

1.

$$\rho_{AB} = \frac{1}{3}(|00\rangle\langle 00| + |11\rangle\langle 00| + |22\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 11| + |22\rangle\langle 11| + |00\rangle\langle 22| + |11\rangle\langle 22| + |22\rangle\langle 22|)$$

$$\rho_A = tr_B(\rho_{AB}) = \frac{I}{3}, \rho_B = tr_A(\rho_{AB}) = \frac{I}{3}$$

2.

(1).

$$(H_1 \otimes H_1)^{-1} C_{NOT} (H_1 \otimes H_1) = H_2 C_{NOT} H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

(2).

$$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} & 0 \\ 0 & 0 & 0 & e^{i\alpha} \end{bmatrix} = \text{左边}$$