

命题逻辑

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灵感来源于生活

- 人是动物
 - 小孩是人
 - 小孩是动物?
-
- 如果今天下雨我就在家里
 - 今天下雨
 - 我在家里

灵感来源于生活

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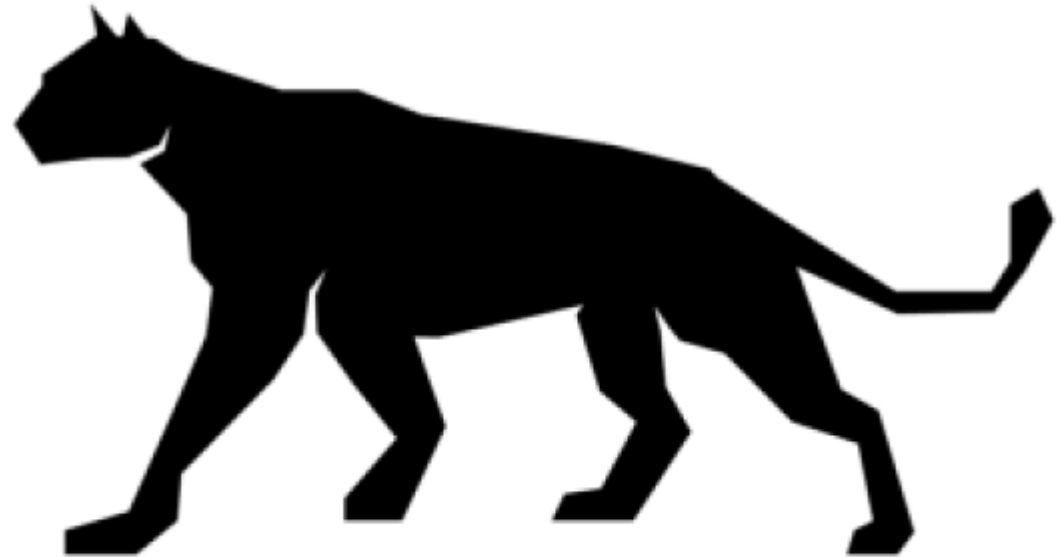
我们是如何做推理的?

剧本杀初级版

- 爱丽丝、爱丽丝的丈夫、他们的儿子、女儿和爱丽丝的兄弟卷入了一起谋杀案。五人中的一人杀死了另外四人中的一人。以下事实与提到的五个人有关：
 1. 案发时，一男一女在一家酒吧里。
 2. 谋杀案发生时，受害者和凶手一起在海滩上。
 3. 谋杀发生时，爱丽丝的两个孩子中的一个独自一人。
 4. 爱丽丝和她的丈夫在谋杀发生时并不在一起。
 5. 受害者的双胞胎不是凶手。
 6. 凶手比受害者年轻。
- 五人中的哪一位是受害者？

The Lady and the Tiger

- Two doors containing either Ladies or Tigers



The Lady and the Tiger

- You will be shown two doors, to two rooms.
 - Each could contain either a lady or a tiger ...
 - It could be that **both** rooms contain a lady, or that **both** rooms contain a tiger!

The Lady and the Tiger

Q1

One of these is true...

**In this room, there is a lady,
and in the other room there is
a tiger.**

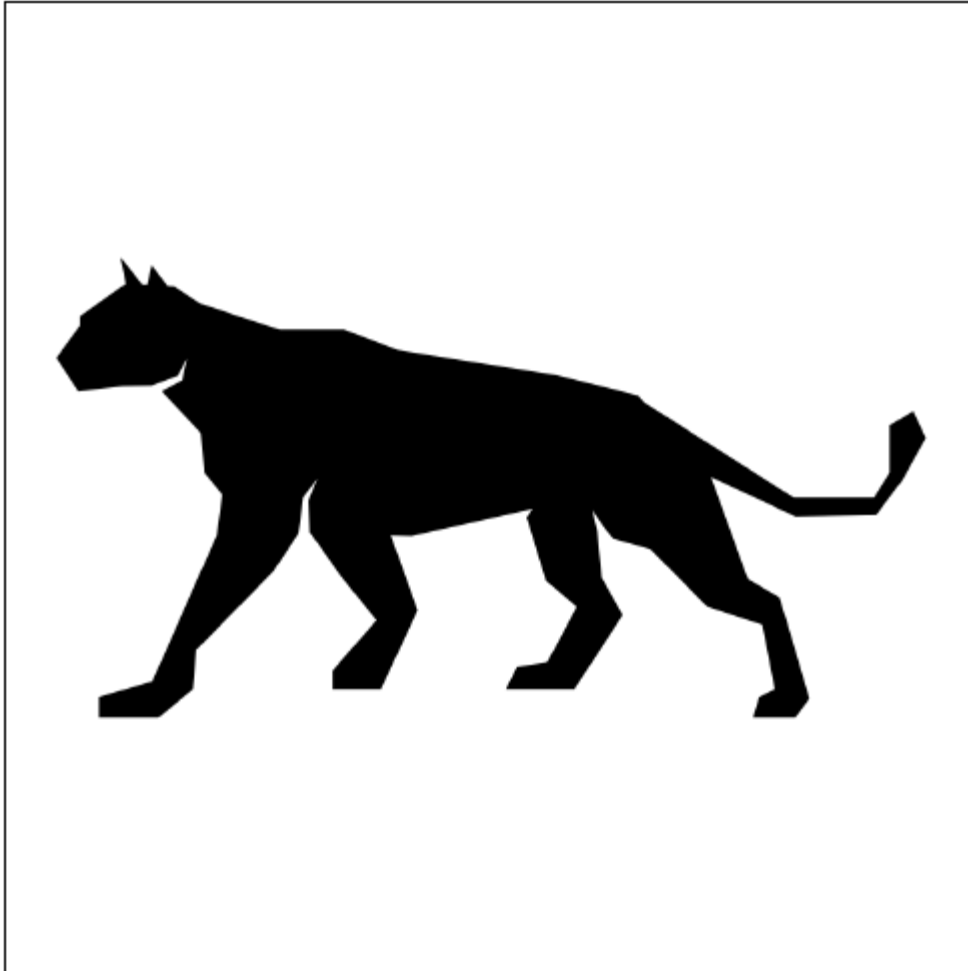
The other is false...

**In one of these rooms there is
a lady, and in one of these
rooms there is a tiger.**

The Lady and the Tiger

Q1

One of these is true...



The other is false...



布尔逻辑公式

1. 一个布尔原子 $p \in \{0,1\}$ 是一个布尔逻辑公式;
2. 如果 φ 是一个布尔逻辑公式, 那么 $(\varphi), \neg\varphi, \varphi \& \varphi, \varphi | \varphi, \varphi \Rightarrow \varphi, \varphi \Leftrightarrow \varphi$ 也是布尔逻辑公式。

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$a | (b \& c)$

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$$a | (b \& c)$$

$$(a | c) \& (\neg a | \neg c)$$

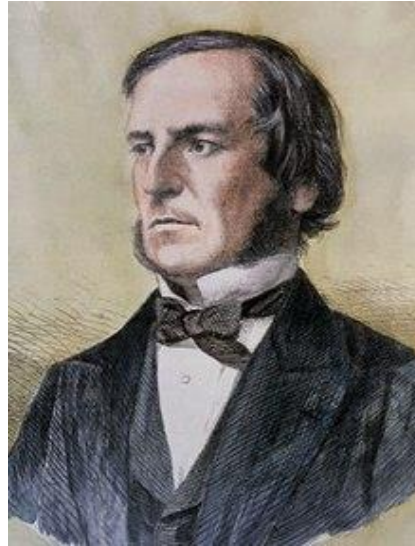
思考

给定一个字符串，如何判定它是否是一个布尔逻辑公式？

布尔逻辑的历史



G. W. Leibniz
(1646 - 1716)



George Boole
(1815 - 1864)



Augustus De Morgan
(1806 - 1871)

The craters Boole and De Morgan on the Moon are named after them.

布尔逻辑公式的解

给布尔公式中每个变量赋值0或1，若可以使得公式最后赋值为1，那么变量赋值的组合就称为它的一个解。

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$a \mid (b \ \& \ c)$

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$a \mid (b \ \& \ c)$

$\{a=1, b = 1, c = 1\}$

布尔逻辑公式的解

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$$a \mid (b \ \& \ c)$$

$$\{a=1, b = 1, c = 1\}$$

$$(a \mid c) \ \& \ (\neg a \mid \neg c)$$

布尔逻辑公式的解

给布尔公式中每个变量赋值0或1，若可以使得公式最后赋值为1，那么变量赋值的组合就称为它的一个解。

$$a \mid (b \ \& \ c)$$

$$\{a=1, b = 1, c = 1\}$$

$$(a \mid c) \ \& \ (\neg a \mid \neg c)$$

$$\{a=1, c=0\}$$

课堂练习

请给出一下布尔公式的一个解：

(1) $(a \mid b) \& (!a \mid !b)$

(2) $(a \& !b) \rightarrow (b \& !c)$

(3) $(a \& b) \rightarrow ((b \mid !c) \& (a \& !b \& c))$

布尔公式求解

| a | b | c | $a \mid (b \ \& \ c)$ |
|---|---|---|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

真值表法：采用穷举思想

布尔公式求解

| a | b | c | $a \mid (b \ \& \ c)$ |
|---|---|---|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

真值表法：采用穷举思想

状态空间
爆炸!

