FYS4411/9411 March 7

Secant method

Derivative is approximated

$$f'(x_k) = \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

Back to Newton's method

$\nabla_\alpha E(E_\alpha)$

$$x_{k+1} = x_k - f(x_k) / f'(x_k)$$

with secant approx

$$x_{k+1} = x_k - f(x_k)\left(\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}\right)$$

Newton's method

$$x_{k+1} = x_k - J^{-1}(x_k)\, f(x_k)$$

Normal not to invert but solve

$$J(x_k)\, s_k = -f(x_k)$$

$$\left(x_{k+1} = x_k - J^{-1} f(x_k) \right)$$

$$x_{k+1} = x_k + s_k$$

<u>Algorithm</u>

Define x_0 (initial guess) and initial Jacobian B_0

For $k = 0$, Maxiter

solve $B_k s_k = -f(x_k)$

$$x_{k+1} = x_k + s_k$$

$$y_k = f(x_{k+1}) - f(x_k)$$

$$B_{k+1} = B_k + \left[\frac{(y_k - B_k s_k) s^T}{s^T s} \right]$$

end for

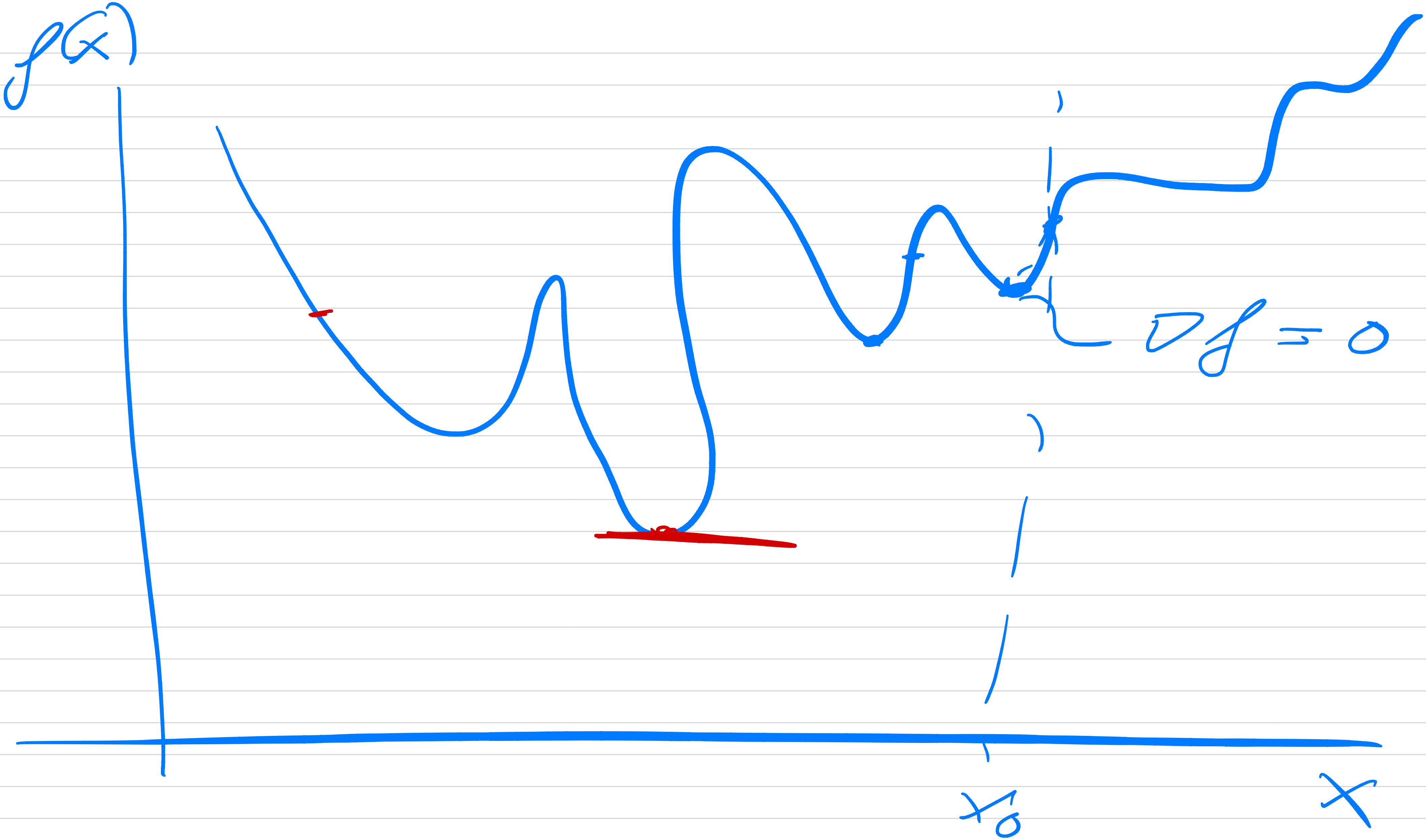
$$B_k s_k = -f(x_k)$$

improving GD with momentum

$$\alpha_{k+1} = \alpha_k - \gamma_k \nabla_\alpha f(\alpha_k)$$

could weight previous iteration with a constant value

$$\alpha_k \longrightarrow \boxed{(1-\rho)\alpha_k + \rho\,\alpha_{k-1}}$$



$f(x)$
$\nabla f = 0$
x_0
x

Automatic differentiation

– Not symbolic

$$f(x) = \sqrt{x^2 + \exp x^2}$$

10 Flops

$$f'(x) = \frac{\left(x + x e^{x^2}\right)}{\sqrt{x^2 + \exp x^2}}$$

4 Flops

1 Flops

5 Flops

1 Flops

2 Flops

$$\boxed{a = x^2} \qquad b = \exp x^2 = \boxed{\exp a}$$

1 Flop

$$c = \boxed{a + b}$$

1 Flop

$$d = \boxed{\sqrt{c}} = f(x)$$

; 5 Flops

$$f'(x) = \frac{x + x \cdot b}{d}$$

2 Flops

1 Flops

= 3 Flops

not finite difference

$$\frac{d^2 u(x)}{dx^2} \approx \frac{u(x+h) + u(x-h) - 2(x)}{h^2}$$

AD:

repeated usage of the chain rule on simple functions like x, x^2, $\sin(x)$, $\cos(x)$, $\exp(x)$ ...