PAPER_1

SECTION 1 (Maximum Marks : 12)

• This section contains FOUR (04) questions.

• Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.

• For each question, choose the option corresponding to the correct answer.

• Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If ONLY the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

1. Let $f(x)$ be a continuously differentiable function on the interval $(0,\infty)$ such that $f(1)=2$ and $\lim_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$ for each $x>0$. Then, for all $x>0,\ f(x)$ is equal to

(A) $\frac{31}{11x}-\frac{9}{11}x^{10}$

(B) $\frac{9}{11x}+\frac{13}{11}x^{10}$

(C) $\frac{-9}{11x}+\frac{31}{11}x^{10}$

(D) $\frac{13}{11x}+\frac{9}{11}x^{10}$

Ans. **(B)**

Sol. $\text{limit}_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$

$\text{limit}_{t\to x}\frac{10t^9f(x)-f'(t)x^{10}}{9t^8}=1$

$\Rightarrow 10x^9f(x)-f(x)x^{10}=9x^8$

$\Rightarrow f'(x)-\frac{10}{x}f(x)=-\frac{9}{x^2}$

$\text{IF }=e^{-\int\frac{10}{x}dx}=\frac{1}{x^{10}}$

$\therefore$ Soln

$$\frac{y}{x^{10}} = \int -\frac{9}{x^{10}} \times \frac{1}{x^2} dx$$

$$= -9\int x^{-12} dx$$

$$\frac{y}{x^{10}} = \frac{9}{11} x^{-11} + C$$

$$\because y(1) = 2 \Rightarrow C = \frac{13}{11}$$

$$\Rightarrow y = \frac{9}{11x} + \frac{13}{11} x^{10}$$

2. **A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is**

(A) $\frac{1}{12}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{5}{7}$ **(D)** $\frac{5}{12}$

Answer (C)

Sol. Let $P(\text{ knows answer }) = k$

$$P\text{ (guesses) } = 1 - k$$

$$P\left(\frac{\text{correct ans}}{\text{guessed}}\right) = \frac{1}{2}$$

$$P\left(\frac{\text{guessed}}{\text{correct answer}}\right) = \frac{P(\text{ guessed })P\left(\frac{\text{correct ans}}{\text{guessed}}\right)}{P(\text{ guessed })P\left(\frac{\text{correct ans}}{\text{guessed}}\right) + P(\text{ knows })P\left(\frac{\text{correct ans}}{\text{knows}}\right)}$$

$$= \frac{(1-k)\left(\frac{1}{2}\right)}{(1-k)\left(\frac{1}{2}\right) + k(1)} = \frac{1}{6}$$

$$\Rightarrow (3-3k)=\frac{1}{2}+\frac{k}{2}$$

$$\Rightarrow \frac{5}{2}=\frac{7k}{2}\Rightarrow k=\frac{5}{7}$$

3. Let $\frac{\pi}{2}<x<\pi$ be such that $\cot x=\frac{-5}{\sqrt{11}}$. Then $\left(\sin\frac{11x}{2}\right)(\sin 6x-\cos 6x)+\left(\cos\frac{11x}{2}\right)(\sin 6x+\cos 6x)$ is equal to

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$ (B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$ (C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$ (D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

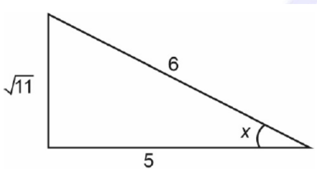
Answer (B)

Sol. Let $E=\sin 6x\cos\frac{11x}{2}-\cos 6x\sin\frac{11x}{2}+\cos 6x\cos\frac{11x}{2}+\sin 6x\sin\frac{11x}{2}$

$$E=\sin\frac{x}{2}+\cos\frac{x}{2}$$

Now, $E^2=1+\sin x$ $\quad\quad \because \cot x=\frac{-5}{\sqrt{11}}$

$$=1+\frac{\sqrt{11}}{6}$$



$$\therefore \quad E=\sqrt{\frac{6+\sqrt{11}}{6}}$$

$$=\sqrt{\frac{12+2\sqrt{11}}{12}}$$

$$=\frac{\sqrt{11}+1}{2\sqrt{3}}$$

4. Consider the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let $S(p,q)$ be a point in the first quadrant such that $\frac{p^2}{9}+\frac{q^2}{4}>1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the

minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, then which of the following options is correct?

