## **Contents**

	Acknowledgments		page x
	Intr	roduction	xi
1	The	1	
	1.1	Complex Numbers	1
	1.2	Dirac Notation, Bras, and Kets	2
	1.3	Tensor Product	5
	1.4	Unitary and Hermitian Matrices	6
	1.5	Hermitian Adjoint of Expressions	6
	1.6	Eigenvalues and Eigenvectors	7
	1.7	Trace of a Matrix	8
2	Qua	9	
	2.1	Tensors	9
	2.2	Qubits	13
	2.3	States	15
	2.4	Helper Functions	24
	2.5	Operators	25
	2.6	Single-Qubit Gates	33
	2.7	Controlled Gates	46
	2.8	Quantum Circuit Notation	51
	2.9	Bloch Sphere	55
	2.10	O Global Phase	59
	2.11	1 Entanglement	60
	2.12	2 No-Cloning Theorem	65
		3 Uncomputation	66
	2.14	4 Reduced Density Matrix and Partial Trace	68
	2.15	5 Measurement	72
3	Simple Algorithms		78
	3.1	Random Number Generator	78
	3.2	Gate Equivalences	79
	3.3	Classical Arithmetic	89
	3.4	Swap Test	93

	3.5	Quantum Teleportation	97
	3.6	Superdense Coding	102
	3.7	Bernstein–Vazirani Algorithm	105
	3.8	Deutsch's Algorithm	108
	3.9	Deutsch-Jozsa Algorithm	118
4	Scala	122	
	4.1	Simulation Complexity	122
	4.2	Quantum Registers	124
	4.3	Circuits	126
	4.4	Fast Gate Application	134
	4.5	Accelerated Gate Application	139
	4.6	Sparse Representation	145
5	Beyo	149	
	5.1	10,000 Years, 2 Days, or 200 Seconds	150
	5.2	Quantum Random Circuit Algorithm	150
	5.3	Circuit Construction	152
	5.4	Estimation	155
	5.5	Evaluation	158
6	Com	160	
	6.1	Phase Kick	161
	6.2	Quantum Fourier Transform	163
		Quantum Arithmetic	172
	6.3		1,2
	6.3 6.4		180
		Phase Estimation	
	6.4 6.5 6.6	Phase Estimation Shor's Algorithm Order Finding	180
	6.4 6.5 6.6	Phase Estimation Shor's Algorithm	180 189
	6.4 6.5 6.6	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm	180 189 196
	6.4 6.5 6.6 6.7	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification	180 189 196 210
	6.4 6.5 6.6 6.7 6.8 6.9	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification	180 189 196 210 227
	6.4 6.5 6.6 6.7 6.8 6.9 6.10	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting	180 189 196 210 227 230
	6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting Quantum Random Walk	180 189 196 210 227 230 234
	6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11 6.12	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting Quantum Random Walk Variational Quantum Eigensolver	180 189 196 210 227 230 234 240
	6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11 6.12 6.13	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting Quantum Random Walk Variational Quantum Eigensolver Quantum Approximate Optimization Algorithm	180 189 196 210 227 230 234 240 253
	6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11 6.12 6.13 6.14	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting Quantum Random Walk Variational Quantum Eigensolver Quantum Approximate Optimization Algorithm Maximum Cut Algorithm	180 189 196 210 227 230 234 240 253 254
7	6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11 6.12 6.13 6.14 6.15	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting Quantum Random Walk Variational Quantum Eigensolver Quantum Approximate Optimization Algorithm Maximum Cut Algorithm Subset Sum Algorithm	180 189 196 210 227 230 234 240 253 254 262
7	6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11 6.12 6.13 6.14 6.15	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting Quantum Random Walk Variational Quantum Eigensolver Quantum Approximate Optimization Algorithm Maximum Cut Algorithm Subset Sum Algorithm Solovay–Kitaev Theorem and Algorithm	180 189 196 210 227 230 234 240 253 254 262
7	6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11 6.12 6.13 6.14 6.15	Phase Estimation Shor's Algorithm Order Finding Grover's Algorithm Amplitude Amplification Quantum Counting Quantum Random Walk Variational Quantum Eigensolver Quantum Approximate Optimization Algorithm Maximum Cut Algorithm Subset Sum Algorithm Solovay–Kitaev Theorem and Algorithm	180 189 196 210 227 230 234 240 253 254 262 266

			Contents	- IX
8	Qua	ntum Languages, Compilers, and Tools		292
	8.1	Challenges for Quantum Compilation		293
	8.2	Quantum Programming Model		294
	8.3	Quantum Programming Languages		294
	8.4	Compiler Optimization		303
	8.5	Transpilation		311
Appendix:	Spai	rse Implementation		322
	Refe	erences		335
	Inde	ex		343