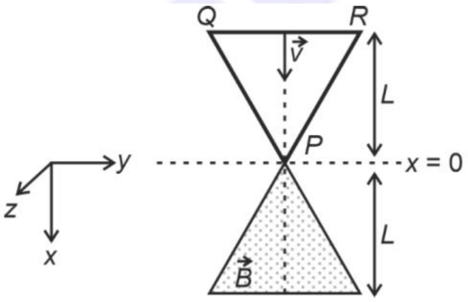
SECTION 1 (Maximum Marks: 12)

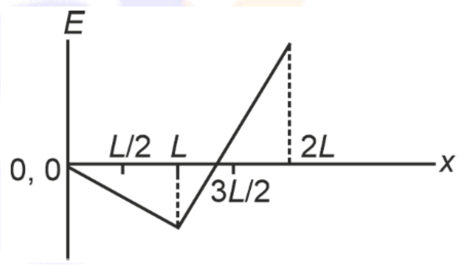
- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

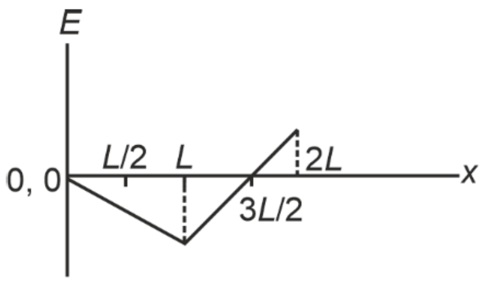
Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

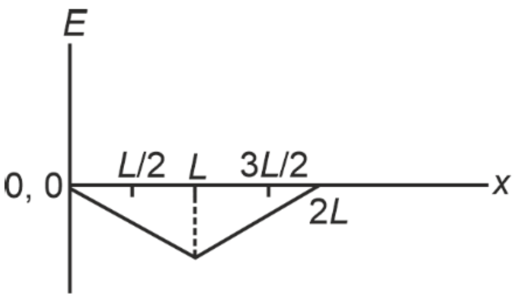
1. A region in the form of an equilateral triangle (in $x-y$ plane) of height L has a uniform magnetic field $\vec{B}$ pointing in the $+z$-direction. A conducting loop PQR, in the form of an equilateral triangle of the same height L, is placed in the $x-y$ plane with its vertex P at $x=0$ in the orientation shown in the figure. At $t=0$, the loop starts entering the region of the magnetic field with a uniform velocity $\vec{v}$ along the $+x$-direction. The plane of the loop and its orientation remain unchanged throughout its motion.



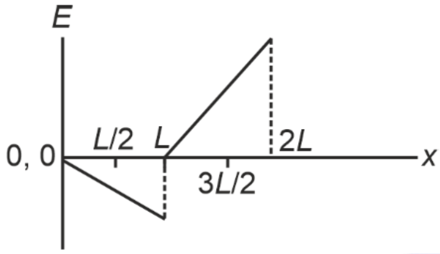
Which of the following graph best depicts the variation of the induced emf (E) in the loop as a function of the distance (x) starting from x = 0?

(A)


(B)




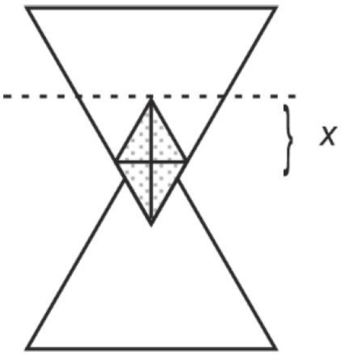
(C)



(D)

Ans. **(A)**

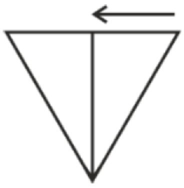
Sol. For x < L



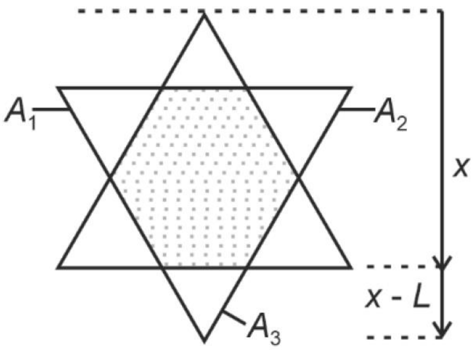
$$\text{Area } = \frac{x}{2}\frac{x}{2}\tan 30\times 4\times\frac{1}{2} = \frac{1}{2}x^2\tan 30$$

$$\phi' = B_0 x\tan 30V \qquad \varepsilon \propto x$$

$$L\tan 30^\circ$$



$x \geq L$



$$\text{Area } = A_0 - A_1 - A_2 - A_3$$

$$= A_0 - 2A_1 - (x-L)(x-L)\tan 30$$

$$= A_0 - (x-L)^2\tan 30 - \left\{L\tan 30 - (x-L)\tan 30^\circ\right\}^2 \frac{1}{2}\times\frac{1}{2}\tan 60^\circ\times 2$$

