```
      double occupied[] = new double[numberOfBins];
      double number[] = new double[numberOfBins];
      double binSize = 1.0/numberOfBins;
      int minX = Lx/3;
      int maxX = 2*minX;
      for(int x = minX;x<=maxX;x++) {
         for(int y = 0;y<Ly;y++) {
            int bin = (int) (numberOfBins*(site[x][y]%1));
            number[bin]++;
            if((site[x][y]>1)&&(site[x][y]<2)) {
               numberOccupied++;
               occupied[bin]++;
            }
         }
      }
      data.setMessage("Number occupied = "+numberOccupied);
      for(int bin = 0;bin<numberOfBins;bin++) {
         data.append(0, (bin+0.5)*binSize, occupied[bin]/number[bin]);
      }
   }
}
```

Problem 13.7 Invasion percolation

(a) Use class `Invasion` to generate an invasion percolation cluster on a 20×40 lattice and describe the qualitative nature of the cluster.

(b) Compute $M(L)$, the number of sites occupied by the invader in the central $L \times L$ region of the $L \times 2L$ lattice when the invader first reaches the right edge. Average over at least twenty configurations. Assume that $M(L) \sim L^D$ and estimate D from a plot of $\ln M$ versus $\ln L$. Compare your estimate for D with the fractal dimension of site percolation clusters at $p = p_c$. (The first published results for $M(L)$ by Wilkinson and Willemsen were for 2000 realizations each for L in the range 20 to 100.)

(c) Determine the probability $P(r)\,\Delta r$ that a site with a random number between r and $r + \Delta r$ is occupied. Choose $\Delta r = 0.01$. Plot $P(r)$ versus r for $L = 20$ and for values of L up to about $L \geq 50$. Is there a value of r near which $P(r)$ changes rapidly? How does this value of r compare to the value of p_c for site percolation on the square lattice? On the basis of your numerical estimate for the exponent D found in part (b) and the qualitative behavior of $P(r)$, make a hypothesis about the relation between the nature of the geometrical properties of the invasion percolation cluster and the spanning percolation cluster at $p = p_c$.

(d) Explain the nature of the two search algorithms given in class `Invasion`. Which method yields the fastest results on a 30×60 lattice? Verify that the CPU time for a linear and binary search is proportional to n and $\log n$, respectively, where n is the number of items in the list to be searched. Hence, for sufficiently large n, a binary search usually is preferred.

*(e) Modify your program so that the invasion percolation clusters are grown from a seed at the origin. Grow a cluster until it reaches a boundary of the lattice. Estimate the fractal dimension as you did for the spanning percolation clusters in Problem 13.3 and compare your two estimates. On the basis of this estimate and your results from parts (b) and (c), can you conclude that the spanning cluster in invasion percolation is a fractal? ■

Diffusion in disordered media. In Chapter 7 we considered random walks on perfect lattices. We found that the mean square displacement of a random walker $\langle R^2(t)\rangle$ is proportional to the time t for sufficiently large t. (For a simple random walk, this relation holds for all t.) Now let us suppose that the random walker is restricted to a disordered lattice, for example, the occupied sites of a percolation cluster. What is the asymptotic t-dependence of $\langle R^2(t)\rangle$ in this case? This model of a random walk on a percolation cluster is known as the "ant in the labyrinth."

Just as a random walk on a lattice is a simple example of diffusion, a random walk on a disordered lattice is a simple example of the general problem of diffusion and transport in disordered media. Because many materials of interest are noncrystalline and disordered, there are many physical phenomena that can be related to the motion of an ant in the labyrinth.

In the usual formulation of the ant in the labyrinth, we place a walker (ant) at random on one of the occupied sites of a percolation cluster that has been generated with probability p. At each time step, the ant tosses a coin with four possible outcomes (for a square lattice). If the outcome corresponds to a step to an occupied site, the ant moves; otherwise, it remains at its present position. In either case, the time t is increased by one unit.

The main quantity of interest is $R^2(t)$, the square of the distance between the ant's position at $t = 0$ and its position at time t. We can generate many walks with different initial positions on the same cluster and average over many percolation clusters to obtain the ant's mean square displacement $\langle R^2(t)\rangle$. How does $\langle R^2(t)\rangle$ depend on p and t? We consider this question in Problem 13.8.

Problem 13.8 The ant in the labyrinth

(a) For $p = 1$, the ant walks on a perfect lattice, and hence, $\langle R^2(t)\rangle = 2dDt$. Suppose that an ant does a random walk on a spanning cluster with $p > p_c$ on a square lattice. Assume that $\langle R^2(t)\rangle \to 4D_s(p)\,t$ for $p > p_c$ and sufficiently long times. We have denoted the diffusion coefficient by D_s because we are considering random walks only on spanning clusters and are not considering walks on the finite clusters that also exist for $p > p_c$. Generate a cluster at $p = 0.7$ using the single cluster growth algorithm considered in Problem 13.3. Choose the initial position of the ant to be the seed site and modify your program to observe the motion of the ant on the screen. Use $L \geq 101$ and average over at least 100 walkers for t up to 500. Where does the ant spend much of its time? If $\langle R^2(t)\rangle \propto t$, what is $D_s(p)/D(p = 1)$?

(b) As in part (a) compute $\langle R^2(t)\rangle$ for $p = 1.0, 0.8, 0.7, 0.65$, and 0.62 with $L = 101$. If time permits, average over several clusters. Make a log-log plot of $\langle R^2(t)\rangle$ versus t. What is the qualitative t-dependence of $\langle R^2(t)\rangle$ for relatively short times? Is $\langle R^2(t)\rangle$ proportional to t for longer times? (Remember that the maximum value of $\langle R^2\rangle$ is bounded by the finite size of the lattice.) If $\langle R^2(t)\rangle \propto t$, estimate $D_s(p)$. Plot $D_s(p)/D(p = 1)$ as a function of p and discuss its qualitative dependence.

(c) Compute $\langle R^2(t)\rangle$ for $p = 0.4$ and confirm that for $p < p_c$, the clusters are finite, $\langle R^2(t)\rangle$ is bounded, and diffusion is impossible.