with $0 < e < 1$. (Choose the point P at $x = 0$, $y = b$.) A special case is $b = a$, for which the ellipse reduces to a circle and $e = 0$.

5.4 ■ ASTRONOMICAL UNITS

It is convenient to choose a system of units in which the magnitude of the product GM is not too large and not too small. To describe the Earth's orbit, the convention is to choose the length of the Earth's semimajor axis as the unit of length. This unit of length is called the *astronomical unit* (AU) and is

$$1\,\mathrm{AU} = 1.496 \times 10^{11}\,\mathrm{m}. \tag{5.15}$$

The unit of time is assumed to be one year or 3.15×10^7 s. In these units the period of the Earth is $T = 1$ year and its semimajor axis is $a = 1$ AU. Hence, from (5.13)

$$GM = \frac{4\pi^2 a^3}{T^2} = 4\pi^2\ \mathrm{AU}^3/\mathrm{years}^2 \quad \text{(astronomical units)}. \tag{5.16}$$

As an example of the use of astronomical units, a program distance of 1.5 would correspond to $1.5 \times (1.496 \times 10^{11}) = 2.244 \times 10^{11}$ m.

5.5 ■ LOG-LOG AND SEMILOG PLOTS

The values of T and a for our solar system are given in Table 5.1. We first analyze these values and determine if T and a satisfy a simple mathematical relationship.

Suppose we wish to determine whether two variables y and x satisfy a functional relationship, $y = f(x)$. To simplify the analysis, we ignore possible errors in the measurements of y and x. The simplest relation between y and x is linear, that is, $y = mx + b$. The existence of such a relation can be seen by plotting y versus x and finding if the plot is linear. From Table 5.1 we see that T is not a linear function of a. For example, an increase in T from 0.24 to 1, an increase of approximately 4, yields an increase in T of approximately 2.5.

For many problems, it is reasonable to assume an exponential relation

$$y = Ce^{rx}, \tag{5.17}$$

or a power law relation

$$y = Cx^n, \tag{5.18}$$

where C, r, and n are unknown parameters.

If we assume the exponential form (5.17), we can take the natural logarithm of both sides to find

$$\ln y = \ln C + rx. \tag{5.19}$$

Hence, if (5.17) is applicable, a plot of $\ln y$ versus x would yield a straight line with slope r and intercept $\ln C$.

Table 5.1 The period T and semimajor axis a of the planets. The unit of length is the astronomical unit (AU). The unit of time is one (Earth) year.

Planet	T (Earth Years)	a (AU)
Mercury	0.241	0.387
Venus	0.615	0.723
Earth	1.0	1.0
Mars	1.88	1.523
Jupiter	11.86	5.202
Saturn	29.5	9.539
Uranus	84.0	19.18
Neptune	165	30.06
Pluto	248	39.44

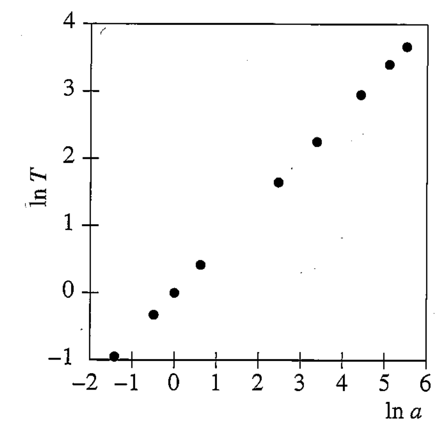

Figure 5.3 Plot of $\ln T$ versus $\ln a$ using the data in Table 5.1. Verify that the slope is 1.50.

The natural logarithm of both sides of the power law relation (5.18) yields

$$\ln y = \ln C + n \ln x. \tag{5.20}$$

If (5.18) applies, a plot of $\ln y$ versus $\ln x$ yields the exponent n (the slope), which is the usual quantity of physical interest if a power law dependence holds.

We illustrate a simple analysis of the data in Table 5.1. Because we expect that the relation between T and a has the power law form $T = Ca^n$, we plot $\ln T$ versus $\ln a$ (see Figure 5.3). A visual inspection of the plot indicates that a linear relationship between $\ln T$ and $\ln a$ is reasonable and that the slope is approximately 1.50 in agreement with Kepler's second law. In Chapter 7 we will discuss the least squares method for fitting a straight line through a number of data points. With a little practice, you can do a visual analysis that is nearly as good.

The `PlotFrame` class contains the axes and titles needed to produce linear, log-log, and semilog plots. It also contains the methods needed to display data in a table format. This table can be displayed programmatically or by right-clicking (control-clicking) at runtime. Listing 5.1 shows a short program that produces the log-log plot of the semimajor axis of the planets versus the orbital period. The arrays `a` and `T` contain the semimajor axis of the