(e) How much must you change the factor `dampingCoef` in the damping method before you can visually see a difference in the simulation? What problems occur when the damping is removed?

*(f) The amplitude of the fields far from the current loop should be characteristic of radiation fields for which the amplitude falls off as $1/r$, where r is the distance from the current loop to the observation point. Do a simulation to detect this dependence if you have sufficient computer resources. ■

Problem 10.24 Microwave cavity resonators

(a) Cavity resonators are a practical way of storing energy in the form of oscillating electric and magnetic fields without losing as much energy as would be dissipated in a resonant LC circuit. Consider a cubical resonator of linear dimension L whose walls are made of a perfectly conducting material. The tangential components of $\mathbf{E}$ and the normal component of $\mathbf{B}$ vanish at the walls. Standing microwaves can be set up in the box of the form (cf. Reitz et al.)

$$E_x = E_{x0} \cos k_x x \sin k_y y \sin k_z z \, e^{i\omega t} \tag{10.59a}$$

$$E_y = E_{y0} \cos k_y y \sin k_x x \sin k_z z \, e^{i\omega t} \tag{10.59b}$$

$$E_z = E_{z0} \cos k_z z \sin k_x x \sin k_y y \, e^{i\omega t}. \tag{10.59c}$$

The wave vector $\mathbf{k} = (k_x, k_y, k_z) = (m_x\pi/L, m_y\pi/L, m_z\pi/L)$, where m_x, m_y, and m_z are integers. A particular mode is labeled by the integers (m_x, m_y, m_z). The initial electric field is perpendicular to $\mathbf{k}$, and $\omega = ck$. Implement the boundary conditions at $(x = 0, y = 0, z = 0)$ and $(x = L, y = L, z = L)$. Set $\Delta t = 0.05$, $\Delta l = 0.1$, and $L = 1$. At $t = 0$, set $\mathbf{B} = 0$, $\mathbf{j} = 0$ (there are no currents within the cavity), and use (10.59) with $(m_x, m_y, m_z) = (0, 1, 1)$ and $E_{x0} = 1$. Plot the field components at specific positions as a function of t and find the resonant frequency ω. Compare your computed value of ω with the analytic result. Do the magnetic fields change with time? Are they perpendicular to $\mathbf{k}$ and $\mathbf{E}$?

(b) Repeat part (a) for two other modes.

(c) Repeat part (a) with a uniform random noise added to the initial field at all positions. Assume the amplitude of the noise is δ and describe the resulting fields for $\delta = 0.1$. Are they similar to those without noise? What happens for $\delta = 0.5$? More quantitative results can be found by computing the power spectrum $|E(\omega)|^2$ for the electric field at a few positions. What is the order of magnitude of δ for which the maximum of $|E(\omega)|^2$ at the standing wave frequency is swamped by the noise?

(d) Change the shape of the container slightly by removing a 0.1×0.1 cubical box from each of the corners of the original resonator. Do the standing wave frequencies change? Determine the standing wave frequency by adding noise to the initial fields and looking at the power spectrum. How do the standing wave patterns change?

(e) Change the shape of the container slightly by adding a 0.1×0.1 cubical box at the center of one of the faces of the original resonator. Do the standing wave frequencies change? How do the standing wave patterns change?

(f) Cut a 0.2×0.2 square hole in a face in the yz-plane and double the computational region in the x direction. Begin with a (0, 1, 1) standing wave and observe how the fields "leak" out of the hole. ■

Problem 10.25 Billiard microwave cavity resonators

(a) Repeat Problem 10.24a for $L_x = L_y = 2$, $L_z = 0.2$, $\Delta l = 0.1$, and $\Delta t = 0.05$. Indicate the magnitude of the electric field in the $L_z = 0.1$ plane by a color code. Choose an initial normal mode field distribution and describe the pattern that you obtain. Then repeat your calculation for a random initial field distribution.

(b) Place an approximately circular conductor in the middle of the cavity of radius $r = 0.4$. Describe the patterns that you see. Such a geometry leads to chaotic trajectories for particles moving within such a cavity (see Project 6.26). Is there any evidence of chaotic behavior in the field pattern?

(c) Repeat part (b) with the circular conductor placed off center. ■

10.9 ■ PROJECTS

Part of the difficulty in understanding electromagnetic phenomena is visualizing its three-dimensional nature. Many interesting problems can be posed based on the simple, but nontrivial question of how three-dimensional electromagnetic fields can be best represented visually in various contexts (cf. Belcher and Olbert). However, we have not suggested projects in this area because of their difficulty.

Many of the techniques used in this chapter, for example, the random walk method and the relaxation method for solving Laplace's equation, have applications in other fields, especially problems in fluid flow and transport. Similarly, the multigrid method, discussed in Project 10.26, has far reaching applications.

Project 10.26 Multigrid method

In general, the relaxation method for solving Laplace's equation is very slow even when using overrelaxation. The reason is that the local updates of the relaxation method cannot quickly take into account effects at very large length scales. The *multigrid method* greatly improves performance by using relaxation at many length scales. The important idea is to use a relaxation method to find the values of the potential on coarser and coarser grids, and then to use the coarse grid values to determine the fine grid values. The fine grid relaxation updates take into account effects at short length scales. If we define the initial grid by a lattice spacing $b = 1$, then the coarser grids are characterized by $b = 2^n$, where n determines the coarseness of the grid and is known as the grid level. We need to decide how to use the fine grid values of the potential to assign values to a coarser grid, and then how to use a coarse grid to assign values to a finer grid. The first step is called *prolongation* and the second step is called *restriction*. There is some flexibility on how to do these two operations. We discuss one approach.

We define the centers of the sites of the coarse grid to be located at the centers of every other site of the fine grid. That is, if the set $\{i, j\}$ represents the positions of the sites of the