Tutorial 2b - Gaussian Elimination (answers)

COMP1046 - Maths for Computer Scientists

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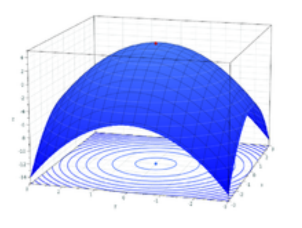

Exercise 1

Use Gaussian Elimination to solve this system of linear equations:

$$x_1 - x_2 = 1$$

$$2x_1 + x_2 = 8$$

Exercise 1 answer

Add -2 times first equation to second equation:

$$x_1 - x_2 = 1$$

$$(2x_1 + x_2) - 2(x_1 - x_2) = 8 - 2 \times 1$$

Second equation now gives

$$3x_2 = 6$$

so $x_2 = 2$. Substitute back into first row: $x_1 = 3$.

Exercise 2

Solve the following system of linear equations using the Gaussian Elimination Algorithm. Show your working using row vector notation.

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 1 \\ x_1 + x_2 - 2x_3 &= 4 \\ 3x_1 - 2x_2 - x_3 &= 7 \end{aligned}$$

From Lecture 5.

Exercise 2 answer

The complete matrix is $\left(\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 1 & 1 & -2 & 4 \\ 3 & -2 & -1 & 7 \end{array}\right)$.

Then rows are

$$\begin{aligned} \mathbf{r}_\mathbf{1}^{(\mathbf{1})} &= (2, -1, 3, 1) \\ \mathbf{r}_\mathbf{2}^{(\mathbf{1})} &= (1, 1, -2, 4) \\ \mathbf{r}_\mathbf{3}^{(\mathbf{1})} &= (3, -2, -1, 7). \end{aligned}$$

For $k = 1$,

$$\begin{aligned} \mathbf{r}_\mathbf{1}^{(\mathbf{2})} &= \mathbf{r}_\mathbf{1}^{(\mathbf{1})} \\ \mathbf{r}_\mathbf{2}^{(\mathbf{2})} &= \mathbf{r}_\mathbf{2}^{(\mathbf{1})} + \tfrac{-1}{2}\mathbf{r}_\mathbf{1}^{(\mathbf{1})} = (0, \tfrac{3}{2}, \tfrac{-7}{2}, \tfrac{7}{2}) \\ \mathbf{r}_\mathbf{3}^{(\mathbf{2})} &= \mathbf{r}_\mathbf{3}^{(\mathbf{1})} + \tfrac{-3}{2}\mathbf{r}_\mathbf{1}^{(\mathbf{1})} = (0, \tfrac{-1}{2}, \tfrac{-11}{2}, \tfrac{11}{2}). \end{aligned}$$

Exercise 2 answer

For $k = 2$,

$$\mathbf{r}_1^{(3)} = \mathbf{r}_1^{(2)}$$
$$\mathbf{r}_2^{(3)} = \mathbf{r}_2^{(2)}$$
$$\mathbf{r}_3^{(3)} = \mathbf{r}_3^{(2)} + \tfrac{1}{3}\mathbf{r}_2^{(2)} = (0, 0, \tfrac{-20}{3}, \tfrac{20}{3})$$

which gives the complete matrix $\left(\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & \frac{3}{2} & \frac{-7}{2} & \frac{7}{2} \\ 0 & 0 & \frac{-20}{3} & \frac{20}{3} \end{array}\right)$.

Working backwards from row 3, we get

$x_3 = -1$, $x_2 = 0$ and $x_1 = 2$.

Exercise 3

Use Gaussian Elimination to solve this system of linear equations:

$$
\begin{aligned}
&-x_1 + 2x_2 - x_4 = 1\\
&x_1 + x_2 + x_3 + 2x_4 = 2\\
&2x_1 - 4x_2 + x_3 = -8\\
&3x_1 + x_3 + 3x_4 = 1
\end{aligned}
$$

Exercise 3 answer

Form the complete matrix: $\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & 1 \\ 1 & 1 & 1 & 2 & 2 \\ 2 & -4 & 1 & 0 & -8 \\ 3 & 0 & 1 & 1 & 1 \end{array}\right)$

Add 1 * row 1 to row 2

Add 2 * row 1 to row 3

Add 3 * row 1 to row 4

After step 2: $\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & 1 \\ 0 & 3 & 1 & 1 & 3 \\ 0 & 0 & 1 & -2 & -6 \\ 0 & 6 & 1 & 0 & 4 \end{array}\right)$

Exercise 3 answer

Add 0 * row 2 to row 3

Add -2 * row 2 to row 4

After step 3: $\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & 1 \\ 0 & 3 & 1 & 1 & 3 \\ 0 & 0 & 1 & -2 & -6 \\ 0 & 0 & -1 & -2 & -2 \end{array}\right)$

Exercise 3 answer

Add 1 * row 3 to row 4

After step 4: $\left(\begin{array}{cccc|c} -1 & 2 & 0 & -1 & 1 \\ 0 & 3 & 1 & 1 & 3 \\ 0 & 0 & 1 & -2 & -6 \\ 0 & 0 & 0 & -4 & -8 \end{array}\right)$

Work backwards for substitution:

Solution: $(-1, 1, -2, 2)$.