SKS Kernel

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1 Born Approximation

In frequency domain, the wave equation is :

$$-\rho\omega^2 u_i = (c_{ijkl}u_{k,l})_{,j} + f_i \tag{1.1}$$

and the solution $u_i(x)$ can be expressed by Green's function (Aki & Richards, 2002):

$$u_n(x) = \int f_i(x')G_{ni}(x,x')\ d^3x' = \int f_i(y)G_{ni}^{xy}\ d^3y \tag{1.2}$$

where the Green's function satisfies:

$$-\rho\omega^2 G_{in}^{xs} = \delta_{in}\delta(x-s) + (c_{ijkl}G_{kn,l})_j \tag{1.3}$$

And it also satisfies the reciprocity theorem:

$$G_{in}^{xy} = G_{ni}^{yx} \tag{1.4}$$

When there is some small perturbation δc_{ijkl} over elastic parameter c_{ijkl}, the wavefield will also be perturbed, and we can get the first order equations:

$$\rho\omega^2\delta u_n = (c_{ijkl}\delta u_{k,l})_j + (\delta c_{ijkl}u_{k,l})_j \tag{1.5}$$

By utilizing (1.2), we can think $(\delta c_{ijkl}u_{k,l})_j$ as the new force, and express the first-order field by using Green's function and Gauss's theorem:

$$\delta u_n = -\int \delta c_{ijkl}(y)u_{k,l}(y)G_{ni,j}^{xy}\ d^3y \tag{1.6}$$

2 Point Source Solution

when the source is a point moment source (we set the delta function as the source time function now), $f_i = -M_{ij}\frac{\partial}{\partial x_j^s}\delta(x-s)$. Then insert it into (1.2), and utilizing the reciprocity of Green's function, we can get:

$$u_n^{xs} = u_n(x) = M_{pq}\frac{\partial}{\partial x_q^s}G_{np}^{xs} = M_{pq}\frac{\partial}{\partial x_q^s}G_{pn}^{sx} \tag{2.1}$$

Insert the equation above into (1.6):

$$\delta u_n = -\int \delta c_{ijkl}(x) G_{in,j}^{xr}\frac{\partial}{\partial x_l}\left(M_{pq}\frac{\partial}{\partial x_q^s}G_{pk}^{sx}\right)\ d^3x \tag{2.2}$$

note the symmetric property of c_{ijkl} and M_{ij}, we can substitute the displacement gradient with the corresponding strain:

$$\begin{aligned} M_{pq}\frac{\partial}{\partial x_q^s}G_{pn}^{sx} &= M_{pq}E_{pqk}^{sx} \\ \delta c_{ijkl}(x)G_{in,j}^{xr} &= \delta c_{ijkl}(x)E_{ijn}^{xr} \end{aligned} \tag{2.3}$$

where $E_{ijn} = 1/2(G_{in,j} + G_{jn,i})$ is the strain tensor of the Green's function. Then we can rewrite the equation as:

$$\delta u_n = -\int \delta c_{ijkl}(x) E_{ijn}^{xr}\frac{\partial}{\partial x_l}\left(M_{pq}E_{pqk}^{sx}\right)\ d^3x \tag{2.4}$$

If we go further to define a temporary field $\phi_n^{sx} = M_{pq}E_{pqk}^{sx}$ and it's symmetric gradient $F_{ij} = 1/2(\phi_{i,j} + \phi_{j,i})$, and utilizing the symmetry of c_{ijkl}, we can rewrite the equation as:

$$\delta u_n = -\int \delta c_{ijkl}(x) E_{ijn}^{xr} F_{kl}^{sx}\ d^3x \tag{2.5}$$

3 SKS Splitting Intensity

In isotropic media, the SV wave is in the R component. But in weak HTI media, the shear wave will split into fast and slow SV waves with time delay

δt. If we assume the isotropic SKS signal is $R^0(t)$, we can obtain the R and T components by:

$$\begin{aligned} R(t) &= R^0(t)\cos^2\beta + R^0(t-\delta t)\sin^2\beta \\ T(t) &= \frac{1}{2}\left[R^0(t) - R^0(t-\delta t)\right]\sin 2\beta \end{aligned} \tag{3.1}$$

where β is the angle between the fast axis and R axis. When the anisotropy is weak, we can approximate $R(t)$ and $T(t)$ as:

$$\begin{aligned} R(t) &= R^0(t) \\ T(t) &= \frac{1}{2}\sin 2\beta\delta t\dot{R}^0(t) \end{aligned} \tag{3.2}$$

The the splitting intensity (directly related to δt), can be defined as:

$$S = 2\frac{\int \dot{R}(t)T(t)\ dt}{\int \dot{R}(t)^2\ dt} \tag{3.3}$$

Note that $T(t)$ is small due to weak anisotropy, we can rewrite the equation as:

$$S = 2\frac{\int \dot{u_R}(t)\delta u_T(t)\ dt}{\int \dot{u}_R(t)^2\ dt} \tag{3.4}$$

Then we can insert (1.6) into (3.4):

$$S = \frac{-2}{\int \dot{u}_R(t)^2\ dt}\int\ dt\ \dot{u}_R(t)\int\ d^3x\ T_n\delta c_{ijkl}(x)E^{xr}_{ijn}F^{sx}_{kl} \tag{3.5}$$

where T_n is the n-th component of the T axis.

4 HTI Media

for HTI media, the perturbation of elastic tensor can be written as

(4.1)

5 AxiSEM Database

References

Aki, K. & Richards, P. G., 2002. *Quantitative seismology*.