练习

用真值表给出布尔逻辑公式 $(a \ \& \ b) \rightarrow ((b \mid !c) \ \& \ (a \ \& \ !b \ \& \ c))$ 的解。

思考题

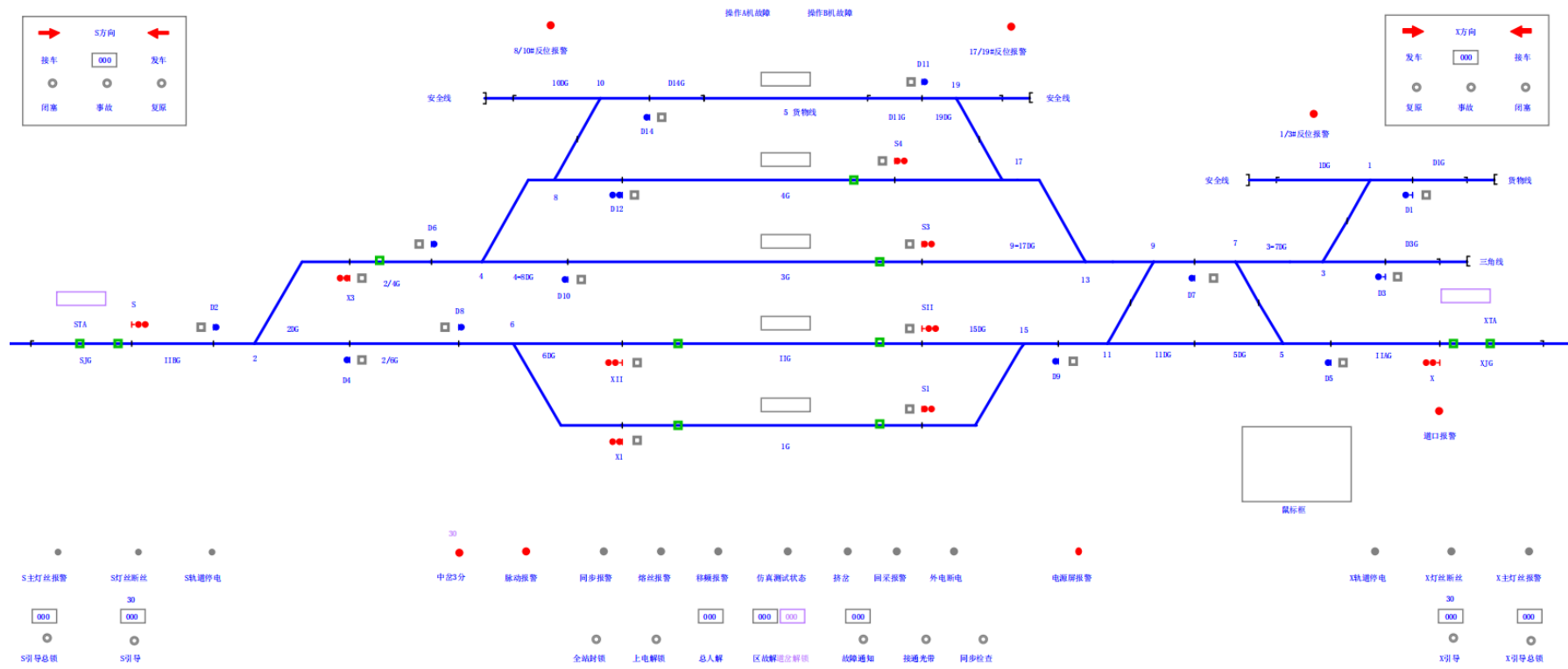
- 如何求解一个布尔公式的所有解？
- 如何高效的求解一个布尔公示的所有解？

从布尔逻辑公式到硬件电路 (1940)



底层电路都是布尔逻辑。

信号系统



The Lady and the Tiger

Q2

Either both signs are false...

**At least one of these rooms
contains a lady**

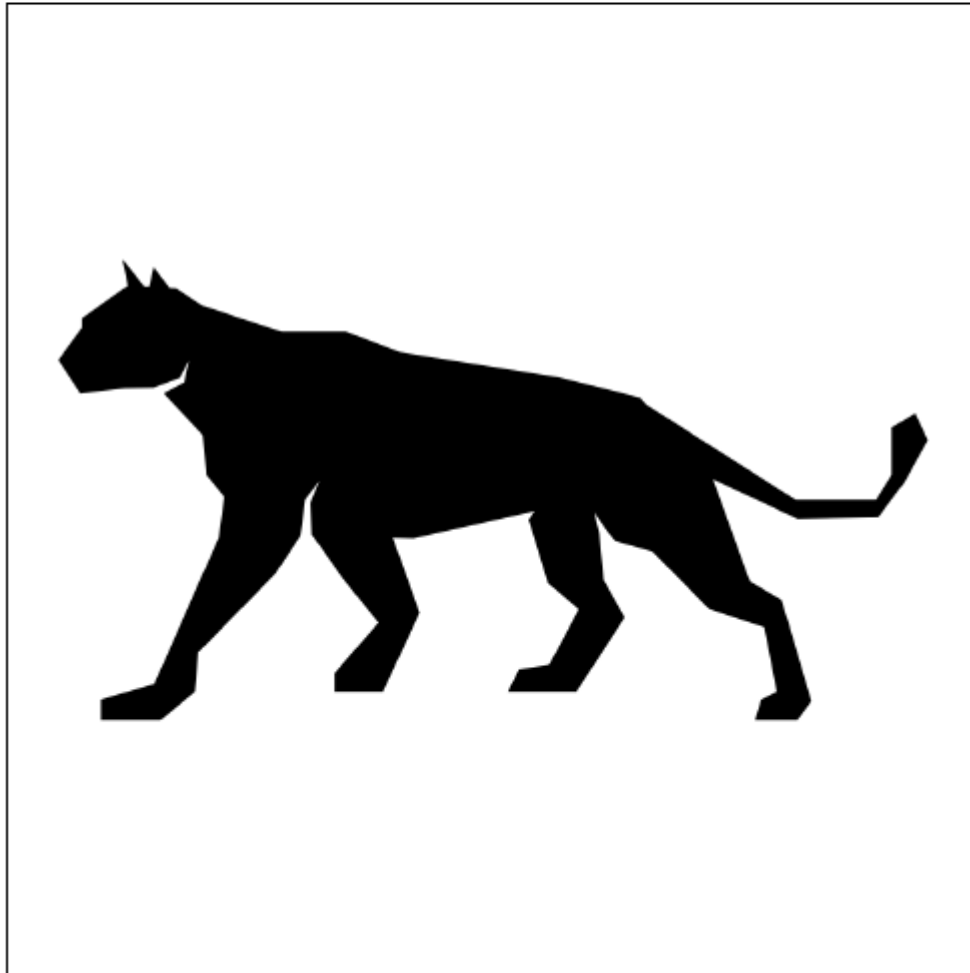
Or both are true...

A tiger is in the other room...

The Lady and the Tiger

Q2

Either both signs are false...



Or both are true...



The Lady and the Tiger

Q3

Either both signs are false...

**Either a tiger is in this room,
or a lady is in the other room.**

Or both are true...

An lady is in the other room.

The Lady and the Tiger

Q3

Either both signs are false...



Or both are true...



The Lady and the Tiger

- Once again, you'll have to tell Ladies from Tigers
- A new twist is being added.
 - Two doors
 - If a lady is in Room I, then the sign on the door is true. If a tiger is in Room I, then the sign on the door is false.
 - The opposite is true for Room II.
- **Problem... the signs fell off the doors. We can't remember which one goes where!!!**

The Lady and the Tiger

Q1

Room ??

This room
contains a
tiger.

Room ??

Both rooms
contain tigers

The Lady and the Tiger

Q1

Room II



Room I



The Lady and the Tiger

- Once again, you'll have to tell Ladies from Tigers
- A new twist is being added.
 - **Three** doors!
 - One lady, **TWO** tigers
 - At most one of the signs is true

The Lady and the Tiger

Q2

Room I

A tiger is in this
room.

Room II

A lady is in this
room.

Room III

A tiger is in room II.

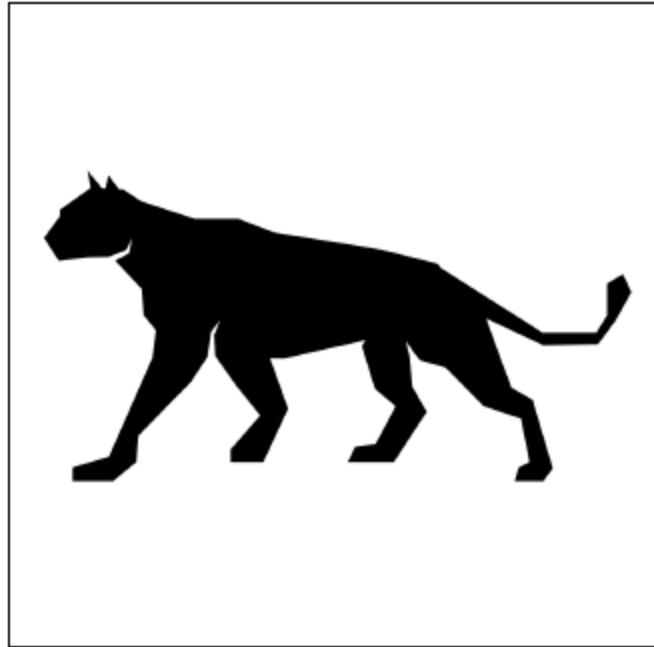
The Lady and the Tiger

Q2

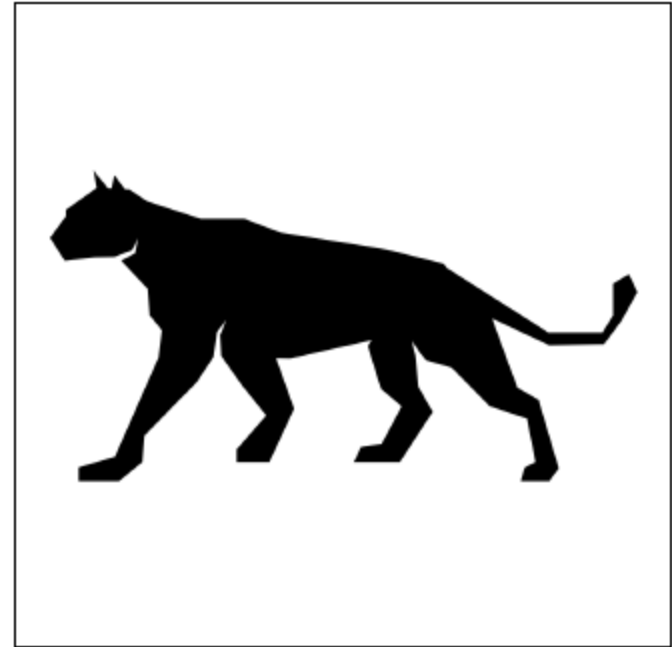
Room I



Room II



Room III



The Lady and the Tiger

- Once again, you'll have to tell Ladies from Tigers
- A new twist is being added.
 - Three doors!
 - One lady, TWO tigers
 - The sign on the door of the room with the lady is true. At least one of the other two signs is false!

The Lady and the Tiger

Q3

Room I

A tiger is in room II.

Room II

A tiger is in this
room.

Room III

A tiger is in room I.

