FACULTAD DE CIENCIAS

Tarea 3

ANÁLISIS NÚMERICO

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1. Problemas de Computadora

Una nota importante es que al inicio de CADA script se incluyen los algoritmos, porfavor cambia la primera linea de cada script para que el path sea el correcto, porfavor.

Esta linea:

```
getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms')
```

Para ejecutar cada uno basta con hacer algo como:

```
exec("/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/23.sce", -1)
```

1.1. 23

Ejecuta los scripts que esta dentro de Code llamado: 23.sce

En este código muestra justo lo que se nos pide, por el método de ecuaciones normales veremos la solución aproximada para cada una de las 2 b que nos dan y vemos a aunque son muy parecidas los resultados son completamente diferentes, y si, aunque puede parecer que o el algoritmo esta mal o algo raro acaba de pasar basta con dar un vistazo a la matriz A y verla como lo que la estamos interpretando, un conjunto de vectores.

Recuerda que lo que estamos haciendo es encontrar una combinación lineal de estos vectores para aproximar a nuestros vectores b.

Pues si nuestros vectores no son ortogonales lo que pasa es que un pequeño cambio en el resultado nos da una combinación lineal muy diferente. Es justo lo que esta pasando estamos trabajando con vectores muy parecidos.

La mejor forma de explicar este fenomeno y mostrar porque la ortogonalidad es tan importante es ver este video que justo habla de porque $1, x, x^2, \ldots$ es una base horrible para los polinomios. El link es *https://youtu.be/pYoGYQOXqTk*

Esto es un pdf y no me puedo quejar del espacio que utilizo, así que dejo el código:

```
// @Author: Rosas Hernandez Oscar Andres
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getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms')
clc;

A23 = [
    0.16 0.10;
    0.17 0.11;
    2.02 1.29;
]

b23 = [
    0.26;
    0.28;
    3.31;
]

b2_23 = [
    0.27;
    0.25;
    3.33;
]

disp("Ax = b")

disp("A:")
disp(A23)

disp("b:")
disp(b23)

x23 = LeastSquares(A23, b23)
disp("x:")
disp(x23)

disp("Ax:")
disp(A23 * x23)

disp("b_2:")
disp(b2_23)

x2_23 = LeastSquares(A23, b2_23)
disp("x_2:")
disp(x2_23)

disp("Ax_2:")
disp(A23 * x2_23)
```

1.2. 24

Antes que nada, espero estar haciendo el problema correcto, este:

3.5. A planet follows an elliptical orbit, which can be represented in a Cartesian (x, y) coordinate system by the equation

$$ay^2 + bxy + cx + dy + e = x^2.$$

(*a*) Use a library routine, or one of your own design, for linear least squares to determine the orbital parameters a, b, c, d, e, given the following observations of the planet's position:

x	1.02	0.95	0.87	0.77	0.67
y	0.39	0.32	0.27	0.22	0.18
x	0.56	0.44	0.30	0.16	0.01
y	0.15	0.13	0.12	0.13	0.15

In addition to printing the values for the orbital parameters, plot the resulting orbit and the given data points in the (x, y) plane.

(*b*) This least squares problem is nearly rank-deficient. To see what effect this has on the solution, perturb the input data slightly by adding

Ahhhh, este problema estuvo demasiado genial, ojala tuviera mas tiempo de intentar más problemas así, pero este si que requiere más explicación.

