Wave function visualizations can be selected at runtime using the `Tools` menu, or they can be selected programmatically using convert methods such as `convertToPostView` and `convertToReImView`. The `Tools` menu also allows the user to select a table view to examine the data being used to draw the wave function and to display a phase legend that shows the color to phase relation.

A `Complex2DFrame` displays a two-dimensional complex scalar field such as a two-dimensional wave function. We instantiate a `Complex2DFrame` and then pass to it a multidimensional array containing the field's real and imaginary components. Listing 16.13 shows how this class is used to show a two-dimensional Gaussian wave packet with a momentum boost.

Listing 16.13 The `Complex2DFrameApp` program displays a two-dimensional Gaussian wave packet with a momentum boost.

```
package org.opensourcephysics.sip.ch16;
import org.opensourcephysics.frames.Complex2DFrame;

public class Complex2DFrameApp {
   public static void main(String[] args) {
      Complex2DFrame frame = new Complex2DFrame("x", "y",
           "Complex field");
      frame.setPreferredMinMax(-1.5, 1.5, -1.5, 1.5);
      double[][][] field = new double[2][32][32];
      frame.setAll(field);
      for(int i = 0, nx = field[0].length;i<nx;i++) {
         double x = frame.indexToX(i);
         for(int j = 0, ny = field[0][0].length;j<ny;j++) {
            double y = frame.indexToY(j);
            double a = Math.exp(-4*(x*x+y*y));
            field[0][i][j] = a*Math.cos(5*x); // real component
            field[1][i][j] = a*Math.sin(5*x); // complex component
         }
      }
      frame.setAll(field);
      frame.setVisible(true);
      frame.setDefaultCloseOperation(javax.swing.JFrame.EXIT_ON_CLOSE);
   }
}
```

The complex field is computed on a n-row by m-column grid and stored in an array with dimensions $2 \times m \times n$. The default visualization uses a grid in which every cell is colored using brightness to show the complex number's magnitude and color to show phase. Other visualizations can be programmed or selected at runtime using the menu.

REFERENCES AND SUGGESTIONS FOR FURTHER READING

The ALPS project, <http://alps.comp-phys.org/>, has open source simulation programs for strongly correlated quantum mechanical systems and C++ libraries for simplifying the development of such code. Although most of the code is beyond the level of this text, this open source project is another example of software for use in both research and education.

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