截图(Alt + A)

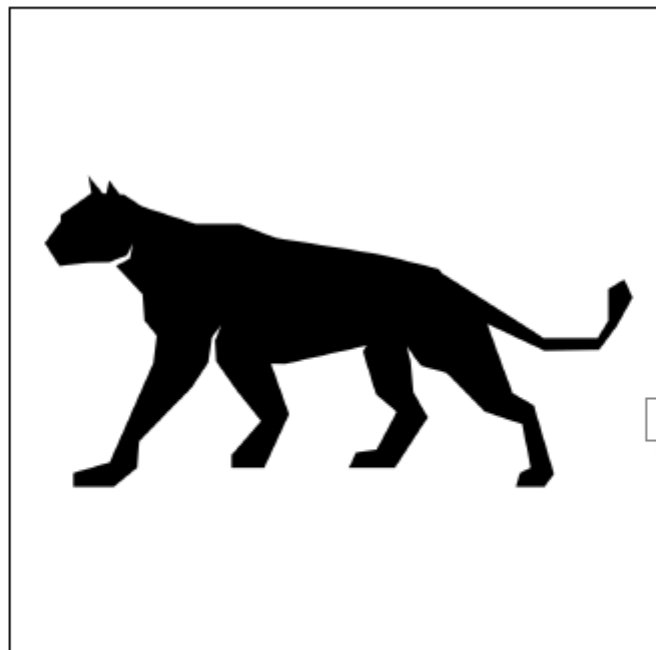
The Lady and the Tiger

Q3

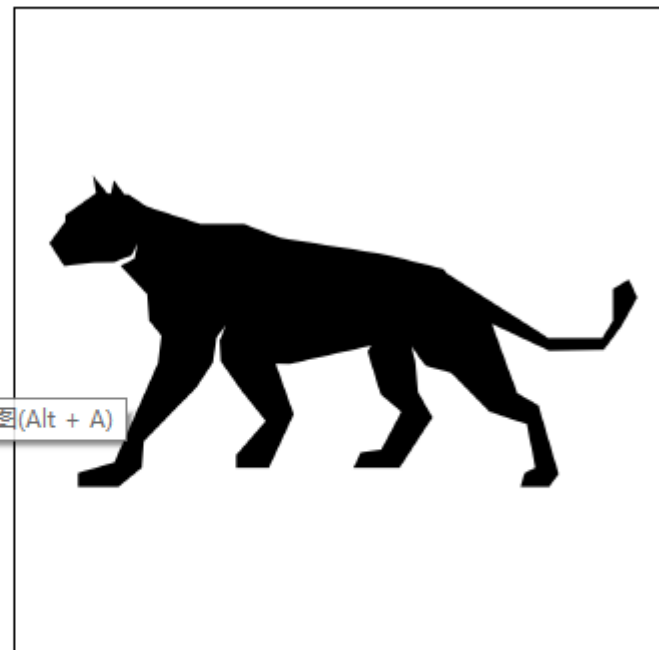
Room I



Room II



Room III



截图(Alt + A)

The Lady and the Tiger

- Only one room contains a lady. Every other room has a tiger, or is empty.
- The sign of the door with the lady is true. The signs of all rooms with tigers are false. Empty room's signs can be true or false.
- This problem would be unsolvable, however, if you knew if Room VIII was empty or not, you'd be able to figure out which room had the lady.

Room I

The lady is in
an odd
numbered
room.

Room II

This room is
empty.

Room III

Either Sign V is
right or Sign VII
is wrong.

Room IV

Sign I is wrong.

Room V

Either Sign II or
Sign IV is right.

Room VI

Sign III is
wrong.

Room VII

The lady is not
in Room I

Room VIII

This room
contains a tiger
and room IX is
empty.

Room IX

This room
contains a tiger
and VI is
wrong.

(=)

等价公式

- 矛盾律: $A \wedge !A = \text{false}$
- 排中律: $A \vee !A = \text{true}$
- 零律: $!A \wedge \text{false} = \text{false}, \quad A \vee \text{true} = \text{true}$
- 同一律: $A \vee \text{false} = A, \quad A \wedge \text{true} = A$
- 双重否定律: $!!A = A$
- 等幂律: $A \wedge A = A, \quad A \vee A = A$

等价公式

- 分配律: $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$
- $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$
- 结合律: $A \wedge (B \wedge C) = (A \wedge B) \wedge C$
- $A \vee (B \vee C) = (A \vee B) \vee C$
- 交换律: $A \vee B = B \vee A, \quad A \wedge B = B \wedge A$
- 德摩根律: $!(A \vee B) = !A \wedge !B, \quad !(A \wedge B) = !A \vee !B$
- 吸收律: $A \vee (A \wedge B) = A, \quad A \wedge (A \vee B) = A$

等价公式

- 蕴含等值律: $A \rightarrow B = \neg A \vee B$
- 等价等值律: $A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A)$
- 假言律: $A \rightarrow B = \neg B \rightarrow \neg A$
- 等价否定律: $A \leftrightarrow B = \neg A \leftrightarrow \neg B$
- 附加律: $A \Rightarrow A \vee B$
- 化简律: $A \wedge B \Rightarrow A$

课堂练习

- 证明: $(A \rightarrow B) \wedge (B \rightarrow C) \Rightarrow (A \rightarrow C)$ (假言三段论)
- 化简: $(A \vee C) \wedge ((A \wedge D) \vee (!A \wedge D)) \vee (A \wedge C) \vee C = ?$
- 化简: $A \vee (!A \wedge B) \vee (!A \wedge !B \wedge C) \vee (!A \wedge !B \wedge !C \wedge D) \vee (!A \wedge !B \wedge !C \wedge !D \wedge E) = ?$

合取范式 (CNF)和析取范式 (DNF)

- 合取范式

具有如 $\bigwedge (\bigvee p)$ 形式的布尔逻辑公式

- 析取范式

具有如 $\bigvee (\bigwedge p)$ 形式的布尔逻辑公式

合取范式 (CNF)和析取范式 (DNF)

原公式: $(a \mid b) \& (c \mid (b \& d))$

合取范式CNF: $(a \mid b) \& (c \mid b) \& (c \mid d)$

析取范式DNF: $(a \& c) \mid (a \& b \& d) \mid (b \& c) \mid (b \& d)$

合取范式 (CNF)和析取范式 (DNF)

- $a \& b \& c$ 是CNF吗? 是DNF吗?

合取范式 (CNF)和析取范式 (DNF)

- $a \& b \& c$ 是CNF吗? 是DNF吗?
- $a \mid b \mid c$ 是CNF吗? 是DNF吗?

合取范式 (CNF)和析取范式 (DNF)

- $a \& b \& c$ 是CNF吗? 是DNF吗?
- $a \mid b \mid c$ 是CNF吗? 是DNF吗?
- $(a \& b) \mid c$ 是CNF吗? 是DNF吗?

合取范式 (CNF)和析取范式 (DNF)

- $a \& b \& c$ 是CNF吗? 是DNF吗?
- $a \mid b \mid c$ 是CNF吗? 是DNF吗?
- $(a \& b) \mid c$ 是CNF吗? 是DNF吗?
- $(a \mid b) \& (b \mid (c \& d))$ 是CNF吗? 是DNF吗?

思考题

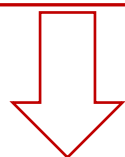
给定任意一个布尔逻辑公式，如何转成它对应的合取（析取）范式？

合取范式 (CNF)和析取范式 (DNF)

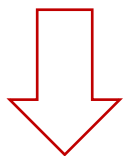
- $(a \mid b) \ \& \ \underline{(b \mid (c \ \& \ d))}$

合取范式 (CNF)和析取范式 (DNF)

- $(a \mid b) \ \& \ (b \mid (c \ \& \ d))$



$$(b \mid c) \ \& \ (b \mid d)$$



$$(a \mid b) \ \& \ (b \mid c) \ \& \ (b \mid d)$$

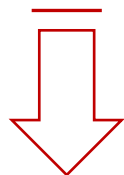
合取范式

合取范式 (CNF)和析取范式 (DNF)

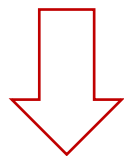
- $(a \mid b) \text{ \underline{\&} } (b \mid (c \& d))$

合取范式 (CNF)和析取范式 (DNF)

- $(a \mid b) \& (b \mid (c \& d))$



$$(a \& (b \mid (c \& d))) \mid (b \& (b \mid (c \& d)))$$



$$(a \& b) \mid (a \& c \& d) \mid b \mid (b \& c \& d)$$

析取范式

课堂练习

求下列公式的合取范式和析取范式

- $(A \rightarrow !B) \leftrightarrow C$
- $(A \leftrightarrow B) \rightarrow (B \wedge C)$

析取范式 (DNF)和解的关系

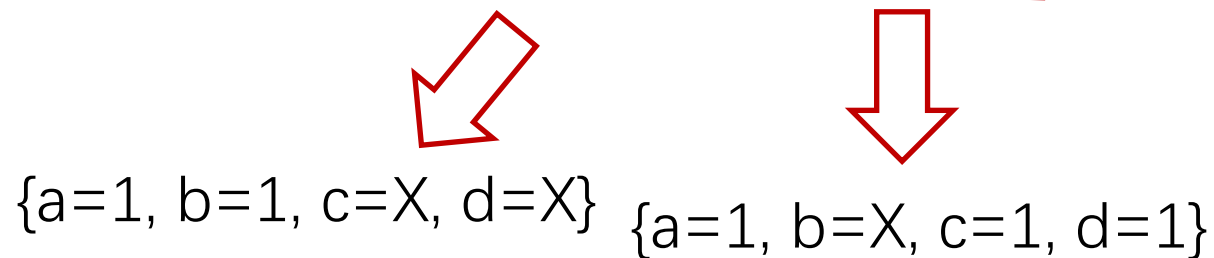
$$(a \& b) \mid (a \& c \& d) \mid b \mid (b \& c \& d)$$



$\{a=1, b=1, c=X, d=X\}$

析取范式 (DNF)和解的关系

$$(a \ \& \ b) \mid (a \ \& \ c \ \& \ d) \mid b \mid (b \ \& \ c \ \& \ d)$$



析取范式 (DNF)和解的关系

$$(a \mid b) \& (b \mid (c \& d)) \quad \longrightarrow \quad (a \& b) \mid (a \& c \& d) \mid b \mid (b \& c \& d)$$

长度指数级增长!



合取范式 (CNF)和解的关系

$(a \mid b) \& (b \mid c) \& (b \mid d)$



合取范式 (CNF)和解的关系

$(a \mid b) \& (b \mid c) \& (b \mid d)$



主流的SAT求解器都是基于CNF输入。

合取范式 (CNF)和解的关系



$(a \mid b) \ \& \ (b \mid (c \ \& \ d))$



$(a \mid b) \ \& \ (b \mid c) \ \& \ (b \mid d)$

合取范式 (CNF)和解的关系

$(a \mid b) \& (b \mid (c \& d))$  $(a \mid b) \& (b \mid c) \& (b \mid d)$

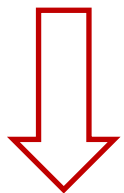
长度指数级增长!