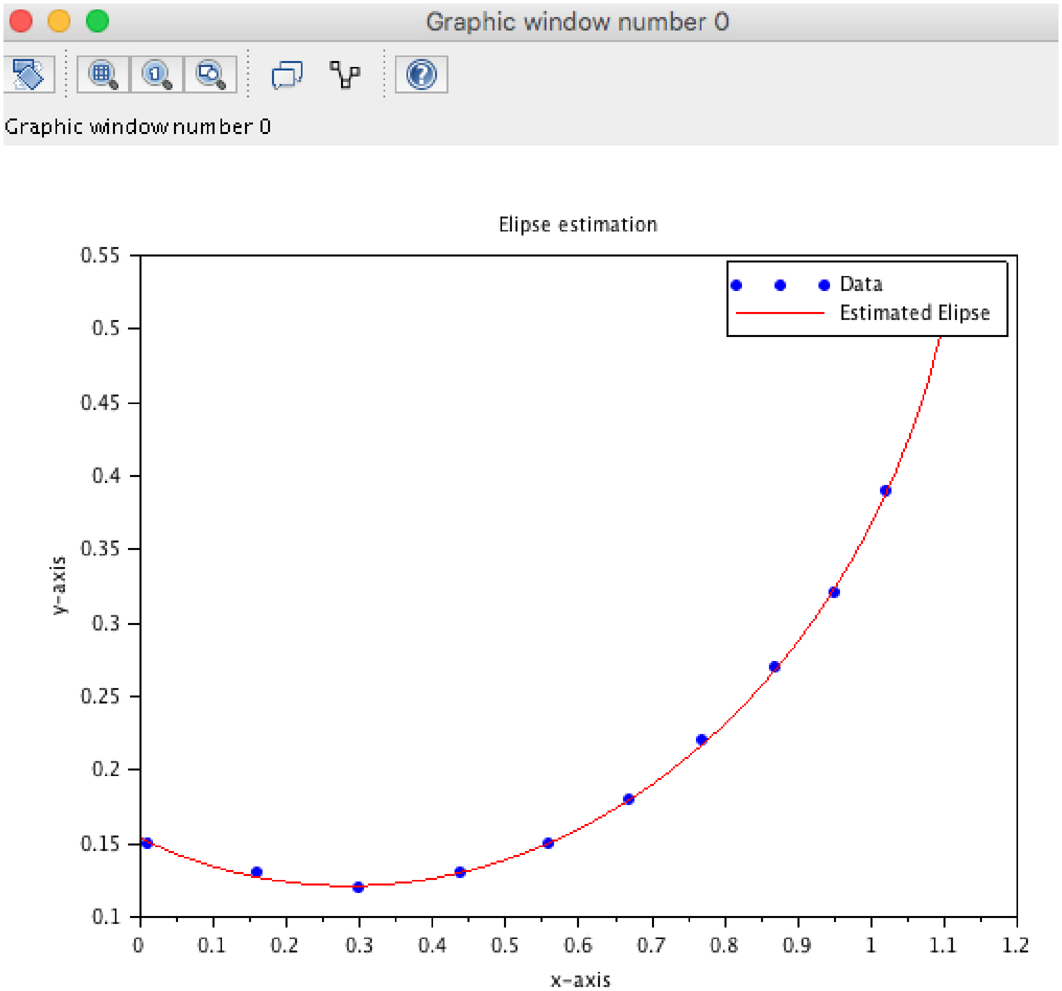
Lo que haremos primero será capturar 2 arreglos con la información que tenemos, después generaremos la matriz, que se basa justo en la ecuación $ay^2 + by + cd + e = x^2$

Ahora podemos resolver nuestro problema usando ecuaciones normales, con ello obtenemos un vector de coheficientes.

Por lo tanto ya nos podemos poner a hacer pronosticos, lo que harmeos será generar un arreglo con puntos equidistantes de $x \in [0, 1]$.

Ahora, solo basta con ver que si x es una constante, la función que hemos calculado ahora nos regresa dos y, nos quedaremos con una y vamos a graficarla, despues de todo con un x constante entonces $ay^2 + by + cd + e = x^2$ es una simple ecuación de segundo grado.

