Problem 5.13 Scattering from a model hydrogen atom

(a) Consider a model of the hydrogen atom for which a positively-charged nucleus of charge $+e$ is surrounded by a uniformly distributed negative charge of equal magnitude. The spherically symmetric negative charge distribution is contained within a sphere of radius a. It is straightforward to show that the force between a positron of charge $+e$ and this model hydrogen atom is given by

$$f(r) = \begin{cases} 1/r^2 - r/a^3 & r \leq a \\ 0 & r > a. \end{cases} \tag{5.30}$$

We have chosen units such that $e^2/(4\pi\epsilon_0) = 1$, and the mass of the positron is unity. What is the ionization energy in these units? Modify the `Scatter` class to incorporate this force. Is the force on the positron from the model hydrogen atom purely repulsive? Choose $a = 1$ and set the beam radius `bmax` $= 1$. Use $E = 0.125$ and $\Delta t = 0.01$. Compute the trajectories for $b = 0.25$, 0.5, and 0.75 and describe the qualitative nature of the trajectories.

(b) Determine the cross section for $E = 0.125$. Choose nine bins so that the angular width of a detector is `delta` $= 20°$, and let `db` $= 0.1, 0.01$, and 0.002. How does the accuracy of your results depend on the number of bins? Determine the differential cross section for different energies and explain its qualitative energy dependence.

(c) What is the value of σ_T for $E = 0.125$? Does σ_T depend on E? The total cross section has units of area, but a point charge does not have an area. To what area does it refer? What would you expect the total cross section to be for scattering from a hard sphere?

(d) Change the sign of the force so that it corresponds to electron scattering. How do the trajectories change? Discuss the change in $\sigma(\theta)$. ■

Problem 5.14 Rutherford scattering

(a) One of the most famous scattering experiments was performed by Geiger and Marsden who scattered a beam of alpha particles on a thin gold foil. Based on these experiments, Rutherford deduced that the positive charge of the atom is concentrated in a small region at the center of the atom rather than distributed uniformly over the entire atom. Use a $1/r^2$ force in class `Scatter` and compute the trajectories for $b = 0.25$, 0.5, and 0.75 and describe the trajectories. Choose $E = 5$ and $\Delta t = 0.01$. The default value of x_0, the initial x-coordinate of the beam, is $x_0 = -5$. Is this value reasonable?

(b) For $E = 5$ determine the cross section with `numberOfBins` $= 18$. Choose the beam width `bmax` $= 2$. Then vary `db` (or `numberOfBins`) and compare the accuracy of your results to the analytical result for which $\sigma(\theta)$ varies as $[\sin(\theta/2)]^{-4}$. How do your computed results compare with this dependence on θ? If necessary, decrease `db`. Are your results better or worse at small angles, intermediate angles, or large angles near 180°? Explain.

(c) Because the Coulomb force is long range, there is scattering at all impact parameters. Increase the beam radius and determine if your results for $\sigma(\theta)$ change. What happens to the total cross section as you increase the beam width?

(d) Compute $\sigma(\theta)$ for different values of E and estimate the dependence of $\sigma(\theta)$ on E. ■

Problem 5.15 Scattering by other potentials

(a) A simple phenomenological form for the effective interaction between electrons in metals is the screened Coulomb (or Thomas–Fermi) potential given by

$$V(r) = \frac{e^2}{4\pi\epsilon_0 r} e^{-r/a}. \tag{5.31}$$

The range of the interaction a depends on the density and temperature of the electrons. The form (5.31) is known as the Yukawa potential in the context of the interaction between nuclear particles and as the Debye potential in the context of classical plasmas. Choose units such that $a = 1$ and $e^2/(4\pi\epsilon_0) = 1$. Recall that the force is given by $f(r) = -dV/dr$. Incorporate this force law into class `Scatter` and compute the dependence of $\sigma(\theta)$ on the energy of the incident particle. Choose the beam width equal to 3. Compare your results for $\sigma(\theta)$ with your results from the Coulomb potential.

(b) Modify the force law in `Scatter` so that $f(r) = 24(2/r^{13} - 1/r^7)$. This form for $f(r)$ is used to describe the interactions between simple molecules (see Chapter 8). Describe some typical trajectories and compute the differential cross section for several different energies. Let `bmax` $= 2$. What is the total cross section? How do your results change if you vary `bmax`? Choose a small angle as the minimum scattering angle. How sensitive is the total cross section to this minimum angle? Does the differential cross section vary for any other angles besides the smallest scattering angle? ■

5.11 ■ THREE-BODY PROBLEMS

Poincaré showed that it is impossible to obtain an analytical solution for the unrestricted motion of three or more objects interacting under the influence of gravity. However, solutions are known for a few special cases, and it is instructive to study the properties of these solutions.

The `ThreeBody` class computes the trajectories of three particles of equal mass moving in a plane and interacting under the influence of gravity. Both the physics and the drawing are implemented in the `ThreeBody` class shown in Listing 5.9. Note that the `getRate` and `computeForce` methods compute trajectories for an arbitrary number of masses. Note how the `computeForce` method uses the `arraycopy` method to quickly zero the arrays. To simplify the drawing of the particle trajectories, the `ThreeBody` class uses an inner class that extends a `Circle` and contains a `Trail`.