Exercise 5.7 Custom buttons
Use a custom button in Problem 5.6 rather than a mouse click to apply an impulsive force to the planet. ■

5.8 ■ VELOCITY SPACE

In Problem 5.6 your intuition might have been incorrect. For example, you might have thought that the orbit would elongate in the direction of the kick. In fact the orbit does elongate but in a direction perpendicular to the kick. Do not worry; you are in good company! Few students have a good qualitative understanding of Newton's law of motion, even after taking an introductory course in physics. A qualitative way of stating Newton's second law is

Forces act on the trajectories of particles by changing velocity, not position.

If we fail to take into account this property of Newton's second law, we will encounter physical situations that appear counterintuitive.

Because force acts to change velocity, it is reasonable to consider both velocity and position on an equal basis. In fact position and momentum are treated in such a manner in advanced formulations of classical mechanics and in quantum mechanics.

In Problem 5.8 we explore some of the properties of orbits in velocity space in the context of the bound motion of a particle in an inverse-square force. Modify your program so that the path in velocity space of the Earth is plotted. That is, plot the point (v_x, v_y) the same way you plotted the point (x, y). The path in velocity space is a series of successive values of the object's velocity vector. If the position space orbit is an ellipse, what is the shape of the orbit in velocity space?

Problem 5.8 Properties of velocity space orbits

(a) Modify your program to display the orbit in position space and in velocity space at the same time. Verify that the velocity space orbit is a circle, even if the orbit in position space is an ellipse. Does the center of this circle coincide with the origin $(v_x, v_y) = (0, 0)$ in velocity space? Choose the same initial conditions that you considered in Problems 5.2 and 5.3.

*(b) Let **u** denote the radius vector of a point on the velocity circle and **w** denote the vector from the origin in velocity space to the center of the velocity circle (see Figure 5.5). Then the velocity of the particle can be written as

$$\mathbf{v} = \mathbf{u} + \mathbf{w}. \tag{5.23}$$

Compute **u** and verify that its magnitude is given by

$$u = GMm/L, \tag{5.24}$$

where L is the magnitude of the angular momentum. Note that L is proportional to m so that it is not necessary to know the magnitude of m.

*(c) Verify that at each moment in time, the planet's position vector **r** is perpendicular to **u**. Explain why this relation holds. ■

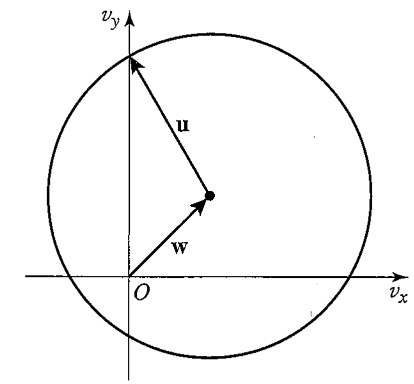

Figure 5.5 The orbit of a particle in velocity space. The vector **w** points from the origin in velocity space to the center of the circular orbit. The vector **u** points from the center of the orbit to the point (v_x, v_y).

Problem 5.9 Effect of impulses in velocity space
How does the velocity space orbit change when an impulsive kick is applied in the tangential or in the radial direction? How do the magnitude and direction of **w** change? From the observed change in the velocity orbit and the above considerations, explain the observed change of the orbit in position space. ■

5.9 ■ A MINI-SOLAR SYSTEM

So far our study of planetary orbits has been restricted to two-body central forces. However, the solar system is not a two-body system because the planets exert gravitational forces on one another. Although the interplanetary forces are small in magnitude in comparison to the gravitational force of the Sun, they can produce measurable effects. For example, the existence of Neptune was conjectured on the basis of a discrepancy between the experimentally measured orbit of Uranus and the predicted orbit calculated from the known forces.

The presence of other planets implies that the total force on a given planet is not a central force. Furthermore, because the orbits of the planets are not exactly in the same plane, an analysis of the solar system must be extended to three dimensions if accurate calculations are required. However, for simplicity, we will consider a model of a two-dimensional solar system with two planets in orbit about a fixed sun.

The equations of motion of two planets of mass m_1 and mass m_2 can be written in vector form as (see Figure 5.6)

$$m_1\frac{d^2\mathbf{r}_1}{dt^2} = -\frac{GMm_1}{r_1^{\,3}}\mathbf{r}_1 + \frac{Gm_1m_2}{r_{21}^{\,3}}\mathbf{r}_{21} \tag{5.25a}$$

$$m_2\frac{d^2\mathbf{r}_2}{dt^2} = -\frac{GMm_2}{r_2^{\,3}}\mathbf{r}_2 - \frac{Gm_1m_2}{r_{21}^{\,3}}\mathbf{r}_{21}, \tag{5.25b}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are directed from the sun to planets 1 and 2, respectively, and $\mathbf{r}_{21} = \mathbf{r}_2 - \mathbf{r}_1$ is the vector from planet 1 to planet 2. It is convenient to divide (5.25a) by m_1 and (5.25b)