```
            case 1:
               x--;
               break;
            case 2:
               y++;
               break;
            case 3:
               y--;
            }
         }              // end else if
      } while(true); // end do loop
   }

   public static void main(String[] args) {
      SimulationControl.createApp(new DLAApp());
   }
}
```

Problem 13.9 Diffusion limited aggregation

(a) DLAApp generates diffusion limited aggregation clusters on a square lattice. Each walker begins at a random site on a launching circle of radius $r = R_{\max} + 2$, where $R_{\max}$ is the maximum distance of any particle in the cluster from the origin. To save computer time, we remove a walker that reaches a distance $2R_{\max}$ from the seed site and place a new walker at random on the circle of radius r. If the clusters appear to be fractals, make a visual estimate of the fractal dimension. Choose a lattice of linear dimension $L \geq 61$. (Experts can make a visual estimate of D to within a few percent.) Modify DLAApp by color coding the sites in the cluster according to their time of arrival; for example, color the first group of sites white, the next group blue, the next group red, and the last group sites green. (Your choice of the size of the group depends in part on the total size of your cluster.) Which parts of the cluster grow faster? Do any of the late arriving green particles reach the center?

(b) At $t = 0$, the four perimeter (growth) sites on the square lattice each have a probability $p_i = 1/4$ of becoming part of the cluster. At $t = 1$, the cluster has mass two and six perimeter sites. Identify the perimeter sites and convince yourself that their growth probabilities are not the same. Do a Monte Carlo simulation and verify that two perimeter sites have growth probabilities $p = 2/9$ and the other four have $p = 5/36$. We discuss a more direct way of determining the growth probabilities in Problem 13.10.

(c) DLAApp generates clusters inefficiently because most of the CPU time is spent while the random walker is wandering far from the perimeter sites of the cluster. There are several ways of making your program more efficient. One way is to let the walker take bigger steps the further it is from the cluster. For example, if the walker is a distance $R > R_{\max}$, a step of length greater than or equal to $R - R_{\max} - 1$ may be permitted if this distance is greater than one lattice unit. If the walker is very close to the cluster, the step length is one lattice unit. Make this modification to class DLA and estimate the fractal dimension of diffusion limited clusters generated on a square lattice by computing $M(r)$, the number of sites in the cluster within a radius r centered at the seed site. Because very large clusters are needed to accurately estimate the fractal dimension, you will obtain only approximate results. Other possible modifications to make the implementation of the algorithm are discussed in Project 13.17 and by Meakin (see references).

*(d) Each time we grow a DLA cluster (and other clusters in which a perimeter site is selected at random), we obtain a slightly different cluster if we use a different random number sequence. One way of reducing this "noise" is to use "noise reduction," that is, a perimeter site is not occupied until it has been visited m times. Each time the random walker lands on a perimeter site, the number of visits for this site is increased by one until the number of visits equals m and then the site is occupied. The reason for doing this is that noise reduction accelerates the approach to the asymptotic scaling behavior. Consider $m = 2$, 3, 4, and 5 and grow DLA clusters on the square lattice. Are there any qualitative differences between the clusters for different values of m?

*(e) In Chapter 12 we found that the exponents describing the percolation transition are independent of the symmetry of the lattice; for example, the exponents for the square and triangular lattices are the same. We might expect that the fractal dimension of DLA clusters would also show such universal behavior. However, the presence of a lattice introduces a small anisotropy that becomes apparent only when very large clusters with the order of 10^6 sites are grown. Modify your program so that DLA clusters are generated on a triangular lattice. Do the clusters have the same visual appearance as on the square lattice? Estimate the fractal dimension and compare your estimate to your result for the square lattice. The best estimates of D for the square and triangular lattices are $D \approx 1.5$ and $D \approx 1.71$, respectively. We are reminded of the difficulty of extrapolating the asymptotic behavior from finite clusters. We consider the growth of diffusion limited aggregation clusters in the continuum in Project 13.16. ■

Laplacian growth model. As we discussed in Section 10.6, we can formulate the solution of Laplace's equation in terms of a random walk. We now do the converse and formulate the DLA algorithm in terms of a solution to Laplace's equation. Consider the probability $P(\mathbf{r})$ that a random walker reaches a site $\mathbf{r}$ starting from the external boundary. This probability satisfies the relation

$$P(\mathbf{r}) = \frac{1}{4} \sum_{\mathbf{a}} P(\mathbf{r} + \mathbf{a}), \tag{13.10}$$

where the sum in (13.10) is over the four nearest neighbor sites (on a square lattice). If we set $P = 1$ on the boundary and $P = 0$ on the cluster, then (13.10) also applies to sites that are neighbors of the external boundary and the cluster. A comparison of the form of (13.10) with the form of (10.12) shows that the former is a discrete version of Laplace's equation $\nabla^2 P = 0$. Hence, $P(\mathbf{r})$ has the same behavior as the electrical potential between two electrodes connected to the outer boundary and the cluster, and the growth probability at a perimeter site of the cluster is proportional to the value of the potential at that site.