Erasmus+ lecture on Machine Learning, December 13, 2023

$$\frac{dg}{dx} = h(x) = g'(x)$$

$$f(x) = \frac{dg}{dx} - g'(x) = 0$$

$$f(x, g(x), g'(x), g''(x), \dots) = 0$$

guess on $g(x)$

$$g_t(x) = h_1(x) + h_2(x, N(x;\epsilon))$$

$N(x;\epsilon)$ is a neural network

x = input to $N(x; \theta)$

θ = network parameters

$$\theta = \{ (w^{(1)}, b^{(1)}), (w^{(2)}, b^{(2)}) \ldots (w^{(L)}, b^{(L)}) \}$$

$$C(x; \theta) = \frac{1}{n} \sum_{i=1}^{N} \left(f(x, g(x), g'(x), \ldots) \right)^2$$

Example

constant

$$g'(x) = -\gamma g(x)$$

$$g(x) = g_0 \exp(-\gamma x)$$

$$g_0 = g(x=0)$$

Trial function

$$g_t(x) = h_1(x) + h_2(x, N(x; \in))$$

$$\gamma = 2 \qquad g_0 = 10$$

$$g_t(x) = g_0 + x N(x; \in)$$

can compare with analytical solutions and standard ODE solvers

$$x \in [0, 1]$$

$$x \Rightarrow x_i = x_0 + i \Delta x \qquad i = 0, 1, 2, \ldots M$$

$$\Delta x = \frac{x_n - x_0}{n} \qquad x_0 = 0$$

$$g(x) \Rightarrow g(x_i) = g_i'$$

$$g_i' = \left.\frac{dg}{dx}\right|_{x=x_i'} \Leftarrow \frac{g_{i+1}' - g_i'}{\Delta x}$$

