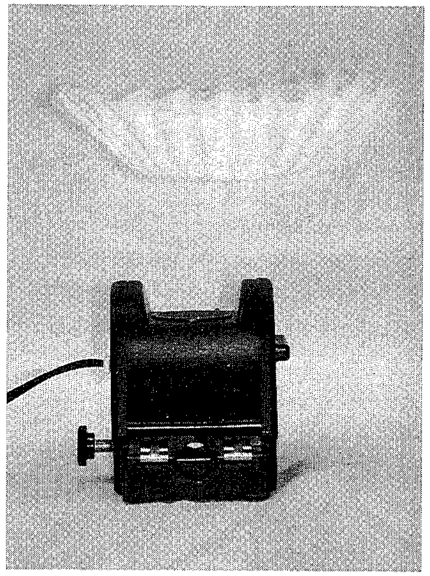

Figure 3.2 A falling coffee filter does not fall with constant acceleration due to the effects of air resistance. The motion sensor below the filter is connected to a computer which records position data and stores it in a text file.

to remain in the air for a time sufficiently long so that the effect of the drag force is appreciable. A reasonable choice is $v(t=0) = 50$ m/s. You might find it convenient to express the drag force in a form proportional to `-v* Math.abs(v)`. One way to determine the maximum height of the pebble is to use the statement

```
if (v*vold < 0) {
   control.println("maximum height = " + y);
}
```

where $\texttt{v} = v_{n+1}$ and $\texttt{vold} = v_n$. Why is this criterion preferable to other criteria that you might imagine using? ■

3.8 ■ TWO-DIMENSIONAL TRAJECTORIES

You are probably familiar with two-dimensional trajectory problems in the absence of air resistance. For example, if a ball is thrown in the air with an initial velocity $\mathbf{v}_0$ at an angle θ_0 with respect to the ground, how far will the ball travel in the horizontal direction, and what is its maximum height and time of flight? Suppose that a ball is released at a nonzero height h above the ground. What is the launch angle for the maximum range? Are your answers still applicable if air resistance is taken into account? We consider these and similar questions in the following.

Consider an object of mass m whose initial velocity $\mathbf{v}_0$ is directed at an angle θ_0 above the horizontal (see Figure 3.3(a)). The particle is subjected to gravitational and drag forces of magnitude mg and F_d; the direction of the drag force is opposite to $\mathbf{v}$ (see Figure 3.3(b)).

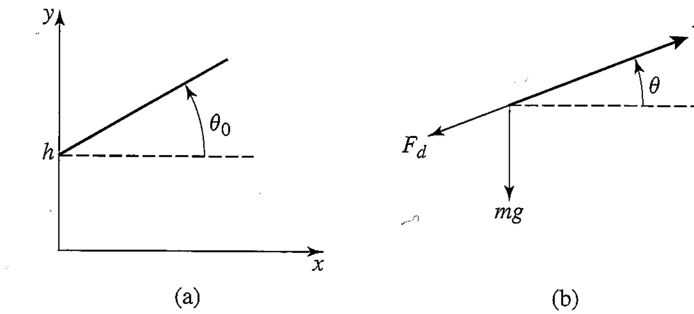

Figure 3.3 (a) A ball is thrown from a height h at a launch angle θ_0 measured with respect to the horizontal. The initial velocity is $\mathbf{v}_0$. (b) The gravitational and drag forces on a particle.

Newton's equations of motion for the x and y components of the motion can be written as

$$m\frac{dv_x}{dt} = -F_d\cos\theta \tag{3.17a}$$

$$m\frac{dv_y}{dt} = -mg - F_d\sin\theta. \tag{3.17b}$$

For example, let us maximize the range of a round steel ball of radius 4 cm. A reasonable assumption for a steel ball of this size and typical speed is that $F_d = C_2v^2$. Because $v_x = v\cos\theta$ and $v_y = v\sin\theta$, we can rewrite (3.17) as

$$m\frac{dv_x}{dt} = -C_2vv_x \tag{3.18a}$$

$$m\frac{dv_y}{dt} = -mg - C - 2vv_y. \tag{3.18b}$$

Note that $-C_2vv_x$ and $-C_2vv_y$ are the x and y components of the drag force $-C_2v^2$. Because (3.18a) and (3.18b) for the change in v_x and v_y involve the square of the velocity, $v^2 = v_x^2 + v_y^2$, we cannot calculate the vertical motion of a falling body without reference to the horizontal component, that is, the motion in the x and y direction is *coupled*.

Problem 3.10 Trajectory of a steel ball

(a) Use `Projectile` and `ProjectileApp` to compute the two-dimensional trajectory of a ball moving in air without air friction and plot y as a function of x. Compare your computed results with the exact results. For example, assume that a ball is thrown from ground level at an angle θ_0 above the horizontal with an initial velocity of $v_0 = 15$ m/s. Vary θ_0 and show that the maximum range occurs at $\theta_0 = \theta_{\max} = 45°$. What is $R_{\max}$, the maximum range, at this angle? Compare your numerical result to the analytic result $R_{\max} = v_0^2/g$.

(b) Suppose that a steel ball is thrown from a height h at an angle θ_0 above the horizontal with the same initial speed as in part (a). If you neglect air resistance, do you expect $\theta_{\max}$ to be larger or smaller than 45°? What is $\theta_{\max}$ for $h = 2$ m? By what percent is the range R changed if θ is varied by 2% from $\theta_{\max}$?