(A) $q=2, p=3\sqrt{3}$

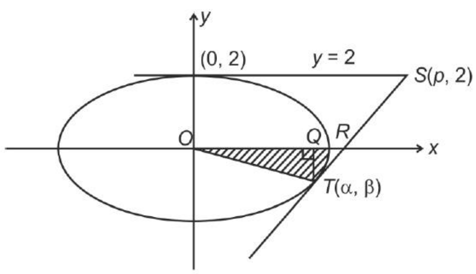
(B) $q=2, p=4\sqrt{3}$

(C) $q=1, p=5\sqrt{3}$

(D) $q=1, p=6\sqrt{3}$

Answer (A)

Sol.



q = 2

$$\text{Area }(ORT)=\frac{3}{2}$$

$$\Rightarrow\left|\frac{1}{2}\times OR\times QT\right|=\frac{3}{2}$$

$$\Rightarrow\left|\frac{1}{2}\times 3\times \beta\right|=\frac{3}{2}$$

$$\Rightarrow \beta=-1$$

$$\therefore \frac{\alpha^2}{9}+\frac{\beta^2}{4}=1$$

$$\Rightarrow \frac{\alpha^2}{9}=1-\frac{1}{4}=\frac{3}{4}$$

$$\Rightarrow \alpha^2=\frac{27}{4}\Rightarrow \alpha=\frac{3\sqrt{3}}{2}$$

Tangent at T

T = 0

$$\frac{x \cdot \frac{3\sqrt{3}}{2}}{9} + \frac{y(-1)}{4} = 1\Big|_{(p,2)}$$

$$\Rightarrow \frac{p\sqrt{3}}{6} - \frac{1}{2} = 1 \Rightarrow \frac{p\sqrt{3}}{6} = \frac{3}{2} \Rightarrow p = 3\sqrt{3}$$

$$\therefore p = 3\sqrt{3}, q = 2$$

SECTION 2 (Maximum Marks : 12)

- This section contains THREE (03) questions.
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are)

correct answer(s).

- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

FULL MARKS : +4 ONLY if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen;

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −2 In all other cases.

5. Let $S = \{a + b\sqrt{2} : a, b \in Z\}, T_1 = \left\{(-1+\sqrt{2})^n : n \in N\right\}$ and $T_2 = \left\{(1+\sqrt{2})^n : n \in N\right\}$. Then which of the following statements is (are) TRUE?

(A) $Z \cup T_1 \cup T_2 \subset S$

(B) $T_1 \cap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set

(C) $T_2 \cap (2024, \infty) \neq \phi$

(D) For any given $a, b \in Z, \cos(\pi(a + b\sqrt{2})) + i\sin(\pi(a + b\sqrt{2})) \in Z$ if and only if $b = 0$, where $i = \sqrt{-1}$

Ans (A, C, D)

Sol. $S=\{a+b\sqrt{2}:a,b\in Z\}$

For $b=0; Z\subset S$

$T_1=\left\{(-1+\sqrt{2})^n:n\in N\right\}$ and $T_2=\left\{(1+\sqrt{2})^n:n\in N\right\}$

For $n\in N$ elements of T_1 and T_2 are of the form $a+b\sqrt{2}$

Hence $Z\cup T_1\cup T_2\subset S$

- Now, $-1+\sqrt{2}<1$ and its higher powers decreases

$\Rightarrow(-1+\sqrt{2})^n<1$ and can be made in $\left(0,\dfrac{1}{2024}\right)$ for some higher n.

- $1+\sqrt{2}>1$ and its higher power increases

$\Rightarrow(1+\sqrt{2})^n$ can be made in $(2024,\infty)$ for some higher n.