$$= A_0 - (x-L)^2 \tan 30^\circ - \tan 30^\circ \{2L - x\}^2 \frac{1}{2}$$

$$\varepsilon' = -2(x-L)\tan 30V - \tan 302(2L-x)(-)V$$

$$= (4L - x - 2x + 2L)\tan 30^\circ \text{V}$$

$$= (4L - 3x)V$$

$$= 0 \text{ at } x = \frac{4L}{3}$$

From 1 & 2

1.33 < 1.5

2. A particle of mass m is under the influence of the gravitational field of a body of mass M (>> m). The particle is moving in a circular orbit of radius r_0 with time period T_0 around the mass M. Then, the particle is subjected to an additional central force, corresponding to the potential energy $V_c(r) = m\alpha / r^3$, where α is a positive constant of suitable dimensions and r is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius r_0 in the combined gravitational potential due to M and $V_c(r)$, but with a new time period T_1, then $\left(T_1^2 - T_0^2\right)/T_1^2$ is given by [G is the gravitational constant.]

(A) $\frac{3\alpha}{GMr_0^2}$ (B) $\frac{\alpha}{2GMr_0^2}$ (C) $\frac{\alpha}{GMr_0^2}$ (D) $\frac{2\alpha}{GMr_0^2}$

Ans. **(A)**

Sol. $$\frac{Gmm}{r_0^2} - \frac{3\alpha m}{r_0^4} = \frac{mv^2}{r_0}$$

$$T = \frac{2\pi r_0}{\sqrt{\frac{Gmr_0^2 - 3\alpha}{r_0^3}}}$$

$$T_0^2 = \frac{4\pi^2}{Gm} r_0^3$$

$$\frac{T^3 - T_0^2}{T_1^2} = 1 - \frac{T_0^2}{T_1^2}$$

$$= 1 - \frac{4\pi^2}{Gm} \frac{r_0^3}{4\pi^2 r_0^2} \frac{Gmr_0^2 - 3\alpha}{r_0^3}$$

$$= 1 - 1 + \frac{3\alpha}{Gmr_0^2}$$

$$= \frac{3\alpha}{GMr_0^2}$$

3. A metal target with atomic number $Z = 46$ is bombarded with a high energy electron beam. The emission of X-rays from the target is analyzed. The ratio r of the wavelengths of the K_α-line and the cut-off is found to be $r = 2$. If the same electron beam bombards another metal target with $Z = 41$, the value of r will be

(A) 2.53 (B) 1.27 (C) 2.24 (D) 1.58

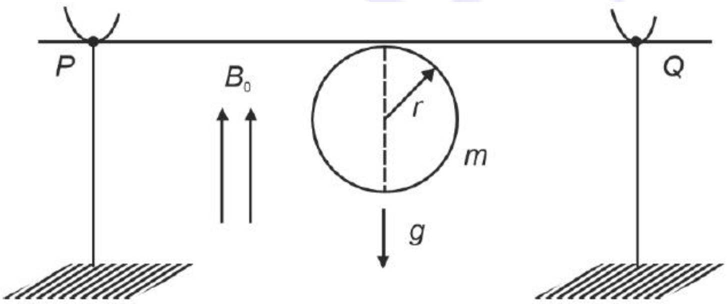
Ans. (A)

Sol. $\frac{1}{\lambda_\alpha} = \frac{3}{4}R(Z-1)^2 p \Rightarrow \lambda_{\text{cut}} = \frac{hc}{\text{eV}}$

$\Rightarrow$ Ratio $\propto \frac{1}{(Z-1)^2}$ for same beam

$\frac{Z}{x} = \frac{40^2}{45^2} \Rightarrow x = \frac{45^2}{40^2} \cdot 2 \approx 2.53$

4. A thin stiff insulated metal wire is bent into a circular loop with its two ends extending tangentially from the same point of the loop. The wire loop has mass m and radius r and it is in a uniform vertical magnetic field B_0, as shown in the figure. Initially, it hangs vertically downwards, because of acceleration due to gravity g, on two conducting supports at P and Q. When a current / is passed through the loop, the loop turns about the line PQ by an angle θ given by

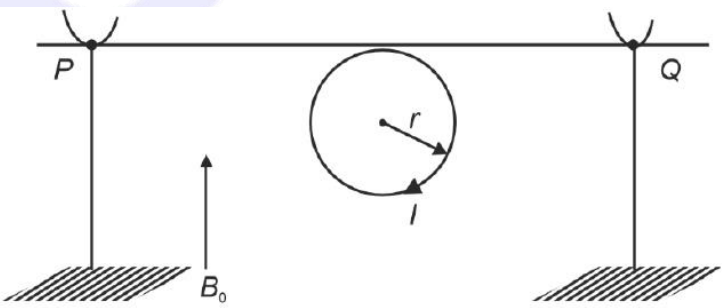


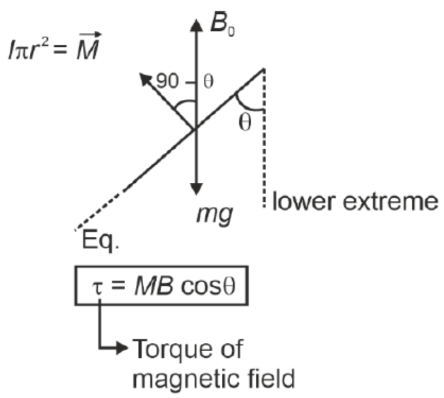
(A) $\tan\theta = \pi r / B_0 / (mg)$ (B) $\tan\theta = 2\pi r / B_0 / (mg)$

