```
      lattice.newLattice();
      for(int i = 0;i<lattice.N;i++) {
         lattice.addRandomSite();
         meanClusterSize[i] += (double) lattice.getMeanClusterSize();
         P_infinity[i] += (double) lattice.getSpanningClusterSize()
                          /lattice.numSitesOccupied;
         P_span[i] += (lattice.getSpanningClusterSize()==0 ? 0 : 1);
         if((int) (pDisplay*lattice.N)==i) {
            for(int j = 0;j<lattice.N;j++) {
               numClustersAccum[j] += lattice.numClusters[j];
            }
            displayLattice();
         }
      }
      // display accumulated results
      numberOfTrials++;
      plotAverages();
   }

   private void plotAverages() {
      plot1.clearData();
      plot2.clearData();
      plot3.clearData();
      plot4.clearData();
      for(int i = 0;i<lattice.N;i++) {
         double p = (double) i/lattice.N; // occupation probability
         plot1.append(0, p, meanClusterSize[i]/numberOfTrials);
         plot2.append(0, p, P_infinity[i]/numberOfTrials);
         plot3.append(0, p, P_span[i]/numberOfTrials);
         if(numClustersAccum[i+1]>0) {
            plot4.append(0, i+1, numClustersAccum[i+1]/numberOfTrials);
         }
      }
   }

   private void displayLattice() {
      double display[] = new double[lattice.N];
      for(int s = 0;s<lattice.N;s++) {
         display[s] = lattice.getClusterSize(s);
      }
      grid.setAll(display);
   }

   public void reset() {
      control.setValue("Lattice size L", 128);
      control.setValue("Display lattice at this p", 0.5927);
   }

   public static void main(String args[]) {
      SimulationControl.createApp(new ClustersApp());
   }
}
```


Problem 12.7 Qualitative behavior of various percolation quantities

(a) Read the code for class `Clusters` and explain how the Newman–Ziff algorithm is implemented.

(b) Collect data for $P_\infty(p)$, the probability that an occupied site belongs to the spanning cluster, $S(p)$, the mean cluster size, and $P_{\rm span}(p)$, the probability of a spanning cluster. Consider $L = 8$, 32, 128, and 256 and average over at least 100 configurations. How does the qualitative behavior of these quantities change with increasing L? Discuss the qualitative dependence of P_∞ and $S(p)$ on p for the largest lattice that you can simulate in a reasonable time.

(c) At what value of p is $P_{\rm span} \approx 0.5$ for each value of L? Call this value $p_c(L)$. How strongly does $p_c(L)$ depend on L? Extrapolate your results for $p_c(L)$ to $L \to \infty$. For example, try fitting your data for $p_c(L)$ to the form $p_c(L) = p_c - cL^{-x}$, where p_c, c, and x are fitting parameters. Because you will likely have insufficient data to determine three parameters with reasonable accuracy, take $x = 3/4$ and plot $p_c(L)$ versus $L^{-3/4}$. How sensitive is your result for the intercept p_c on the assumed value of x? A more sophisticated analysis is discussed in Project 12.13.

(d) Consider the cluster distribution $n_s(p)$. Why is n_s a decreasing function of s? Does n_s decrease more quickly for $p = p_c$ or for $p \neq p_c$? Why is there so much scatter in n_s for large s? Plot $\ln n_s$ versus s and $\ln n_s$ versus $\ln s$ for each value of p. Which form fits best? Assume that a power law (straight line on a log-log plot) works for s less than some cutoff. Estimate the cutoff as a function of p and show that this cutoff diverges as $p \to p_c$. ■

12.4 ■ CRITICAL EXPONENTS AND FINITE SIZE SCALING

We are familiar with different phases of matter from our everyday experience. The most familiar example is water which can exist as a gas, liquid, or solid. It is well known that water changes from one phase to another at a well-defined temperature and pressure; for example, the transition from ice to liquid water occurs at 0 °C at atmospheric pressure. Such a change of phase is an example of a *thermodynamic phase transition.* Most substances also exhibit a *critical point.* For example, beyond a particular temperature and pressure, it is not possible to distinguish between the liquid and gaseous phases, and the phase boundary terminates.

Another example of a critical point occurs in magnetic systems at the Curie temperature T_c and zero magnetic field. We know that at low temperatures some substances such as iron exhibit ferromagnetism, a spontaneous magnetization in the absence of an external magnetic field. If we raise the temperature of a ferromagnet, the spontaneous magnetization decreases and vanishes continuously at a critical temperature T_c. For $T > T_c$, the system is a paramagnet. In Chapter 15 we will use Monte Carlo methods to investigate the behavior of a magnetic system near the magnetic critical point.

In the following, we will find that the properties of the *geometrical* phase transition in percolation are qualitatively similar to the properties of the critical point in thermodynamic transitions. We will see that in the vicinity of a critical point, the qualitative behavior of the system is governed by the occurrence of long-range correlations.