ENtanglement-enhanced QUantum sensIng fRom topologY (ENQUIRY) Project description

Young CAS Grant 2025 Proposal, Gunnar Felix Lange

Background and motivation

Background: The United Nations has declared 2025 as the International Year of Quantum Science and Technology in honor of the centenary of Werner Heisenberg's seminal formulation of quantum mechanics. Heisenberg's discovery heralded the birth of modern quantum theory, and concurrently sounded the death knell for many of the established truths of pre-quantum, classical, physics. The remainder of the 20th century was the age of foundational quantum theory, as physicists forged new fundamental epistemological and mathematical frameworks. Today, in the 21st century, we are harnessing this foundational knowledge to create technologies that would be unimaginable without the principles of quantum theory, making this the century of quantum technology. Quantum technology in turn necessitates the re-evaluation of the foundational framework of many other fields:

1) Quantum **computing** challenges the foundations of classical computing, as posited by Turing.
2) Quantum **communication** challenges the foundations of classical information theory, as posited by Shannon.
3) Quantum **sensing** challenges the foundations of classical estimation theory as posited by Cramér and Rao.

In each of these fields, quantum effects allow the violation of theorems and bounds previously thought to be fundamental. This has far-reaching epistemological consequences, as it challenges our basic notions of what computation, information and estimation entails. It also has far-reaching technological consequences, as these violations can be used in targeted ways to enable novel devices and infrastructure, which outperform any comparable classical technology. This is why McKinsey last year estimated the market value of quantum technology to be over $170 billion by 2040, with the total potential impact in the trillions of dollars [1].

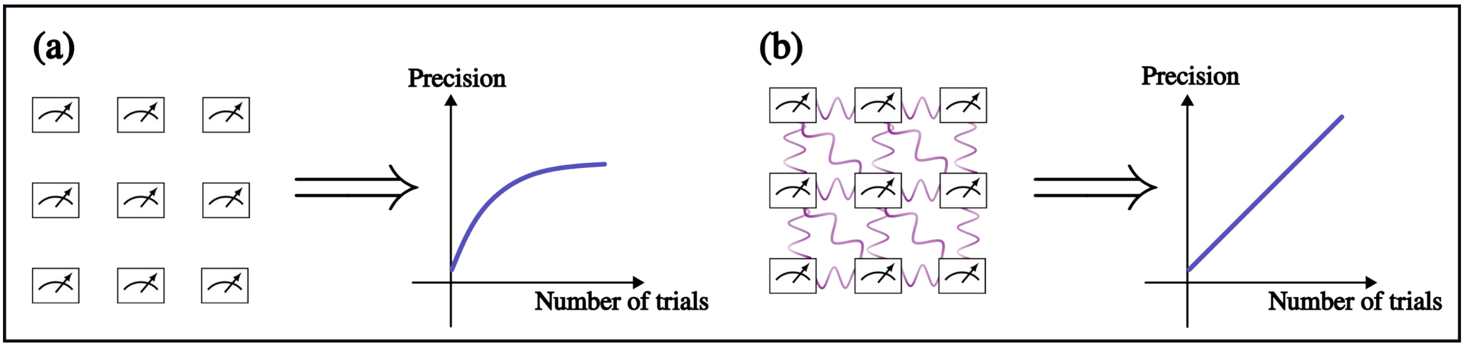

Fig. 1: Fundamentals of quantum sensing. *Classical sensors **(a)** struggle to achieve high precision with repeated measurements. Quantum entangled sensors **(b)** enable better scaling, facilitating ultra-precise sensing.*

Quantum sensing: The Young CAS project ENQUIRY focuses on the foundations of quantum sensing. As illustrated schematically in Fig. 1, the basic question in sensing is how precisely a physical quantity, such as pressure, temperature or electric field, can be estimated through repeated measurements. Classically, the fundamental limit, derived from parameter estimation theory, is given by the **Crámer-Rao bound**. This bound states that the required number of measurements to achieve a given precision scales **quadratically** as shown in Fig. 1(a). Thus a tenfold increase in precision classically requires at least 100 additional measurements. By contrast, when using quantum mechanics, the **quantum Crámer-Rao** bound states that the maximum attainable precision scales **linearly** with the number of measurements, as shown in Fig. 1(b), so that a tenfold increase in precision can be obtained with only 10 additional measurements.

