# Numbers

## 1 Counting

Our conventional system is *decimal*, which is base-10. We use the following ten symbols to represent natural numbers:

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

To begin counting, we start with the symbol 0 to represent zero, then move to the next symbol to represent the next number, and so on.

| **Number** | **Base-10 Representation** |
|---|---|
| Zero | 0 |
| One | 1 |
| Two | 2 |
| Three | 3 |
| Four | 4 |
| Five | 5 |
| Six | 6 |
| Seven | 7 |
| Eight | 8 |
| Nine | 9 |

At this point, we have run out of symbols! With every number system:

**When you run out of symbols, start over, and add one to the left**

For the next number, ten, we would go from 9 back to 0 and add one to the left of it to get

$$10$$

Then we can go through all of the symbols again until we get to 19 and repeat the process to get 20.

## 2 Binary

*Binary* is a base-2 number system which uses the symbols:

$$0, 1$$

Let's count in binary.

| **Number** | **Base-2 Representation** |
|---|---|
| Zero | 0 |
| One | 1 |

We ran out of symbols much more quickly this time! No worries. We just start over and add one to the left. Repeating this process, we'll get:

| Number | Base-2 Representation |
|---|---|
| Zero | 0 |
| One | 1 |
| Two | 10 |
| Three | 11 |
| Four | 100 |
| Five | 101 |
| Six | 110 |
| Seven | 111 |
| Eight | 1000 |
| Nine | 1001 |

Some very nice patterns emerge. In fact, once we notice patterns in base-2, we might go back and look for similar patterns in base-10.

# 3 Place-Value

## Decimal

A number is composed of multiple *digits* where each digit has a place-value. For base-10, a number like 7248 has its digits in the following place-values:

| Place-Value | $10^3$ | $10^2$ | $10^1$ | $10^0$ |
|---|---|---|---|---|
| Digit | 7 | 2 | 4 | 8 |

This is one way to see that this number is

$$7 \cdot 10^3 + 2 \cdot 10^2 + 4 \cdot 10^1 + 8 \cdot 10^0$$

or

7 thousands, 2 hundreds, 4 tens, and 8 ones.

## Binary

In binary, we have numbers that look like

10111

which has place values:

| Place-Value | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$ |
|---|---|---|---|---|---|
| Digit | 1 | 0 | 1 | 1 | 1 |

This is one way to see that this number is

$$1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$

or

1 sixteen, 0 eights, 1 four, 1 two, and 1 one.

which in decimal is

$$16 + 4 + 2 + 1 = 23$$

# 4 Hexadecimal

Hexadecimal is base-16 and uses the symbols

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F$$

Notice that we won't run out of symbols until we reach $F$ (fifteen).

| **Number** | **Base-16 Representation** |
|---|---|
| Zero | 0 |
| One | 1 |
| Two | 2 |
| Three | 3 |
| Four | 4 |
| Five | 5 |
| Six | 6 |
| Seven | 7 |
| Eight | 8 |
| Nine | 9 |
| Ten | $A$ |
| Eleven | $B$ |
| Twelve | $C$ |
| Thirteen | $D$ |
| Fourteen | $E$ |
| Fifteen | $F$ |
| Sixteen | 10 |
| Seventeen | 11 |

Consider the hexidecimal number $3B7E$ with place-values:

| Place-Value | $16^3$ | $16^2$ | $16^1$ | $16^0$ |
|---|---|---|---|---|
| Digit | 3 | $B$ | 7 | $E$ |

which is

$$3 \cdot 16^3 + 11 \cdot 16^2 + 7 \cdot 16^1 + 14 \cdot 16^0$$

or

3 four thousand ninety-sixes, 11 two hundred fifty-sixes, 7 sixteens, and 14 ones

which in decimal is 15230

# 5 Colors

A hexadecimal color code specifies red-green-blue (RGB) intensities. It is written

$$\#\underbrace{\texttt{RR}}_{\text{Red}}\underbrace{\texttt{GG}}_{\text{Green}}\underbrace{\texttt{BB}}_{\text{Blue}}$$

where:

- **RR** represents the intensity of the red component.
- **GG** represents the intensity of the green component.
- **BB** represents the intensity of the blue component.

For example, the color #00D712 has

$$\underbrace{00}_{\text{Red (0 in decimal)}}\ \underbrace{D7}_{\text{Green (215 in decimal)}}\ \underbrace{12}_{\text{Blue (18 in decimal)}}$$

This number has no red, a lot of green, and a tiny bit of blue. Basically, it looks green!

The table below has several examples of colors and their hexidecimal representation.

| Color | Hex Color Code |
|---|---|
| Red | #FF0000 |
| Green | #00FF00 |
| Blue | #0000FF |
| Yellow | #FFFF00 |
| Cyan | #00FFFF |
| Magenta | #FF00FF |
| Gray | #808080 |
| Black | #000000 |
| White | #FFFFFF |

Table 1: Common Colors and Their Hexadecimal Codes

## Exercises

1. Convert the following binary numbers to decimal:

   (a) 111 (c) 1010101 (e) 110011
   (b) 10001 (d) 11100001 (f) 100011

2. Convert the following decimal numbers to binary:

   (a) 7 (c) 63 (e) 15
   (b) 100 (d) 64 (f) 16

3. Demonstrate how to use the standard algorithm to add 486 to 351 by hand.

4. The standard algorithm for addition works in every base. Use it to add the binary numbers 101 and 11. To check your answer, convert the numbers to decimal and check the result. If that goes alright, try adding 100111 to 1010.

5. Convert the following hexadecimal numbers to decimal:

   (a) B (c) 10 (e) ABC
   (b) 64 (d) A1 (f) 3CD0

6. Use your understanding of hex color codes to come up with educated guesses for the following colors. Check your colors with a hex color utility online.

   (a) Dark blue (c) Pink (e) Brown
   (b) Purple (d) Orange (f) Gold