probabilities for $\lambda = 0.01, 0.05, 0.1, 1$, and 10. Begin with 1000 incident neutrons and increase this number until satisfactory statistics are obtained. Give a qualitative explanation of your results.

(b) Choose $t = 1$, $p_c = p_s$, and $\lambda = 0.05$ and compare your results with the analogous case considered in part (a).

(c) Repeat part (b) with $t = 2$ and $\lambda = 0.1$. Do the various probabilities depend on λ and t separately or only on their ratio? Answer this question before doing the simulation.

(d) Draw some typical paths of the neutrons. From the nature of these paths, explain the results in parts (a)–(c). For example, how does the number of scattering events change as the absorption probability changes? ■

Problem 11.20 Inelastic neutron scattering

(a) In Problem 11.19 we assumed elastic scattering; that is, no energy is lost during scattering. Here we assume that some of the neutron energy E is lost, and that the mean free path is proportional to the speed and hence to $\sqrt{E}$. Modify your program so that a neutron loses a fraction f of its energy at each scattering event and assume that $\lambda = \sqrt{E}$. Consider $f = 0.05$, 0.1, and 0.5 and compare your results with those found in Problem 11.19a using the values for λ in Problem 11.19a to determine the initial values for E.

(b) Make a histogram for the path lengths between scattering events and plot the path length distribution function for $f = 0.1$, 0.5, and 0 (elastic scattering). ■

This procedure for simulating neutron scattering and absorption is more computer intensive than necessary. Instead of considering a single neutron at a time, we can consider a collection of M neutrons. Then, instead of determining whether one neutron is captured or scattered, we determine the number that is captured and the number that is scattered. For example, at the first scattering site, $p_c M$ of the neutrons are captured and $p_s M$ are scattered. We also assume that all the scattered neutrons move in the same direction with the same path length, both of which are generated at random as before. At the next scattering site, there are $p_s^2 M$ scattered neutrons and $p_s p_c M$ captured neutrons. At the end of m steps, the number of neutrons remaining is $w = p_s^m M$, and the number of captured neutrons is $(p_c + p_c p_s + p_c p_s^2 + \cdots + p_c p_s^{m-1})M$. If the new position at the mth step is at $z < 0$, we add w to the sum for the reflected neutrons; if $z > t$, we add w to the neutrons transmitted. When the neutrons are reflected or transmitted, we start over again at $z = 0$ with another collection of neutrons.

Problem 11.21 More efficient neutron scattering method

Apply the improved Monte Carlo method to neutron transmission through a plate. Repeat the simulations suggested in Problem 11.19 and compare your new and previous results. Also, compare the computational times for the two approaches to obtain comparable statistics. ■

The power of the Monte Carlo method becomes apparent for more complicated geometries or when the material is spatially nonuniform so that the cross sections vary from point to point. A problem of current interest is the absorption of various forms of radiation in the human body.

Problem 11.22 Transmission through layered materials

Consider two plates with the same thickness $t = 1$ that are stacked on top of one another with no space between them. For one plate $p_c = p_s$, and for the other $p_c = 2p_s$; that is, the top plate is a better absorber. Assume that $\lambda = 1$ in both plates. Find the transmission, reflection, and absorption probabilities for elastic scattering. Does it matter which plate receives the incident neutrons? ■

APPENDIX 11A: ERROR ESTIMATES FOR NUMERICAL INTEGRATION

We derive the dependence of the truncation error on the number of intervals for the numerical integration methods considered in Sections 11.1 and 11.3. These estimates are based on the assumed adequacy of the Taylor series expansion of the integrand $f(x)$:

$$f(x) = f(x_i) + f'(x_i)(x - x_i) + \frac{1}{2}f''(x_i)(x - x_i)^2 + \cdots, \tag{11.64}$$

and the integration of (11.1) in the interval $x_i \le x \le x_{i+1}$:

$$\int_{x_i}^{x_{i+1}} f(x)\,dx = f(x_i)\Delta x + \frac{1}{2}f'(x_i)(\Delta x)^2 + \frac{1}{6}f''(x_i)(\Delta x)^3 + \cdots. \tag{11.65}$$

We first estimate the error associated with the rectangular approximation with $f(x)$ evaluated at the left side of each interval. The error Δ_i in the interval $[x_i, x_{i+1}]$ is the difference between (11.65) and the estimate $f(x_i)\Delta x$:

$$\Delta_i = \left[\int_{x_i}^{x_{i+1}} f(x)\,dx\right] - f(x_i)\Delta x \approx \frac{1}{2}f'(x_i)(\Delta x)^2. \tag{11.66}$$

We see that to leading order in Δx, the error in each interval is order $(\Delta x)^2$. Because there are a total of n intervals and $\Delta x = (b - a)/n$, the total error associated with the rectangular approximation is $n\Delta_i \sim n(\Delta x)^2 \sim n^{-1}$.

The estimated error associated with the trapezoidal approximation can be found in the same way. The error in the interval $[x_i, x_{i+1}]$ is the difference between the exact integral and the estimate $\frac{1}{2}[f(x_i) + f(x_{i+1})]\Delta x$:

$$\Delta_i = \left[\int_{x_i}^{x_{i+1}} f(x)\,dx\right] - \frac{1}{2}[f(x_i) + f(x_{i+1})]\Delta x. \tag{11.67}$$

If we use (11.65) to estimate the integral and (11.64) to estimate $f(x_{i+1})$ in (11.67), we find that the term proportional to f' cancels, and that the error associated with one interval is order $(\Delta x)^3$. Hence, the total error in the interval $[a, b]$ associated with the trapezoidal approximation is order n^{-2}.

Because Simpson's rule is based on fitting $f(x)$ in the interval $[x_{i-1}, x_{i+1}]$ to a parabola, error terms proportional to f'' cancel. We might expect that error terms of order $f'''(x_i)(\Delta x)^4$ contribute, but these terms cancel by virtue of their symmetry. Hence, the $(\Delta x)^4$ term of the Taylor expansion of $f(x)$ is adequately represented by Simpson's rule. If we retain the $(\Delta x)^4$ term in the Taylor series of $f(x)$, we find that the error in the