Such improved scaling has enormous implication for any field where precise and/or rapid measurements are required, such as in GPS systems, medical imaging and atomic clocks. Additionally, quantum sensors are expected to be more robust to drift than classical sensors, allowing higher precision measurements over longer times. This is why many large national and international actors, including Bosch, Kongsberg and SINTEF, are currently investing heavily into quantum sensing.

Achieving such a dramatic increase in precision requires the targeted use of **quantum entanglement**; correlations in the sensor that have no classical counterpart. Usually, entanglement has to be carefully engineered in such systems, as is done in e.g. squeezed photon sensors. This can be laborious, and fickle, significantly limiting the applicability of quantum sensors.

Critical quantum sensing: More recently, a different idea has come to the fore: **critical quantum sensing**. Just as water spontaneously changes from liquid to gas phase at the critical value of 100°C, quantum systems may undergo a **quantum phase transition** at a critical value of some external parameter (such as strain, pressure, electric field, etc.), suddenly changing the pattern of correlation among the components of the system. Close to such a critical value, even small changes in the external parameter can be detected, as they lead to large changes in the quantum state of the sensor. This has been shown theoretically to enable quantum enhanced sensitivity [2], i.e. a precision beyond the classical Crámer-Rao bound, as illustrated in Fig. 2. Quantum sensors based on phase transitions will likely require less careful engineering of entanglement, making them less fickle.

Quantum phase transitions, however, usually involve rather complicated many-body phenomena and formally only exist at zero Kelvin. Additionally, extracting the relevant information from the sensor requires the sensors to stay in its lowest energy state throughout the transition, necessitating a slow approach to the critical point, potentially limiting the sensors ability to measure rapidly varying signals. There are however certain classes of phase transitions, **topological quantum phase transitions,** which do not suffer from many of these drawbacks.

Topological quantum sensing: Topological phenomena in materials describe unusual robust quantum correlations in a material, which manifest as anomalous surface signatures and bulk conductivities. At a topological phase transitions, the global pattern of correlations reorganizes, as shown schematically in Fig. 2, potentially allowing for critical quantum sensing. Such topological phase transitions may well constitute the **ideal platform** for critical quantum sensing because they:

1) Occur quite generically in many real materials,
2) Can be controlled using various relevant external parameters,
3) Do not require complicated many-body electron correlations,
4) Can be stabilized at temperatures above 0 Kelvin.

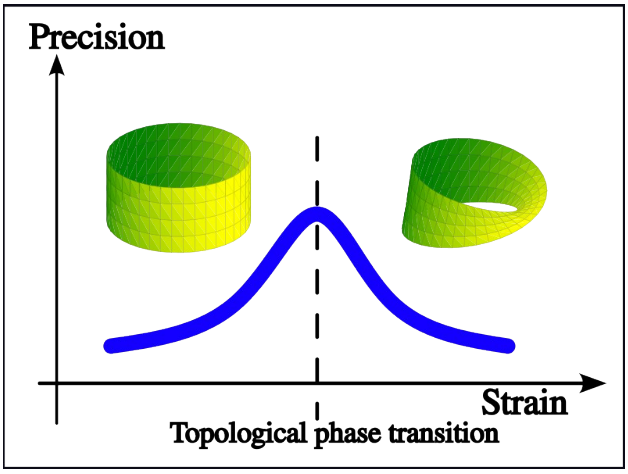

Fig. 2: Illustration of topological critical strain sensing. *At some critical value of an external parameter, such as strain, the sensor undergoes a quantum topological phase transition, leading to a precision beyond the classical limit.*