(C) $\tan\theta = \pi r / B_0 / (2mg)$ (D) $\tan\theta = mg / (\pi r l B_0)$

Ans. **(A)**

Sol.





Now $\dfrac{\text{For equilibrium}}{\tau = mgr\sin\theta}$

$$I\pi r^2 B_0 \cos\theta = mgr\sin\theta$$

$$\tan\theta = \frac{I\pi r B_0}{mg}$$

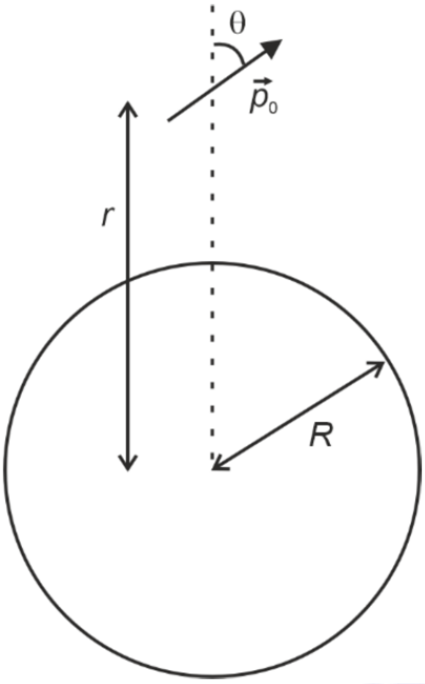
SECTION 2 (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : + 2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If unanswered;
  Negative Marks : –2 In all other cases.

---

5. A small electric dipole $\vec{p}_0$, having a moment of inertia *I* about its center, is kept at a distance r from the center of a spherical shell of radius R. The surface charge density σ is uniformly distributed on the spherical shell. The dipole is initially oriented at a small angle θ as shown in the figure. While staying at a distance r, the dipole is free to rotate about its center.



If released from rest, then which of the following statement(s) is (are) correct? [ε_0 is the permittivity of free space.]

(A) The dipole will undergo small oscillations at any finite value of r.

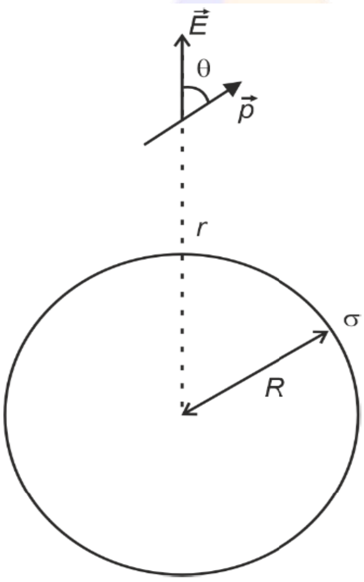
(B) The dipole will undergo small oscillations at any finite value of $r > R$.

(C) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{2\sigma p_0}{\varepsilon_0 I}}$ at $r = 2R$

(D) The dipole will undergo small oscillations with an angular frequency of $\sqrt{\frac{\sigma p_0}{100\varepsilon_0 I}}$ at $r = 10R$

Ans. **(B, D)**

Sol.



$\tau = |\vec{p} \times \vec{E}|$

$I\alpha = p_0 E \sin\theta$

$$\alpha = \frac{p \cdot \theta}{I}\left(\frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{r^2}\right) \quad \Rightarrow \quad \alpha = \left(\frac{p_0 \sigma R^2}{I\varepsilon_0 r^2}\right)\cdot\theta$$

$$\therefore \omega = \sqrt{\frac{p_0 \sigma R^2}{I\varepsilon_0 r^2}}$$

For $r = 2R$

$$\omega = \sqrt{\frac{p_0 \sigma}{4/\varepsilon_0}}$$

(C is incorrect)

Also, for $r = 10R$

$$\omega = \sqrt{\frac{p_0 \sigma}{4/(100)}}$$ (D is correct)

It will oscillate for any finite value of $r > R$. (B is correct)

6. A table tennis ball has radius $(3/2)\times 10^{-2}\,\mathrm{m}$ and mass $(22/7)\times 10^{-3}\,\mathrm{kg}$. It is slowly pushed down into a swimming pool to a depth of $d = 0.7\,\mathrm{m}$ below the water surface and then released from rest. It emerges from the water surface at speed v, without getting wet, and rises up to a height H. Which of the following option(s) is (are) correct?