- $\cos\pi(a+b\sqrt{2})+i\sin\pi(a+b\sqrt{2})\in Z$ if

$a+b\sqrt{2}$ is an integer $\Rightarrow b=0$

6. Let R^2 denote $R\times R$. Let

$S=\left\{(a,b,c):a,b,c\in R \text{ and } ax^2+2bxy+cy^2>0 \text{ for all } (x,y)\in R^2-\{(0,0)\}\right\}$.

Then which of the following statements is (are) TRUE?

(A) $\left(2,\dfrac{7}{2},6\right)\in S$

(B) If $\left(3,b,\dfrac{1}{12}\right)\in S$, then $|2b|<1$

(C) For any given $(a,b,c)\in S$, the system of linear equations $ax+by=1$ $bx+cy=-1$ has a unique solution.

(D) For any given $(a,b,c)\in S$, the system of linear equations $(a+1)x+by=0$ $bx+(c+1)y=0$ has a unique solution.

Ans **(B,C,D)**

Sol. $ax^2+2bxy+cy^2>0$

$y,x\in R-\{(0,0)\}$

$\Rightarrow c\left(\dfrac{y}{x}\right)^2+2b\left(\dfrac{y}{x}\right)+a>0$

$\Rightarrow c>0, D<0$

$4b^2-4ac<0$

$\Rightarrow b^2 < ac$

(A) $\left(2, \frac{7}{2}, 6\right)$

$\left(\frac{7}{2}\right)^2 > 2 \times 6$

$\therefore$ option A is incorrect

(B) If $\left(3, b, \frac{1}{12}\right) \in S$

$\Rightarrow b^2 < 3 \cdot \frac{1}{12}$

$\Rightarrow b^2 < \frac{1}{4}$

$\Rightarrow 4b^2 < 1$

$\Rightarrow |2b| < 1$ option

$\Rightarrow |2b| < 1$ option B is correct

(C) $ax + by = 1$

$bx + cy = -1$

D = $\begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 \neq 0$

$\therefore$ unique solution option C is correct.

(D) $(a+1)x + by = 0$

$bx + (c+1)y = 0$

$\begin{vmatrix} (a+1) & b \\ b & (c+1) \end{vmatrix}$

$= (a+1)(c+1) - b^2$

$\Rightarrow ac - b^2 + a + c + 1$

$b^2 < ac \Rightarrow ac$ is $+ve$

$\Rightarrow a$ and c are positive then $\left(ac - b^2\right) + a + c + 1 > 0$

$\therefore$ unique solution

$\therefore$ option D is correct.

7. Let R^3 denote the three-dimensional space. Take two points $P=(1,2,3)$ and $Q=(4,2,7)$. Let dist (X,Y) denote the distance between two points X and Y in R^3. Let

$S=\left\{X\in R^3:(\text{dist}(X,P))^2-(\text{dist}(X,Q))^2=50\right\}$ and

$T=\left\{Y\in R^3:(\text{dist}(Y,Q))^2-(\text{dist}(Y,P))^2=50\right\}$

Then which of the following statements is (are) TRUE?

(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48 , two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48 , two of whose vertices are from S and the other two vertices are from T.

Ans **(A,B,C)**

Sol. $S:\left\{\left((x-1)^2+(y-2)^2+(z-3)^2\right)-\left((x-4)^2+(y-2)^2+(z-7)^2\right)=50\right\}$

$\Rightarrow S:\{6x+8z-105=0\}$

Similarly $T=\{6x+8z-5=0\}$

S represents a plane. So it will contain a triangle of area 1. So (A) is correct.

T represents a plane. So (B) is correct.

ST are two parallel planes at a distance of 10 units from each other.

$\therefore$ (C) is correct and (D) is incorrect.

SECTION 3 (Maximum Marks : 24)

- This section contains SIX (06) questions.
- The answer to each question is a NON-NEGATIVE INTEGER.
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : $+4$ If ONLY the correct integer is entered;

  Zero Marks : 0 In all other cases.

