Plot your data for the dependence of the period T on the semimajor axis a and verify Kepler's third law. Given the ratio of T^2/a^3 that you found, determine the numerical value of this ratio in SI units for our solar system.

(c) The force center is at $(x, y) = (0, 0)$ and is one focus. Find the second focus by symmetry. Compute the sum of the distances from each point on the orbit to the two foci and verify that the orbit is an ellipse.

(d) According to Kepler's second law, the orbiting object sweeps out equal areas in equal times. If we use an algorithm with a fixed time step Δt, it is sufficient to compute the area of the triangle swept in each time step. This area equals one-half the base of the triangle times its height, or $\frac{1}{2}\Delta t\,(\mathbf{r} \times \mathbf{v}) = \frac{1}{2}\Delta t\,(xv_y - yv_x)$. Is this area a constant? This constant corresponds to what physical quantity?

*(e) Show that algorithms with a fixed value of Δt break down if the "planet" is too close to the sun. What is the cause of the failure of the method? What advantage might there be to using a variable time step? What are the possible disadvantages? (See Project 5.19 for an example where a variable time step is very useful.) ■

Problem 5.4 Noninverse square forces

(a) Consider the dynamical effects of a small change in the attractive inverse-square force law; for example, let the magnitude of the force equal $Cm/r^{2+\delta}$, where $\delta \ll 1$. For simplicity, take the numerical value of the constant C to be $4\pi^2$ as before. Consider the initial conditions $x(t = 0) = 1$, $y(t = 0) = 0$, $v_x(t = 0) = 0$, and $v_y(t = 0) = 5$. Choose $\delta = 0.05$ and determine the nature of the orbit. Does the orbit of the planet retrace itself? Verify that your result is not due to your choice of Δt. Does the planet spiral away from or toward the sun? The path of the planet can be described as an elliptical orbit that slowly rotates or *precesses* in the same sense as the motion of the planet. A convenient measure of the precession is the angle between successive orientations of the semimajor axis of the ellipse. This angle is the rate of precession per revolution. Estimate the magnitude of this angle for your choice of δ. What is the effect of decreasing the semimajor axis for fixed δ? What is the effect of changing δ for fixed semimajor axis?

(b) Einstein's theory of gravitation (the general theory of relativity) predicts a correction to the force on a planet that varies as $1/r^4$ due to a weak gravitational field. The result is that the equation of motion for the trajectory of a particle can be written as

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{GM}{r^2}\left[1 + \alpha\left(\frac{GM}{c^2}\right)^2 \frac{1}{r^2}\right]\hat{\mathbf{r}}, \tag{5.21}$$

where the parameter α is dimensionless. Take $GM = 4\pi^2$ and assume $\alpha = 10^{-3}$. Determine the nature of the orbit for this potential. (For our solar system, the constant α is a maximum for the planet Mercury, but is much smaller than 10^{-3}.)

(c) Suppose that the attractive gravitational force law depends on the inverse cube of the distance, Cm/r^3. What are the units of C? For simplicity, take the numerical value of C to be $4\pi^2$. Consider the initial condition $x(t = 0) = 1$, $y(t = 0) = 0$, $v_x(t = 0) = 0$ and determine analytically the value of $v_y(t = 0)$ required for a circular orbit. How small a value of Δt is needed so that the simulation yields a circular orbit over several periods? How does this value of Δt compare with the value needed for the inverse-square force law?

(d) Vary $v_y(t = 0)$ by approximately 2% from the circular orbit condition that you determined in part (c). What is the nature of the new orbit? What is the sign of the total energy? Is the orbit bound? Is it closed? Are all bound orbits closed? ■

Problem 5.5 Effect of drag resistance on a satellite orbit

Consider a satellite in orbit about the Earth. In this case it is convenient to measure distances in terms of the radius of the Earth, $R = 6.37 \times 10^6$ m, and the time in terms of hours. Because the force on the satellite is proportional to Gm, where $m = 5.99 \times 10^{24}$ kg is the mass of the Earth, we need to evaluate the product Gm in Earth units (EU). In these units the value of Gm is given by

$$\begin{aligned} Gm &= 6.67 \times 10^{-11}\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\left(\frac{1\,\text{EU}}{6.37 \times 10^6\,\text{m}}\right)^3 (3.6 \times 10^3\,\text{s/h})^2 (5.99 \times 10^{24}\,\text{kg}) \\ &= 20.0\,\text{EU}^3/\text{h}^2 \quad \text{(Earth units).} \end{aligned} \tag{5.22}$$

Modify the `Planet` class to incorporate the effects of drag resistance on the motion of an orbiting Earth satellite. Assume that the drag force is proportional to the square of the speed of the satellite. To be able to observe the effects of air resistance in a reasonable time, take the magnitude of the drag force to be approximately one-tenth of the magnitude of the gravitational force. Choose initial conditions such that a circular orbit would be obtained in the absence of drag resistance and allow at least one revolution before "switching on" the drag resistance. Describe the qualitative change of the orbit due to drag resistance. How does the total energy and the speed of the satellite change with time? ■

5.7 ■ IMPULSIVE FORCES

What happens to the orbit of an Earth satellite when it is hit by space debris? We now discuss the modifications we need to make in `Planet` and `PlanetApp` so that we can apply an impulsive force (a kick) by a mouse click. If we apply a vertical kick when the position of the satellite is as shown in Figure 5.4a, the impulse would be tangential to the orbit. A radial kick can be applied when the satellite is as shown in Figure 5.4b.

User actions, such as mouse clicks or keyboard entries, are passed from the operating system to Java *event listeners*. Although this standard Java framework is straightforward, we have simplified it to respond to mouse actions within the Open Source Physics panels and frames.[1] In order for an Open Source Physics program to respond to mouse actions, the program implements the `InteractiveMouseHandler` interface and then registers its ability to process mouse actions with the `PlotFrame`. This procedure is demonstrated in the following test program. You can copy the `handleMouseAction` code into your program and replace

[1] See the *Open Source Physics: A User's Guide with Examples* for an extensive discussion of interactive drawing panels.