Problem 4.11 Trajectory of a pendulum in phase space

(a) Modify your program from Problem 4.10 so that the phase space trajectories (ω versus θ) of $N = 16$ pendula with different initial conditions can be compared. Plot several phase space trajectories for different values of the total energy. Are the phase space trajectories closed? Does the shape of the trajectory depend on the total energy?

(b) Choose a set of initial conditions that form a rectangle in phase space and plot the state of each pendulum as a circle. Does the shape of this area change with time? What happens to the total area? ■

4.5 ■ ELECTRICAL CIRCUIT OSCILLATIONS

In this section we discuss several electrical analogues of the mechanical systems that we have considered. Although the equations of motion are similar in form, it is convenient to consider electrical circuits separately, because the nature of the questions of interest is somewhat different.

The starting point for electrical circuit theory is Kirchhoff's loop rule, which states that the sum of the voltage drops around a closed path of an electrical circuit is zero. This law is a consequence of conservation of energy, because a voltage drop represents the amount of energy that is lost or gained when a unit charge passes through a circuit element. The relations for the voltage drops across each circuit element are summarized in Table 4.1.

Imagine an electrical circuit with an alternating voltage source $V_s(t)$ attached in series to a resistor, inductor, and capacitor (see Figure 4.5). The corresponding loop equation is

$$V_L + V_R + V_C = V_s(t). \tag{4.21}$$

The voltage source term V_s in (4.21) is the *emf* and is measured in units of volts. If we substitute the relationships shown in Table 4.1, we find

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_s(t), \tag{4.22}$$

where we have used the definition of current $I = dQ/dt$. We see that (4.22) for the series RLC circuit is identical in form to the damped harmonic oscillator (4.17). The analogies between ideal electrical circuits and mechanical systems are summarized in Table 4.2.

Although we are already familiar with (4.22), we first consider the dynamical behavior of an RC circuit described by

$$RI(t) = R\frac{dQ}{dt} = V_s(t) - \frac{Q}{C}. \tag{4.23}$$

Two RC circuits corresponding to (4.23) are shown in Figure 4.6. Although the loop equation (4.23) is identical regardless of the order of placement of the capacitor and resistor in Figure 4.6, the output voltage measured by the oscilloscope in Figure 4.6 is different. We will see in Problem 4.12 that these circuits act as filters that pass voltage components of certain frequencies while rejecting others.

An advantage of a computer simulation of an electrical circuit is that the measurement of a voltage drop across a circuit element does not affect the properties of the circuit. In fact,

Table 4.1 The voltage drops across the basic electrical circuit elements. Q is the charge (coulombs) on one plate of the capacitor, and I is the current (amperes).

Element	Voltage Drop	Symbol	Units
resistor	$V_R = IR$	resistance R	ohms (Ω)
capacitor	$V_C = Q/C$	capacitance C	farads (F)
inductor	$V_L = L\,dI/dt$	inductance L	henries (H)

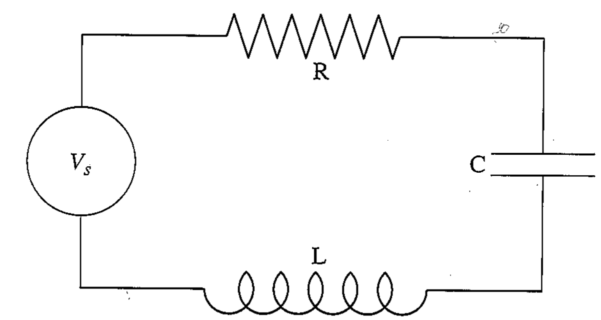

Figure 4.5 A simple series RLC circuit with a voltage source V_s.

Table 4.2 Analogies between electrical parameters and mechanical parameters.

Electric Circuit	Mechanical System
charge Q	displacement x
current $I = dQ/dt$	velocity $v = dx/dt$
voltage drop	force
inductance L	mass m
inverse capacitance $1/C$	spring constant k
resistance R	damping γ

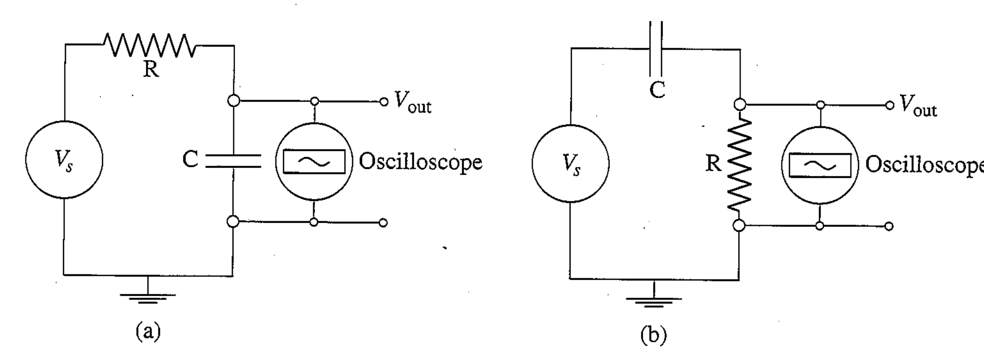

Figure 4.6 Examples of RC circuits used as low and high pass filters. Which circuit is which?