Topological sensing has only been proposed theoretically very recently [3], though there are already initial promising experimental results [4]. Additionally, my collaborators and myself recently proposed a further tantalizing possibility in Refs. [5,6]: **spin topological phases**. Topological phase transitions between spin topological phases can occur in the spin, rather than the electronic, degrees of freedom of the sensor. The advantages of this are twofold:

1) It widens the class of possible topological phases, making more systems available for critical quantum sensing.
2) It suggests that it may be possible to undergo a phase transition in the spin degrees of freedom *only*, potentially avoiding the necessity to approach the critical point slowly.

Taken together, this suggests that topological phases, and particularly spin topology phases, may be ideally suited for critical quantum sensing. The ENQUIRY Young CAS project therefore proposes to investigate the use of topological, and particularly spin topological, phases for quantum sensing.

The ENQUIRY Young CAS proposal

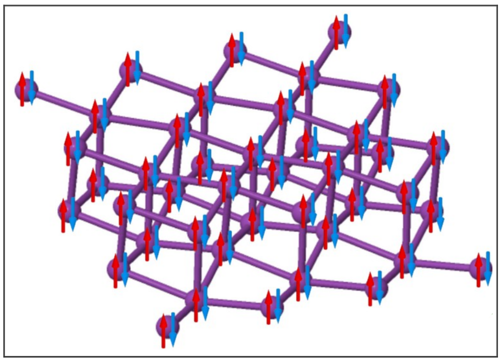

Fig. 3: Spin structures in ultrathin ***Bismuth*** *(taken from Ref. [5]).*

Research goals: In Ref. [5] we proved mathematically that the sensitivity of spin topological systems increase with the amount of topology (i.e. the amount of correlation) in the system, suggesting that such topological phases naturally realize good quantum sensors, and that transitions between phases with high amounts of topology may lead to enhanced sensitivity. Furthermore, we showed in Ref. [6] that phase transitions of spin topological materials can be engineered using light in ultrathin Bismuth, as illustrated in Fig. 3, and my collaborators and myself have recently found similar results for phase transitions using strain, in multiple other materials. This suggests the possibility to use these ideas to build real light and strain sensors. Thus, the two main Research Objectives (ROs) for the ENQUIRY project, following from Refs. [5,6], are:

RO1: Deepen the understanding of the interplay of critical quantum sensing, topology and entanglement.

RO2: Investigate how to use topology and entanglement in material candidates for topological quantum sensors.

RO1: Understanding the interplay of critical quantum sensing with topology and entanglement
It is widely believed that sensors which go beyond the classical Crámer-Rao bound, i.e. achieve quantum-enhanced sensitivity, require the use of entanglement as a resource. For critical quantum sensing, in particular when mediated by a topological phase transition, the arguments for the presence of quantum-enhanced sensitivity are heuristic, and no clear mapping to entanglement arguments exist. Properly understanding the role of entanglement will put critical quantum sensing on a more robust theoretical footing, and will allow the targeted use of entanglement as a **non-classical resource**. The aforementioned spin topological phases are ideal for this, as they sensitively depend on the entanglement between the spin and orbital degrees of freedom of the electrons in the material. Furthermore, the same idea could also be extended to other degrees of freedom, pointing towards a broader understanding of the interplay of entanglement and topology in materials. With this in mind, the first Research Question (RQ1) of the ENQUIRY project is:

RQ1: What is the interplay of entanglement and topology, particularly at spin topological phase transitions?

<u>Task 1.1 – Defining entanglement for spin topology</u>. To address RQ1, we begin by combining the methods from Refs. [7,8] with those from Ref. [5] to define spin entanglement spectra, and investigate how these can be related to the quantum Fisher information, which in turn is related to quantum sensitivity for a potential sensor.

