(ii) If $x^2 > \epsilon$, then

$$q_0 = 0 \tag{17.53a}$$

$$q_1 = \sqrt{x^2} \tag{17.53b}$$

$$q_2 = r_{0,1}/2q_1 \tag{17.53c}$$

$$q_3 = r_{0,2}/2q_1, \tag{17.53d}$$

else compute $y^2 = 1/[2(1 - r_{2,2})]$.

(iii) If $y^2 > \epsilon$, then

$$q_0 = 0 \tag{17.54a}$$

$$q_1 = 0 \tag{17.54b}$$

$$q_2 = \sqrt{y^2} \tag{17.54c}$$

$$q_3 = r_{1,2}/2q_2, \tag{17.54d}$$

else

$$q_0 = 0 \tag{17.55a}$$

$$q_1 = 0 \tag{17.55b}$$

$$q_2 = 0 \tag{17.55c}$$

$$q_3 = 1. \tag{17.55d}$$

Euler angles to matrix. Euler angles are generally described in physics texts (see Goldstein) as a group of three rotations about a set of body frame axes. An object is created with the body frame aligned with the space frame. The first rotation is about the body frame's $\hat{3}$-axis by an angle ϕ; the second rotation is about the new x-axis by an angle θ, and the third rotation is about the new z-axis by an angle ϕ. Other definitions of Euler angles are possible. For example, the Java 3D API defines Euler angles as three rotations about a set of axes fixed in space. All possible positions of an object can be represented using either of these conventions.

The first rotation is about z-axis and is given by

$$\mathcal{A}(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{17.56}$$

The second rotation is about the new x-axis and is given by

$$\mathcal{B}(\theta) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}. \tag{17.57}$$

The last rotation is again about the new z-axis and is given by

$$\mathcal{C}(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{17.58}$$

The application of the three Euler rotation matrices $\mathcal{C}(\psi)$, $\mathcal{B}(\theta)$, and $\mathcal{A}(\phi)$ in this order produces the transformation

$$\mathcal{R}(\psi,\theta,\phi) = \begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi & \sin\psi\sin\theta \\ -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix}. \tag{17.59}$$

Euler angles to quaternion. There are many possible conventions for the Euler angles. We again use the definition found in Goldstein:

$$q_0 = \cos\theta/2\cos\frac{1}{2}(\phi+\psi) \tag{17.60a}$$

$$q_1 = \sin\theta/2\cos\frac{1}{2}(\phi-\psi) \tag{17.60b}$$

$$q_2 = \sin\theta/2\sin\frac{1}{2}(\phi-\psi) \tag{17.60c}$$

$$q_3 = \sin\theta/2\sin\frac{1}{2}(\phi+\psi). \tag{17.60d}$$

Matrix to Euler angles. The conversion from matrix elements to Euler angles is ill-defined because inverse trigonometric functions do not uniquely specify the resulting quadrant. From (17.59) we see that $\cos\theta = r_{2,2}$. We then use $\sin\theta = \sqrt{1-\cos^2\theta}$ to compute $\sin\theta$ to within a sign. As in the matrix to quaternion conversion, we again use if statements to avoid dividing a number less than the machine precision ϵ:

If $|r_{2,2}| > \epsilon$, then

$$\cos\theta = r_{2,2} \tag{17.61a}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} \tag{17.61b}$$

$$\cos\phi = r_{1,0}/\sin\theta \tag{17.61c}$$

$$\sin\phi = -r_{2,0}/\sin\theta \tag{17.61d}$$

$$\cos\psi = r_{1,2}/\sin\theta \tag{17.61e}$$

$$\sin\psi = r_{0,2}/\sin\theta, \tag{17.61f}$$

else

$$\cos\theta = 0 \tag{17.62a}$$

$$\sin\theta = 1 \tag{17.62b}$$

$$\cos\phi = r_{1,0} \tag{17.62c}$$

$$\sin\phi = -r_{2,0} \tag{17.62d}$$

$$\cos\psi = 1 \tag{17.62e}$$

$$\sin\psi = 0. \tag{17.62f}$$