The `ThreeBodyApp` class in Listing 5.10 is the target class for the three-body program. The `doStep` method merely increments the model's differential equations solver and repaints the view.

Listing 5.10 A program that displays the trajectories of three bodies interacting via gravitational forces.

```
package org.opensourcephysics.sip.ch05;
import org.opensourcephysics.controls.*;
import org.opensourcephysics.frames.*;

public class ThreeBodyApp extends AbstractSimulation {
   PlotFrame frame = new PlotFrame("x", "y", "Three-Body Orbits");
   ThreeBody trajectory = new ThreeBody();

   public ThreeBodyApp() {
      frame.addDrawable(trajectory);
      frame.setSquareAspect(true);
      frame.setSize(450, 450);
   }

   public void initialize() {
      trajectory.odeSolver.setStepSize(control.getDouble("dt"));
      trajectory.initialize(ThreeBodyInitialConditions.MONTGOMERY);
      frame.setPreferredMinMax(-1.5, 1.5, -1.5, 1.5);
   }

   public void reset() {
      control.setValue("dt", 0.1);
      enableStepsPerDisplay(true);
      initialize();
   }

   protected void doStep() {
      trajectory.doStep();
      frame.setMessage("t="+decimalFormat.format(trajectory.state[4]));
   }

   public static void main(String[] args) {
      SimulationControl.createApp(new ThreeBodyApp());
   }
}
```

Problem 5.16 Stability of solutions to the three-body problem

Examine the stability of the three solutions to the three-body problem by slightly varying the initial velocity of one of the masses. Before passing your new initial state to `trajectory.initialize`, calculate the center of mass velocity and subtract this velocity from every object. Show that any instability is due to the physics and not to the numerical differential equation solver. Which of the three analytic solutions is stable? Check conservation of the total energy and angular momentum. ∎

5.12 ■ PROJECTS

Project 5.17 Effect of a "solar wind"

(a) Assume that a satellite is affected not only by the Earth's gravitational force, but also by a weak uniform "solar wind" of magnitude W acting in the horizontal direction. The equations of motion can be written as

$$\frac{d^2x}{dt^2} = -\frac{GMx}{r^3} + W \tag{5.32a}$$

$$\frac{d^2y}{dt^2} = -\frac{GMy}{r^3}. \tag{5.32b}$$

Choose initial conditions so that a circular orbit would be obtained for $W = 0$. Then choose a value of W whose magnitude is about 3% of the acceleration due to the gravitational field and compute the orbit. How does the orbit change?

(b) Determine the change in the velocity space orbit when the solar wind (5.32) is applied. How does the total angular momentum and energy change? Explain in simple terms the previously observed change in the position space orbit. See Luehrmann for further discussion of this problem. ∎

Project 5.18 Resonances and the asteroid belt

(a) A histogram of the number of asteroids versus their distance from the Sun shows some distinct gaps. These gaps, called the *Kirkwood gaps*, are due to resonance effects. That is, if asteroids were in these gaps, their periods would be simple fractions of the period of Jupiter. Modify class `Planet2` so that planet two has the mass of Jupiter by setting `GM1 = 0.001*GM`. Because the asteroid masses are very small compared to that of Jupiter, the gravitational force on Jupiter due to the asteroids can be neglected. The initial conditions listed in `Planet2` are approximately correct for Jupiter. The initial conditions for the asteroid (planet one in `Planet2`) correspond to the 1/3 resonance (the period of the asteroid is one third that of Jupiter). Run the program with these changes and describe the orbit of the asteroid.

(b) Use Kepler's third law, $T^2/a^3 =$ constant, to determine the values of a, the asteroid's semimajor axis, such that the ratio of its period of revolution about the Sun to that of Jupiter is 1/2, 3/7, 2/5, and 2/3. Set the initial value of `x(1)` equal to a for each of these ratios and choose the initial value of `vy(1)` so that the asteroid would have a circular orbit if Jupiter was not present. Describe the orbits that you obtain.

(c) It is instructive to plot a as a function of time. However, because it is not straightforward to measure a directly in the simulation, it is more convenient to plot the quantity $-2GMm/E$, where E is the total energy of the asteroid and m is the mass of the asteroid. Because E is proportional to m, the quantity $-2GMm/E$ is independent of m. If the interaction of the asteroid with Jupiter is ignored, it can be shown that $a = -2GMm/E$, where E is the asteroid kinetic energy plus the asteroid-Sun potential energy. Derive this result for circular orbits. Plot the quantity $-2GMm/E$ versus time for about thirty revolutions for the initial conditions in Problem 5.18b.