(c) Use your modified program from part (b) to draw the field lines for the two-dimensional analogs of the distributions considered in Problem 10.3. Compare the results for two and three dimensions and discuss any qualitative differences.

(d) Can your program be used to demonstrate Gauss's law using point charges? What about line charges? ■

10.4 ■ ELECTRIC POTENTIAL

It often is easier to analyze the behavior of a system using energy rather than force concepts. We define the electric potential $V(\mathbf{r})$ by the relation

$$V(\mathbf{r}_2) - V(\mathbf{r}_1) = -\int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{E} \cdot d\mathbf{r}, \tag{10.6}$$

or

$$\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r}). \tag{10.7}$$

Only differences in the potential between two points have physical significance. The gradient operator ∇ is given in Cartesian coordinates by

$$\nabla = \frac{\partial}{\partial x}\hat{\mathbf{x}} + \frac{\partial}{\partial y}\hat{\mathbf{y}} + \frac{\partial}{\partial z}\hat{\mathbf{z}}, \tag{10.8}$$

where the vectors $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are unit vectors along the x-, y-, and z-axes, respectively. If V depends only on the magnitude of $\mathbf{r}$, then (10.7) becomes $E(r) = -dV(r)/dr$. Recall that $V(r)$ for a point charge q relative to a zero potential at infinity is given by

$$V(r) = \frac{q}{r} \quad \text{(Gaussian units)}. \tag{10.9}$$

The surface on which the electric potential has an equal value everywhere is called an *equipotential surface* (a curve in two dimensions). Because $\mathbf{E}$ is in the direction in which the electric potential decreases most rapidly, the electric field lines are orthogonal to the equipotential surfaces at any point.

The Open Source Physics frames package contains the `Scalar2DFrame` class to provide graphical representations of scalar fields (see Appendix 9B). Problem 10.5 uses a scalar field plot to show the electric potential. The following code fragment shows how to calculate the electric potential at a grid point.

```
List chargeList = frame.getDrawables(Charge.class);
Iterator it = chargeList.iterator();
while (it.hasNext()) {
   Charge charge = (Charge) it.next();
   double xs = charge.getX(), ys = charge.getY();
   for (int ix = 0; ix < n; ix++) {
      double x = frame.indexToX(ix);
      double dx = (xs - x); // charge gridpoint separation
      for (int iy = 0; iy < n; iy++) {
         double y = frame.indexToY(iy);
         double dy = (ys -y); //charge gridpoint separation
         double r2 = dx * dx + dy * dy;
         double r = Math.sqrt(r2);
         if (r > 0) {
            eField[ix][iy] += charge.q/r;
         }
      }
   }
}
frame.setAll(eField);
```


Problem 10.5 Equipotential contours

(a) Write a program based on `ElectricFieldApp` that draws equipotential lines using the charge distributions considered in Problem 10.3.

(b) Explain why equipotential surfaces (lines in two dimensions) never cross. ■

We can use the orthogonality between the electric field lines and the equipotential lines to modify `FieldLineApp` so that it draws the latter. Because the components of the line segment $\Delta\mathbf{s}$ parallel to the electric field line are given by $\Delta x = \Delta s(E_x/E)$ and $\Delta y = \Delta s(E_y/E)$, the components of the line segment perpendicular to $\mathbf{E}$, and hence parallel to the equipotential line, are given by $\Delta x = -\Delta s(E_y/E)$ and $\Delta y = \Delta s(E_x/E)$. It is unimportant whether the minus sign is assigned to the x- or y-component, because the only difference would be the direction that the equipotential lines are drawn.

Problem 10.6 Equipotential lines

(a) Write a program that is based on `FieldLineApp` and `FieldLine` to draw some of the equipotential lines for the charge distributions considered in Problem 10.3. Use a mouse click to determine the initial position of an equipotential line. The equipotential calculation should stop when the line returns close to the starting point or after an unreasonable number of calculations. The program should also kill the thread when the user moves a charge, hits the Reset button, or when the application terminates.

(b) What would a higher density of equipotential lines mean if we drew lines such that each adjacent line differed from a neighboring one by a fixed potential difference?

(c) Explain why equipotential surfaces never cross. ■

Problem 10.7 The electric potential due to a finite sheet of charge

Consider a uniformly charged nonconducting plate of total charge Q and linear dimension L centered at $(0, 0, 0)$ in the xy-plane. In the limit $L \to \infty$ with the charge density $\sigma = Q/L^2$ a constant, we know that the electric field is normal to the sheet, and its magnitude is given by $2\pi\sigma$ (Gaussian units). What is the electric field due to a finite sheet of charge? A simple method is to divide the plate into a grid of p square regions on a side such that each region is sufficiently small to be approximated by a point charge of magnitude $q = Q/p^2$. Because the potential is a scalar, it is easier to compute the total potential rather than the total electric field from the $N = p^2$ point charges. Use the relation (10.9) for the potential from a point