Tseitin 转换(1968)

$(a \mid b) \ \& \ (b \mid (c \ \& \ d))$

Tseitin 转换(1968)

$(a \mid b) \& (b \mid (c \& d))$



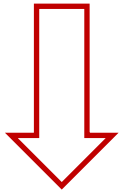
$x1 \& x2 \&$

$x1 \Leftrightarrow (a \mid b) \&$

$x2 \Leftrightarrow (b \mid (c \& d))$

Tseitin 转换(1968)

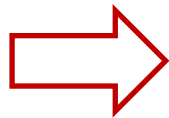
$(a \mid b) \& (b \mid (c \& d))$



$x_1 \& x_2 \&$

$x_1 \Leftrightarrow (a \mid b) \&$

$x_2 \Leftrightarrow (b \mid (c \& d))$



$x_1 \& x_2 \&$

$x_1 \Leftrightarrow (a \mid b) \&$

$x_2 \Leftrightarrow (b \mid x_3) \&$

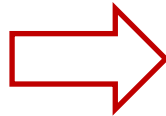
$x_3 \Leftrightarrow c \& d$

Tseitin 转换(1968)

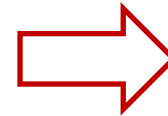
$(a \mid b) \& (b \mid (c \& d))$



$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid (c \& d))$



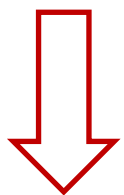
$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid x_3) \&$
 $x_3 \Leftrightarrow c \& d$



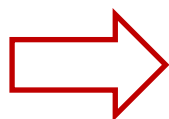
$x_1 \& x_2 \&$
 $(\neg x_1 \mid a \mid b) \&$
 $(\neg a \mid x_1) \&$
 $(\neg b \mid x_1) \&$
 $x_2 \Leftrightarrow (b \mid x_3) \&$
 $x_3 \Leftrightarrow c \& d$

Tseitin 转换(1968)

$(a \mid b) \& (b \mid (c \& d))$



$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid (c \& d))$



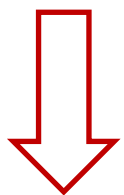
$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid x_3) \&$
 $x_3 \Leftrightarrow c \& d$



$x_1 \& x_2 \&$
 $(\neg x_1 \mid a \mid b) \&$
 $(\neg a \mid x_1) \&$
 $(\neg b \mid x_1) \&$
 $(\neg x_2 \mid b \mid x_3) \&$
 $(\neg b \mid x_2) \&$
 $(\neg x_3 \mid x_2) \&$
 $x_3 \Leftrightarrow c \& d$

Tseitin 转换(1968)

$(a \mid b) \& (b \mid (c \& d))$



$x_1 \& x_2 \&$

$x_1 \Leftrightarrow (a \mid b) \&$

$x_2 \Leftrightarrow (b \mid (c \& d))$



$x_1 \& x_2 \&$

$x_1 \Leftrightarrow (a \mid b) \&$

$x_2 \Leftrightarrow (b \mid x_3) \&$

$x_3 \Leftrightarrow c \& d$



$x_1 \& x_2 \&$

$(\neg x_1 \mid a \mid b) \&$

$(\neg a \mid x_1) \&$

$(\neg b \mid x_1) \&$

$(\neg x_2 \mid b \mid x_3) \&$

$(\neg b \mid x_2) \&$

$(\neg x_3 \mid x_2) \&$

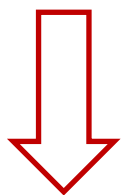
$(x_3 \mid \neg c \mid \neg d) \&$

$(\neg x_3 \mid c) \&$

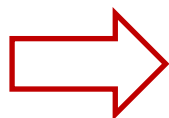
$(\neg x_3 \mid d)$

Tseitin 转换(1968)

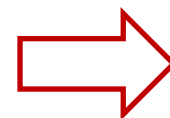
$(a \mid b) \& (b \mid (c \& d))$



$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid (c \& d))$



$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid x_3) \&$
 $x_3 \Leftrightarrow c \& d$



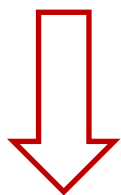
$x_1 \& x_2 \&$
 $(\neg x_1 \mid a \mid b) \&$
 $(\neg a \mid x_1) \&$
 $(\neg b \mid x_1) \&$
 $(\neg x_2 \mid b \mid x_3) \&$
 $(\neg b \mid x_2) \&$
 $(\neg x_3 \mid x_2) \&$
 $(x_3 \mid \neg c \mid \neg d) \&$
 $(\neg x_3 \mid c) \&$
 $(\neg x_3 \mid d)$

等可满足

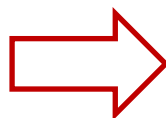


Tseitin 转换(1968)

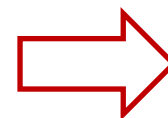
$(a \mid b) \& (b \mid (c \& d))$



$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid (c \& d))$



$x_1 \& x_2 \&$
 $x_1 \Leftrightarrow (a \mid b) \&$
 $x_2 \Leftrightarrow (b \mid x_3) \&$
 $x_3 \Leftrightarrow c \& d$



$x_1 \& x_2 \&$
 $(\neg x_1 \mid a \mid b) \&$
 $(\neg a \mid x_1) \&$
 $(\neg b \mid x_1) \&$
 $(\neg x_2 \mid b \mid x_3) \&$
 $(\neg b \mid x_2) \&$
 $(\neg x_3 \mid x_2) \&$
 $(x_3 \mid \neg c \mid \neg d) \&$
 $(\neg x_3 \mid c) \&$
 $(\neg x_3 \mid d)$

等可满足

长度线性增长

课堂练习

用Tseitin求下列公式对应的CNF (提示: 把 \rightarrow 和 \leftrightarrow 先消去)

- $(A \rightarrow \neg B) \leftrightarrow C$
- $(A \leftrightarrow B) \rightarrow (B \wedge C)$

(≡)

DPLL算法

$x1 \ \&$

$x2 \ \&$

$(!x1 \mid a \mid b) \ \&$

$(!a \mid x1) \ \&$

$(!b \mid x1) \ \&$

$(!x2 \mid b \mid x3) \ \&$

$(!b \mid x2) \ \&$

$(!x3 \mid x2) \ \&$

$(x3 \mid !c \mid !d) \ \&$

$(!x3 \mid c) \ \&$

$(!x3 \mid d)$

DPLL算法

$x1 \ \&$

$x2 \ \&$

$(!x1 \mid a \mid b) \ \&$

$(!a \mid x1) \ \&$

$(!b \mid x1) \ \&$

$(!x2 \mid b \mid x3) \ \&$

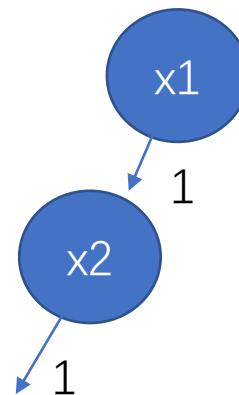
$(!b \mid x2) \ \&$

$(!x3 \mid x2) \ \&$

$(x3 \mid !c \mid !d) \ \&$

$(!x3 \mid c) \ \&$

$(!x3 \mid d)$



DPLL算法

$x1 \ \&$
 $x2 \ \&$
 $(!x1 \mid a \mid b) \ \&$

$(!a \mid x1) \ \&$
 $(!b \mid x1) \ \&$

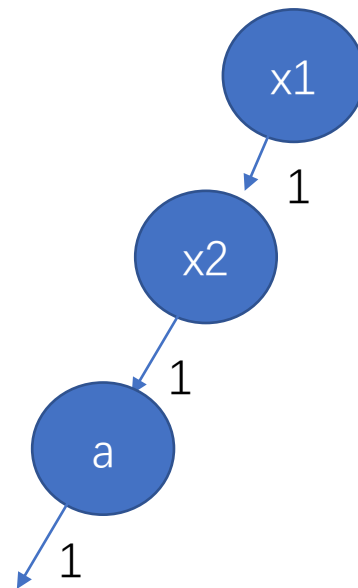
$(!x2 \mid b \mid x3) \ \&$

$(!b \mid x2) \ \&$
 $(!x3 \mid x2) \ \&$

$(x3 \mid !c \mid !d) \ \&$

$(!x3 \mid c) \ \&$

$(!x3 \mid d)$



DPLL算法

$x1 \ \&$
 $x2 \ \&$
 $(!x1 \mid a \mid b) \ \&$

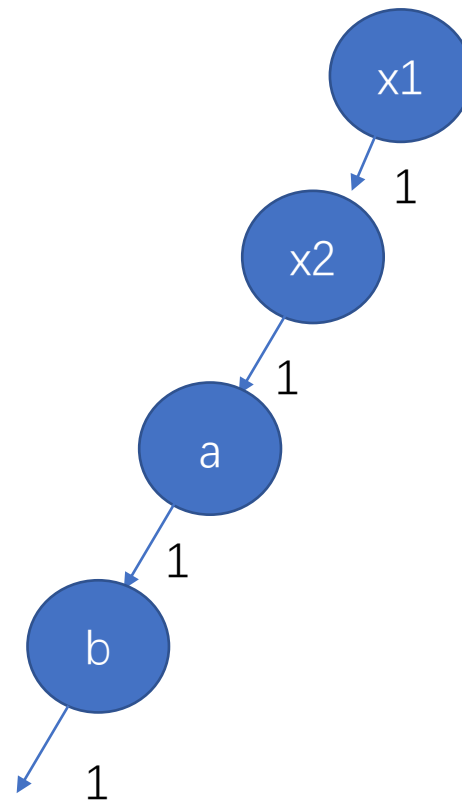
$(!a \mid x1) \ \&$
 $(!b \mid x1) \ \&$
 $(!x2 \mid b \mid x3) \ \&$

$(!b \mid x2) \ \&$
 $(!x3 \mid x2) \ \&$

$(x3 \mid !c \mid !d) \ \&$

$(!x3 \mid c) \ \&$

$(!x3 \mid d)$



DPLL算法

$x1 \ \&$
 $x2 \ \&$
 $(!x1 \mid a \mid b) \ \&$

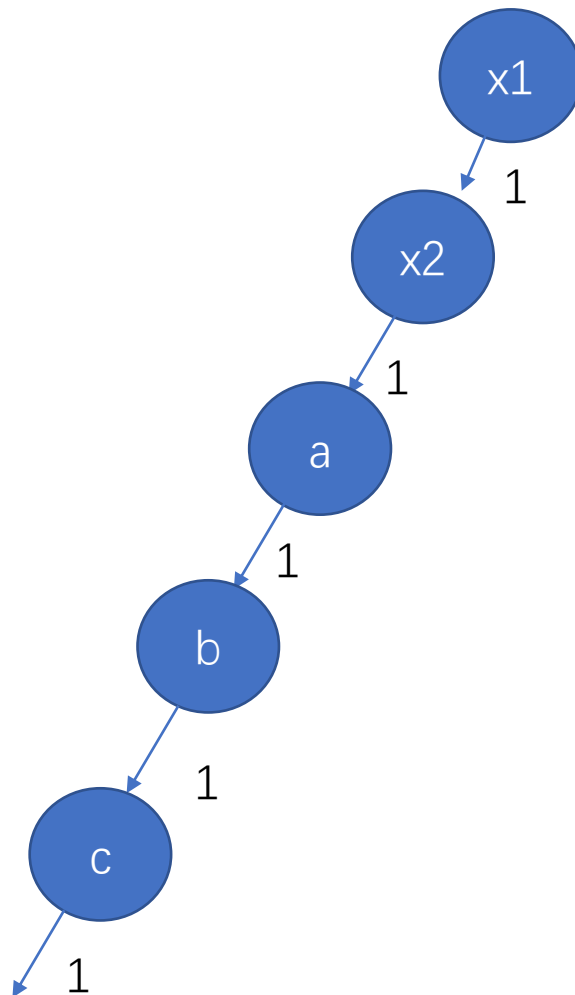
$(!a \mid x1) \ \&$
 $(!b \mid x1) \ \&$
 $(!x2 \mid b \mid x3) \ \&$

