of interaction given by

$$V = \frac{1}{2}\sum_{j=1}^{N}[e^{-(u_j-u_{j-1})} - 1]^2. \tag{9.58}$$

This form of the interaction is known as the Morse potential. All parameters in the potential (such as the overall strength of the potential) have been set to unity. The force on the jth particle is

$$F_j = -\frac{\partial V}{\partial u_j} = Q_j(1 - Q_j) - Q_{j+1}(1 - Q_{j+1}), \tag{9.59a}$$

where

$$Q_j = e^{-(u_j-u_{j-1})}. \tag{9.59b}$$

In linear systems it is possible to set up a pulse of any shape and maintain the shape of the pulse indefinitely. In a nonlinear system, there also exist solutions that maintain their shape, but we will find in Problem 9.30 that not all pulse shapes do so. The pulses that maintain their shape are called *solitons*.

Problem 9.30 Solitons

(a) Modify the program developed in Problem 9.2 so that the force on particle j is given by (9.59). Use periodic boundary conditions. Choose $N \geq 60$ and an initial pulse of the form $u(x,t) = 0.5\,e^{-(x-10)^2}$. You should find that the initial pulse splits into two pulses plus some noise. Describe the motion of the pulses (solitons). Do they maintain their shape, or is this shape modified as they move? Describe the motion of the particles far from the pulse. Are they stationary?

(b) Save the displacements of the particles when the peak of one of the solitons is located near the center of your display. Is it possible to fit the shape of the soliton to a Gaussian? Continue the simulation and after one of the solitons is relatively isolated, set `u(j)` $= 0$ for all `j` far from this soliton. Does the soliton maintain its shape?

(c) Repeat part (b) with a pulse given by $u(x,0) = 0$ everywhere except for $u(20,0) = u(21,0) = 1$. Do the resulting solitons have the same shape as in part (b)?

(d) Begin with the same Gaussian pulse as in part (a) and run until the two solitons are well separated. Then change at random the values of `u(j)` for particles in the larger soliton by about 5% and continue the simulation. Is the soliton destroyed? Increase the perturbation until the soliton is no longer discernible.

(e) Begin with a single Gaussian pulse as in part (a). The two resultant solitons will eventually "collide." Do the solitons maintain their shape after the collision? The principle of superposition implies that the displacement of the particles is given by the sum of the displacements due to each pulse. Does the principle of superposition hold for solitons?

(f) Compute the speeds, amplitudes, and width of the solitons produced from a single Gaussian pulse. Take the amplitude of a soliton to be the largest value of its displacement and the half-width to correspond to the value of x at which the displacement is half its maximum value. Repeat these calculations for solitons of different amplitudes by choosing the initial amplitude of the Gaussian pulse to be 0.1, 0.3, 0.5, 0.7, and 0.9. Plot the soliton speed and width versus the corresponding soliton amplitude.

(g) Change the boundary conditions to free boundary conditions and describe the behavior of the soliton as it reaches a boundary. Compare this behavior with that of a pulse in a system of linear oscillators.

(h) Begin with an initial sinusoidal disturbance that would be a normal mode for a linear system. Does the sinusoidal mode maintain its shape? Compare the behavior of the nonlinear and linear systems. ■

9.8 ■ INTERFERENCE

Interference is one of the most fundamental characteristics of all wave phenomena. The term *interference* is used when there are a small number of sources, and the term *diffraction* is used when the number of sources is large and can be treated as a continuum. Because it is relatively easy to observe interference and diffraction phenomena with light, we discuss these phenomena in this context.

Consider the field from one or more point sources lying in a plane. The electric field at position $\mathbf{r}$ associated with the light emitted from a monochromatic point source at $\mathbf{r}_1$ is a spherical wave radiating from that point. This wave can be thought of as the real part of a complex exponential:

$$E(\mathbf{r},t) = \frac{A}{|\mathbf{r}-\mathbf{r}_1|}e^{i(q|\mathbf{r}-\mathbf{r}_1|-\omega t)}, \tag{9.60}$$

where $|\mathbf{r}-\mathbf{r}_1|$ is the distance between the source and the point of observation, and q is the wavenumber $2\pi/\lambda$. The superposition principle implies that the total electric field at $\mathbf{r}$ from N point sources at $\mathbf{r}_i$ is

$$E(\mathbf{r},t) = e^{-i\omega t}\sum_{n=1}^{N}\frac{A_n}{|\mathbf{r}-\mathbf{r}_n|}e^{i(q|\mathbf{r}-\mathbf{r}_n|)} = e^{-i\omega t}\mathcal{E}(\mathbf{r}). \tag{9.61}$$

The time evolution can be expressed as an oscillatory complex exponential $e^{-i\omega t}$ that multiplies a complex space part $\mathcal{E}(\mathbf{r})$. The spatial part $\mathcal{E}(\mathbf{r})$ is a *phasor* which contains both the maximum value of the electric field and the time within a cycle when the physical field reaches its maximum value. As the system evolves, the complex electric field $E(\mathbf{r},t)$ oscillates between purely real and purely imaginary values. Both the energy density (the energy per unit volume) and the light intensity (the energy passing through a unit area) are proportional to the square of the magnitude of the phasor. Because light fields oscillate at $\approx 6\times10^{14}$ Hz, typical optics experiments observe the time average (rms value) of $\mathcal{E}$ and do not observe the phase angle.

Huygens's principle states that each point on a wavefront (a surface of constant phase) can be treated as the source of a new spherical wave or *Huygens's wavelet*. The wavefront at some later time is constructed by summing these wavelets. The `HuygensApp` program implements Huygens's principle by assuming superposition from an arbitrary number of