[Given: $\pi = 22/7, g = 10\,\mathrm{ms}^{-2}$, density of water $= 1\times 10^3\,\mathrm{kg\,m}^{-3}$, viscosity of water $= 1\times 10^{-3}\,\mathrm{Pa-s}$.]

(A) The work done in pushing the ball to the depth d is $0.077\,\mathrm{J}$.

(B) If we neglect the viscous force in water, then the speed $v = 7\,\mathrm{m/s}$.

(C) If we neglect the viscous force in water, then the height $H = 1.4\,\mathrm{m}$.

(D) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is $500/9$.

Ans. **(A, B, D)**

Sol. Work done in pushing the ball

$W = (v\rho g)d - (v\sigma g)d$

Where $\rho \rightarrow$ Density of water

$\sigma \rightarrow$ Density of ball

$$\Rightarrow \quad W = \frac{4}{3}\pi R^3 \times 10 \times 0.7\left[1000 - \frac{3}{4}\times\frac{10^{-3}}{R^3}\right]$$

$W = 0.077\,\mathrm{J}$ [A is correct]

$\Rightarrow$ When ball is released at bottom same work (i.e. $0.077\,\mathrm{J}$) is done on ball.

$\therefore \quad \frac{1}{2}mv^2 = 0.077 \quad \Rightarrow \quad v = \sqrt{\frac{0.077 \times 2}{\frac{22}{7} \times 10^{-3}}} = 7\,\text{m/s}$

[B is correct]

$\Rightarrow$ also, $H = \frac{v^2}{2g} = \frac{7 \times 7}{2 \times 10} = 2.45\,\text{m}$[C is incorrect]

$\Rightarrow$ Net force $F_{\text{net}} = v\sigma g - v\sigma g = 0.11\,\text{N}$

Also, viscous force is maximum when $v = 7\,\text{m/s}$.

$\therefore \quad (F_v)_{\max} = 6\pi\eta rv$

$= 6 \times \frac{22}{7} \times 10^{-3}\left(\frac{3}{2} \times 10^{-2}\right) \times 7$

$= 18 \times 11 \times 10^{-5}\,\text{N}$

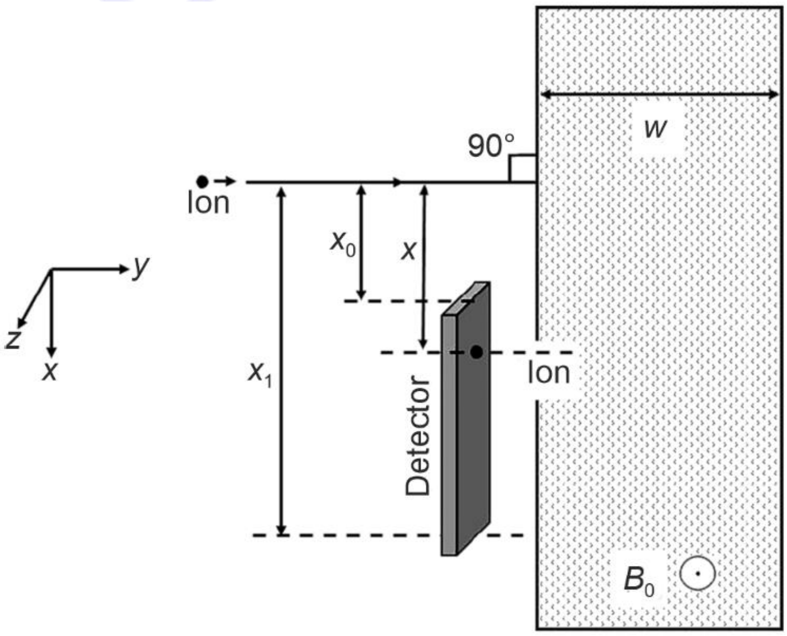
Now,

$\frac{F_{\text{net}}}{(F_v)_{\max}} = \frac{500}{9}$

[D is correct]

7. A positive, singly ionized atom of mass number A_M is accelerated from rest by the voltage $192\,\text{V}$. Thereafter, it enters a rectangular region of width w with magnetic field $\vec{B}_0 = 0.1\hat{k}$ Tesla, as shown in the figure. The ion finally hits a detector at the distance x below its starting trajectory.

[Given: Mass of neutron/proton $= (5/3) \times 10^{-27}\,\text{kg}$, charge of the electron $= 1.6 \times 10^{-19}\,\text{C}$.]



(A) The value of x for H^+ ion is 4cm.

(B) The value of x for an ion with $A_M = 144$ is 48cm.

(C) For detecting ions with $1 \le A_M \le 196$, the minimum height $(x_1 - x_0)$ of the detector is 55cm.

(D) The minimum width w of the region of the magnetic field for detecting ions with $A_M = 196$ is 56cm.