$(!b \mid x2) \ \&$
 $(!x3 \mid x2) \ \&$

$(x3 \mid !c \mid !d) \ \&$

$(!x3 \mid c) \ \&$

$(!x3 \mid d)$



DPLL算法

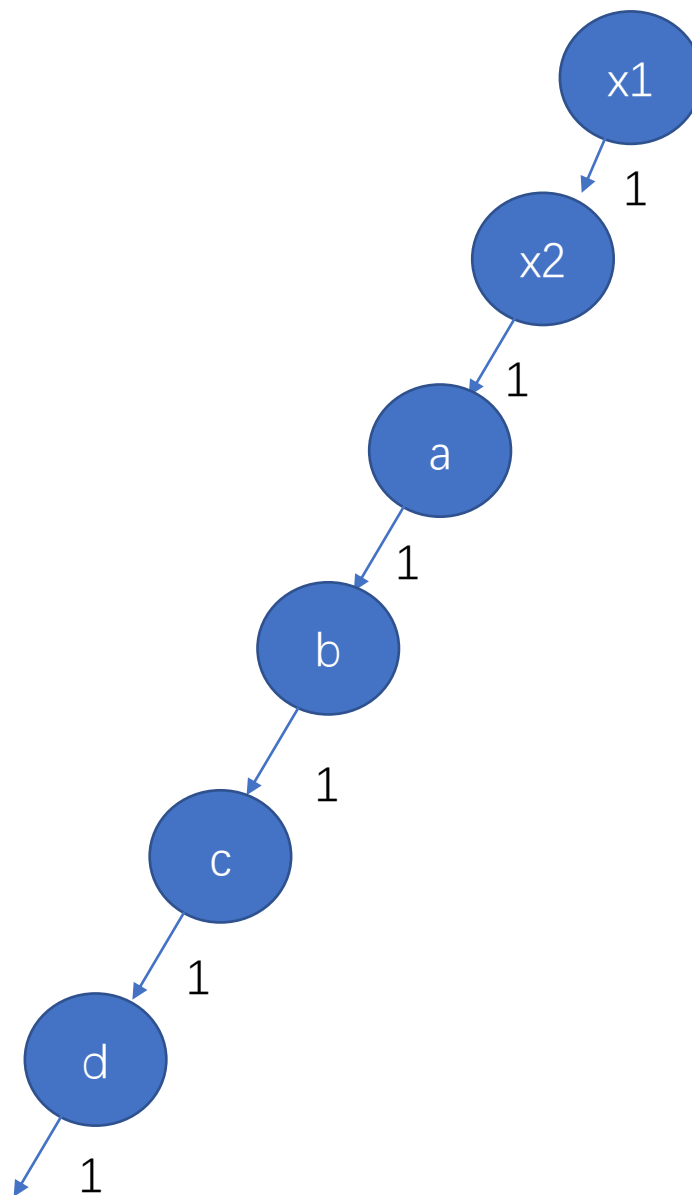
$x1 \ \&$
 $x2 \ \&$
 $(!x1 \mid a \mid b) \ \&$

$(!a \mid x1) \ \&$
 $(!b \mid x1) \ \&$
 $(!x2 \mid b \mid x3) \ \&$

$(!b \mid x2) \ \&$
 $(!x3 \mid x2) \ \&$

$(x3 \mid !c \mid !d) \ \&$

$(!x3 \mid c) \ \&$
 $(!x3 \mid d)$



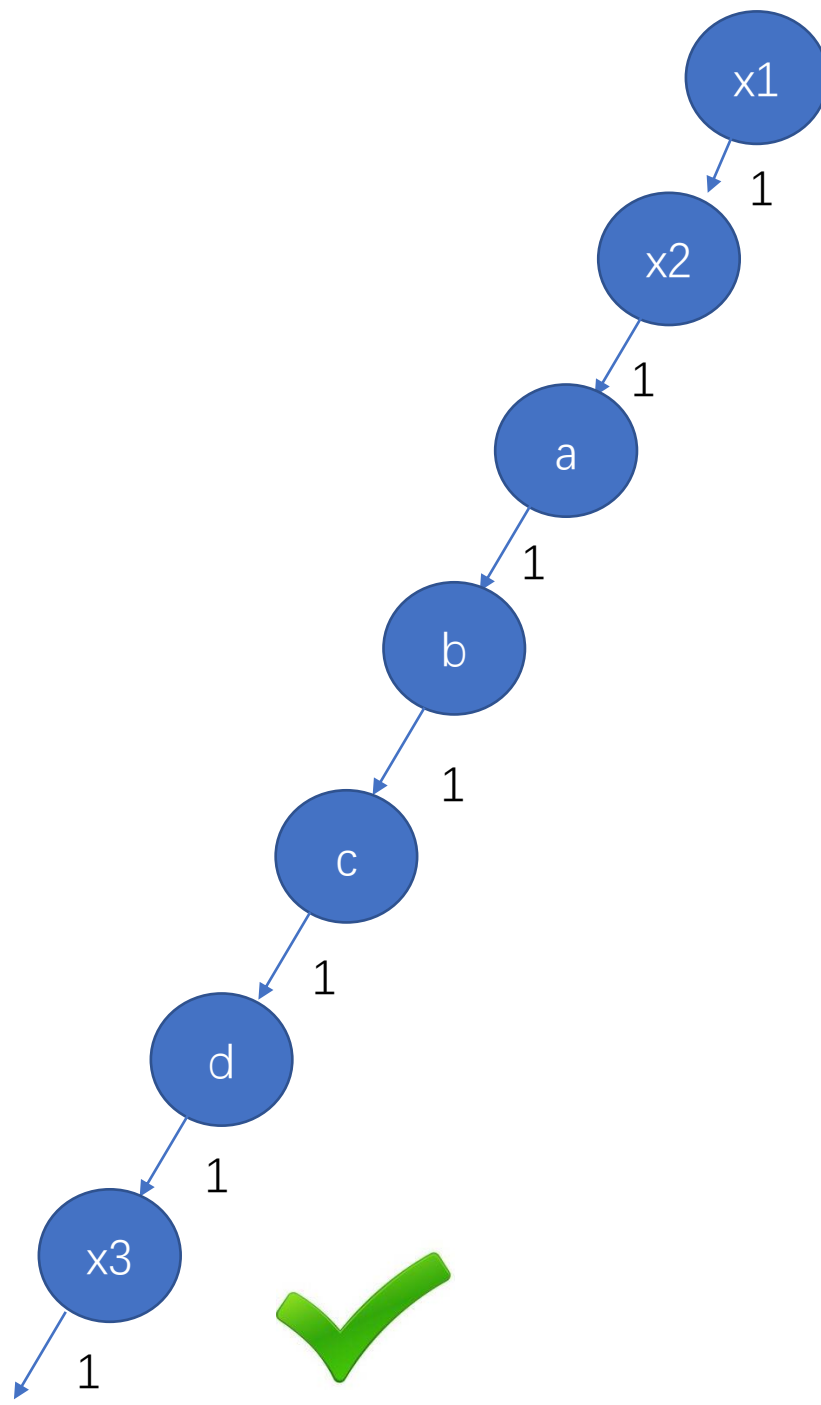
DPLL算法

$x1 \ \&$
 $x2 \ \&$
 $(!x1 \mid a \mid b) \ \&$

$(!a \mid x1) \ \&$
 $(!b \mid x1) \ \&$
 $(!x2 \mid b \mid x3) \ \&$

$(!b \mid x2) \ \&$
 $(!x3 \mid x2) \ \&$

$(x3 \mid !c \mid !d) \ \&$
 $(!x3 \mid c) \ \&$
 $(!x3 \mid d)$



DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$
 $(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$

DPLL算法

$(\neg a \mid b \mid c) \&$

$(a \mid c \mid d) \&$

$(a \mid c \mid \neg d) \&$

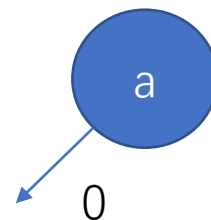
$(a \mid \neg c \mid d) \&$

$(a \mid \neg c \mid \neg d) \&$

$(\neg b \mid \neg c \mid d) \&$

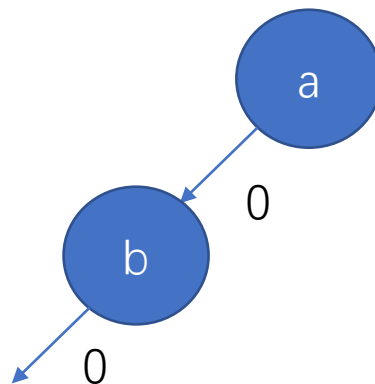
$(\neg a \mid b \mid \neg c) \&$

$(\neg a \mid \neg b \mid c)$



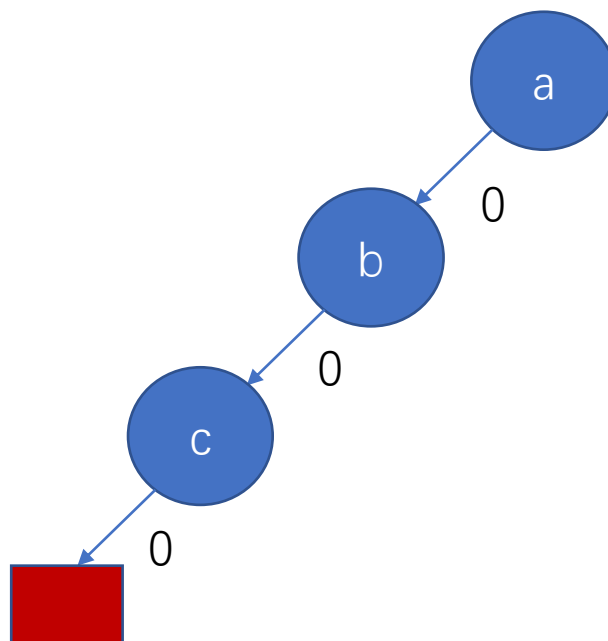
DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$
 $(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$



DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$
 $(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$

