CHAPTER

2 Tools for Doing Simulations

We introduce some of the core syntax of Java in the context of simulating the motion of falling particles near the Earth's surface. A simple algorithm for solving first-order differential equations numerically is also discussed.

2.1 ■ INTRODUCTION

If you were to take a laboratory-based course in physics, you would soon be introduced to the oscilloscope. You would learn the function of many of the knobs, how to read the display, and how to connect various devices so that you could measure various quantities. If you did not know already, you would learn about voltage, current, impedance, and AC and DC signals. Your goal would be to learn how to use the oscilloscope. In contrast, you would learn only a little about the inner workings of the oscilloscope.

The same approach can be easily adopted with an object-oriented language such as Java. If you are new to programming, you will learn how to make Java do what you want, but you will not learn everything about Java. In this chapter, we will present some of the essential syntax of Java and introduce the Open Source Physics library, which will facilitate writing programs with a graphical user interface and visual output such as plots and animations.

One of the ways that science progresses is by making models. If the model is sufficiently detailed, we can determine its behavior and then compare the behavior with experiment. This comparison might lead to verification of the model, changes in the model, and further simulations and experiments. In the context of computer simulation, we usually begin with a set of initial conditions, determine the dynamical behavior of the model numerically, and generate data in the form of tables of numbers, plots, and animations. We begin with a simple example to see how this process works.

Imagine a particle such as a ball near the surface of the Earth subject to a single force, the force of gravity. We assume that air friction is negligible, and the gravitational force is given by

$$F_g = -mg, \tag{2.1}$$

where m is the mass of the ball and $g = 9.8$ N/kg is the gravitational field (force per unit mass) near the Earth's surface. To make our example as simple as possible, we first assume that there is only vertical motion. We use Newton's second law to find the motion of the ball:

$$m\frac{d^2y}{dt^2} = F, \tag{2.2}$$

where y is the vertical coordinate defined so that up is positive, t is the time, F is the total force on the ball, and m is the inertial mass (which is the same as the gravitational mass in (2.1)). If we set $F = F_g$, (2.1) and (2.2) lead to

$$\frac{d^2y}{dt^2} = -g. \tag{2.3}$$

Equation (2.3) is a statement of a model for the motion of the ball. In this case the model is in the form of a second-order differential equation.

You are probably familiar with the model summarized in (2.3) and know the analytical solution:

$$y(t) = y(0) + v(0)t - \frac{1}{2}gt^2 \tag{2.4a}$$

$$v(t) = v(0) - gt. \tag{2.4b}$$

Nevertheless, we will determine the motion of a freely falling particle numerically in order to introduce the tools that we will need in a familiar context.

We begin by expressing (2.3) as two first-order differential equations:

$$\frac{dy}{dt} = v \tag{2.5a}$$

$$\frac{dv}{dt} = -g, \tag{2.5b}$$

where v is the vertical velocity of the ball. We next *approximate* the derivatives by small (finite) differences:

$$\frac{y(t+\Delta t) - y(t)}{\Delta t} = v(t) \tag{2.6a}$$

$$\frac{v(t+\Delta t) - v(t)}{\Delta t} = -g. \tag{2.6b}$$

Note that in the limit $\Delta t \to 0$, (2.6) reduces to (2.5). We can rewrite (2.6) as

$$y(t+\Delta t) = y(t) + v(t)\Delta t \tag{2.7a}$$

$$v(t+\Delta t) = v(t) - g\Delta t. \tag{2.7b}$$

The finite difference approximation we used to obtain (2.7) is an example of the *Euler* algorithm. Equation (2.7) is an example of a *finite difference* equation, and Δt is the time step.

Now we are ready to follow $y(t)$ and $v(t)$ in time. We begin with an initial value for y and v and then *iterate* (2.7). If Δt is sufficiently small, we will obtain a numerical answer that is close to the solution of the original differential equations in (2.6). In this case we know the answer, and we can test our numerical results directly.

Exercise 2.1 A simple example

Consider the first-order differential equation

$$\frac{dy}{dx} = f(x), \tag{2.8}$$