# MCS: Tutorial 1
# Proposition and Predicate Logic

# Proposition

- A ***proposition*** is simply a statement. ***Propositional logic*** studies the ways statements can interact with each other. It is important to remember that propositional logic does not really care about the content of the statements. For example, in terms of propositional logic, the claims, “if the moon is made of cheese then basketballs are round,” and “if spiders have eight legs then Sam walks with a limp” are exactly the same. They are both implications: statements of the form, $P \rightarrow Q$.

Suppose P and Q are the statements: P: Jack passed math. Q: Jill passed math.

a. Translate “Jack and Jill both passed math” into symbols.
b. Translate “If Jack passed math, then Jill did not” into symbols.
c. Translate “$P \vee Q$” into English.
d. Translate “$\neg(P \wedge Q) \rightarrow Q$” into English.
e. Suppose you know that if Jack passed math, then so did Jill. What can you conclude if you know that:
    i. Jill passed math?
    ii. Jill did not pass math?

# Truth table

- Make a truth table for the statement $\neg P \lor Q$.

# Logical Equivalence

- Two statements P and Q are ***logically equivalent*** provided P is true precisely when Q is true. That is, P and Q have the same truth value under any assignment of truth values to their atomic parts.
- To verify that two statements are logically equivalent, you can make a truth table for each and check whether the columns for the two statements are identical.

# Examples

- 1. Prove that the statements $\neg(P \rightarrow Q)$ and $P \wedge \neg Q$ are logically equivalent without using truth tables.
- 2. Are the statements $(P \vee Q) \rightarrow R$ and $(P \rightarrow R) \vee (Q \rightarrow R)$ logically equivalent?

# Examples

3. Simplify the following statements (so that negation only appears right before variables).

a. $\neg(P \to \neg Q)$
b. $(\neg P \vee \neg Q) \to \neg(\neg Q \wedge R)$.
c. $\neg((P \to \neg Q) \vee \neg(R \wedge \neg R))$.
d. It is false that if Sam is not a man then Chris is a woman, and that Chris is not a woman.

# Beyond Propositions

- Not every statement can be analyzed using logical connectives alone. For example, we might want to work with the statement:

  All primes greater than 2 are odd.

- To write this statement symbolically, we must use quantifiers. We can translate as follows

$$\forall x((\ P(x) \wedge \ \ x>2\ ) \rightarrow O(x))$$

- Use P(x) denote “x is prime” and O(x) to denote “x is odd.” These are not propositions, since their truth value depends on the input x. Better to think of P and O as denoting *properties* of their input. The technical term for these is ***predicates*** and when we study them in logic, we need to use ***predicate logic***.

- It is important to stress that predicate logic *extends* propositional logic.
- You will notice that our statement above still used the (propositional) logical connectives. Everything that we learned about logical equivalence and deductions still applies.
- However, predicate logic allows us to analyze statements at a higher resolution, digging down into the individual propositions P, Q, etc.

# Predicates

- Suppose we claim that there is no smallest number. We can translate this into symbols as

$$\neg\exists x\forall y\ (x\leq y)$$

(literally, “it is not true that there is a number x such that for all numbers y, x is less than or equal to y”).

- However, we know how negation interacts with quantifiers: we can pass a negation over a quantifier by switching the quantifier type (between universal and existential). So the statement above should be *logically equivalent* to

$$\forall x\exists y\ (y<x)$$

- Notice that $y<x$ is the negation of $x\leq y$. This literally says, “for every number x there is a number y which is smaller than x.” We see that this is another way to make our original claim.

# Examples

1. Use quantifiers to express the statement that “There is a woman who has taken a flight on every airline in the world.”

# Question 1

2. Use quantifiers to express the statement that “There does not exist a woman who has taken a flight on every airline in the world.”

# More Exercises (Not required)!!

- No solution will be provided!
- Textbook
  - Section 1.4
    - 7-10, 43, 44, 46-51, 59-62
  - Section 1.5
    - 3, 4, 7-9, 14-17, 29-32, 48*, 49*, 52*
- Supplementary questions: Q1-Q19