Así que mostramos, nuestra predicción, mira que hermoso, hasta te puse una foto de como se ve.




```
// @Author: Rosas Hernandez Oscar Andres
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// @Author: Laurrabaquio Rodríguez Miguel Salvador
// @Author: Pahua Castro Jesús Miguel Ángel

getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms')
clc;

x24 = [
      1.02;
      0.95;
      0.87;
      0.77;
      0.67;
      0.56;
      0.44;
      0.30;
      0.16;
      0.01;
]

y24 = [
      0.39;
      0.32;
      0.27;
      0.22;
      0.18;
      0.15;
      0.13;
      0.12;
      0.13;
      0.15;
```
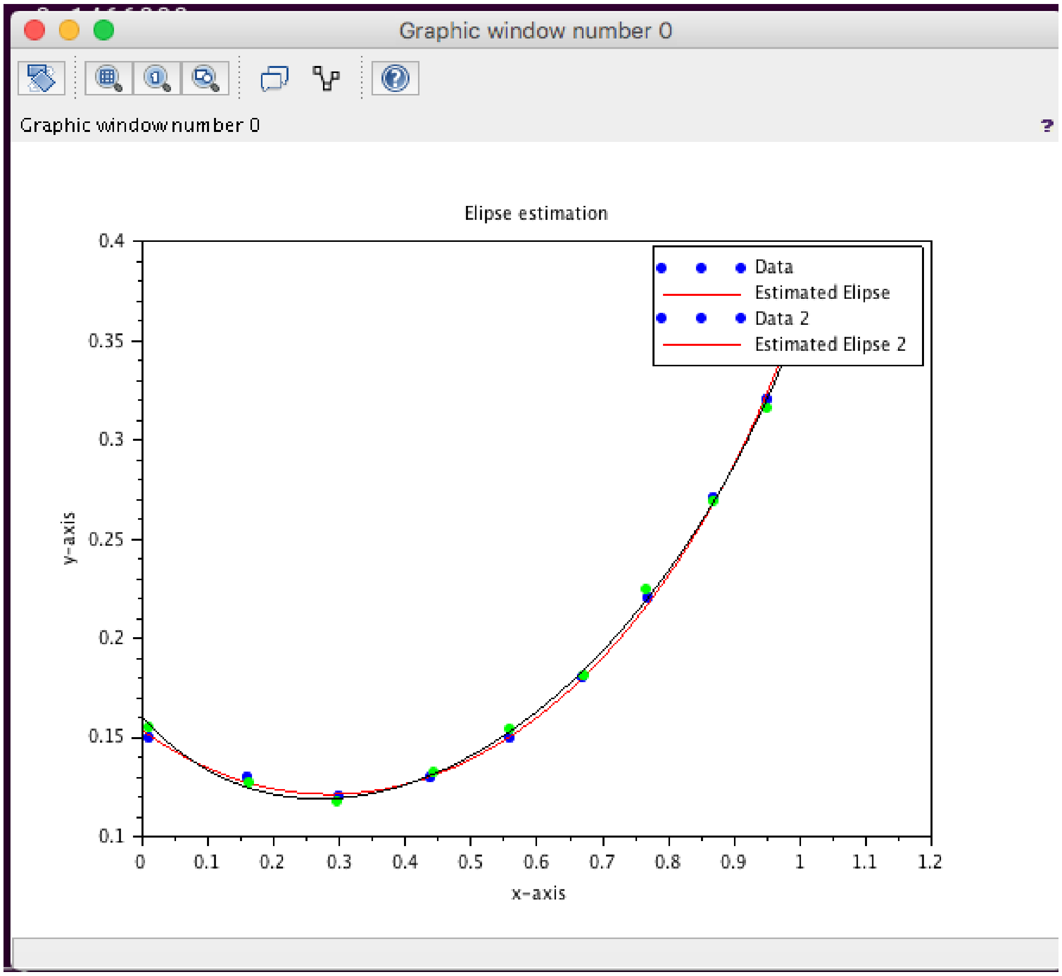

```
]

A24 = eye(10, 5)
b24 = x24

for i = (1 : 10)
    A24(i, 1) = y24(i) * y24(i)
    A24(i, 2) = x24(i) * y24(i)
    A24(i, 3) = x24(i)
    A24(i, 4) = y24(i)
    A24(i, 5) = 1

    b24(i) = x24(i) * x24(i)
end

disp("Ax = b")

disp("xs:")
disp(x24)

disp("ys:")
disp(y24)

disp("A:")
disp(A24)

disp("b:")
disp(b24)

estimedx24 = LeastSquares(A24, b24)
disp("x:")
disp(estimedx24)

disp("Ax:")
disp(A24 * estimedx24)

clf()

function [x] = solveEquation(a, b, c)
    x = (-1 * b + sqrt(b*b - 4*a*c)) / (2 * a)
endfunction

function [y] = solve(coefficients, x)
    a = coefficients(1)
    b = coefficients(2)
    c = coefficients(3)
    d = coefficients(4)
    e = coefficients(5)

    reala = (a)
    realb = (b * x  + d)
    realc = (c * x  + e - x * x)

    y = solveEquation(reala, realb, realc)
endfunction

someX = linspace(0, 1.10, 50)
someY = someX

for i = (1 : 50)
    someY(i) = solve(estimedx24, someX(i))
end

plot(x24, y24, '.b')
plot(someX, someY, 'r-')
hl=legend(['Data'; 'Estimated Elipse']);
xtitle("Elipse estimation", "x-axis", "y-axis")
```

