Boolean Logic and Digital Circuits

When working with binary, a single bit is either TRUE (1), or FALSE (0). We can operate on the value of binary bits with logical operations. Three fundamental logical operations are AND, OR, and NOT.

NOT

The simplest operation is NOT. Given a boolean variable A, where A is either 0 or 1, the NOT operation negates the current value of A. The digital circuit diagram for NOT looks like this:

If A is 0, then NOT A is 1

If A is 1, then NOT A is 0

A nice way to represent this is with a truth table. The left-hand side is the input value, and the right-hand side is the output value.

$$\begin{array}{c|c}
A & \neg A \\
\hline
0 & 1 \\
1 & 0
\end{array}$$

Notice the syntax for representing not. We use the expression $\neg A$ to say "NOT A".

AND

Given two input bits A and B, the AND operation evaluates to TRUE when A and B are both true. The digital circuit diagram for AND looks like this:

$$A \longrightarrow Q$$

The truth table for AND is:

A	B	$A \wedge B$
0	0	0
0	1	0
1	0	0
1	1	1

OR

Given input bits A and B, the OR operation evaluates to TRUE when either A or B is true. The digital circuit diagram for OR looks like this:

$$A \longrightarrow Q$$

The truth table for OR is:

A	B	$A \lor B$
0	0	0
0	1	1
1	0	1
1	1	1

For each of the following Boolean expressions, write the truth table and draw the corresponding circuit diagram.

1.
$$A \wedge (A \vee B)$$

7.
$$\neg (A \lor B)$$

13.
$$(A \lor B) \land C$$

2.
$$A \lor (A \land B)$$

8.
$$A \wedge \neg B$$

14.
$$(A \lor B) \land \neg C$$

3.
$$(A \lor B) \land (\neg A \lor B)$$
 9. $\neg (A \land B)$

9.
$$\neg (A \land B)$$

15.
$$\neg (A \land B) \lor (B \land \neg C)$$

4.
$$A \vee \neg A$$

10.
$$\neg (A \land B \lor C)$$

16.
$$\neg (A \lor B) \lor (B \land \neg C)$$

5.
$$\neg (A \land B)$$

11.
$$\neg \neg (A \lor B)$$

17.
$$(A \land \neg B) \lor (\neg A \land B)$$

6.
$$\neg A \lor \neg B$$

12.
$$A \wedge B \wedge C$$

18.
$$A \wedge B$$