8. Let $a=3\sqrt{2}$ and $b=\dfrac{1}{5^{\frac{1}{6}}\sqrt{6}}$. If $x,y\in R$ are such that

$3x+2y=\log_a(18)^{\frac{5}{4}}$ and $2x-y=\log_b(\sqrt{1080})$

then $4x+5y$ is equal to ______.

Ans **(8)**

Sol. $a=3\sqrt{2}\Rightarrow a^2=18$

Notice that $1080=5\cdot 6^3\Rightarrow$

$$5^{\frac{1}{6}}\cdot 6^{\frac{1}{2}}=(1080)^{\frac{1}{6}}=\frac{1}{b}\Rightarrow 1080^{\frac{1}{2}}=\frac{1}{b^3}$$

$$\Rightarrow 3x+2y=\log_a\left(a^2\right)^{\frac{5}{4}}=\frac{5}{2}\quad ...(i)$$

$$2x-y=\log_b\frac{1}{b^3}=\log_b b^{-3}=-3$$

$\Rightarrow$ Solving (i) & (ii)

$$\Rightarrow x=\frac{-1}{2},y=2\Rightarrow 4x+5y=8$$

9. Let $f(x)=x^4+ax^3+bx^2+c$ be a polynomial with real coefficients such that $f(1)=-9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3+3ax^2+2bx=0$, where $i=\sqrt{-1}$. If $\alpha_1,\alpha_2,\alpha_3$, and α_4 are all the roots of the equation $f(x)=0$, then $|\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2+|\alpha_4|^2$ is equal to

Ans **(20)**

Sol. $\because f(1)=-9\Rightarrow 1+a+b+c=-9$

$$4x^3+3ax^2+2bx=0$$

$$\Rightarrow x=0,\quad 4x^2+3ax+2b=0$$

$$\Rightarrow \sqrt{3}i \text{ and } -\sqrt{3}i \text{ are roots of (2)}$$

$$\Rightarrow \sqrt{3}i-\sqrt{3}i=\frac{-3a}{4},\sqrt{3}i(-\sqrt{3}i)=\frac{2b}{4}$$

$$\Rightarrow a=0,b=6,c=-16$$

$$\Rightarrow f(x)=0\Rightarrow x^4+6x^2-16=0$$

$$\Rightarrow x^2=\frac{-6\pm\sqrt{36+64}}{2}=-3\pm 5=2,-8$$

$$x=-\sqrt{2},+\sqrt{2},-2\sqrt{2}i,2\sqrt{2}i$$

$$\Rightarrow |\alpha_1|^2+|\alpha_2|^2+|\alpha_3|^2+|\alpha_4|^2=20$$

10. Let $S=\left\{A=\begin{pmatrix}0 & 1 & c\\ 1 & a & d\\ 1 & b & e\end{pmatrix}: a,b,c,d,e\in\{0,1\} \text{ and } |A|\in\{-1,1\}\right\}$, where $|A|$ denotes the determinant of A. Then the number of elements in S is ______.

Ans **(16)**

Sol. $|A|=-(e-d)+c(b-a)=\pm 1$

$\text{Case }(i): c=0\Rightarrow (e,d)=(1,0),(0,1)\rightarrow 2\text{ ways}$

Case (i) : $c=0\Rightarrow (e,d)=(1,0),(0,1)\rightarrow 2$ ways

b and a can be each 2 ways

$\Rightarrow$ Total = 8 ways

Case (ii): $c\neq 0\Rightarrow c=1$

$\Rightarrow$ d – e + b – a = $\pm$1

$$\left.\begin{matrix}1 & 1 & 1 & 0\\ 1 & 1 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{matrix}\right\}\rightarrow 4\times 2=8 \text{ ways}$$

Total = 16 ways

11. A group of 9 students, $s_1,s_2,\ldots,s_9$, is to be divided to form three teams X, Y and, Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to form such teams, is____

