effect, known as *frame dragging*, occurs near any spinning gravitational object, including Earth.

Problem 18.17 shows that frame dragging becomes dramatic as the falling particle approaches the horizon for the extreme black hole, $r_h = M$. (The horizon is where the metric coefficient of dr^2 becomes infinite.) Note that the coefficient of the dt^2 term goes to zero at $r = r_s = 2M$. This value is called the *static limit*. The space between the static limit and the horizon is dragged along in the direction of rotation of the black hole so that an observer cannot remain at a fixed angle no matter how powerful the rockets are.

18.9 ■ PROJECTS

Numerical relativity is still in its infancy but is making progress in simulating astrophysical scenarios such as binary black hole mergers, binary neutron star mergers, and supernova core collapse. A key problem is achieving long-time stability of the numerical solutions. A search on "numerical relativity" will yield many interesting websites and entrees to current research.

Project 18.18 Three-dimensional rapidly moving objects
Extend the analysis in Section 18.1 to three-dimensional objects and model their appearance as seen by a single observer at the origin using the transformation and rendering techniques described in Chapter 17. Does a sphere appear to be a sphere even when it passes by an observer? Does a cube appear to be a cube? ■

Project 18.19 Light Links

(a) Imagine two stationary observers near a black hole wishing to establish a communication link using a laser beam. In what direction should the laser be pointed to establish such a link? Simulate this scenario using two dragable objects on a Schwarzschild map and draw the light ray representing the communication link. Use a root finding algorithm, such as the bisection method introduced in Chapter 6, to determine the proper launch angle. Calculate and display the proper distance along this light path and study how this distance changes as the light path grazes the event horizon.

(b) Construct a light triangle connecting three observers. Display the sum of the interior angles as measured by the observers to simulate Gauss's mountain top experiment. ■

Project 18.20 Seeing orbits
Viewing an orbit requires that we calculate the particle's trajectory and the trajectory of the light ray from the particle to the viewer. An added complication arises because the light reaching the view is retarded by the travel time. Write a program that shows an orbiting particle as seen by a stationary observer in the equatorial plane by keeping track of both particle and light-link parameters. ■

REFERENCES AND SUGGESTIONS FOR FURTHER READING

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Victor Weisskopf, "The visual appearance of rapidly moving objects," Physics Today **13** (9), 24–27 (1960).

Most general relativity texts begin with a treatment of tensor analysis. The following two texts present this material using the four-dimensional spacetime metric.

Edwin F. Taylor and John A. Wheeler, *Exploring Black Holes: An Introduction to General Relativity* (Addison–Wesley Longman, 2000).

James Hartle, *Gravity: An Introduction to General Relativity* (Addison–Wesley, 2003).

The website, <`http://archive.ncsa.uiuc.edu/Cyberia/NumRel/NumRelHome.html`>, developed by the National Center for Supercomputing Applications, is one of many that discuss Einstein's contributions and recent progress in numerical relativity.