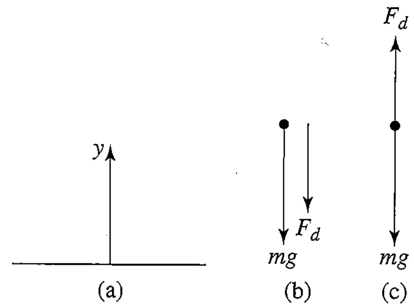

Figure 3.1 (a) Coordinate system with y measured positive upward from the ground. (b) The force diagram for downward motion. (c) The force diagram for upward motion.

with a constant acceleration $g = 9.8\,\text{m/s}^2$. Take $R = 6.37 \times 10^6$ m. Make sure that the one percent difference is due to the physics of the force law and not the accuracy of your algorithm. ■

For particles near the Earth's surface, a more important modification is to include the drag force due to air resistance. The direction of the drag force $F_d(v)$ is opposite to the velocity of the particle (see Figure 3.1). For a falling body, $F_d(v)$ is upward as shown in Figure 3.1(b). Hence, the total force F on the falling body can be expressed as

$$F = -mg + F_d. \tag{3.9}$$

The velocity dependence of $F_d(v)$ is known theoretically in the limit of very low speeds for small objects. In general, it is necessary to determine the velocity dependence of $F_d(v)$ empirically over a limited range of velocities. One way to obtain the form of $F_d(v)$ is to measure y as a function of t and then compute $v(t)$ by calculating the numerical derivative of $y(t)$. Similarly, we can use $v(t)$ to compute $a(t)$ numerically. From this information, it is possible in principle to find the acceleration as a function of v and to extract $F_d(v)$ from (3.9). However, this procedure introduces errors (see Problem 3.8b) because the accuracy of the derivatives will be less than the accuracy of the measured position. An alternative is to reverse the procedure, that is, assume an explicit form for the v dependence of $F_d(v)$ and use it to solve for $y(t)$. If the calculated values of $y(t)$ are consistent with the experimental values of $y(t)$, then the assumed v dependence of $F_d(v)$ is justified empirically.

The two common assumed forms of the velocity dependence of $F_d(v)$ are

$$F_{1,d}(v) = C_1 v, \tag{3.10a}$$

and

$$F_{2,d}(v) = C_2 v^2, \tag{3.10b}$$

where the parameters C_1 and C_2 depend on the properties of the medium and the shape of the object. In general, (3.10a) and (3.10b) are useful *phenomenological* expressions that yield approximate results for $F_d(v)$ over a limited range of v.

Because $F_d(v)$ increases as v increases, there is a limiting or *terminal velocity* (speed) at which the net force on a falling object is zero. This terminal speed can be found from

(3.9) and (3.10) by setting $F_d = mg$ and is given by

$$v_{1,t} = \frac{mg}{C_1} \quad \text{(linear drag)} \tag{3.11a}$$

$$v_{2,t} = \left(\frac{mg}{C_2}\right)^{1/2} \quad \text{(quadratic drag)}, \tag{3.11b}$$

for the linear and quadratic cases, respectively. It is often convenient to express velocities in terms of the terminal velocity. We can use (3.10) and (3.11) to write F_d in the linear and quadratic cases as

$$F_{1,d} = C_1 v_{1,t}\left(\frac{v}{v_{1,t}}\right) = mg\frac{v}{v_{1,t}} \tag{3.12a}$$

$$F_{2,d} = C_2 v_{2,t}{}^2\left(\frac{v}{v_{2,t}}\right)^2 = mg\left(\frac{v}{v_{2,t}}\right)^2. \tag{3.12b}$$

Hence, we can write the net force (per unit mass) on a falling object in the convenient forms

$$F_1(v)/m = -g\left(1 - \frac{v}{v_{1,t}}\right) \tag{3.13a}$$

$$F_2(v)/m = -g\left(1 - \frac{v^2}{v_{2,t}{}^2}\right). \tag{3.13b}$$

To determine if the effects of air resistance are important during the fall of ordinary objects, consider the fall of a pebble of mass $m = 10^{-2}$ kg. To a good approximation, the drag force is proportional to v^2. For a spherical pebble of radius 0.01 m, C_2 is found empirically to be approximately 10^{-2} kg/m. From (3.11b) we find the terminal velocity to be about 30 m/s. Because this speed would be achieved by a freely falling body in a vertical fall of approximately 50 m in a time of about 3 s, we expect that the effects of air resistance would be appreciable for comparable times and distances.

Data is often stored in text files, and it is convenient to be able to read this data into a program for analysis. The `ResourceLoader` class in the Open Source Physics tools package makes reading these files easy. This class can read many different data types, including images and sound. An example of how to use the `ResourceLoader` class to read string data is given in `DataLoaderApp`.

Listing 3.10 Example of the use of the `ResourceLoader` class to read data into a program.

```
package org.opensourcephysics.sip.ch03;
import org.opensourcephysics.tools.*;

public class DataLoaderApp {
   public static void main(String[] args) {
      // reads from directory where DataLoaderApp is located
      String fileName = "falling.txt";
      // gets the data file
      Resource res = ResourceLoader.getResource(fileName,
                        DataLoaderApp.class);
```