Ans. **(A, B)**

Sol. $x = 2R$

$= 2\dfrac{mv}{qB}$

$2\dfrac{\sqrt{2m(e\Delta V)}}{qB}$

For H^+ ion

x = 3.91 cm

$\simeq$ 4 cm (A is correct)

For $m = 144(m_P)$

$= 12(x_{H^+})$

= 48 cm (B is correct)

For $1 \le A_M \le 196$

$\Rightarrow (x_1 - x_0)_{\min} = 2R_{196} - 2R_1$

$= (14 \times 4) - 4$

$= 52\text{cm}$ (C is incorrect)

For $A_M = 196$

$w_{\min} = R_{196} = 28\text{cm}$ (D is incorrect)

SECTION 3 (Maximum Marks: 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If **ONLY** the correct integer is entered;

Zero Marks : 0 In all other cases.

---

8. 8. The dimensions of a cone are measured using a scale with a least count of $2\,\text{mm}$. The diameter of the base and the height are both measured to be $20.0\,\text{cm}$. The maximum percentage error in the determination of the volume is

Ans. **(3)**

Sol. $V = \frac{1}{3}\pi R^2 H \Rightarrow \frac{dV}{V} = 2\cdot\frac{dR}{R} + \frac{dH}{H}$

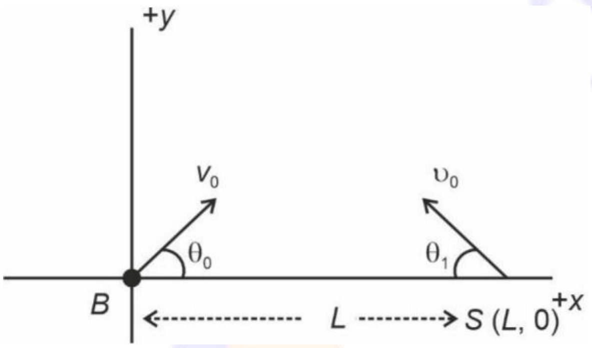
% error in measuring volume

$$= \left[2 \times \frac{0.2}{20} + \frac{0.2}{20}\right] \times 100 = 3$$

9. A ball is thrown from the location $(x_0, y_0) = (0, 0)$ of a horizontal playground with an initial speed v_0 at an angle θ_0 from the $+x$-direction. The ball is to be hit by a stone, which is thrown at the same time from the location $(x_1, y_1) = (L, 0)$. The stone is thrown at an angle $(180 - \theta_1)$ from the $+x$-direction with a suitable initial speed. For a fixed v_0, when $(\theta_0, \theta_1) = (45^\circ, 45^\circ)$, the stone hits the ball after time T_1, and when $(\theta_0, \theta_1) = (60^\circ, 30^\circ)$, it hits the ball after time T_2. In such a case, $(T_1/T_2)^2$ is

Ans. **(2)**

Sol.



Let B : Ball

S : Stone

υ_0 : Initial speed of stone.

Since relative acceleration = zero

$\Rightarrow$ Path seen would be straight line

$\Rightarrow$ To meet, $v_0 \sin\theta_0 = \upsilon_0 \sin\theta_1$

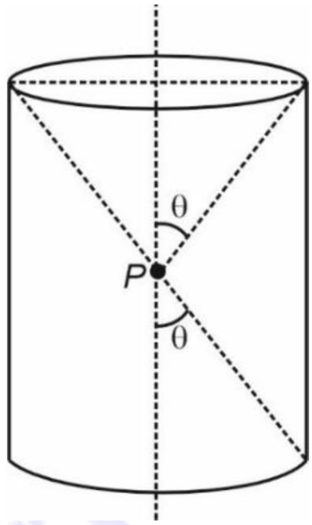
And $\Delta t = \frac{L}{\upsilon_0 \cos\theta_1 + v_0 \cos\theta_0}$

Case I: $\upsilon_0 = v_0 \Rightarrow \Delta t_1 = T_1 = \frac{L}{v_0\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]} = \frac{L}{\sqrt{2}v_0}$

Case II: $\sqrt{3}v_0 = v_0 \Rightarrow \Delta t_2 = T_2 = \dfrac{L}{\sqrt{3}v_0 \cdot \dfrac{\sqrt{3}}{2} + \dfrac{v_0}{2}} = \dfrac{L}{2v_0}$

$$\Rightarrow \left(\frac{T_1}{T_2}\right)^2 = (\sqrt{2})^2 = 2$$

10. A charge is kept at the central point P of a cylindrical region. The two edges subtend a half-angle θ at P, as shown in the figure. When $\theta = 30^\circ$, then the electric flux through the curved surface of the cylinder is Φ If $\theta = 60^\circ$, then the electric flux through the curved surface becomes $\Phi/\sqrt{n}$, where the value of n is_____.