La sección b trata de intentar mover un poco los datos sobre alguna insertudimbre y con eso ver como afecta la orbita y nuestras estimaciones y la verdad... todo Ok




```
// @Author: Rosas Hernandez Oscar Andres
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// @Author: Laurrabaquio Rodríguez Miguel Salvador
// @Author: Pahua Castro Jesús Miguel Ángel

getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms')
clc;

x24 = [
      1.02;
      0.95;
      0.87;
      0.77;
      0.67;
      0.56;
      0.44;
      0.30;
      0.16;
      0.01;
]

y24 = [
      0.39;
      0.32;
      0.27;
      0.22;
      0.18;
      0.15;
```

```
    0.13;
    0.12;
    0.13;
    0.15;
]

A24 = eye(10, 5)
b24 = x24

for i = (1 : 10)
    A24(i, 1) = y24(i) * y24(i)
    A24(i, 2) = x24(i) * y24(i)
    A24(i, 3) = x24(i)
    A24(i, 4) = y24(i)
    A24(i, 5) = 1

    b24(i) = x24(i) * x24(i)
end

clf()

function [x] = solveEquation(a, b, c)
    x = (-1 * b + sqrt(b*b - 4*a*c)) / (2 * a)
endfunction

function [y] = solve(coefficients, x)
    a = coefficients(1)
    b = coefficients(2)
    c = coefficients(3)
    d = coefficients(4)
    e = coefficients(5)

    reala = (a)
    realb = (b * x  + d)
    realc = (c * x  + e - x * x)

    y = solveEquation(reala, realb, realc)
endfunction

someX = linspace(0, 1, 50)'
someY = someX

for i = (1 : 50)
    someY(i) = solve(estimedx24, someX(i))
end

x24_2 = eye(10, 1)
y24_2 = eye(10, 1)

A24_2 = eye(10, 5)
b24_2 = x24_2

for i = (1 : 10)
    x24_2(i) = x24(i) + (rand() * 0.010 - 0.005)
    y24_2(i) = y24(i) + (rand() * 0.010 - 0.005)

    A24_2(i, 1) = y24_2(i) * y24_2(i)
    A24_2(i, 2) = x24_2(i) * y24_2(i)
    A24_2(i, 3) = x24_2(i)
    A24_2(i, 4) = y24_2(i)
    A24_2(i, 5) = 1

    b24_2(i) = x24_2(i) * x24_2(i)
end

estimedx24_2 = LeastSquares(A24_2, b24_2)

clf()

someX_2 = linspace(0, 1, 50)'
someY_2 = someX_2

for i = (1 : 50)
    someY_2(i) = solve(estimedx24_2, someX_2(i))
end


plot(x24, y24, '.b')
plot(someX, someY, 'r-')

disp("====")
disp(someX_2)
disp(someY_2)

plot(x24, y24, '.b')
plot(someX, someY, 'r-')

plot(x24_2, y24_2, '.green')
```

```
plot(someX_2, someY_2, 'black-')

hl=legend(['Data'; 'Estimated Elipse'; 'Data 2'; 'Estimated Elipse 2']);
xtitle("Elipse estimation", "x-axis", "y-axis")
```