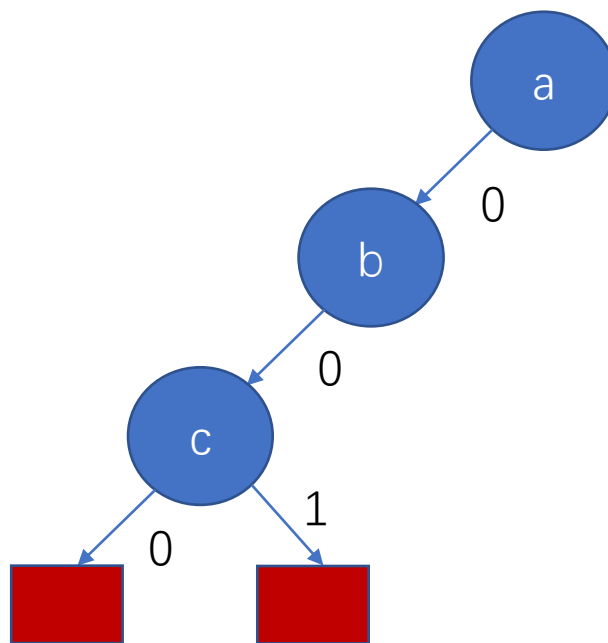


DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$

$(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$

$(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$



DPLL算法

$(\neg a \mid b \mid c) \&$

$(a \mid c \mid d) \&$

$(a \mid c \mid \neg d) \&$

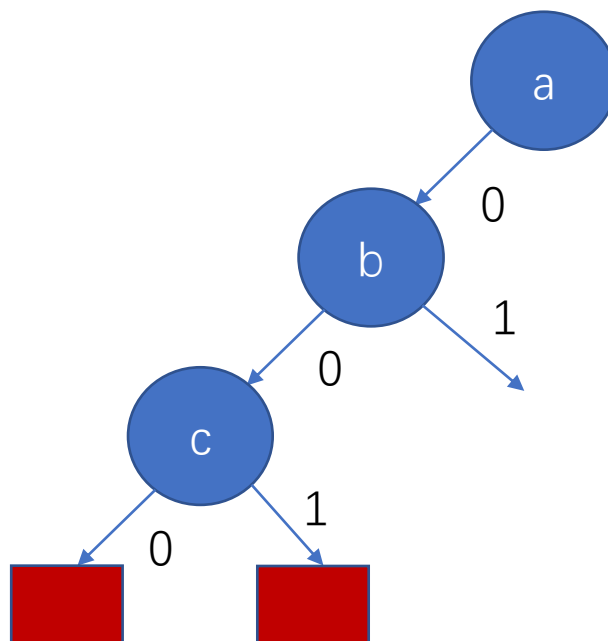
$(a \mid \neg c \mid d) \&$

$(a \mid \neg c \mid \neg d) \&$

$(\neg b \mid \neg c \mid d) \&$

$(\neg a \mid b \mid \neg c) \&$

$(\neg a \mid \neg b \mid c)$

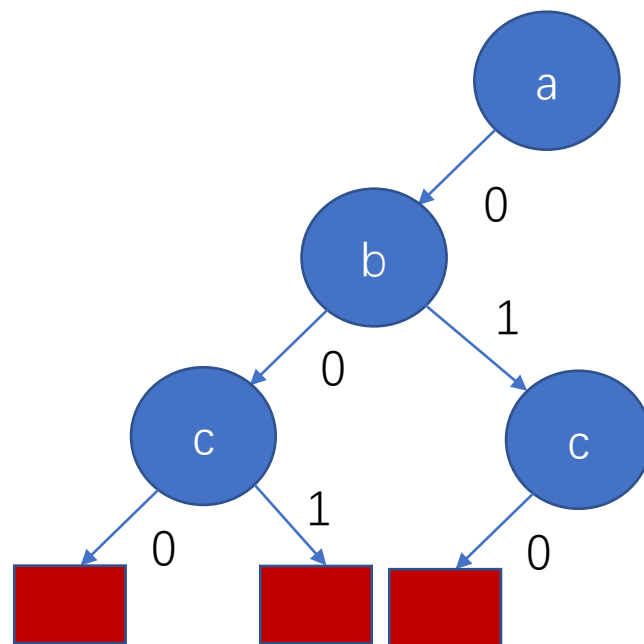


DPLL算法

$(\neg a \mid b \mid c) \&$

$(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$

$(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$

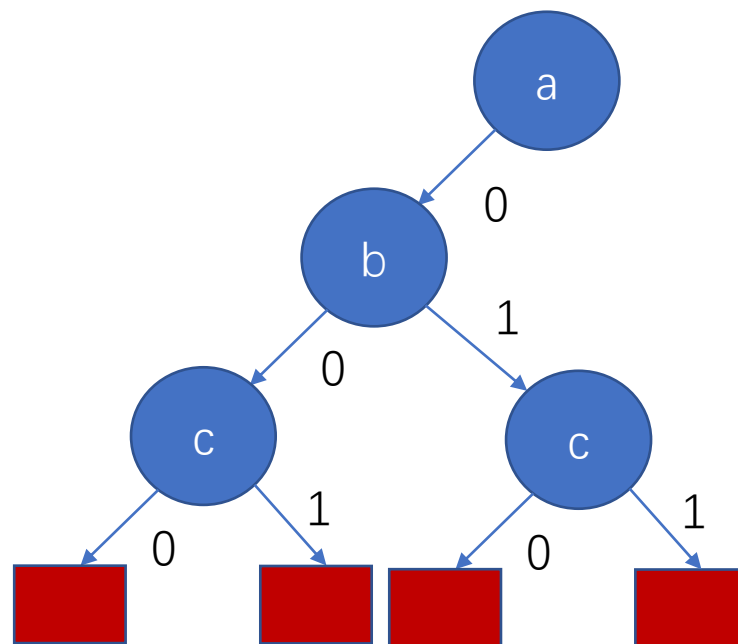


DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$

$(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$

$(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$



DPLL算法

$(!a \mid b \mid c) \&$

$(a \mid c \mid d) \&$

$(a \mid c \mid !d) \&$

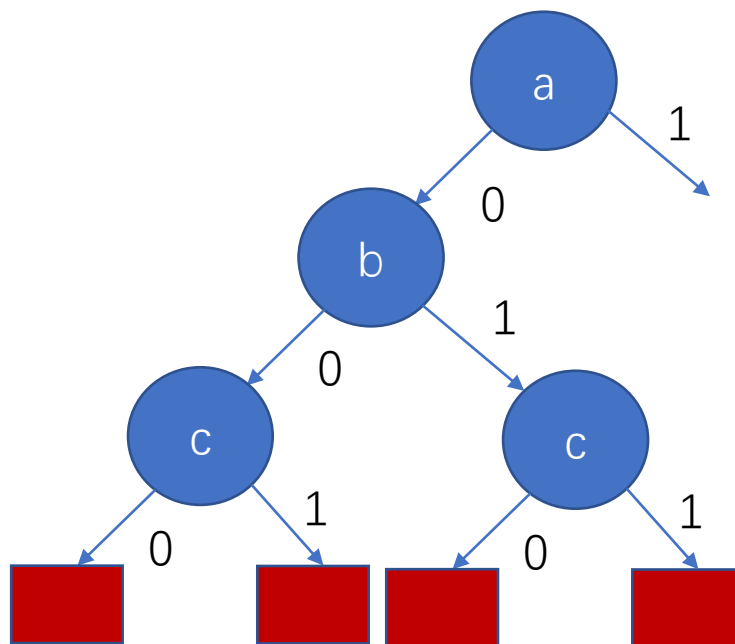
$(a \mid !c \mid d) \&$

$(a \mid !c \mid !d) \&$

$(!b \mid !c \mid d) \&$

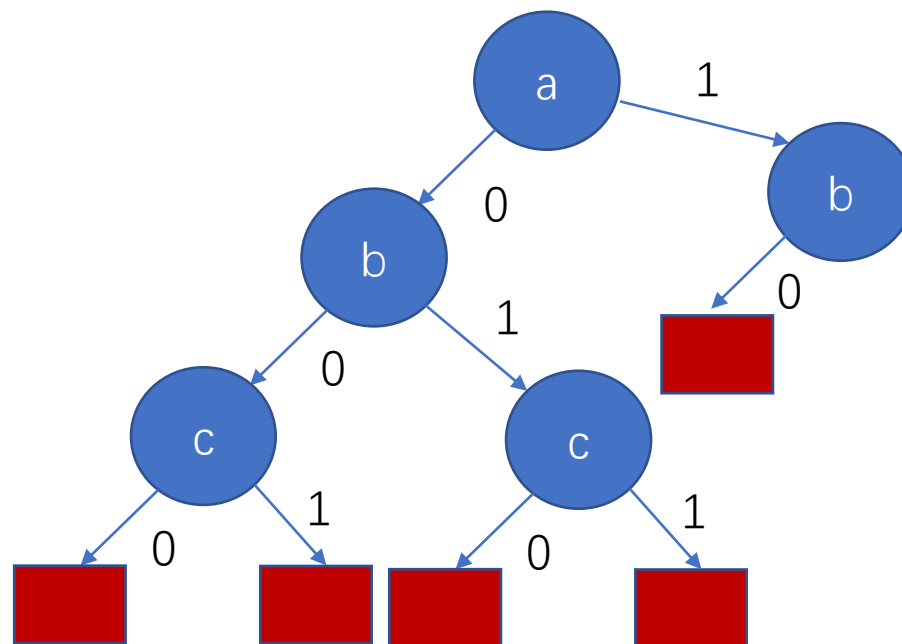
$(!a \mid b \mid !c) \&$

$(!a \mid !b \mid c)$



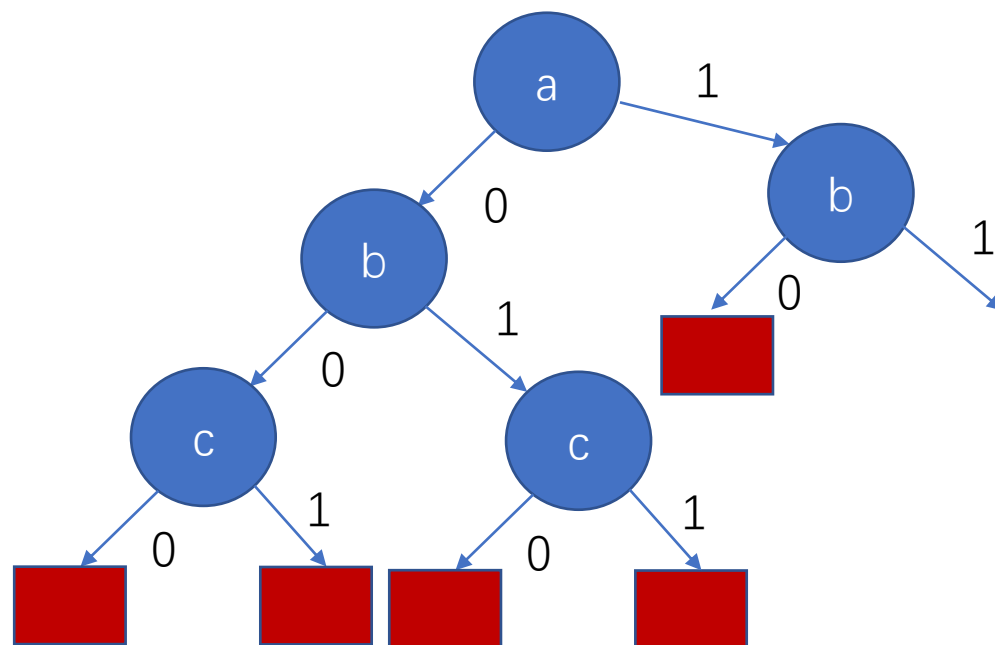
DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$
 $(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$