Ans **(665)**

Sol. Number of required ways

$$=\frac{9!}{2!3!4!}-\left(n\left(s_1\in X\right)+n\left(s_2\in Y\right)-n\left(s_1\in X \text{ and } s_2\in Y\right)\right)$$

$$=\frac{9!}{2!3!4!}-\left(\frac{8!}{1!3!4!}+\frac{8!}{2!2!4!}-\frac{7!}{1!2!4!}\right)$$

= 665

12. Let $\overrightarrow{OP}=\frac{\alpha-1}{\alpha}\hat{i}+\hat{j}+\hat{k},\overrightarrow{OQ}=\hat{i}+\frac{\beta-1}{\beta}\hat{j}+\hat{k}$ and $\overrightarrow{OR}=\hat{i}+\hat{j}+\frac{1}{2}\hat{k}$ be three vectors, where $\alpha,\beta\in R-\{0\}$ and O denotes the origin. If $(\overrightarrow{OP}\times\overrightarrow{OQ})\cdot\overrightarrow{OR}=0$ and the point $(\alpha,\beta,2)$ lies on the plane $3x+3y-z+l=0$, then the value of l is ____

Ans **(5)**

Sol. $(\overrightarrow{OP}\times\overrightarrow{OQ})\cdot\overrightarrow{OR}=0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix}=0$$

$$\Rightarrow \frac{\alpha-1}{\alpha}\left(\frac{\beta-1}{2\beta}-1\right)-\left(\frac{1}{2}-1\right)+1\left(1-\frac{\beta-1}{\beta}\right)=0$$

$$\frac{\alpha-1}{\alpha}\left(\frac{-\beta-1}{2\beta}\right)+\frac{1}{2}+\frac{1}{\beta}=0$$

$$\Rightarrow \frac{\beta+2}{2\beta}=\frac{\alpha\beta+\alpha-\beta-1}{2\alpha\beta}.$$

$$\Rightarrow \alpha\beta+2\alpha=\alpha\beta+\alpha-\beta-1$$

$$\Rightarrow \alpha+\beta+1=0 \qquad ...(1)$$

Now $(\alpha,\beta,2)$ lies on $3x+3y-z+I=0$

$$\Rightarrow 3(\alpha+\beta)-2+I=0 \qquad ...(2)$$

$$\Rightarrow -3-2+I=0 \Rightarrow I=5$$

13. Let X be a random variable, and let P(X = x) denote the probability that X takes the value x. Suppose that the points (x, P(X = x)), x = 0,1,2,3,4, lie on a fixed straight line in the xy-plane, and P(X = x) = 0 for all $x\in R-\ \{0,1,2,3,4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is ____.

Ans **(42)**

Sol. $$\sum_{x=0}^{4} xP(x)=\frac{5}{2}$$

$$\sum_{x=0}^{4} x^2P(x)=?$$

$$(0,P(0)),(1,P(1)),(2,P(2)),(3,P(3)),(4,P(4))$$

$$K=P(1)-P(0)=P(2)-P(1)=P(3)-P(2)=P(4)-P(3)$$

$$P(1)=K+P(0)$$

$$P(2)=2K+P(0)$$

$$P(3) = 3K + P(0)$$

$$P(4) = 4K + P(0)$$

$$P(0) + P(1) + P(2) + P(3) + P(4) = 1$$

$$\Rightarrow 5P(0) + 10K = 1$$

$$K + P(0) + 4K + 2P(0) + 9K + 3P(0) + 16K + 4P(0) = \frac{5}{2}$$

$$30K + 10P(0) = \frac{5}{2}$$

$$\therefore \quad 10K = \frac{1}{2}$$

$$K = \frac{1}{20}, P(0) = \frac{1}{10}$$

$$P(1) = \frac{3}{20}, P(2) = \frac{4}{20}, P(3) = \frac{5}{20}, P(4) = \frac{6}{20}$$

$$\sum_{x=0}^{4} x^2 P(x) = 8$$

$$\therefore \quad \text{Variance } = 8 - \frac{25}{4} = \frac{32-25}{4} = \frac{7}{4}$$

$$\therefore \quad 24\alpha = \frac{24 \times 7}{4} = 42$$

SECTION 4 (Maximum Marks : 12)

