Machine Learning: Bayesian Machine Learning

Master of Science thesis project

Nov 28, 2019

Bayesian Machine Learning, Level Densities and Probability

The level density $\rho(E)$ as function of energy E plays a central in many physics applications, ranging from the modeling of nuclear astrophysics reactions central to the synthesis of the elements to the classification and understanding of phases in condensed matter physics.

In statistical physics it defines the thermodynamical potential in the micro-canonical ensembler and thereby the entropy as

$$S(E) = -k_B \ln(\rho(E)),$$

and the partition function $Z(\beta)$ (with $\beta = 1/k_B T$) as

$$Z(\beta) = \int dE \exp(-\beta E)\rho(E),$$

and the expectation values of various moments of the energy as

$$\mathbb{E}^n(\beta) = \frac{\int dE E^n \exp(-\beta E)\rho(E)}{Z(\beta)}.$$

We can rewrite this equation as

$$\mathbb{E}^n(\beta) = \int dE E^n P(E|\beta),$$

where $P(E|\beta)$ is the likelihood of being in a state with energy E with temperature β. The probability is defined as

$$P(E|\beta) = \frac{\exp(-\beta E)\rho(E)}{Z(\beta)}.$$

With the density of states we can in turn define a probability distribution function (PDF) in say for example the canonical ensemble. Alternatively, if we have the PDF we can find the denisty of states.

Thesis Projects

The aim of this thesis project is employ Bayesian machine learning to define a PDF, either from experiment or from theoretical simulations. Eventually, based on the PDF, can attempt to define a a level density $\rho(E)$, or the other way around. The first step is to use an already available model for extracting the level density from exact diagonalization. These data will then be used to define a posterior distribution based on a Bayesian machine learning approach.

Specific tasks and milestones. The projects can easily be split into several parts and form the basis for collaborations among several students. The milestones are as follows

1. Spring 2020:
2. Fall 2020:
3. Spring 2021:

The thesis is expected to be handed in May/June 2021.

References. Highly relevant articles for possible thesis projects are: