(d) Because there is no diffusion for $p < p_c$, we might expect that D_s vanishes as $p \to p_c$ from above, that is, $D_s(p) \sim (p - p_c)^{\mu_s}$ for $p \gtrsim p_c$. Extend your calculations of part (b) to larger L, more walkers (at least 1000), and more values of p near p_c and estimate the dynamical exponent μ_s.

(e) At $p = p_c$, we might expect $\langle R^2(t)\rangle$ to exhibit a different type of t-dependence, for example, $\langle R^2(t)\rangle \to t^{2/z}$ for large t. Do you expect the exponent z to be greater or less than two? Do a simulation of $\langle R^2(t)\rangle$ at $p = p_c$ and estimate z. Choose $L \geq 201$ and average over several spanning clusters.

(f) The algorithm we have been using corresponds to a "blind" ant because the ant chooses from four outcomes even if some of these outcomes are not possible. In contrast, the "myopic" ant can look ahead and see the number q of nearest neighbor occupied sites. The ant then chooses one of the q possible outcomes and thus always takes a step. Redo the simulations in part (b). Does $\langle R^2(t)\rangle$ reach its asymptotic linear dependence on t earlier or later compared to the blind ant?

*(g) The limitation of approach we have taken so far is that we have to average over different random walks $R^2(t)$ on a given cluster and also average over different clusters. A more efficient way of treating random walks on a random lattice is to use an exact enumeration approach and to consider all possible walks on a given cluster. The idea of the exact enumeration method is that $W_{t+1}(i)$, the probability that the ant is at site i at time $t+1$, is determined solely by the probabilities of the ant being at the neighbors of site i at time t. Store the positions of the occupied sites in an array and introduce two arrays corresponding to $W_{t+1}(i)$ and $W_t(i)$ for all sites i in the cluster. Use the probabilities $W_t(i)$ to obtain $W_{t+1}(i)$ (see Figure 13.10). Spatial averages such as the mean square displacement can be calculated from the probability distribution function at different times. The details of the method and the results are discussed in Majid et al., who used walks of 5000 steps on clusters with $\sim 10^3$ sites and averaged their results over 1000 different clusters.

*(h) Another reason for the interest in diffusion in disordered media is that the diffusion coefficient is proportional to the electrical conductivity of the medium. One of Einstein's many contributions was to show that the mobility, the ratio of the mean velocity of the particles in a system to an applied force, is proportional to the self-diffusion coefficient in the absence of the applied force (see Reif). For a system of charged particles, the mean velocity of the particles is proportional to the electrical current and the applied force is proportional to the voltage. Hence, the mobility and the electrical conductivity are proportional, and the conductivity is proportional to the self-diffusion coefficient.

The electrical conductivity σ vanishes near the percolation threshold as $\sigma \sim (p - p_c)^{\mu}$ with $\mu \approx 1.30$ (see Section 12.1). The difficulty of doing a direct Monte Carlo calculation of σ was considered in Project 12.18. We measured the self-diffusion coefficient D_s by always placing the ant on a spanning cluster rather than on *any* cluster. In contrast, the conductivity is measured for the entire system including all finite clusters. Hence, the self-diffusion coefficient D that enters into the Einstein relation should be determined by placing the ant at random anywhere on the lattice, including sites that belong to the spanning cluster and sites that belong to the many finite clusters. Because only those ants that start on the spanning cluster can contribute to D, D is related to D_s by $D = P_\infty D_s$, where P_∞ is the probability that the ant would land on a spanning cluster. Because P_∞ scales as $P_\infty \sim (p - p_c)^{\beta}$, we have that $(p - p_c)^{\mu} \sim (p - p_c)^{\beta}(p - p_c)^{\mu_s}$ or $\mu = \mu_s + \beta$. Use your result for μ_s found in part (d) and the exact result $\beta = 5/36$ (see Table 12.1) to estimate μ and compare your result to the critical exponent μ for the dc electrical conductivity.

*(i) We can also derive the scaling relation $z = 2 + \mu_s/\nu = 2 + (\mu - \beta)\nu$, where z is defined in part (e). Is it easier to determine μ_s or z accurately from a Monte Carlo simulation on a finite lattice? That is, if your real interest is estimating the best value of the critical exponent μ for the conductivity, should you determine the conductivity directly or should we measure the self-diffusion coefficient at $p = p_c$ or at $p > p_c$? What is your best estimate of the conductivity exponent μ? ■

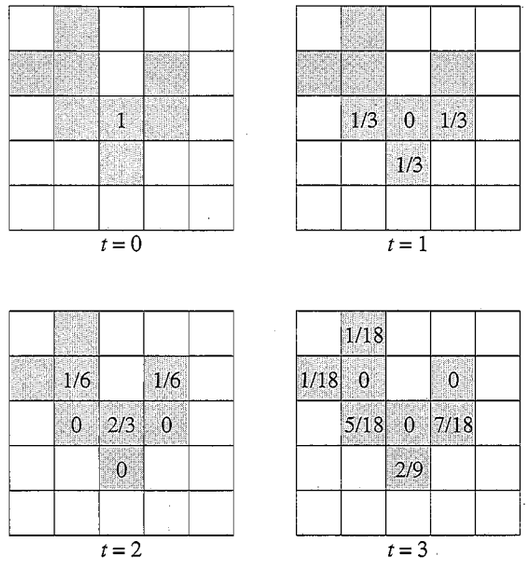

Figure 13.10 The evolution of the probability distribution function $W_t(i)$ for three successive time steps.

Diffusion limited aggregation. Many objects in nature grow by the random addition of subunits. Examples include snow flakes, lightning, crack formation along a geological fault, and the growth of bacterial colonies. Although it might seem unlikely that such phenomena have much in common, the behavior observed in many models gives us clues that these and many other natural phenomena can be understood in terms of a few unifying principles. A popular model that is a good example of how random motion can give rise to beautiful self-similar clusters is known as *diffusion limited aggregation* or DLA.

The first step is to occupy a site with a seed particle. Next, a particle is released at random from a point on the circumference of a large circle whose center coincides with the seed. The particle undergoes a random walk until it reaches a perimeter site of the seed and sticks. Then another random walker is released from the circumference of a large circle and walks until it reaches a perimeter site of one of the two particles in the cluster and sticks. This process is repeated many times (typically on the order of several thousand to several