it is often the case that it would be difficult or impossible to obtain the answers by familiar methods. Still another reason is that writing a program and doing a simulation can aid your intuitive understanding of the system, especially if the questions involve the subtle concept of probability. Probability is an elusive concept in part because it cannot be measured at one time. To reinforce the importance of thinking about how to solve a problem on a computer, we suggest some problems in probability in the following. Does thinking about how to write a program to simulate these problems help you to find a pencil and paper solution?

Problem 7.21 Three boxes: stick or switch?
Suppose that there are three identical boxes, each with a lid. When you leave the room, a friend places a \$10 bill in one of the boxes and closes the lid of each box. The friend knows the location of the \$10 bill, but you do not. You then reenter the room and guess which one of the boxes has the \$10 bill. As soon as you do, your friend opens the lid of a box that is empty. If you have chosen an empty box, your friend will open the lid of the other empty box. If you have chosen the right box, your friend will open the lid of one of the two empty boxes. You now have the opportunity to stay with your original choice or switch to the other unopened box. Suppose that you play this contest many times, and that each time you guess correctly, you keep the money. To maximize your winnings, should you maintain your initial choice or should you switch? Which strategy is better? This contest is known as the Monty Hall problem. Write a program to simulate this game and output the probability of winning for switching and for not switching. It is likely that before you finish your program, the correct strategy will become clear. To make your program more useful, consider an arbitrary number of boxes. ■

Problem 7.22 Conditional probability
Suppose that many people in a community are tested at random for HIV. The accuracy of the test is 87%, and the incidence of the disease in the general population, independent of any test, is 1%. If a person tests positive for HIV, what is the probability that this person really has HIV? Write a program to compute the probability. The answer can be found by using Bayes' theorem (cf. Bernardo and Smith). The answer is much less than 87%. ■

Problem 7.23 The roll of the dice
Suppose that two gamblers each begin with \$100 in capital, and on each throw of a coin, one gambler must win \$1 and the other must lose \$1. How long can they play on the average until the capital of the loser is exhausted? How long can they play if they each begin with \$1000? Neither gambler is allowed to go into debt. The eventual outcome is known as the gambler's ruin. ■

Problem 7.24 The boys of summer
Luck plays a large role in the outcome of any baseball season. The National League Central Division standings for 2004 are given in Table 7.1. Suppose that the teams remain unchanged, and their probability of winning a particular game is given by their 2004 winning percentage. Do a simulation to determine the probability that the Cardinals would lead the division for another season. For simplicity, assume that the teams play only each other. ■

Much of the present day motivation for the development of probability comes from science, rather than from gambling. The next problem has much to do with statistical physics, even though this application is not apparent.

Table 7.1 The National League Central standings for 2004.

Team	Won	Lost	Percentage
St. Louis Cardinals	105	57	0.648
Houston Astros	92	70	0.568
Chicago Cubs	89	73	0.469
Pittsburgh Pirates	72	89	0.447
Cincinnati Reds	76	86	0.426
Milwaukee Brewers	67	94	0.416

Problem 7.25 Money exchange
Consider a line that has been subdivided into bins. There can be an indefinite number of coins in each bin. For simplicity, we initially assign one coin to each bin. The money exchange proceeds as follows. Select two bins at random. If there is at least one coin in the first bin, move one coin to the second bin. If the first bin is empty, then do nothing. After many coin exchanges, how is the occupancy of the bins distributed? Are the coins uniformly distributed as in the initial state or are many bins empty? Write a Monte Carlo program to simulate this money exchange and show the state of the bins visually. Consider a system with at least 256 bins. Plot the histogram $H(n)$ versus n, where $H(n)$ is the number of bins with n coins. Do your results change qualitatively if you consider bigger systems or begin with more coins in each bin? ■

Problem 7.26 Distribution of cooking times
An industrious physics major finds a job at a local fast food restaurant to help her pay her way through college. Her task is to cook 20 hamburgers on a grill at any one time. When a hamburger is cooked, she is supposed to replace it with an uncooked hamburger. However, our physics major does not pay attention to whether the hamburger is cooked or not. Her method is to choose a hamburger at random and replace it by an uncooked one. She does not check if the hamburger that she removes from the grill is ready. What is the distribution of cooking times of the hamburgers that she removes? For simplicity, assume that she replaces a hamburger at regular intervals of thirty seconds and that there is an indefinite supply of uncooked hamburgers. Does the qualitative nature of the distribution change if she cooks 40 hamburgers at any one time? ■

7.6 ■ METHOD OF LEAST SQUARES

In Problem 7.20 we did a simulation of $N(t)$, the number of unstable nuclei at time t. Given the finite accuracy of our data, how do we know if our simulation results are consistent with the exponential relation between N and t? The approach that we have been using is to plot the computed values of $\log N(t)$ as a function of t and to rely on our eye to help us draw the curve that best fits the data points. Such a visual approach works best when the curve is a straight line, that is, when the relation is linear. The advantages of this approach are that it is straightforward and it allows us to see what we are doing. For example, if a data point is far from the straight line or if there is a gap in the data, we will notice it easily. If the analytical relation is not linear, it is likely that we will notice that the data points do not fit a simple straight line but instead show curvature. If we blindly let a computer fit the data to