```
      pendulum.initializeState(new double[] {theta, omega, 0});
      clear();
   }

   public void clear() {
      phaseSpace.clearData();
      poincare.clearData();
      phaseSpace.render();
      poincare.render();
   }

   public static void main(String[] args) {
      SimulationControl control =
            SimulationControl.createApp(new PoincareApp());
      control.addButton("clear", "Clear");
   }
}
```

Problem 6.17 Dynamics of a driven, damped simple pendulum

(a) Use `PoincareApp` to simulate the driven, damped simple pendulum. In the program, $\omega = 2$ so that the period T of the external force equals π. The program also assumes that $\omega_0 = 1$. Use $\gamma = 0.2$ and $A = 0.85$ and compute the phase space trajectory. After the transient, how many points do you see in the Poincaré plot? What is the period of the pendulum? Vary the initial values of θ and $d\theta/dt$. Is the attractor independent of the initial conditions? Remember to ignore the transient behavior.

(b) Modify `PoincareApp` so that it plots θ and $d\theta/dt$ as a function of t. Describe the qualitative relation between the Poincaré plot, the phase space plot, and the t dependence of θ and $d\theta/dt$.

(c) The amplitude A plays the role of the control parameter for the dynamics of the system. Use the behavior of the Poincaré plot to find the value $A = A_c$ at which the (0, 0) attractor becomes unstable. Start with $A = 0.1$ and continue increasing A until the (0, 0) attractor becomes unstable.

(d) Find the period for $A = 0.1$, 0.25, 0.5, 0.7, 0.75, 0.85, 0.95, 1.00, 1.02, 1.031, 1.033, 1.036, and 1.05. Note that for small A, the period of the oscillator is twice that of the external force. The steady state period is 2π for $A_c < A < 0.71$, π for $0.72 < A < 0.79$, and then 2π again.

(e) The first period doubling occurs for $A \approx 0.79$. Find the approximate values of A for further period doubling and use these values of A to compute the exponent δ defined by (6.10). Compare your result for δ with the result found for the one-dimensional logistic map. Are your results consistent with those that you found for the logistic map? An analysis of this system can be found in the article by McLaughlin.

(f) Sometimes a trajectory does not approach a steady state even after a very long time, but a slight perturbation causes the trajectory to move quickly onto a steady state attractor. Consider $A = 0.62$ and the initial condition ($\theta = 0.3$, $d\theta/dt = 0.3$). Describe the behavior of the trajectory in phase space. During the simulation, change θ by 0.1. Does the trajectory move onto a steady state trajectory? Do similar simulations for other values of A and other initial conditions.

(g) Repeat the calculations of parts (b)–(d) for $\gamma = 0.05$. What can you conclude about the effect of damping?

(h) Replace the fourth-order Runge–Kutta algorithm by the Euler–Richardson algorithm which is lower order. Which algorithm gives the better trade-off between accuracy and speed? ■

Problem 6.18 The basin of an attractor

(a) For $\gamma = 0.2$ and $A > 0.79$, the pendulum rotates clockwise or counterclockwise in the steady state. Each of these two rotations is an attractor. The set of initial conditions that lead to a particular attractor is called the basin of the attractor. Modify `PoincareApp` so that the program draws the basin of the attractor with $d\theta/dt > 0$. For example, your program might simulate the motion for about 20 periods and then determine the sign of $d\theta/dt$. If $d\theta/dt > 0$ in the steady state, then the program plots a point in phase space at the coordinates of the initial condition. The program repeats this process for many initial conditions. Describe the basin of attraction for $A = 0.85$ and increments of the initial values of θ and $d\theta/dt$ equal to $\pi/10$.

(b) Repeat part (a) using increments of the initial values of θ and $d\theta/dt$ equal to $\pi/20$ or as small as possible given your computer resources. Does the boundary of the basin of attraction appear smooth or rough? Is the basin of the attractor a single region or is it disconnected into more than one piece?

(c) Repeat parts (a) and (b) for other values of A, including values near the onset of chaos and in the chaotic regime. Is there a qualitative difference between the basins of periodic and chaotic attractors? For example, can you always distinguish the boundaries of the basin? ■

*6.9 ■ HAMILTONIAN CHAOS

Hamiltonian systems are a very important class of dynamical systems. The most familiar are mechanical systems without friction, and the most important of these is the solar system. The linear harmonic oscillator and the simple pendulum that we considered in Chapter 3 are two simple examples. Many other systems can be included in the Hamiltonian framework, for example, the motion of charged particles in electric and magnetic fields and ray optics. The Hamiltonian dynamics of charged particles is particularly relevant to confinement issues in particle accelerators, storage rings, and plasmas. In each case a function of all the coordinates and momenta called the Hamiltonian is formed. For many systems this function can be identified with the total energy. The Hamiltonian for a particle in a potential $V(x, y, z)$ is

$$H = \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) + V(x, y, z). \tag{6.35}$$

Typically we write (6.35) using the notation

$$H = \sum_i \frac{p_i^2}{2m} + V(\{q_i\}), \tag{6.36}$$