1.3. 25

Dejemos que el códgo hable:

```
// @Author: Rosas Hernandez Oscar Andres
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// @Author: Pahua Castro Jesús Miguel Ángel

// Creates a very bad conditioned least squares problem
// @param: m a number, the size of the problem
// @param: n a number, the size of the problem
// @param: epsilon a number, the pertubation size

// @return: Q a matriz in M_{m x25 m} that is ortogonal
// @return: R a matriz in M_{m x25 n} that is triangular superior


function [] = 25(m, n, epsilon)

    // Create the polynomial
    t = zeros(m, 1)
    for(i = 1 : m)
        t(i) = (i-1) / (m-1)
    end

    //Some valuation result
    y25 = zeros(m, 1)
    for(i = 1 : m)
        for(j = 1 : n)
            y25(i) = y25(i) + t(i)^(j-1)
        end
    end

    //Create the least square matrix
    A25 = zeros(m, n)
    for(i = 1 : m)
        for(j = 1 : n)
            A25(i, j) = t(i)^(j - 1)
        end
    end

    //Lets create the perturbations
    y2_25 = zeros(m, 1)
    for(i = 1 : m)
        y2_25(i) = y25(i) + (2*rand() - 1) * epsilon
    end

    //Lets solve
    x25 = LeastSquares(A25, y25)
    z = LeastSquares(A25, y2_25)

    [Q1, R1] = GramSchmidt(A25, 0)
    x2_25 = QRDecomposition(Q1, R1, y25)

    [Q1, R1] = GramSchmidt(A25, 0)
    x2_25 = QRDecomposition(Q1, R1, y2_25)

   disp("a)")
   if(norm(x25-z) < norm(x2_25-z2))
       disp("Least square is the least sensitive to pertubations")
    else
       disp("QR Decomposition is the least sensitive to pertubations")
    end

    disp("c)")
    if(A25*x25 == y25)
        disp("The difference in the solution to least squares do not affect the ajust in the
     polynomial")
    else
        disp("The difference in the solution to least squares do affect the ajust in the
     polynomial")
    end

    if(A25*x2_25 == y25)
        disp("The difference in the solution to QR do not affect the ajust in the polynomial")
    else
        disp("The difference in the solution to QR do affect the ajust in the polynomial")
    end

endfunction
```

