(b) The bond percolation threshold on a square lattice is $p_c = 0.5$. Use your program to compute the conductivity for a $L = 30$ square lattice. Average over at least ten spanning configurations for $p = 0.51, 0.52$, and 0.53. Note that you can eliminate all bonds that are not part of the spanning cluster and all occupied bonds connected to only one other occupied bond. Why? If possible, consider more values of p. Estimate the critical exponent t defined in (12.41).

(c) Fix p at $p = p_c = 1/2$ and use finite size scaling to estimate the conductivity exponent t.

*(d) Use larger lattices and the multigrid method (see Project 10.26) to improve your results. If you have sufficient computing resources, compute t for a simple cubic lattice for which $p_c \approx 0.249$. (In general, t is not the same for lattice and continuum percolation.) ■

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