FYS5419, MARCH 27, 2023

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

Basis states $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$H = H_0 + H_1$$

$$H_0 |0\rangle = \varepsilon_0 |0\rangle$$

$$H_0 |1\rangle = \varepsilon_1 |1\rangle$$

$$\varepsilon = \frac{\varepsilon_0 + \varepsilon_1}{2} \qquad \Omega = \frac{\varepsilon_0 - \varepsilon_1}{2}$$

$$H_0 = \varepsilon I + \Omega \sigma_z$$

$$\sigma_z = Z \qquad \sigma_y = Y \qquad \sigma_x = X$$

$$H_0 = \begin{bmatrix} \varepsilon_0 & 0 \\ 0 & \varepsilon_1 \end{bmatrix}$$

$$Z = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$H_I = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

$$c = \frac{V_{11} + V_{22}}{2} \qquad \omega_z = \frac{(V_{11} - V_{22})}{2}$$

$\omega_x = V_{12} = V_{21}$

$H_I = c I + \omega_z Z + \omega_x X$

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

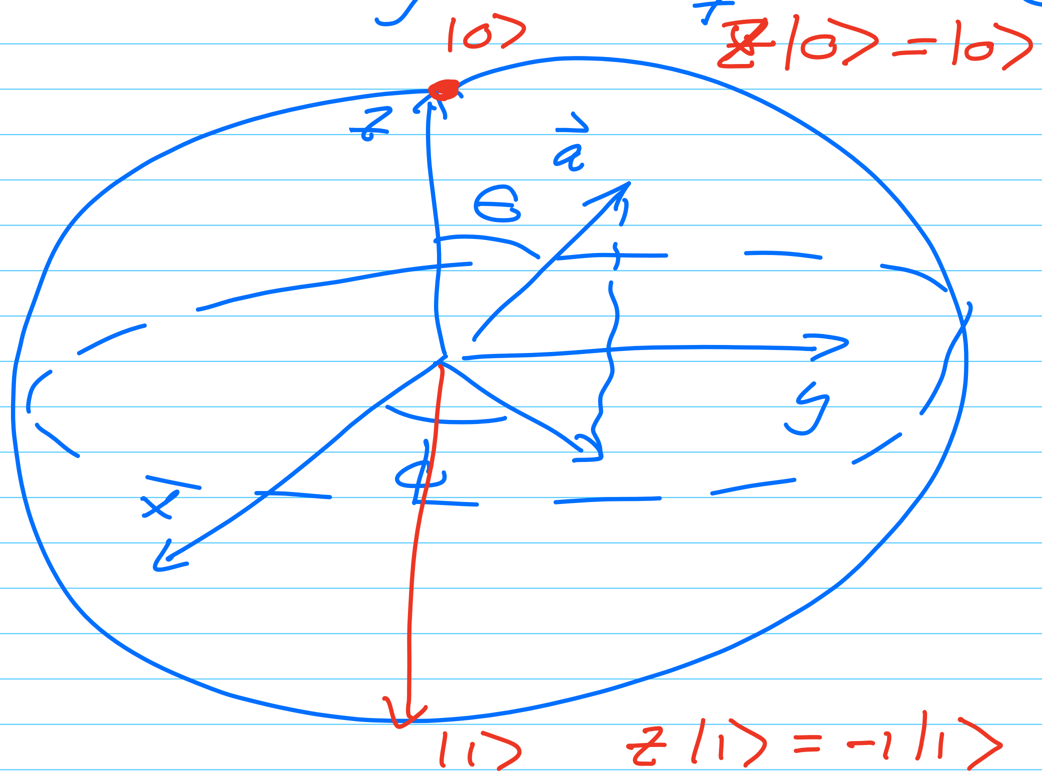
$I^2 = I = XX = YY = Z \cdot Z$

$|0\rangle \rightarrow \boxed{R_x(\theta)} \rightarrow \boxed{R_y(\phi)} \rightarrow |\psi\rangle$

$|\psi\rangle = R_y(\phi) R_x(\theta) |0\rangle$

ansatz.

Bloch sphere can represent an arbitrary one-qubit state



Every qubit system can be visualized as a point on a 3dim sphere with radius 1 (r) and angles ϕ and θ

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

Rotation matrices

$$R_X(\theta) = e^{i\,\theta/2\,X}$$

$$= \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$= \cos\left(\frac{\theta}{2}\right)\mathbb{I} - i\sin\frac{\theta}{2}\,X$$

$$R_y(\theta) = \cos\left(\frac{\theta}{2}\right)\mathbb{I} - i\sin\frac{\theta}{2}\,Y$$

$$R_Z(\theta) = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}$$

with $|\psi\rangle$, we will evaluate

$$\langle\psi_T|H|\psi_T\rangle =$$

$$\langle\psi_T|(\varepsilon+c)\mathbb{1} + (\Omega+\omega_z)Z + \omega_x X|\psi_T\rangle = E_T$$