- This section contains FOUR (04) Matching List Sets.
- Each set has ONE Multiple Choice Question.
- Each set has TWO lists: List-I and List-II.
- List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 ONLY if the option corresponding to the correct combination is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

14. Let α and β be the distinct roots of the equation $x^2+x-1=0$. Consider the set $T=\{1,\alpha,\beta\}$. For a 3×3 matrix $M=\left(a_{ij}\right)_3\times3)$, define $R_i=a_{i1}+a_{i2}+a_\beta$ and $C_j=a_{1j}+a_{2j}+a_{3j}$ for $i=1,2,3$ and $j=1,2,3$

Match each entry in List-I to the correct entry in List-II.

	List-I		List-II
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j is	(2)	12
(R)	Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{\begin{pmatrix}x\\y\\z\end{pmatrix} : x,y.z \in R, M\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}a_{12}\\0\\-a_{23}\end{pmatrix}\right\}$ is	(3)	Infinite
(S)	Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4)	6
		(5)	0

The correct option is

(A) (P) → (4) (Q) → (2) (R) → (5) (S) → (1)

(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (5)

(C) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

(D) (P) → (1) (Q) → (5) (R) → (3) (S) → (4)

Ans. **(C)**

Sol. $x^2+x-1=0 \rightarrow$ roots are α and β

$\alpha+\beta=-1 \quad \alpha\beta=-1$

Set $T=\{1,\alpha,\beta\} M=\left(a_{ij}\right)_{3\times3}$

$R_i=a_{i1}+a_{i2}+a_{i3} \quad C_j=a_{1j}+a_{2j}+a_{3j}$ (P) R_i=C_j = 0 for all i, j

$\alpha+\beta=-1 \quad T=\{1,\alpha,\beta\}$

Number of matrices

$= \underset{\downarrow}{\lfloor 3} \times \underset{\searrow}{2} \times 1 = 12$

Number of ways to arrange 1, α, β in R_1

Number of ways to arrange 1, α, β in R_2

$$\begin{bmatrix} 1 & \alpha & \beta \\ & - & - \end{bmatrix}$$

(Q) Number of symmetric matrices = ?

$C_j = 0 \forall j$

Number of symmetric matrices

$$= \lfloor 3 \times 1 = 6 \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

(R) $M \to$ skew symmetric of 3 × 3

$|M| = 0 \quad a_{ij} \in T$ for $i > j$

$$M \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}$$

$$\begin{bmatrix} 0 & -a_{21} & -a_{31} \\ a_{21} & 0 & -a_{32} \\ a_{31} & a_{32} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{12} \\ 0 \\ -a_{23} \end{bmatrix}$$

As $x, y, z \in R$ and a_{12} $\& a_{23} \in R$ $\& |M| = 0$

$\therefore$ System has infinite solutions

\text { (S) } \begin{aligned}

$R_i = 0 \forall i$

$$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix} C_1 \to C_1 + C_2 + C_3 \, |M| = \begin{vmatrix} 1+\alpha+\beta & \alpha & \beta \\ 1+\alpha+\beta & \beta & 1 \\ 1+\alpha+\beta & 1 & \alpha \end{vmatrix} = 0$$

(P) $\to (2)(Q) \to (4)(R) \to (3)(S) \to (5)$

15. Let the straight line y =2x touch a circle with center $(0, \alpha), \alpha > 0$, and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$. Match each entry in List-I to the correct entry in List-II.

