Mathematics for Computer Scientists COMP1046

Coursework 1

- Deadline for submission: 23:59, 7 November 2024.
- Submit as a PDF file via Moodle. Use student ID as your file name.
- You may type your answer in Word or LATEX. Handwritten solutions are NOT accepted.
- If you wish to type your answer in LATEX, a LATEX template is provided as a .tex file with a symbol list in it.
- The full mark for this coursework is 100.
- Late submission penalty: 5% deducted for each day late.
- Please show working process using the steps we used in the lectures.

1. Assume the following statement is true:

$$\forall x(\exists y(S(x,y) \wedge M(y)) \rightarrow \exists z(P(z) \wedge R(x,z))).$$

Prove that $\neg\exists z P(z) \rightarrow \forall x \forall y(S(x,y) \rightarrow \neg M(y))$ is true.

Please use the format of "Step-Reason" for your proof.

(20 marks)

2. Let $f: X \rightarrow Y$, $g: Y \rightarrow Z$ be two functions, prove that:

(1) If $g \circ f$ is one-to-one, then f is one-to-one;

(2) If $g \circ f$ is onto, then g is onto.

(15 marks)

3. Let R be an equivalent relation on a set A, [a] is the equivalence class of a with respect to relation R.

Prove that:

(1) $\forall a \in A$, $a \in [a]$;

(2) $(a,b) \in R$ if and only if $[a]=[b]$;

(3) If $[a] \neq [b]$, then $[a] \cap [b] = \varnothing$.

(15 marks)

4. Use mathematical induction to show that:

Let S be a set with $|S|=n$, and $\mathrm{P}(S)$ be its power set, then $|\mathrm{P}(S)|=2^n$.

(15 marks)

5. Probability:

(1) If bag A contains n white balls and m red balls, and bag B contains N white balls and M red balls, and now take any ball from bag A and put it in bag B, and then take any ball from bag B, what is the probability of getting (i.e. taking it from bag B) a white ball?

(2) If box C contains 5 red balls and 4 white balls, and box D contains 4 red balls and 5 white balls. First take any 2 balls from box C and put them in box D, and then take 1 ball from box D, what is the probability of getting a white ball.

(20 marks)

6. There are 3 balls and 4 boxes, and the boxes are numbered 1, 2, 3, 4. Place the balls one by one independently and randomly into 4 boxes. Take X to denote the smallest number of the box with at least one ball (e.g., $X = 3$ means box 1, box 2 are empty, and the box 3 has at least one ball), what is the expectation $E(X)$.

(15 marks)