplitude of the input voltage as a function of ω. Use a logarithmic scale for ω. What range of frequencies is passed? Does this circuit act as a high pass or a low pass filter? Answer the same questions for the oscilloscope in Figure 4.6b. Use your results to explain the operation of a high and low pass filter. Compute the value of the cutoff frequency for which the amplitude of the output voltage drops to $1/\sqrt{2}$ (half-power) of the input value. How is the cutoff frequency related to RC?

(c) Plot the voltage drops across the capacitor and resistor as a function of time. The phase difference ϕ between each voltage drop and the source voltage can be found by finding the time t_m between the corresponding maxima of the voltages. Because ϕ is usually expressed in radians, we have the relation $\phi/2\pi = t_m/T$, where T is the period of the oscillation. What is the phase difference ϕ_C between the capacitor and the voltage source and the phase difference ϕ_R between the resistor and the voltage source? Do these phase differences depend on ω? Does the current lead or lag the voltage, that is, does the maxima of $V_R(t)$ come before or after the maxima of $V_s(t)$? What is the phase difference between the capacitor and the resistor? Does the latter difference depend on ω?

(d) Modify your program to find the steady state response of an LR circuit with a source voltage $V_s(t) = \cos \omega t$. Let $R = 100\,\Omega$ and $L = 2 \times 10^{-3}$ H. Because $L/R = 2 \times 10^{-5}$ s, it is convenient to measure the time and frequency in units of $T_0 = L/R$. We write $t^* = t/T_0$, $\omega^* = \omega T_0$, and rewrite the equation for an LR circuit as

$$I(t^*) + \frac{dI(t^*)}{dt^*} = \frac{1}{R} \cos \omega^* t^*. \tag{4.24}$$

Because it will be clear from the context, we now simply write t and ω rather than t^* and ω^*. What is a reasonable value of the step size Δt? Compute the steady state amplitude of the voltage drops across the inductor and the resistor for the input frequencies $f = 10$, 20, 30, 35, 50, 100, and 200 Hz. Use these results to explain how an LR circuit can be used as a low pass or a high pass filter. Plot the voltage drops across the inductor and resistor as a function of time and determine the phase differences ϕ_R and ϕ_L between the resistor and the voltage source and the inductor and the voltage source. Do these phase differences depend on ω? Does the current lead or lag the voltage? What is the phase difference between the inductor and the resistor? Does the latter difference depend on ω? ■

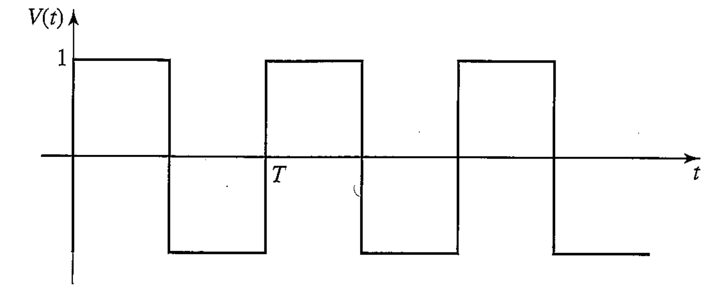

Figure 4.7 Square wave voltage with period T and unit amplitude.

Problem 4.13 Square wave response of an RC circuit
Modify your program so that the voltage source is a periodic square wave as shown in Figure 4.7. Use a $1.0\,\mu$F capacitor and a $3000\,\Omega$ resistor. Plot the computed voltage drop across the capacitor as a function of time. Make sure the period of the square wave is long enough so that the capacitor is fully charged during one half-cycle. What is the approximate time dependence of $V_C(t)$ while the capacitor is charging (discharging)? ■

We now consider the steady state behavior of the series RLC circuit shown in Figure 4.5 and represented by (4.22). The response of an electrical circuit is the current rather than the charge on the capacitor. Because we have simulated the analogous mechanical system, we already know much about the behavior of driven RLC circuits. Nonetheless, we will find several interesting features of AC electrical circuits in the following two problems.

Problem 4.14 Response of an RLC circuit

(a) Consider an RLC series circuit with $R = 100\,\Omega$, $C = 3.0\,\mu$F, and $L = 2$ mH. Modify the simple harmonic oscillator program or the RC filter program to simulate an RLC circuit and compute the voltage drops across the three circuit elements. Assume an AC voltage source of the form $V(t) = V_0 \cos \omega t$. Plot the current I as a function of time and determine the maximum steady state current $I_{\max}$ for different values of ω. Obtain the *resonance curve* by plotting $I_{\max}(\omega)$ as a function of ω and compute the value of ω at which the resonance curve is a maximum. This value of ω is the *resonant frequency*.

(b) The sharpness of the resonance curve of an AC circuit is related to the quality factor or Q value. (Q should not be confused with the charge on the capacitor.) The sharper the resonance, the larger the value of Q. Circuits with high Q (and hence a sharp resonance) are useful for tuning circuits in a radio so that only one station