<u>Task 1.2 – Investigating entanglement spectra at the phase transitions</u>. Having understood how spin entanglement spectra can be defined in materials, we subsequently look at these spectra at a spin topological phase transition, to investigate the scaling of entanglement with system size, and relate it to the Fisher information. This scaling is a proxy for the usefulness of the phase transition for sensing. This will be done using analytical arguments wherever possible, though numerical calculations using the models from Ref. [5] will be used to gain intuition.

<u>Task 1.3 – Generalizing entanglement in materials:</u> Finally, the insights into entanglement at topological phase transitions from the previous two tasks should be generalized beyond spin topological phases, to other degrees of freedom relevant in materials. This could e.g. include orbital angular momentum or sublattice degrees of freedom.

Actually utilizing entanglement for quantum-enhanced sensing, however, requires carefully choosing a measurement protocol, to maximize sensitivity. This leads to the second RQ of the ENQUIRY project:

RQ2: How can we maximize useful entanglement for sensing at topological phase transitions?

Task 2.1 – Investigate sensing protocols for bulk states: To answer this RQ, we must investigate the source of entanglement. Most of the entanglement likely arises from the bulk states involved in the phase transition. Exploiting the entanglement in these states could be done using an adiabatic protocol as in Ref. [9]. For spin topological transitions, it may also be possible to avoid adiabaticity, by focusing on the spin sector only, but this will require choosing a different basis for sensing. This will be investigated by combining the models from Ref. [5] with the methodology of Ref. [9], and the insights gained from answering RQ1.

Task 2.2 – Investigate sensing protocol for surface states: One of the intriguing features of topological phase transitions is that they involve an abrupt change in surface states, which mediate the phase transition. These surface states could also carry entanglement, and be useful for sensing, as explored in [3]. As such, this task seeks to combine the models from Ref. [5] with the methodology from [3] and the insights on entanglement from RQ1.

Lastly, to take these findings closer to real life, the final step of the ENQUIRY project involves investigating realistic material candidates which realize these effect. This leads to the second RO of the ENQUIRY project:

RO2: Applying topology and entanglement in material candidates for topological quantum sensors
This topic is essential to move topological quantum sensors from a theoretical idea to the real world. In the longer run, this topic should be expanded to include RQs on realistic material conditions and experimental collaborations. Within the limited scope of the ENQUIRY project, however, the RQ to be addressed is:

RQ3: Which materials are best suited for topological quantum sensing?

Task 3.1 – Investigate the use of ultrathin Bismuth for quantum sensing: Ultrathin Bismuth was used in Refs. [5,6] to illustrate spin topological phase transitions. It undergoes such transitions as a function of light and strain, and can therefore serve as a realistic novel light and strain sensor. To develop this idea, this task will look at both finite-size scaling of the entanglement, as developed in RQ1, and look at the sensing protocols developed in RQ2, from the perspective of Bismuth. This will be done with state of the art **Density Functional Theory** (DFT).

Task 3.2 – Investigate the use of fullerne networks for quantum sensing: Fullerene networks are emerging as versatile tools for material science, as they offer a high degree of tunability [10]. Some of our preliminary results suggest that such networks also undergo spin topological phase transitions under strain, making them a natural candidate to investigate along the same lines as Bismuth in Task 3.1.

Task 3.3 – Look for other material candidates: This task is explorative, using DFT and database searches to look for and evaluate other material candidates for spin topological phase transitions and sensing. The relevant CAS fellows have access to the required computational resources, and therefore do not need additional compute time.

Summary
Taken together, these two ROs and three RQs will significantly advance the nascent field of topological quantum sensing. Because the project spans all the way from purely theoretical studies to concrete material realizations, it will be of great fundamental interest, whilst simultaneously facilitating real sensing technologies. The funding from CAS will allow gathering a range of top researcher in multiple disciplines from internationally leading universities to collaboratively advance the feasibility of quantum sensing.

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