

# Introduction to Computational Physics Lectures, FYS4411/9411

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Nov 18, 2018

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# Aims

- ▶ Be able to apply central many-particle methods like the Variational Monte Carlo method to properties of many-fermion systems and many-boson systems.
- ▶ Understand how to simulate quantum mechanical systems with many interacting particles. The methods are relevant for atomic, molecular, solid state, materials science, nanotechnology, quantum chemistry and nuclear physics.
- ▶ Learn to manage and structure larger projects, with unit tests, object orientation and writing clean code
- ▶ Learn about a proper statistical analysis of large data sets
- ▶ Parallelization and code optimizations

# Lectures and ComputerLab

- ▶ Lectures: Thursday (2.15pm-4pm), remotely. First time January 18.
- ▶ Computerlab: Thursday (4.15pm-7pm), first time January 17, last lab session May 9.
- ▶ Weekly plans and all other information are on the webpage
- ▶ Intensive week starts February 4 and ends February 8, week 7.
- ▶ **First project to be handed in March 18.**
- ▶ **Second and final project to be handed in June 1.**
- ▶ There is no final exam, only project work.

# Course Format

- ▶ Two compulsory projects. Electronic reports only. You are free to choose your format. We use devilry to hand in the projects.
- ▶ Evaluation and grading: The two projects count 1/2 each of the final mark. No exam.
- ▶ The computer lab (room FV329) consists of 16 Linux PCs, but many prefer own laptops. C/C++ is the default programming language, but Fortran2008 and Python are also used. All source codes discussed during the lectures can be found at the webpage of the course. We recommend either C/C++, Fortran2008 or Python as programming languages.

# Topics covered in this course

- ▶ Parallelization (MPI and OpenMP), high-performance computing topics. Choose between Python, Fortran2008 and/or C++ as programming languages.
- ▶ Algorithms for Monte Carlo Simulations (multidimensional integrals), Metropolis-Hastings and importance sampling algorithms. Improved Monte Carlo methods.
- ▶ Statistical analysis of data from Monte Carlo calculations, blocking method.
- ▶ Eigenvalue solvers
- ▶ For project 2 there will be three variants:
  1. Variational Monte Carlo for fermions
  2. Coupled cluster theory for fermions (iterative methods)
  3. Neural networks and Machine Learning to solve the same problems as in project 1

## Topics covered in this course

- ▶ Search for minima in multidimensional spaces (conjugate gradient method, steepest descent method, quasi-Newton-Raphson, Broyden-Jacobian).
- ▶ Iterative methods for solutions of non-linear equations.
- ▶ Object orientation
- ▶ Data analysis and resampling techniques
- ▶ Variational Monte Carlo (VMC) for 'ab initio' studies of quantum mechanical many-body systems.
- ▶ Simulation of two-dimensional systems like quantum dots (project 2).
- ▶ Simulation of trapped bosons using VMC (project 1)
- ▶ Machine learning and neural networks (project 2)
- ▶ Coupled cluster theory (project 2)

# Quantum Monte Carlo Motivation

Most quantum mechanical problems of interest in for example atomic, molecular, nuclear and solid state physics consist of a large number of interacting electrons and ions or nucleons.

The total number of particles  $N$  is usually sufficiently large that an exact solution cannot be found.

Typically, the expectation value for a chosen hamiltonian for a system of  $N$  particles is

$$\langle H \rangle = \frac{\int d\mathbf{R}_1 d\mathbf{R}_2 \dots d\mathbf{R}_N \Psi^*(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N) H(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N) \Psi(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N)}{\int d\mathbf{R}_1 d\mathbf{R}_2 \dots d\mathbf{R}_N \Psi^*(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N) \Psi(\mathbf{R}_1, \mathbf{R}_2, \dots, \mathbf{R}_N)}$$

an in general intractable problem.

This integral is actually the starting point in a Variational Monte Carlo calculation. **Gaussian quadrature: Forget it!** Given 10 particles and 10 mesh points for each degree of freedom and an ideal 1 Tflops machine (all operations take the same time), how long will it take to compute the above integral? The lifetime of the universe is of the order of  $10^{17}$  s.

# Quantum Monte Carlo Motivation

As an example from the nuclear many-body problem, we have Schroedinger's equation as a differential equation

$$\hat{H}\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A, \alpha_1, \dots, \alpha_A) = E\Psi(\mathbf{r}_1, \dots, \mathbf{r}_A, \alpha_1, \dots, \alpha_A)$$

where

$$\mathbf{r}_1, \dots, \mathbf{r}_A,$$

are the coordinates and

$$\alpha_1, \dots, \alpha_A,$$

are sets of relevant quantum numbers such as spin and isospin for a system of  $A$  nucleons ( $A = N + Z$ ,  $N$  being the number of neutrons and  $Z$  the number of protons).



# Quantum Monte Carlo Motivation

There are

$$2^A \times \binom{A}{Z}$$

coupled second-order differential equations in  $3A$  dimensions.

For a nucleus like beryllium-10 this number is **215040**. This is a truly challenging many-body problem.

Methods like partial differential equations can at most be used for 2-3 particles.

## Various many-body methods

- ▶ Monte-Carlo methods
- ▶ Renormalization group (RG) methods, in particular density matrix RG
- ▶ Large-scale diagonalization (Iterative methods, Lanczo's method, dimensionalities  $10^{10}$  states)
- ▶ Coupled cluster theory, favoured method in quantum chemistry, molecular and atomic physics. Applications to ab initio calculations in nuclear physics as well for large nuclei.
- ▶ Perturbative many-body methods
- ▶ Green's function methods
- ▶ Density functional theory/Mean-field theory and Hartree-Fock theory

The physics of the system hints at which many-body methods to use.

# Quantum Monte Carlo Motivation

## Pros and Cons of Monte Carlo

- ▶ Is physically intuitive.
- ▶ Allows one to study systems with many degrees of freedom. Diffusion Monte Carlo (DMC) and Green's function Monte Carlo (GFMC) yield in principle the exact solution to Schroedinger's equation.
- ▶ Variational Monte Carlo (VMC) is easy to implement but needs a reliable trial wave function, can be difficult to obtain. This is where we will use Hartree-Fock theory to construct an optimal basis.
- ▶ DMC/GFMC for fermions (spin with half-integer values, electrons, baryons, neutrinos, quarks) has a sign problem. Nature prefers an anti-symmetric wave function. PDF in this case given distribution of random walkers ( $p \geq 0$ ).
- ▶ The solution has a statistical error, which can be large.
- ▶ There is a limit for how large systems one can study, DMC needs a huge number of random walkers in order to achieve

# Quantum Monte Carlo Motivation

## Where and why do we use Monte Carlo Methods in Quantum Physics

- ▶ Quantum systems with many particles at finite temperature: Path Integral Monte Carlo with applications to dense matter and quantum liquids (phase transitions from normal fluid to superfluid). Strong correlations.
- ▶ Bose-Einstein condensation of dilute gases, method transition from non-linear PDE to Diffusion Monte Carlo as density increases.
- ▶ Light atoms, molecules, solids and nuclei.
- ▶ Lattice Quantum-Chromo Dynamics. Impossible to solve without MC calculations.
- ▶ Simulations of systems in solid state physics, from semiconductors to spin systems. Many electrons active and possibly strong correlations.