frequency dependence of the amplitude of $|E|$ due to a charge oscillating at the frequency $\omega$. It is shown in standard textbooks that the power associated with radiation from an oscillating dipole is proportional to $\omega^4$. How does the $\omega$-dependence that you measured compare to that for dipole radiation? Repeat for a much bigger value of $R$ and explain any differences.

(b) Repeat part (a) for a charge moving in a circle. Are there any qualitative differences?

■

## *10.8 ■ MAXWELL'S EQUATIONS

In Section *10.7 we found that accelerating charges produce electric and magnetic fields that depend on position and time. We now investigate the direct relation between changes in **E** and **B** given by the differential form of Maxwell's equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{c}\nabla \times \mathbf{E} \tag{10.46}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c\nabla \times \mathbf{B} - 4\pi \mathbf{j}, \tag{10.47}$$

where **j** is the electric current density. We can regard (10.46) and (10.47) as the basis of electrodynamics. In addition to (10.46) and (10.47), we need the relation between **j** and the charge density $\rho$ that expresses the conservation of charge:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{j}. \tag{10.48}$$

A complete description of electrodynamics requires (10.46), (10.47), (10.48), and the initial values of all currents and fields.

For completeness, we obtain the Maxwell's equations that involve $\nabla \cdot \mathbf{B}$ and $\nabla \cdot \mathbf{E}$ by taking the divergence of (10.46) and (10.47), substituting (10.48) for $\nabla \cdot \mathbf{j}$, and then integrating over time. If the initial fields are zero, we obtain (using the relation $\nabla \cdot (\nabla \times \mathbf{a}) = 0$ for any vector **a**)

$$\nabla \cdot \mathbf{E} = 4\pi\rho \tag{10.49}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{10.50}$$

If we introduce the electric and magnetic potentials, it is possible to convert the first-order equations (10.46) and (10.47) to second-order differential equations. However, the familiar first-order equations are better suited for numerical analysis. To solve (10.46) and (10.47) numerically, we need to interpret the curl and divergence of a vector. As its name implies, the curl of a vector measures how much the vector twists around a point. A coordinate free definition of the curl of an arbitrary vector **W** is

$$(\nabla \times \mathbf{W}) \cdot \hat{\mathbf{S}} = \lim_{S \to 0} \frac{1}{S} \oint_C \mathbf{W} \cdot d\mathbf{l}, \tag{10.51}$$



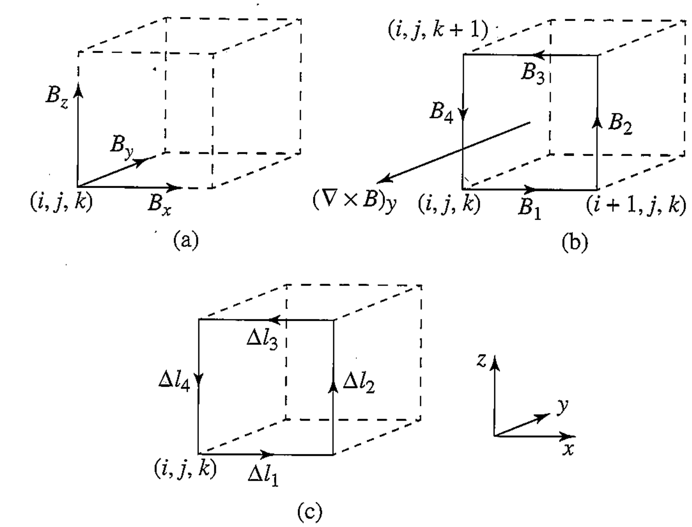


**Figure 10.6** Calculation of the curl of **B** defined on the edges of a cube. (a) The edge vector **B** associated with cube $(i, j, k)$. (b) The components $B_i$ along the edges of the front face of the cube. $B_1 = B_x(i, j, k)$, $B_2 = B_z(i+1, j, k)$, $B_3 = -B_x(i, j, k+1)$, and $B_4 = -B_z(i, j, k)$. (c) The vector components $\Delta \mathbf{l}_i$ on the edges of the front face. (The $y$-component of $\nabla \times \mathbf{B}$ defined on the face points in the negative $y$ direction.)

where **S** is the area of any surface bordered by the closed curve $C$, and $\hat{\mathbf{S}}$ is a unit vector normal to the surface $S$.

Equation (10.51) gives the component of $\nabla \times \mathbf{W}$ in the direction of $\hat{\mathbf{S}}$ and suggests a way of computing the curl numerically. We divide space into cubes of linear dimension $\Delta l$. The rectangular components of **W** can be defined either on the edges or on the faces of the cubes. We compute the curl using both definitions. We first consider a vector **B** that is defined on the edges of the cubes so that the curl of **B** is defined on the faces. (We use the notation **B** because we will find that it is convenient to define the magnetic field in this way.) Associated with each cube is one edge vector and one face vector. We label the cube by the coordinates corresponding to its lower left front corner; the three components of **B** associated with this cube are shown in Figure 10.6a. The other edges of the cube are associated with $B$ vectors defined at neighboring cubes.

The discrete version of (10.51) for the component of $\nabla \times \mathbf{B}$ defined on the front face of the cube $(i, j, k)$ is

$$(\nabla \times \mathbf{B}) \cdot \hat{\mathbf{S}} = \frac{1}{(\Delta l)^2} \sum_{i=1}^{4} B_i \Delta l_i, \tag{10.52}$$

where $S = (\Delta l)^2$, and $B_i$ and $l_i$ are shown in Figures 10.6b and 10.6c, respectively. Note that two of the $B_i$ are associated with neighboring cubes.

The components of a vector can also be defined on the faces of the cubes. We call this vector **E** because it will be convenient to define the electric field in this way. In Figure 10.7a we show the components of **E** associated with the cube $(i, j, k)$. Because **E** is normal to a cube face, the components of $\nabla \times \mathbf{E}$ lie on the edges. The components $E_i$ and $l_i$ are shown in Figures 10.7b and 10.7c, respectively. The form of the discrete version of $\nabla \times \mathbf{E}$