From the variational principle

$$E_T \geq E_0 \text{ (exact lowest state)}$$

$|0\rangle$ and $|1\rangle$

$$|\psi\rangle = R_y(\phi) R_x(\theta) |0\rangle$$

$$H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$$

$$\frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

$$R_y(\phi) R_x(\theta) |0\rangle =$$

$$\alpha(\phi)\beta(\theta)|0\rangle +$$

$$\delta(\phi)\gamma(\theta)|1\rangle$$

$$X = \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\sigma_z} \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{H}$$

$$= \frac{1}{2} \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} =$$

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = X$$

$$\hat{H} = \alpha \mathbb{I} + \beta Z + \gamma X$$

$$= \alpha \mathbb{I} + \beta Z + \gamma HZH$$

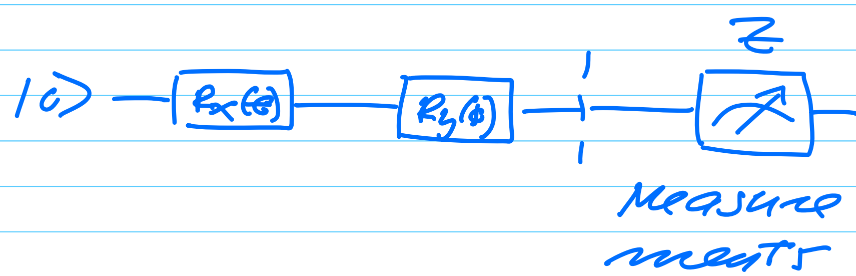
$$\langle \psi_T | \hat{H} | \psi_T(\phi, \theta) \rangle = \langle \psi_T | \hat{H} | \psi_T \rangle$$

$$= \alpha \langle \psi_T | \mathbb{I} | \psi_T \rangle \quad (1)$$

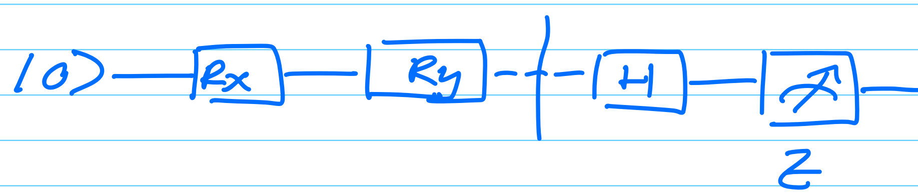
$$+ \beta \langle \psi_T | Z | \psi_T \rangle \quad (2)$$

$$+ \gamma \langle \psi_T H | Z | H \psi_T \rangle \quad (3)$$

(2)



(3)



$$R_x(\theta)|0\rangle =$$

$$\left(\cos(\theta/2)\, \mathbb{I} - i \sin \theta/2 \, X\right)|0\rangle$$

$\cos\theta/2\,|0\rangle - i\sin\frac{\theta}{2}\,|1\rangle$

$X\,|0\rangle = |1\rangle$

$R_y(\phi) = \cos(\phi/2)\,\mathbb{I} - i\sin\phi/2\,Y$

$R_y(\phi)\,R_x(\theta)\,|0\rangle$

$\cos\theta/2\,\cos\phi/2\,\mathbb{I}\,|0\rangle$

$-\cos\phi/2\,\mathbb{I}\,X\,(i\sin\theta/2)\,|1\rangle$

$-i\sin\phi/2\,Y\cos\theta/2\,|0\rangle$

$-\sin\theta/2\cdot\sin\phi/2\,Y\,|1\rangle$

$= (\cos\theta/2\,\cos\phi/2)\,|0\rangle$

$-(\cos\phi/2\,i\sin\theta/2)\,|1\rangle$

$+\sin\phi/2\,\cos\theta/2\,|1\rangle$

$-i\sin\theta/2\,\sin\phi/2\,|0\rangle$

Two-qubit case

$$Z_{2\times2}\otimes I_{2\times2} = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & -1 & \\ 0 & & & -1 \end{bmatrix}$$

in the Lipkin model

$\sum_{j=1}^{N} \sigma_i z_i$ acts only on one qubit at the time,

In general when doing a measurement we need the following expression

$$U^{\dagger} (Z \otimes I) U$$

For $Z \otimes I$ $\quad U = I \otimes I$

$X_i X_j$ and $Y_i Y_j$

$XX = X \otimes X \qquad YY = Y \otimes Y$

$Z \cdot Z = Z \otimes Z$

$Z \otimes Z \quad : \quad U = CX_{10}$

$$CX_{10} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

XX has $U = CX_{10}(H \otimes H)$

$$XX = U^{\dagger} (Z \otimes Z) U$$

YY has $U = CX_{10} (\underbrace{HS^{\dagger}}_{2\times 2} \otimes HS^{\dagger})$

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$