Ans. (3)

Sol. For any θ, let us first find the flux inside a cone of half angle θ. we know that for such a cone, solid angle subtended at centre is

$\Omega = 2\pi[1 - \cos\theta]$

$\Rightarrow$ Flux through 1 cone $= \phi_0 = \dfrac{\Omega}{4\pi} \cdot \dfrac{Q}{\varepsilon_0} = \dfrac{Q}{2\varepsilon_0}[1 - \cos\theta]$

$\Rightarrow$ Flux through curved surface

$= \dfrac{Q}{\varepsilon_0} - 2\phi_0$

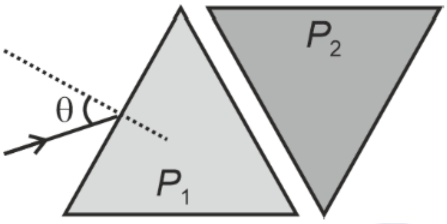
$= \dfrac{Q}{\varepsilon_0} - \dfrac{Q}{\varepsilon_0}[1 - \cos\theta] = \dfrac{Q}{\varepsilon_0}\cos\theta$

$\Rightarrow \phi = \dfrac{Q}{\varepsilon_0} \cdot \dfrac{\sqrt{3}}{2}$

And $\dfrac{\phi}{\sqrt{n}} = \dfrac{Q}{\varepsilon_0} \cdot \dfrac{1}{2}$

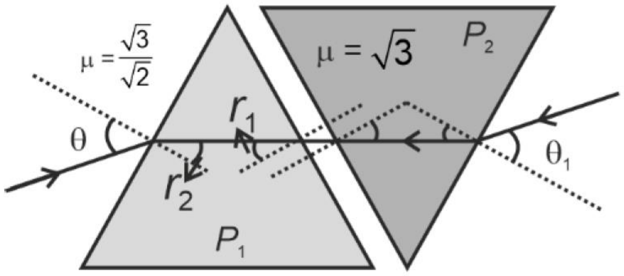
$\Rightarrow \sqrt{n} = \sqrt{3} \Rightarrow n = 3$

11. Two equilateral-triangular prisms P_1 and P_2 are kept with their sides parallel to each other, in vacuum, as shown in the figure. A light ray enters prism P_1 at an angle of incidence θ such that the outgoing ray undergoes minimum deviation in prism P_2. If the respective refractive indices of P_1 and P_2 are $\sqrt{\frac{3}{2}}$ and $\sqrt{3}, \theta = \sin^{-1}\left[\sqrt{\frac{3}{2}}\sin\left(\frac{\pi}{\beta}\right)\right]$, where the value of β is .



Ans. (12)

Sol. By using optical reversibility principle



For prism P_2

→ Minimum deviation

$$1 \times \sin\theta_1 = \sqrt{3}\sin r \qquad r_1 = r_2 = \frac{A}{2}$$

$$\sin\theta_1 = \sqrt{3} \times \frac{1}{2} \qquad r_1 = r_2 = 30^\circ$$

$$\Rightarrow i = e = 60^\circ$$

For prism P_1

Incident angle will be 60°

$$1 \times \sin 60^\circ = \frac{\sqrt{3}}{\sqrt{2}}\sin r_1$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}\sin r_1$$

$$r_1 + r_2 = 60^\circ$$

$$\sin r_1 = \frac{1}{\sqrt{2}}$$

$r_1 = 45^\circ$

$r_2 = 15^\circ$

$$\frac{\sqrt{3}}{\sqrt{2}}\sin\left(45^\circ\right) = 1\times\sin\theta$$

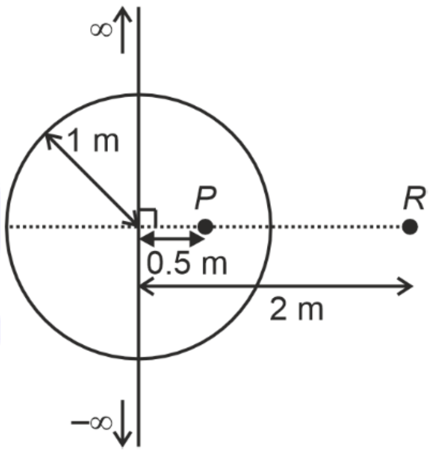
$$15^\circ = \frac{\pi\times15}{180}\text{rad} = \frac{\pi}{12}\text{rad}$$

$$\theta = \sin^{-1}\left[\frac{\sqrt{3}}{\sqrt{2}}\sin\left(\frac{\pi}{12}\right)\right]$$

$\beta = 12$

12. An infinitely long thin wire, having a uniform charge density per unit length of $5\text{nC}/\text{m}$, is passing through a spherical shell of radius 1m, as shown in the figure. A 10nC charge is distributed uniformly over the spherical shell. If the configuration of the charges remains static, the magnitude of the potential difference between points P and R, in Volt, is

[Given: $\ln$ SI units $\frac{1}{4\pi\epsilon_0} = 9\times10^9, \ln 2 = 0.7$. Ignore the area pierced by the wire.]