	List-I		List-II
(P)	α equals	(1)	(–2, 4)
(Q)	r equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	(–2, 6)
(S)	B_1 equals	(4)	5
		(5)	(2, 4)

The correct option is

(A) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

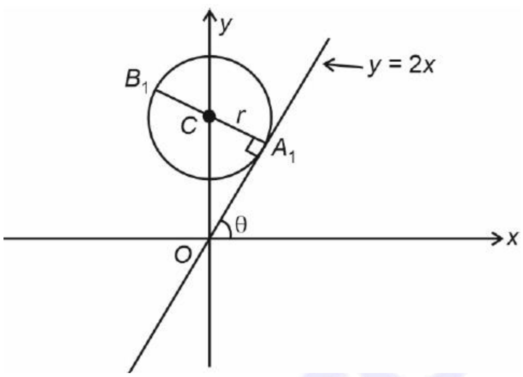
(B) (P) → (2) (Q) → (4) (R) → (1) (S) → (3)

(C) (P) → (4) (Q) → (2) (R) → (5) (S) → (3)

(D) (P) → (2) (Q) → (4) (R) → (3) (S) → (5)

Ans. **(C)**

Sol.



Slope of line $= 2 \Rightarrow \tan\theta = 2$

$C(0,\alpha) \quad \alpha > 0$

$\alpha + r = 5 + \sqrt{5}$

Line $y = 2x$ is tangent to the circle

$\therefore \left|\frac{0-\alpha}{\sqrt{4+1}}\right| = r$

$\Rightarrow |-\alpha| = r\sqrt{5}$

$\Rightarrow \alpha = r\sqrt{5} \quad$ as $\alpha > 0$

From equation (1) $r\sqrt{5} + r = 5 + \sqrt{5}$

$\Rightarrow \quad r(\sqrt{5}+1) = \sqrt{5}(\sqrt{5}+1)$

$\Rightarrow \quad r = \sqrt{5}$

And $\alpha = r\sqrt{5} = \sqrt{5}\times\sqrt{5} = 5$

Centre $C(0,5)$

OC = 5 $\qquad A_1C = \sqrt{5}$

$\therefore \quad OA_1 = \sqrt{25-5} = \sqrt{20} = 2\sqrt{5}$

$\tan\theta = 2$ (from figure)

$\cos\theta = \dfrac{1}{\sqrt{5}} \quad \sin\theta = \dfrac{2}{\sqrt{5}}$

$A_1\left(0+OA_1\cos\theta, 0+OA_1\sin\theta\right)$

$A_1\left(2\sqrt{5}\times\dfrac{1}{\sqrt{5}}, 2\sqrt{5}\times\dfrac{2}{\sqrt{5}}\right)$

$A_1(2,4)$

Let $B_1(x_1, y_1)$

$\therefore \quad \dfrac{x_1+2}{2} = 0$ and $\dfrac{y_1+4}{2} = 5$

$x_1 = -2$

$B_1(-2,6) \quad y_1 = 6$

$\alpha = 5 \quad r = \sqrt{5} \quad A_1(2,4) \quad B_1(-2,6)$

16. Let $\gamma \in R$ be such that the lines $L_1 : \dfrac{x+11}{1} = \dfrac{y+21}{2} = \dfrac{z+29}{3}$ and $L_2 : \dfrac{x+16}{3} = \dfrac{y+11}{2} = \dfrac{z+4}{\gamma}$ intersect. Let R_1 be the point of intersection of L_1 and L_2. Let $O=(0,0,0)$, and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2.

Match each entry in List-I to the correct entry in List-II.

	List-I		List-II
(P)	γ equals	(1)	$-\hat{i}-\hat{j}+\hat{k}$
(Q)	A possible choice for $\hat{n}$ is	(2)	$\sqrt{\frac{3}{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1}\cdot\hat{n}$ is	(4)	$\frac{1}{\sqrt{6}}\hat{i}-\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\frac{2}{3}}$

The correct option is

(A) (P) → (3) (Q) → (4) (R) → (1) (S) → (2)

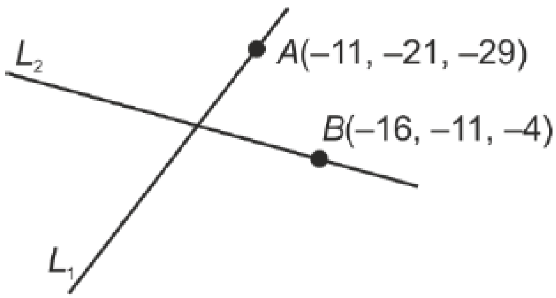
(B) (P) → (5) (Q) → (4) (R) → (1) (S) → (2)

