Markov Chain Example: Banking & Default Risk

Follow a customer through repayment of a bank loan:

Consider three states: 1=good customer, 2=delinquent, 3=default.

Transition probabilities for each month are given as:

- from good customer to delinquent is 0.05;
- from delinquent to good customer is 0.1;
- from delinquent to default is 0.02.
- It is impossible to move from being a good customer to default in one period.
- Also, it is impossible to move out of default.

Then the transition matrix is

$$P = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.1 & 0.88 & 0.02 \\ 0 & 0 & 1 \end{pmatrix}$$

Suppose a customer starts as good:

$$S_{[0]} = (1 \quad 0 \quad 0)$$

Then at the next month:

$$S_{[1]} = S_{[0]}P = (0.95 \quad 0.05 \quad 0)$$

and after another two month:

$$S_{[2]} = S_{[1]}P = (0.9075 \quad 0.0915 \quad 0.001)$$

$$S_{[3]} = S_{[2]}P = (0.871275 \quad 0.125895 \quad 0.00283)$$

This means that after 3 months, there is only 0.283% chance the customer is in default.

This information is useful for banks to model risk and expected losses.