Ans. **(17)**

Sol. $$E_{\text{Line charge}} = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$\Rightarrow \Delta V_{\text{Line charge}} = \int_{0.5}^{2}\frac{\lambda}{2\pi\epsilon_0 r}dr = \frac{\lambda}{2\pi\epsilon_0}\ln 4 \qquad \text{.............(i)}$$

$$\Delta V_{\text{Sphere}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} - \frac{1}{4\pi\epsilon_0}\frac{Q}{2R} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2} \qquad \text{.........(ii)}$$

$$\Rightarrow \Delta V_{\text{Net}} = \frac{\lambda}{2\pi\epsilon_0}\ln 4 + \frac{1}{4\pi\epsilon_0}\frac{Q}{2} = 171\text{ Volts}$$

13. A spherical soap bubble inside an air chamber at pressure $P_0 = 10^5\,\text{Pa}$ has a certain radius so that the excess pressure inside the bubble is $\Delta P = 144\,\text{Pa}$. Now, the chamber pressure is reduced to $8P_0/27$ so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure ΔP in both the cases to be much smaller than the chamber pressure. The new excess pressure ΔP in Pa is

Ans. **(96)**

Sol. Since the situation follow isothermal condition.

$$P_1V_1 = P_2V_2$$

$$V_1 = \frac{4}{3}\pi R_1^3, V_2 = \frac{4}{3}\pi R_2^3$$

$$P_1 = P_0 + \Delta P_1, \Delta P_1 = \frac{4T}{R_1}$$

and $P_2 = \frac{8P_0}{27} + \Delta P_2, \Delta P_2 = \frac{4T}{R_2}$

So for isothermal condition

$$(P_0 + \Delta P_1)\times\frac{4}{3}\pi R_1^3 = \left(\frac{8P_0}{27} + \Delta P_2\right)\times\frac{4}{3}\pi R_2^3$$

here $P_0 = 10^5\,\text{Pa}$

$$\Delta P_1 = 144\,\text{Pa}$$

and $\Delta P_1 \ll P_0$

So $(P_0 + \Delta P_1)\left(\frac{4T}{\Delta P_1}\right)^3 = \left(\frac{8P_0}{27} + \Delta P_2\right)\left(\frac{4T}{\Delta P_2}\right)^3$

$$\frac{P_0}{(\Delta P_1)^3} \approx \frac{8P_0}{27}\times\frac{1}{(\Delta P_2)^3}$$

$$\Delta P_2 = \frac{2}{3}\Delta P_1 = \frac{2}{3}\times(144\,\text{Pa})$$

$$\Delta P_2 = 96\,\text{Pa}$$

SECTION 4 (Maximum Marks: 12)

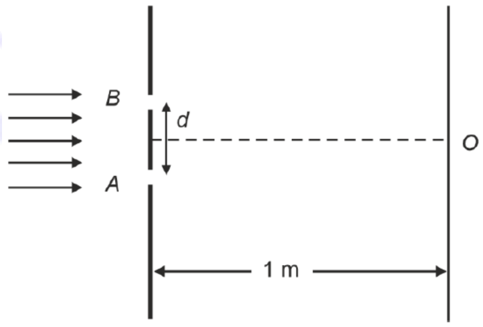
- This section contains **TWO (02)** paragraphs.
- Based on each paragraph, there are **TWO (02)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct numerical value is entered in the designated place;

Zero Marks : 0 In all other cases.

PARAGRAPH I (14 to 15)

In a Young's double slit experiment, each of the two slits A and B, as shown in the figure, are oscillating about their fixed center and with a mean separation of 0.8mm. The distance between the slits at time t is given by $d = (0.8 + 0.04\sin\omega t)\text{mm}$, where $\omega = 0.08\text{rads}^{-1}$. The distance of the screen from the slits is 1m and the wavelength of the light used to illuminate the slits is $6000\mathring{A}$. The interference pattern on the screen changes with time, while the central bright fringe (zeroth fringe) remains fixed at point O.



14. The 8^{th} bright fringe above the point O oscillates with time between two extreme positions. The separation between these two extreme positions, in micrometer (μm), is

Ans. **(601.50)**

Sol. As central bright fringe position is not changing, the two slits are oscillating with a phase diff of π.

For 8^{th} bright fringe

$$y = \frac{8\lambda D}{(0.8 + 0.04\sin\omega t)} \times 10^3$$

$$= \frac{8 \times 6000 \times 10^{-10} \times 10^{3}}{(0.8 + 0.04 \sin \omega t)}$$

$$y = \frac{48 \times 10^{-4}}{(0.8 + 0.04 \sin \omega t)}$$

d varies from 0.84 mm to 0.76 mm

$\Delta y = 6.015 \times 10^{-4}$

= 601.50 μm

15. The maximum speed in $\mu m / s$ at which the 8^{th} bright fringe will move is.

Ans. **(24.00)**

Sol. Finding speed

$$\frac{\delta y}{\delta t} = \frac{\delta}{\delta t}\left(\frac{8\lambda D}{d}\right)$$

