# Mathematics for Computer Scientists COMP1046

## Coursework 1

• Deadline for submission: 23:59, 5 November 2024.

• Submit as a PDF file via Moodle. Use student ID as your file name.

• You may type your answer in Word or LATEX. Handwritten solutions are NOT accepted.

• If you wish to type your answer in LATEX, a LATEX template is provided as a .tex file with a symbol list in it.

• The full mark for this coursework is 100.

• Late submission penalty: 5% deducted for each day late.

• Please show working process using the steps we used in the lectures.

1. Assume the following statement is true:

$$\forall x(\exists y\big(S(x,y) \wedge M(y)\big) \rightarrow \exists z(P(z) \rightarrow R(x,z)) ).$$

Prove that $\neg\exists zP(z) \rightarrow \forall x\forall y(S(x,y) \rightarrow \neg M(x))$ is true.

Please use the format of "Step-Reason" for your proof.

(20 marks)

2. Let $f : X \rightarrow Y$, $g : Y \rightarrow Z$ be two functions, prove that:

(1) If $g \circ f$ is one-to-one, then $f$ is one-to-one;

(2) If $g \circ f$ is onto, then $g$ is onto.

(15 marks)

3. Let *R* be an equivalent relation on a set *A*, [a] is the equivalence class of a with respect to relation *R*.

Prove that:

(1) $\forall a \in A$, $a \in [a]$;

(2) $(a,b) \in R$ if and only if $[a]=[b]$;

(3) If $[a] \neq [b]$, then $[a] \cap [b] = \varnothing$.

(15 marks)

4. Use mathematical induction to show that:

Let S be a set with $|S| = n$, and $\mathcal{P}(S)$ be its power set, then $|\mathcal{P}(S)| = 2^n$.

(15 marks)

5. Probability:

(1) If bag *A* contains *n* white balls and *m* red balls, and bag *B* contains *N* white balls and *M* red balls, and now take any ball from bag *A* and put it in bag *B*, and then take any ball from bag *B*, what is the probability of getting (i.e. taking it from bag *B*) a white ball?

(2) If box *C* contains 5 red balls and 4 white balls, and box *D* contains 4 red balls and 5 white balls. First take any 2 balls from box *C* and put them in box *D*, and then take 1 ball from box *D*, what is the probability of getting a white ball.

(20 marks)

6. There are 3 balls and 4 boxes, and the boxes are numbered 1, 2, 3, 4. Place the balls one by one independently and randomly into 4 boxes. Take $X$ to denote the smallest number of the box with at least one ball (e.g., $X = 3$ means box 1, box 2 are empty, and the box 3 has at least one ball), what is the exception $E(X)$.

(15 marks)