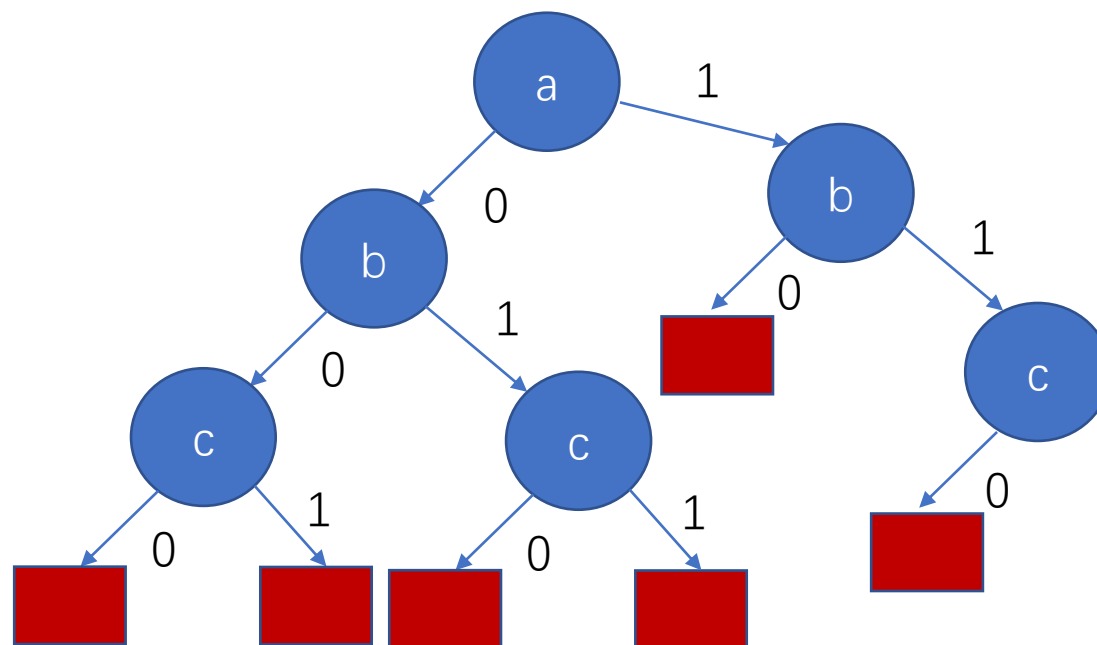
DPLL算法

$(!a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid !d) \&$
 $(a \mid !c \mid d) \&$
 $(a \mid !c \mid !d) \&$
 $(!b \mid !c \mid d) \&$
 $(!a \mid b \mid !c) \&$
 $(!a \mid !b \mid c)$



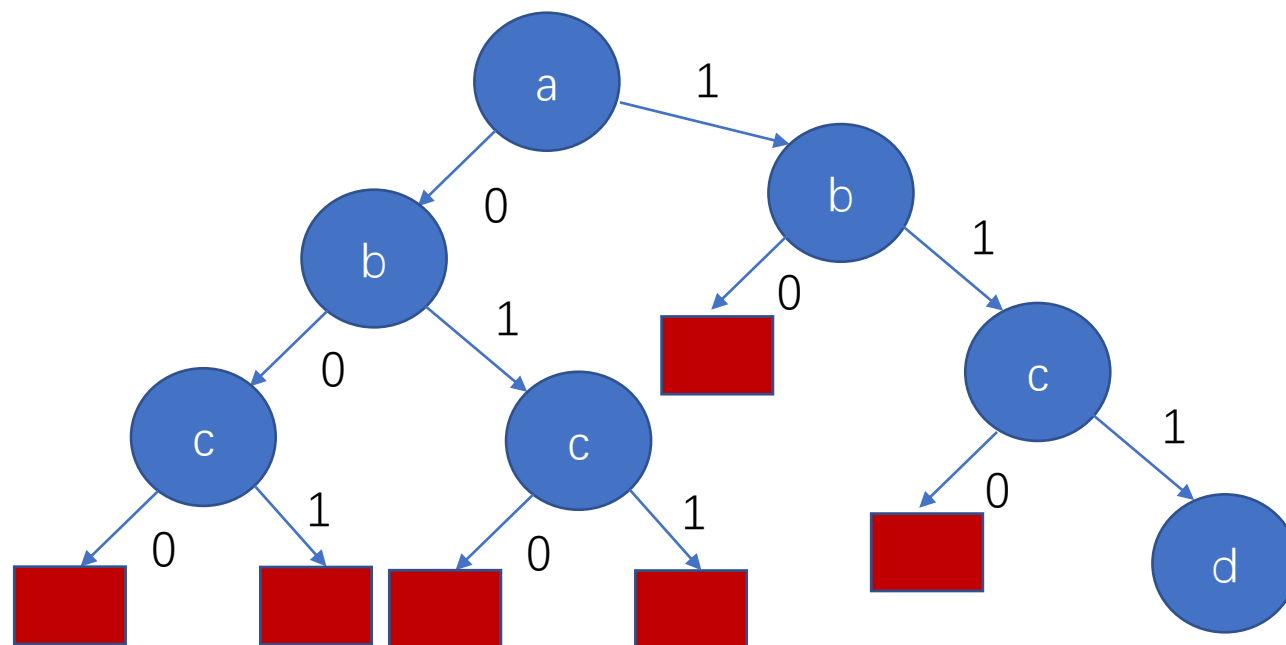
DPLL算法

$(!a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid !d) \&$
 $(a \mid !c \mid d) \&$
 $(a \mid !c \mid !d) \&$
 $(!b \mid !c \mid d) \&$
 $(!a \mid b \mid !c) \&$
 $(!a \mid !b \mid c)$



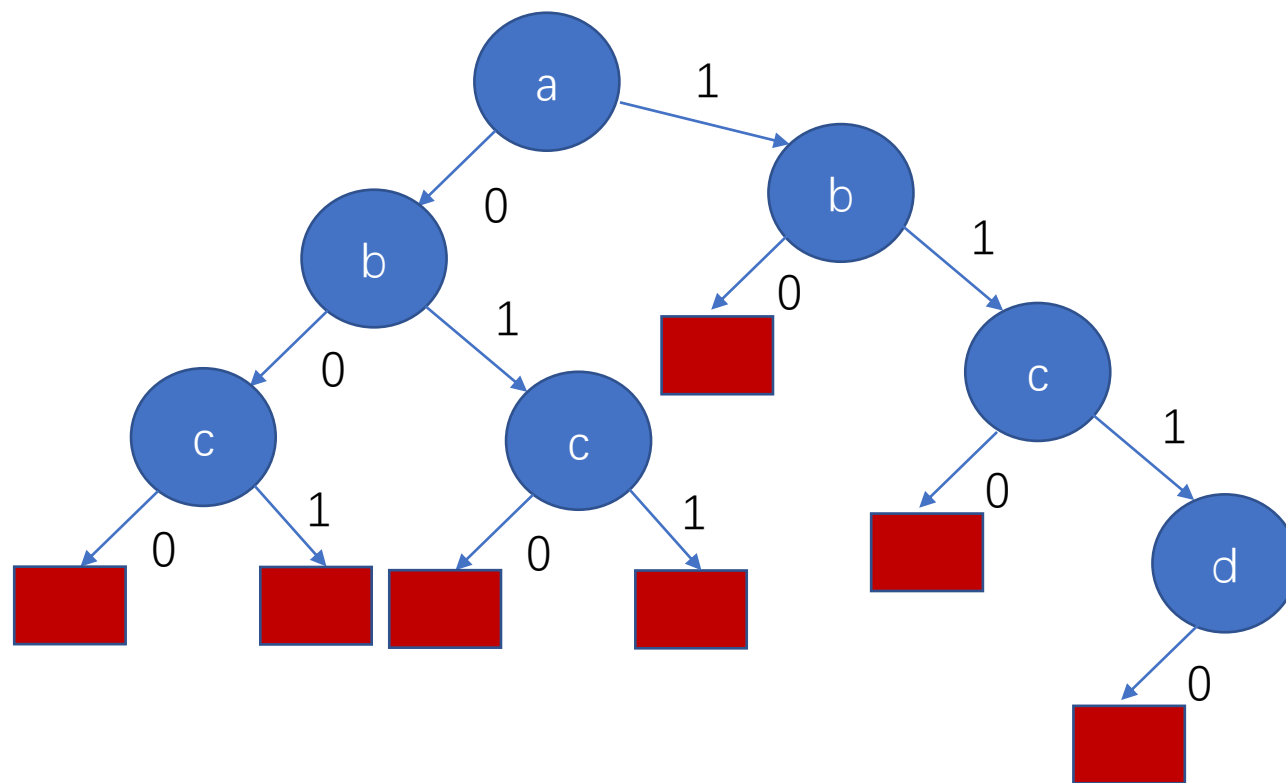
DPLL算法

$(!a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid !d) \&$
 $(a \mid !c \mid d) \&$
 $(a \mid !c \mid !d) \&$
 $(!b \mid !c \mid d) \&$
 $(!a \mid b \mid !c) \&$
 $(!a \mid !b \mid c)$



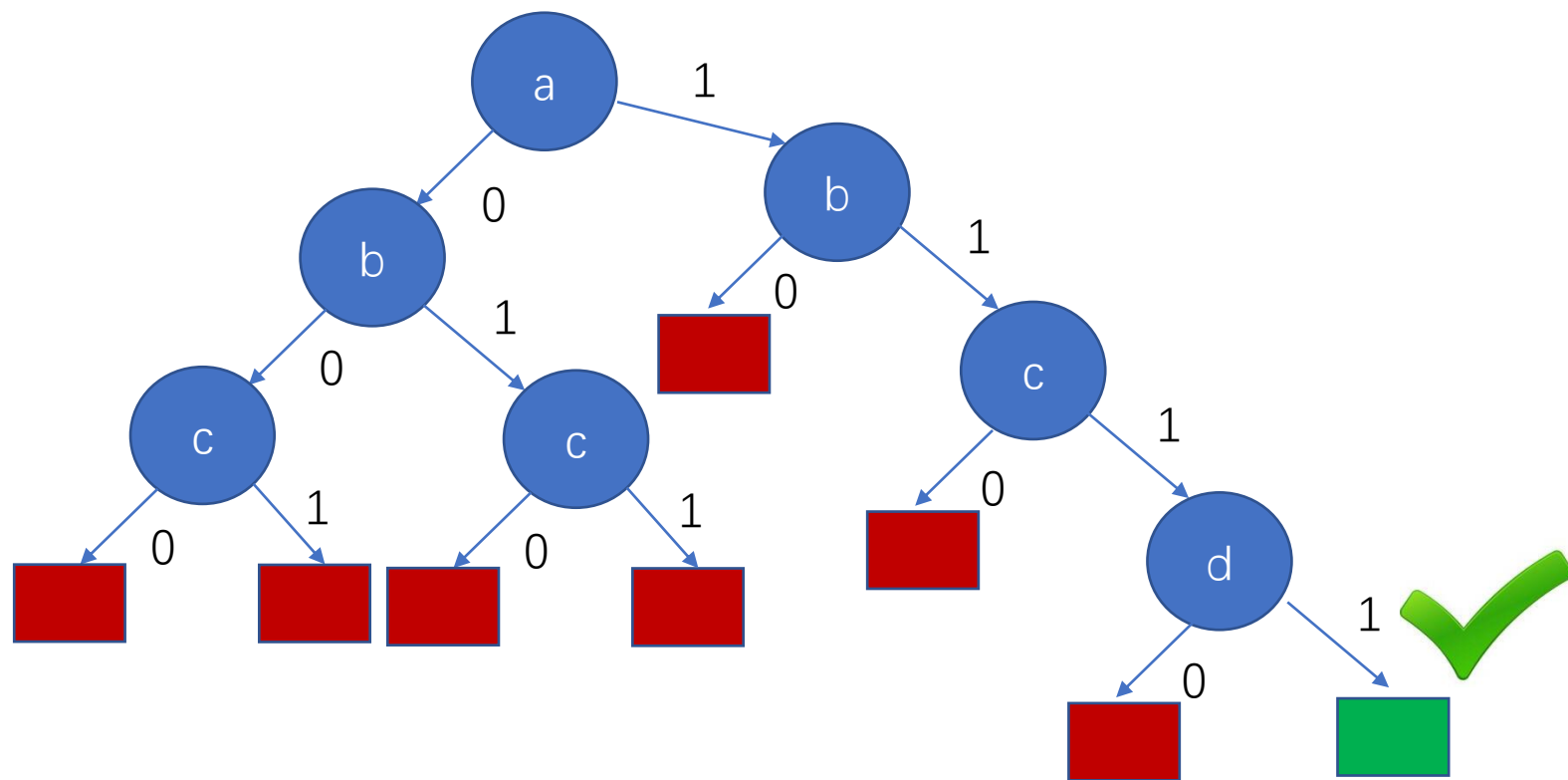
DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$
 $(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$