(C) (P) → (3) (Q) → (4) (R) → (1) (S) → (5)

(D) (P) → (3) (Q) → (1) (R) → (4) (S) → (5)

Ans **(C)**

Sol. Vector parallel to the line $L_1\left(\text{ say } \vec{b}_1\right) == \hat{i} + 2\hat{i} + 3\hat{k}$



Normal vector of plane $(\vec{n})$ containing L_1 and L_2 will be perpendicular to both $\vec{b}_1$ and $\overrightarrow{AB}$

$$\Rightarrow \vec{n} = p(\overrightarrow{AB} \times \vec{n}) = p(5\hat{i} - 10\hat{j} - 25\hat{k}) \times (i + 2\hat{j} + 3\hat{k})$$

$$= p(20\hat{i} - 40\hat{j} + 20\hat{k})$$

$$\Rightarrow \hat{n} = \frac{1}{\sqrt{6}}\hat{i} - \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

Now, vector parallel to L_2 (say $\vec{b}_2$) is perpendicular to $\vec{n} \Rightarrow \vec{b}_2 \cdot \vec{n} = 0$

$$(3\hat{i} + 2\hat{j} + \gamma\hat{k}) \cdot p(20\hat{i} - 40\hat{j} + 20\hat{k}) = 0$$

$$\Rightarrow \gamma = 1$$

Now, for point of intersection (POI)

$$L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = \lambda \text{ and } L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma} = u$$

Comparing x and y coordinates, $-11+\lambda = -16+3u$ and $-21+2\lambda = -11+2u$ $\Rightarrow \lambda = 10, u = 5$

$\Rightarrow$ POI i.e., $\overrightarrow{OR_1} : (-\hat{i} - \hat{j} + \hat{k})$ and $\overrightarrow{OR} \cdot \hat{n} = \sqrt{\frac{2}{3}}$

17. Let $f: R \to R$ and $g: R \to R$ be functions defined by $f(x)=\begin{cases} x|x|\sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x=0, \end{cases}$ and

$$g(x)=\begin{cases} 1-2x, & 0 \le x \le \frac{1}{2}, \\ 0, & \text{otherwise} \end{cases}$$

Let $a, b, c, d \in R$. Define the function $h: R \to R$ by

$$h(x)=af(x)+b\left(g(x)+g\left(\frac{1}{2}-x\right)\right)+c(x-g(x))+dg(x), x \in R$$

Match each entry in List-I to the correct entry in List-II.

	List-I		List-II
(P)	If *a* = 0, *b* = 1, *c* = 0 and *d* = 0, then	(1)	*h* is one-one
(Q)	If *a* = 1, *b* = 0, *c* = 0 and *d* = 0, then	(2)	*h* is onto.
(R)	If *a* = 0, *b* = 0, *c* = 1 and *d* = 0, then	(3)	*h* is differentiable on $\mathbb{R}$.
(S)	If *a* = 0, *b* = 0, *c* = 0 and *d* = 1, then	(4)	the range of *h* is [0, 1]
		(5)	the range of *h* is {0, 1}

The correct option is

(A) (P) → (4) (Q) → (3) (R) → (1) (S) → (2)

(B) (P) → (5) (Q) → (2) (R) → (4) (S) → (3)

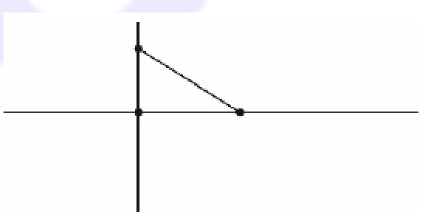
(C) (P) → (5) (Q) → (3) (R) → (2) (S) → (4)

(D) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

Ans **(C)**

Sol.

$$g(x)=\begin{cases} 1-2x, & 0 \le x \le \frac{1}{2} \\ 0, & otherwise \end{cases}$$



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