$$g_{i\pm i}' = g(x_i \pm \Delta x)$$

$$g_{i+1}' = g_i' + \Delta x \cdot g_i'$$

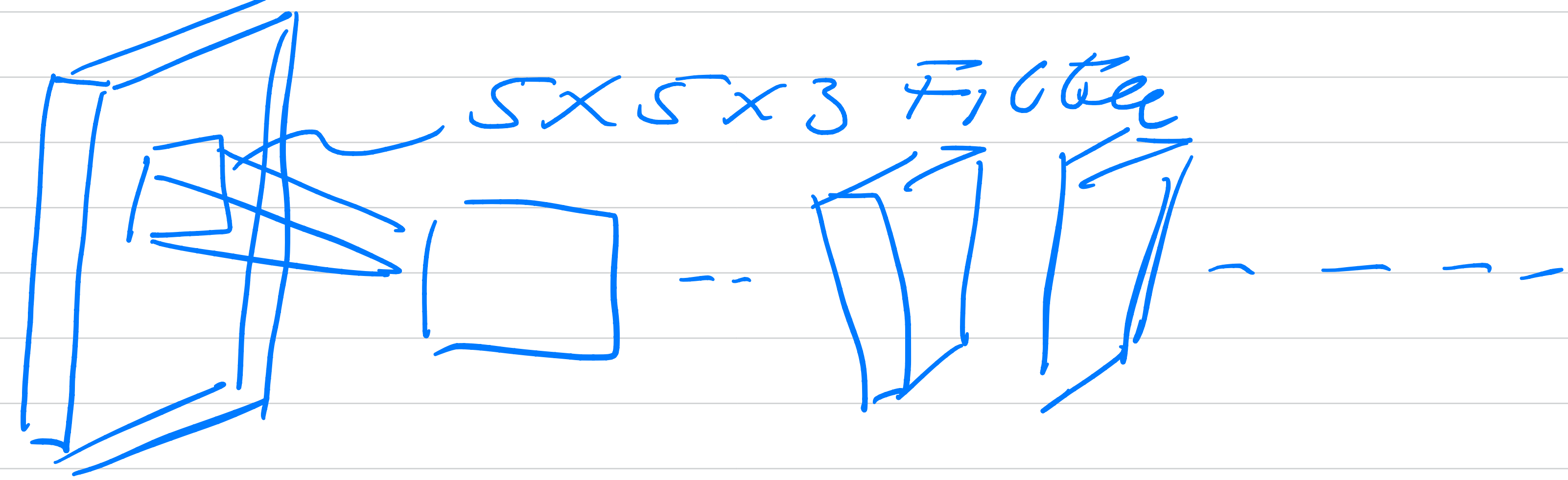
Euler's forward (explicit) method

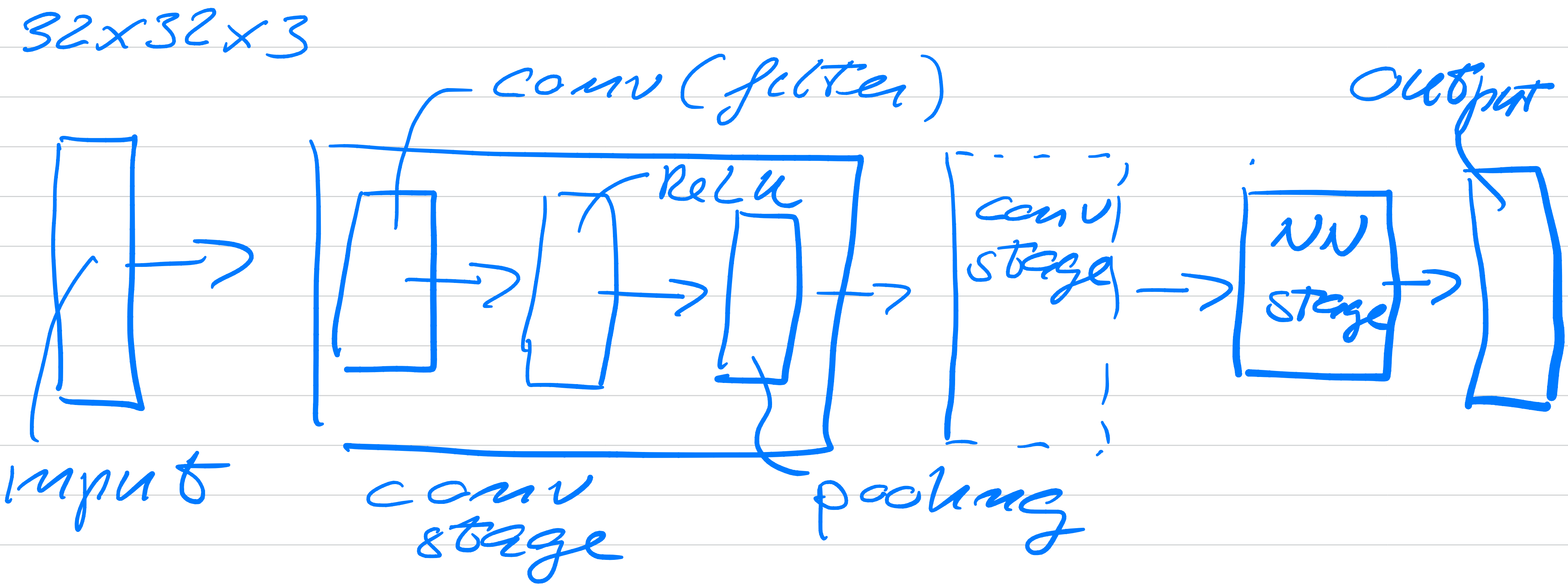
our NN is defined by

$$\hat{\theta} = \underset{\theta}{\arg\min} \; C(x;\theta)$$

$$= \underset{\theta}{\arg\min} \; \frac{1}{n} \sum_{i=1}^{n} f^2$$

CNN


5x5x3 Filter


32x32x3
conv (filter)
ReLU
conv stage
NN stage
output
input
conv stage
pooling