$$= -\frac{8\lambda D}{d^2} \frac{\delta d}{(\delta t)}$$

$$v = -\frac{8\lambda D}{d^2}(0.04\omega \cos \omega t) \times 10^{-3}$$

$$v_{max} = \frac{8\lambda D}{d^2} \times 4\omega \times 10^{-5}$$

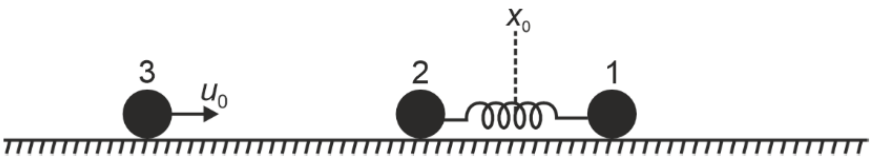
$$= \frac{8 \times 6 \times 10^{-7} \times 1 \times 4 \times 8 \times 10^{-7}}{64 \times 10^{-8}}$$

$$= 24 \times 10^{-6}$$

$$= 24 \mu m / s$$

PARAGRAPH II (16 to 17)

Two particles, 1 and 2, each of mass m, are connected by a massless spring, and are on a horizontal frictionless plane, as shown in the figure. Initially, the two particles, with their center of mass at x_0, are oscillating with amplitude a and angular frequency ω. Thus, their positions at time t are given by $x_1(t) = (x_0 + d) + a \sin \omega t$ and $x_2(t) = (x_0 - d) -$ a $\sin \omega t$, respectively, where $d > 2a$. Particle 3 of mass m moves towards this system with speed $u_0 = a\omega / 2$, and undergoes instantaneous elastic collision with particle 2, at time t_0. Finally, particles 1 and 2 acquire a center of mass speed v_{cm} and oscillate with amplitude b and the same angular frequency ω.



16. If the collision occurs at time $t_0 = 0$, the value of $v_{cm}/(a\omega)$ will be

Ans. **(00.75)**

Sol. At $t = 0, 2$ is at mean position

$\therefore u_2 = a\omega$ towards left after collision, velocity will exchange

$\therefore \quad v_2 = \dfrac{a\omega}{2}$ towards right

$u_1 = a\omega$ towards right

$\therefore \quad v_{cm} = \dfrac{3a\omega}{4}$

$\dfrac{v_{cm}}{a\omega} = \dfrac{3}{4} = 0.75$

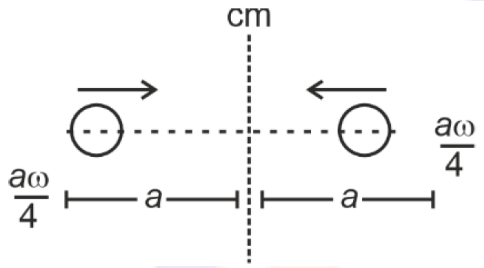
At $t = \dfrac{\pi}{2\omega}, u_2 = 0$

After collision, $v_2 = \dfrac{a\omega}{2}$ towards right

$u_1 = 0$

$\therefore \quad v_{cm} = \dfrac{a\omega}{4}$ towards right

w.r.t. centre of mass



$v = \omega\sqrt{A^2 - x^2}$

$\dfrac{a\omega}{4} = \omega\sqrt{A^2 - a^2}$

$\dfrac{a^2}{16} + a^2 = A^2$

$\dfrac{17}{16}a^2 = A^2 = b^2$

$\therefore \quad b^2 = \dfrac{17}{16}a^2$

$\dfrac{4b^2}{a^2} = \dfrac{17}{4} = 4.25$

17. If the collision occurs at time $t_0 = \pi / (2\omega)$, then the value of $4b^2 / a^2$ will be

Ans. **(04.25)**

Sol. At $t = 0, 2$ is at mean position

$\therefore \quad u_2 = a\omega$ towards left after collision, velocity will exchange

$\therefore \quad v_2 = \dfrac{a\omega}{2}$ towards right

$u_1 = a\omega$ towards right

$\therefore \quad v_{\text{cm}} = \dfrac{3a\omega}{4}$

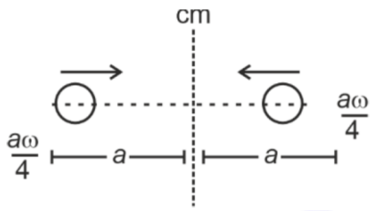
$\dfrac{v_{\text{cm}}}{a\omega} = \dfrac{3}{4} = 0.75$

At $t = \dfrac{\pi}{2\omega}, u_2 = 0$

After collision, $v_2 = \dfrac{a\omega}{2}$ towards right $u_1 = 0$

$\therefore \quad v_{\text{cm}} = \dfrac{a\omega}{4}$ towards right

w.r.t.



$V = \omega\sqrt{A^2 - x^2}$

$\dfrac{a\omega}{4} = \omega\sqrt{A^2 - a^2}$

$\dfrac{a^2}{16} + a^2 = A^2$

$\dfrac{17}{16}a^2 = A^2 = b^2$

$\therefore \quad b^2 = \dfrac{17}{16}a^2$

$\dfrac{4b^2}{a^2} = \dfrac{17}{4} = 4.25$