

MCS Tutorial 4

Relations and Counting

Huan Jin

Relations

1. Consider a relation R on the set \mathbb{Z}^+ defined as

$$R = \{(x, y) \mid x + y \text{ is even}\}$$

Show whether R is reflexive, symmetric, antisymmetric and/or transitive.

2. Let R and S be the following relations:

- $R = \{(1,1), (1,2), (2,4), (3,2), (4,3)\}$
- $S = \{(1,0), (2,4), (3,1), (3,2), (4,1)\}$

What is the composite of the relations R and S , $S \circ R$?

3. Let $R = \{(1,1), (2,4), (3,4), (4,2)\}$. Find the powers R^2, R^3, R^4, \dots

4. On the set of real numbers \mathbb{R} define the relation $S = \{(x, y) : x, y \in \mathbb{R}, \text{ and } x - y \text{ is an integer}\}$

(1) Show that S is an equivalence relation on \mathbb{R}

(2) What is the equivalence class for each $x \in \mathbb{R}$

Equivalence classes

5. Consider the power set of $X = \{a, b, c\}$ and define R on the power set as follows: URV iff U and V have the same cardinality. Find the equivalence classes of R

Counting

1. Recall that a bit string is an ordered list of characters using only the digits 0 and 1.
 - a) How many bit strings of length ten are there?
 - b) How many bit strings of length ten have exactly three 1s?
 - c) How many bit strings of length ten have exactly three 1s and none of these 1s are adjacent to each other?



2. You have a combination lock with four digits (0 to 9). You set the lock so that you do not use the same digit more than once.
- a) How many ways are there to set the lock?
 - b) If you additionally do not use four consecutive digits (ie increasing by 1 in each place; eg 1,2,3,4 or 3,4,5,6), how many ways do you have for setting the lock?
 - c) You now think that you will **also** allow for **any** four-digits made from the digits 0 to 4. How many ways are there to set the lock now?



Question 3

Consider sets A and B with $|A|=10$ and $|B|=17$.

- How many functions $f:A \rightarrow B$ are there?
- How many functions $f:A \rightarrow B$ are injective?

Further practice and homework

Many exercises in Rosen's textbook:-

- For Counting, try exercises in Section 6.1;
- For Permutations and Combinations, try exercises in Section 6.3.