```
   int count = 0;
   int maxCount =
      (int) (Math.log(Math.abs(x2 - x1)/tolerance)/Math.log(2));
   maxCount = Math.max(MAX_ITERATIONS, maxCount) + 2;
   double y1 = f.evaluate(x1), y2 = f.evaluate(x2);
   if (y1 * y2 > 0) { // y1 and y2 must have opposite sign
      return Double.NaN; // interval does not contain a root
   }
   while (count < maxCount) {
      double x = (x1 + x2) / 2;
      double y = f.evaluate(x);
      if (Math.abs(y) < tolerance) return x;
      if (y * y1 > 0) { // replace the endpoint that has the same sign
         x1 = x;
         y1 = y;
      }
      else {
         x2 = x;
         y2 = y;
      }
      count++;
   }
   return Double.NaN; // did not converge in max iterations
}
```

The bisection algorithm is guaranteed to converge if you can find an interval where the function changes sign. However, it is slow. Newton's algorithm is very fast but may not converge. We develop an algorithm in the following problem that combines these two approaches.

Problem 6.32 Finding roots
Modify Newton's algorithm to keep track of the interval between the minimum and the maximum of x while iterating (6.79). If the iterate x_{n+1} jumps outside this interval, interrupt Newton's method and use the bisection algorithm for one iteration. Test the root at the end of the iterative process to check that the algorithm actually found a root. Test your algorithm on the function in (6.78). ■

REFERENCES AND SUGGESTIONS FOR FURTHER READING

Books

Ralph H. Abraham and Christopher D. Shaw, *Dynamics: The Geometry of Behavior*, 2nd ed. (Addison–Wesley, 1992). The authors use an abundance of visual representations.

Hao Bai-Lin, *Chaos II* (World Scientific, 1990). A collection of reprints on chaotic phenomena. The following papers were cited in the text. James P. Crutchfield, J. Doyne Farmer, Norman H. Packhard, and Robert S. Shaw, "Chaos," Sci. Am. **255** (6), 46–57 (1986); Mitchell J. Feigenbaum, "Quantitative universality for a class of nonlinear transformations," J. Stat. Phys. **19**, 25–52 (1978); M. Hénon, "A two-dimensional mapping with a strange attractor," Commun. Math. Phys. **50**, 69–77 (1976); Robert M. May, "Simple mathematical models with very complicated dynamics," Nature **261**, 459–467 (1976);

Robert Van Buskirk and Carson Jeffries, "Observation of chaotic dynamics of coupled nonlinear oscillators," Phys. Rev. A **31**, 3332–3357 (1985).

G. L. Baker and J. P. Gollub, *Chaotic Dynamics: An Introduction*, 2nd ed. (Cambridge University Press, 1995). A good introduction to chaos with special emphasis on the forced, damped, nonlinear harmonic oscillator. Several programs are given.

Pedrag Cvitanovic, *Universality in Chaos*, 2nd ed. (Adam–Hilger, 1989). A collection of reprints on chaotic phenomena including the articles by Hénon and May also reprinted in the Bai-Lin collection and the chaos classic, Mitchell J. Feigenbaum, "Universal behavior in nonlinear systems," Los Alamos Sci. **1**, 4–27 (1980).

Robert Devaney, *A First Course in Chaotic Dynamical Systems* (Addison–Wesley, 1992). This text is a good introduction to the more mathematical ideas behind chaos and related topics.

Jan Fröyland, *Introduction to Chaos and Coherence* (Institute of Physics Publishing, 1992). See Chapter 7 for a simple model of Saturn's rings.

Martin C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer–Verlag, 1990). A good introduction to problems in quantum chaos for the more advanced student.

Robert C. Hilborn, *Chaos and Nonlinear Dynamics* (Oxford University Press, 1994). An excellent pedagogically oriented text.

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E. Atlee Jackson, *Perspectives of Nonlinear Dynamics*, Vols. 1 and 2 (Cambridge University Press, 1989, 1991). An advanced text that is a joy to read.

R. V. Jensen, "Chaotic scattering, unstable periodic orbits, and fluctuations in quantum transport," Chaos **1**, 101–109 (1991). This paper discusses the quantum version of systems similar to those discussed in Projects 6.28 and 6.26.

Francis C. Moon, *Chaotic and Fractal Dynamics, An Introduction for Applied Scientists and Engineers* (Wiley, 1992). An engineering oriented text with a section on how to build devices that demonstrate chaotic dynamics.

Edward Ott, *Chaos in Dynamical Systems* (Cambridge University Press, 1993). An excellent textbook on chaos at the upper undergraduate to graduate level. See also E. Ott, "Strange attractors and chaotic motions of dynamical systems," Rev. Mod. Phys. **53**, 655–671 (1981).

Edward Ott, Tim Sauer, and James A. Yorke, eds., *Coping with Chaos* (John Wiley & Sons, 1994). A reprint volume emphasizing the analysis of experimental time series from chaotic systems.

Heinz–Otto Peitgen, Hartmut Jürgens, and Dietmar Saupe, *Fractals for the Classroom*, Part II (Springer–Verlag, 1992). A delightful book with many beautiful illustrations. Chapter 11 discusses the nature of the bifurcation diagram of the logistic map.

Ian Percival and Derek Richards, *Introduction to Dynamics* (Cambridge University Press, 1982). An advanced undergraduate text that introduces phase trajectories and the theory of stability. A derivation of the Hamiltonian for the driven damped pendulum considered in Section 6.4 is given in Chapter 5, example 5.7.