points each and 10 sets of 100 points each. If the sets of 50 points each are statistically independent (that is, if τ is significantly smaller than 50), then your estimate of the error for the two groupings should be approximately the same. The importance of correlations between sampled points is discussed further in Section 15.7. ■

*11.8 ■ NEUTRON TRANSPORT

We consider the application of a nonuniform probability distribution to the simulation of the transmission of neutrons through bulk matter, one of the original applications of a Monte Carlo method. Suppose that a neutron is incident on a plate of thickness t. We assume that the plate is infinite in the x and y directions and the z-axis is normal to the plate. At any point within the plate, the neutron can either be captured with probability p_c or scattered with probability p_s. These probabilities are proportional to the capture cross section and scattering cross section, respectively. If the neutron is scattered, we need to find its new direction as specified by the polar angle θ (see Figure 11.5). Because we are not interested in how far the neutron moves in the x or y direction, the value of the azimuthal angle ϕ is irrelevant.

If the neutrons are scattered equally in all directions, then the probability $p(\theta, \phi)\, d\theta\, d\phi$ equals $d\Omega/4\pi$, where $d\Omega$ is an infinitesimal solid angle and 4π is the total solid angle. Because $d\Omega = \sin\theta\, d\theta\, d\phi$, we have

$$p(\theta, \phi) = \frac{\sin\theta}{4\pi}. \tag{11.57}$$

We can find the probability density for θ and ϕ separately by integrating over the other angle. For example,

$$p(\theta) = \int_0^{2\pi} p(\theta, \phi)\, d\phi = \frac{1}{2}\sin\theta, \tag{11.58}$$

and

$$p(\phi) = \int_0^{\pi} p(\theta, \phi)\, d\theta = \frac{1}{2\pi}. \tag{11.59}$$

Because the point probability $p(\theta, \phi)$ is the product of the probabilities $p(\theta)$ and $p(\phi)$, θ and ϕ are independent variables. Although we do not need to generate a random angle ϕ, we note that because $p(\phi)$ is a constant, ϕ can be found from the relation

$$\phi = 2\pi r. \tag{11.60}$$

To find θ according to the distribution (11.58), we substitute (11.58) in (11.30) and obtain

$$r = \frac{1}{2}\int_0^{\theta} \sin x\, dx. \tag{11.61}$$

The integration in (11.61) gives

$$\cos\theta = 1 - 2r. \tag{11.62}$$

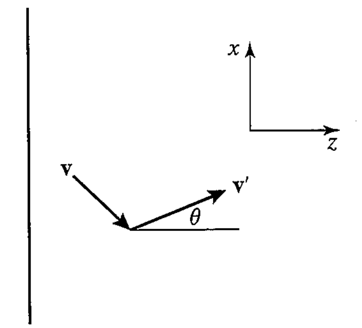

Figure 11.5 The definition of the scattering angle θ. The velocity before scattering is $\mathbf{v}$ and the velocity after scattering is $\mathbf{v}'$. The scattering angle θ is independent of $\mathbf{v}$ and is defined relative to the z-axis.

Note that (11.60) implies that ϕ is uniformly distributed between 0 and 2π, and (11.62) implies that $\cos\theta$ is uniformly distributed between -1 and $+1$. We could invert the cosine in (11.62) to solve for θ. However, to find the z-component of the path of the neutron through the plate, we need to multiply the path length ℓ by $\cos\theta$, and hence we need $\cos\theta$ rather than θ.

The path length, which is the distance traveled between subsequent scattering events, is obtained from the exponential probability density $p(\ell) \propto e^{-\ell/\lambda}$ (see (11.34)). From (11.36) we have

$$\ell = -\lambda \ln r, \tag{11.63}$$

where λ is the mean free path.

Now we have all the necessary ingredients for calculating the probabilities for a neutron to pass through the plate, to be reflected off the plate, or to be captured and absorbed in the plate. The input parameters are the thickness of the plate t, the capture probability p_c, and the mean free path λ. The scattering probability is $p_s = 1 - p_c$. We begin with $z = 0$ and implement the following steps:

1. Determine if the neutron is captured or scattered. If it is captured, then add one to the number of captured neutrons and go to step 5.
2. If the neutron is scattered, compute $\cos\theta$ from (11.62) and ℓ from (11.63). Change the z-coordinate of the neutron by $\ell\cos\theta$.
3. If $z < 0$, add one to the number of reflected neutrons. If $z > t$, add one to the number of transmitted neutrons. In either case skip to step 5 below.
4. Repeat steps 1–3 until the fate of the neutron has been determined.
5. Repeat steps 1–4 with additional incident neutrons until sufficient data has been obtained.

Problem 11.19 Elastic neutron scattering

(a) Write a program to implement the above algorithm for neutron scattering through a plate. Assume $t = 1$ and $p_c = p_s/2$. Find the transmission, reflection, and absorption