DPLL算法

$(\neg a \mid b \mid c) \&$
 $(a \mid c \mid d) \&$
 $(a \mid c \mid \neg d) \&$
 $(a \mid \neg c \mid d) \&$
 $(a \mid \neg c \mid \neg d) \&$
 $(\neg b \mid \neg c \mid d) \&$
 $(\neg a \mid b \mid \neg c) \&$
 $(\neg a \mid \neg b \mid c)$



课堂练习

使用DPLL算法求下面公式的解

- $(A \vee B) \wedge (B \vee !C \vee D) \wedge (!A \vee !B) \wedge (!A \vee !C \vee !D) \wedge A$
- $(A \wedge B) \vee (!B \wedge !C) \vee (A \wedge !C \wedge D)$ (提示: 先用Tsitin转换)

SAT (Boolean SATisfiability Problem)

给定一个布尔逻辑公式，判定其是否存在解的过程。

SAT是第一个被证明的NP完全问题。（1970）

很多问题都可以转化为SAT问题解决，比如验证问题。

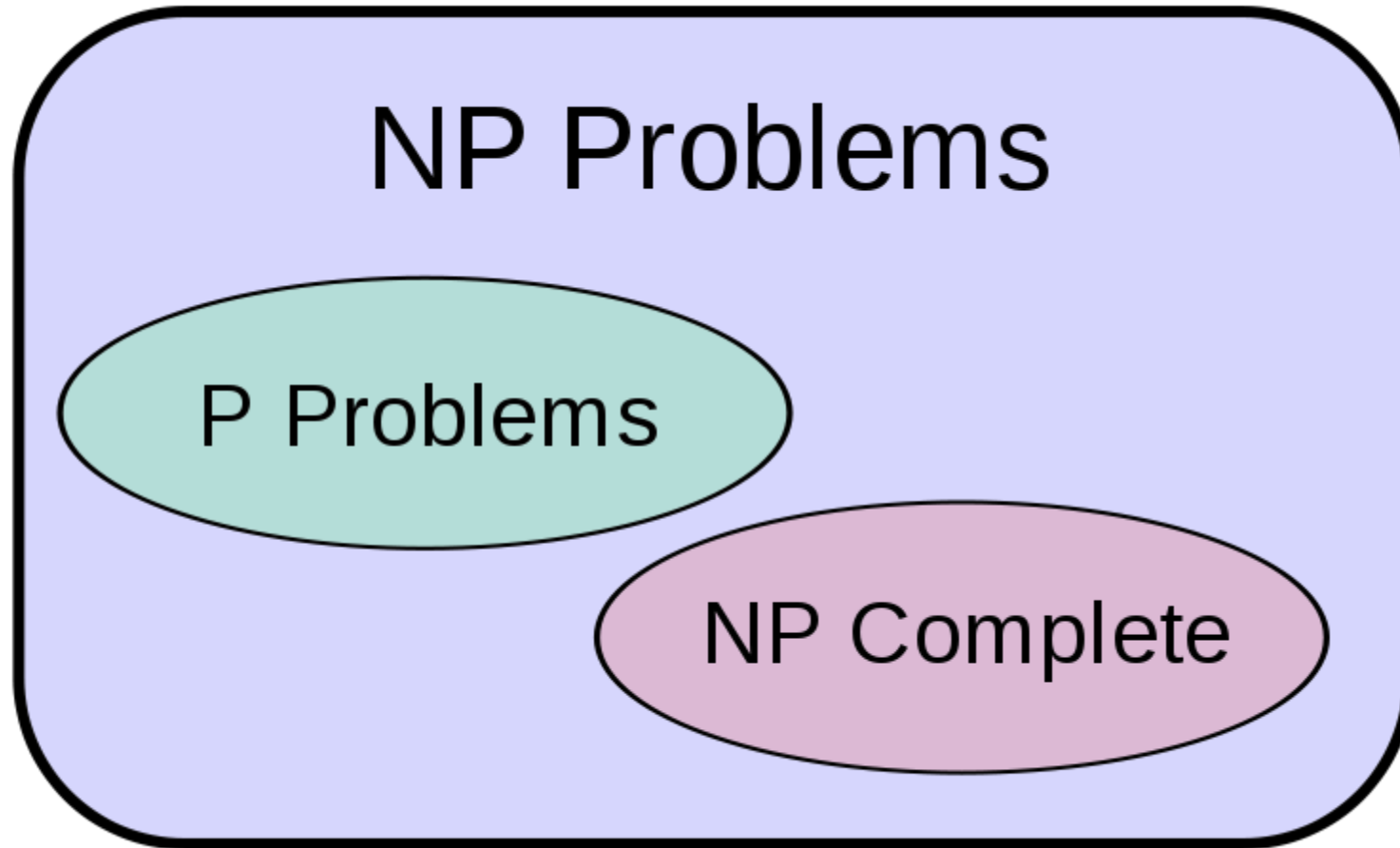
P = NP?

数学界7大未解决问题之一：100万美金的奖励。

P: 给定一个问题和解，判定解是否为该问题的真实解。

NP: 给定一个问题，寻找该问题的一个真实解。

$P = NP?$



思考题

有没有更高效的SAT求解算法？

DIMACS: 基于CNF的SAT求解器输入

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

$(a \mid c \mid d) \ \&$

$(a \mid c \mid \neg d) \ \&$

$(a \mid \neg c \mid d) \ \&$

$(a \mid \neg c \mid \neg d) \ \&$

$(\neg b \mid \neg c \mid d) \ \&$

$(\neg a \mid b \mid \neg c) \ \&$

$(\neg a \mid \neg b \mid c)$

p cnf 4 8

-1 2 3 0

1 3 4 0

1 3 -4 0

1 -3 4 0

1 -3 -4 0

-2 -3 4 0

-1 2 -3 0

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

p cnf 4 8

文件头p cnf, 变量
和项个数

(!a | b | c) &

-1 2 3 0

(a | c | d) &

1 3 4 0

(a | c |!d) &

1 3 -4 0

(a | !c | d) &

1 -3 4 0

(a | !c | !d) &

1 -3 -4 0

(!b | !c | d) &

-2 -3 4 0

(!a | b | !c) &

-1 2 -3 0

(!a | !b | c)

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

p cnf 4 8

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

-1 2 3 0

$(a \mid c \mid d) \ \&$

1 3 4 0

$(a \mid c \mid \neg d) \ \&$

1 3 -4 0

$(a \mid \neg c \mid d) \ \&$

1 -3 4 0

$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

-1 2 -3 0

$(\neg a \mid \neg b \mid c)$

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

p cnf 4 8

-1 2 3 0

$(a \mid c \mid d) \ \&$

1 3 4 0

$(a \mid c \mid \neg d) \ \&$

1 3 -4 0

$(a \mid \neg c \mid d) \ \&$

1 -3 4 0

$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

-1 2 -3 0

$(\neg a \mid \neg b \mid c)$

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

p cnf 4 8

-1 2 3 0

$(a \mid c \mid d) \ \&$

1 3 4 0

$(a \mid c \mid \neg d) \ \&$

1 3 -4 0

$(a \mid \neg c \mid d) \ \&$

1 -3 4 0

$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

-1 2 -3 0

$(\neg a \mid \neg b \mid c)$

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

p cnf 4 8

-1 2 3 0

$(a \mid c \mid d) \ \&$

1 3 4 0

$(a \mid c \mid \neg d) \ \&$

1 3 -4 0

$(a \mid \neg c \mid d) \ \&$

1 -3 4 0

$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

-1 2 -3 0

$(\neg a \mid \neg b \mid c)$

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

p cnf 4 8

-1 2 3 0

$(a \mid c \mid d) \ \&$

1 3 4 0

$(a \mid c \mid \neg d) \ \&$

1 3 -4 0

$(a \mid \neg c \mid d) \ \&$

1 -3 4 0

$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

-1 2 -3 0

$(\neg a \mid \neg b \mid c)$

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

p cnf 4 8

-1 2 3 0

$(a \mid c \mid d) \ \&$

1 3 4 0

$(a \mid c \mid \neg d) \ \&$

1 3 -4 0

$(a \mid \neg c \mid d) \ \&$

1 -3 4 0

$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

-1 2 -3 0

$(\neg a \mid \neg b \mid c)$

-1 -2 3 0

c a=1, b=2, c=3, d=4

DIMACS: 基于CNF的SAT求解器输入

文件头p cnf, 变量
和项个数

$(\neg a \mid b \mid c) \ \&$

p cnf 4 8

-1 2 3 0

$(a \mid c \mid d) \ \&$

1 3 4 0

$(a \mid c \mid \neg d) \ \&$

1 3 -4 0

$(a \mid \neg c \mid d) \ \&$

1 -3 4 0

$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

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$(a \mid \neg c \mid \neg d) \ \&$

1 -3 -4 0

$(\neg b \mid \neg c \mid d) \ \&$

-2 -3 4 0

$(\neg a \mid b \mid \neg c) \ \&$

-1 2 -3 0

$(\neg a \mid \neg b \mid c)$

-1 -2 3 0

c a=1, b=2, c=3, d=4

注释

国际SAT年度竞赛： <http://www.satcompetition.org/>

课程小结—布尔逻辑

- 布尔逻辑基本概念介绍
- 布尔逻辑公式的合取范式(CNF)和析取范式(DNF)
- 布尔公式求解 (SAT) DPLL算法简介