Evaluation of Functions

Find the smallest number of the computer

```
      X = 0.0
10    X = x + 0.1
      if (X .ne. 1.0) go to 10
```

```
PROGRAM EPS_test
IMPLICIT NONE
INTEGER :: n
REAL :: eps,X

n=0
eps=1.0
do while ( 1.0+eps > 1.0 )
     n=n+1
     eps=eps/2.0
end do

print *, n, 2.0*eps

X=0.16749E–3s00
print 100,x

100 format(E13.4)

END PROGRAM EPS_test
```

Machine round-off error

```
PROGRAM RNDOFF
IMPLICIT NONE
   integer i
   real h,tmp1,tmp2,tmp3
   real xv(100)

open (unit = 2, file = 'rnd.dat', status = 'unknown')

   h = 1.0
   do i = 1,21
      tmp1 = 10.0+h
      tmp2 = tmp1 – 10.0
      tmp3 = abs(tmp2 – h)/h
      write(2,*) i,h,tmp3
      h = h/10.0
      xv(i) = h
   end do

   close (unit = 2)

END PROGRAM RNDOFF
```

Polynomials - *Polynomial Evaluation*

Q: how to evaluate the following polynomial?

$p = c(0) + c(1)x + c(2)x^2 + c(3)x^3 + c(4)x^4$

```
p = c(0) + c(1)*x + c(2)*x**2 + c(3)*x**3 + c(4)*x**4
```

You will be found guilty of such criminal behavior and summarily executed, but your program won't!

Nested multiplication:

```
p = c(0) + x*( c(1) + x*( c(2) + x*( c(3) + x*c(4) )))
```

or

```
p = ((( c(4)*x + c(3) )*x + c(2) )*x + c(1) )*x + c(0)
```

This uses only *n* multiplications.

```
p = c(n)
do j = n – 1, 0, – 1
   p = p*x + c(j)
end do
```

Polynomials - *Polynomial Evaluation*

One may also evaluate a polynomial $P(x)$ and its derivative $dP(x)/dx$ simultaneously:

```
p = c(n)
dp = 0
do j = n – 1, 0, –1
   dp = dp*x + p
   p = p*x + c(j)
end dos
```

Polynomials - *Multiply a polynomial by* $(x - a)$

```
c(n+1) = c(n)
do j = n, 1, –1
    c(j) = c(j – 1) – c(j)*a
end do
c(0) = –c(0)*a
```

Polynomials - *Evaluation a rational function*

$$R(x) = \frac{P_\mu(x)}{Q_\nu(x)}$$

$$= \frac{p_0 + p_1 x + \cdots + p_\mu x^\mu}{q_0 + q_1 x + \cdots + q_\nu x^\nu}$$

Just do it as two separate polynomials followed by a division.

Usually one scales q_0 to 1.

Horner's method, synthetic division, etc.

Some complex arithmetic

- Complex modulus: $|a+ib| = \sqrt{a^2+b^2}$?

  You may get *overflow* message!

$$|a+ib| = \begin{cases} |a|\sqrt{1+(b/a)^2}, & |a| \geq |b| \\ |b|\sqrt{1+(a/b)^2}, & |a| < |b| \end{cases}$$

- Complex division

$$\frac{a+ib}{c+id} = \begin{cases} \dfrac{[a+b(d/c)]+i[b-a(d/c)]}{c+d(d/c)}, & |c| \geq |d| \\ \dfrac{[a(c/d)+b]+i[b(c/d)-a]}{c(c/d)+d}, & |c| < |d| \end{cases}$$

  to prevent avoidable overflows, underflows, or loss of precision.

- Complex square root?

Special functions

- Gamma Function, Beta Function, Factorials, Binomial Coefficients
- Incomplete Gamma Function, Error Function, Chi-Square Probability Function, Cumulative Poisson Function
- Exponential Integrals: $E_n(x) = \int_0^\infty \mathrm{d}t \frac{e^{-xt}}{t^n}, \quad x > 0, \quad n = 0,1,...$
- Incomplete Beta Function, Student's Distribution, F-Distribution, Cumulative Binomial Distribution
- Bessle Functions: Integer order, Fractional order, Modified, Spherical, Airy functions, etc.
- Elliptic Integrals and Jacobian Elliptic Functions
- Hypergeometric Functions

Understand their properties and usages.

Call Math libraries in your programs.