AE1MCS: Tutorial 2
Inference Rules and Proofs

Rule of inferences

Given that $\forall x(L(x) \rightarrow F(x))$ and $\exists x(F(x) \wedge \neg C(x))$, prove that $\exists x(F(x) \wedge \neg C(x))$

All lions are fierce.	$\forall x\left(L(x) \rightarrow F(x)\right)$
Some lions do not drink coffee.	$\exists x\left(L(x) \wedge \neg C(x)\right)$
Some fierce creatures do not drink coffee.	$\exists x\left(F(x) \wedge \neg C(x)\right)$

Proof methods

Proving conditional statements

- 1. Show that if a and b are real numbers and $a \neq 0$, then there is a unique real number r such that $ar + b = 0$.

Prove by contradiction

- 2.Prove that at least one of the real numbers a_1, a_2,⋯, a_n is greater than or equal to the average of these numbers.

Prove of Equivalence

- 3. Prove that if n is an integer, these four statements are equivalent: (1) n is even, (2) $n+1$ is odd, (3) $3n+1$ is odd, (4) $3n$ is even.

Proof by (counter) Example

- 4. For all integers a and b, prove or disprove if a is odd or b is odd, then a+b is odd.

Mathematical Induction

5. Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer n.

Strong Induction

6. Show that if n is an integer greater than 1, then n can be written as the product of primes.

More Exercises in the Textbook

- Section 1.6
  - 3, 5, 7, 13, 15, 17-20, 23-29, 33, 34-35*
- Section 1.7
  - 13, 14, 16, 19-25, 34, 35, 38-40
- Section 1.8
  - 3, 4, 7, 15, 29-32
- Section 5.1
  - 3-17, 18, 19
- Section 5.2
  - 1-4
- Supplementary questions: Q23-Q32