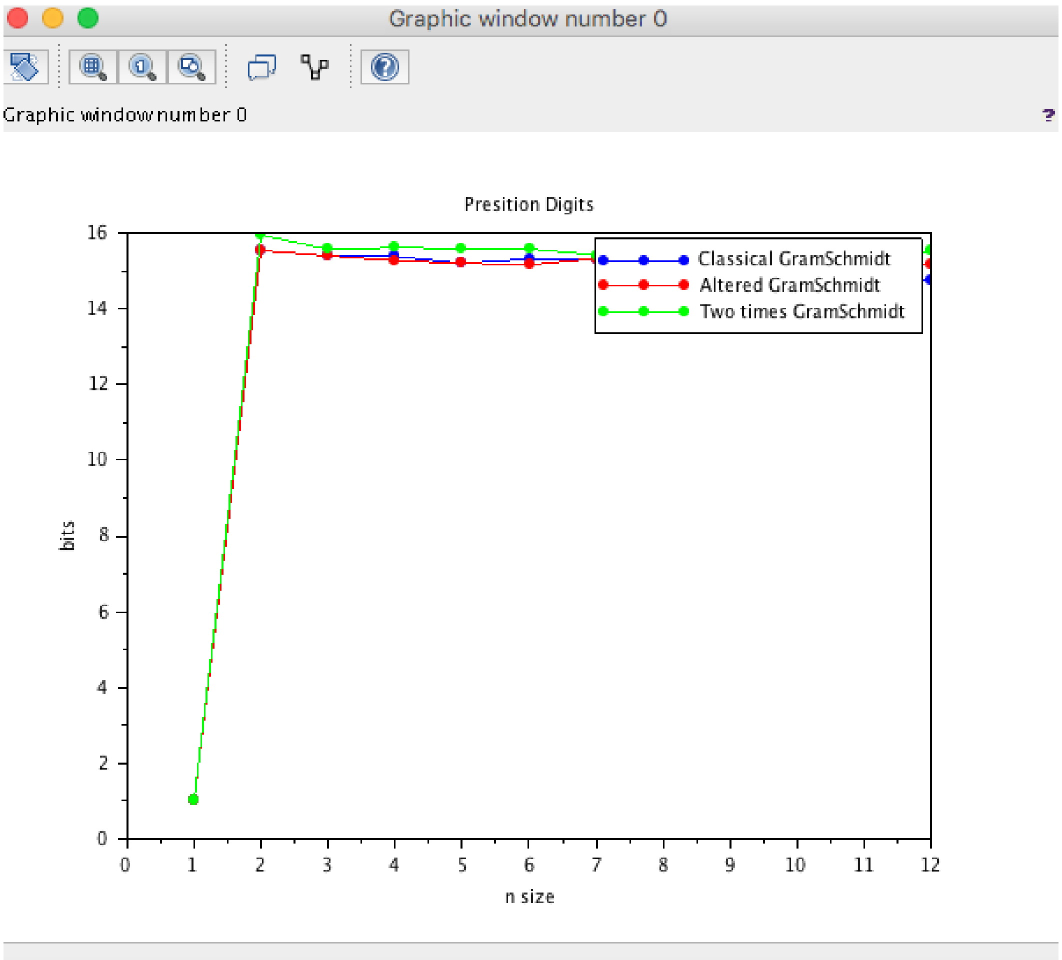
1.4. 26

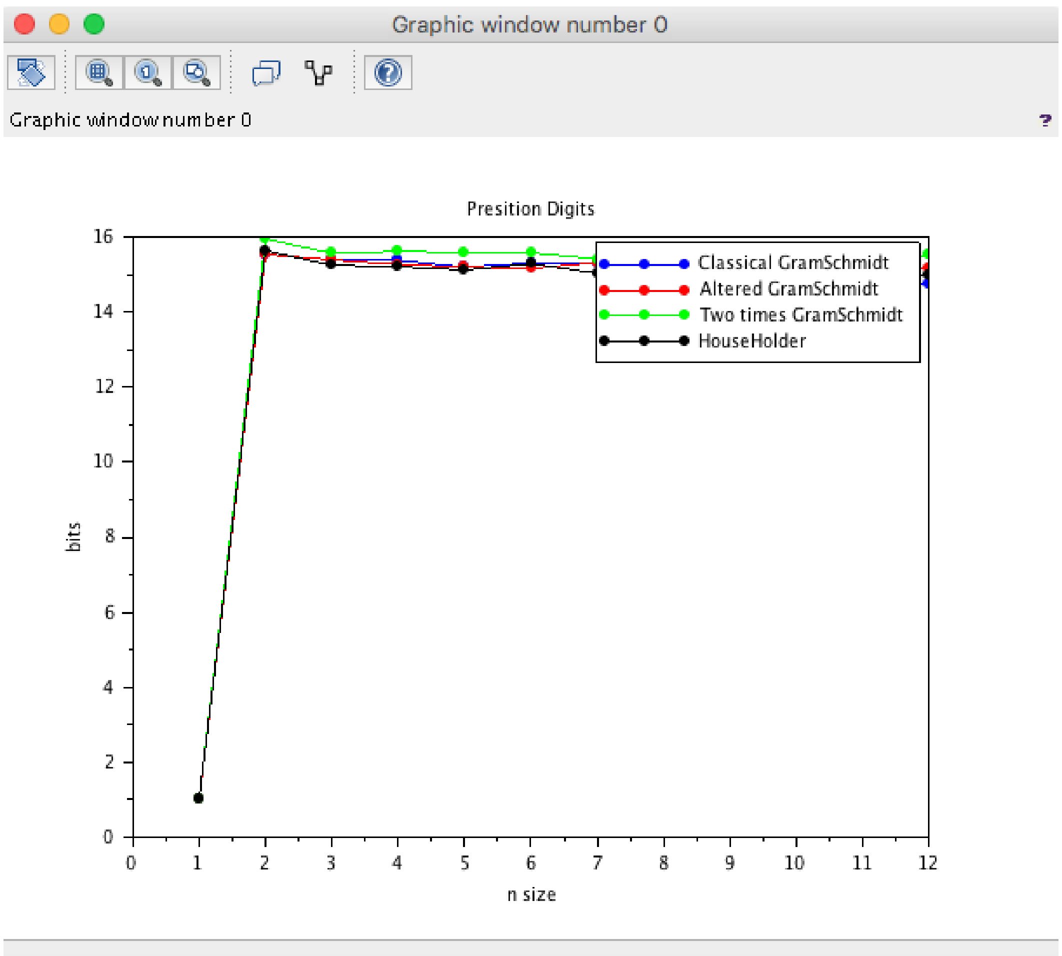
Admiremos antes que nada lo bonita que es la matriz de Hilbert:

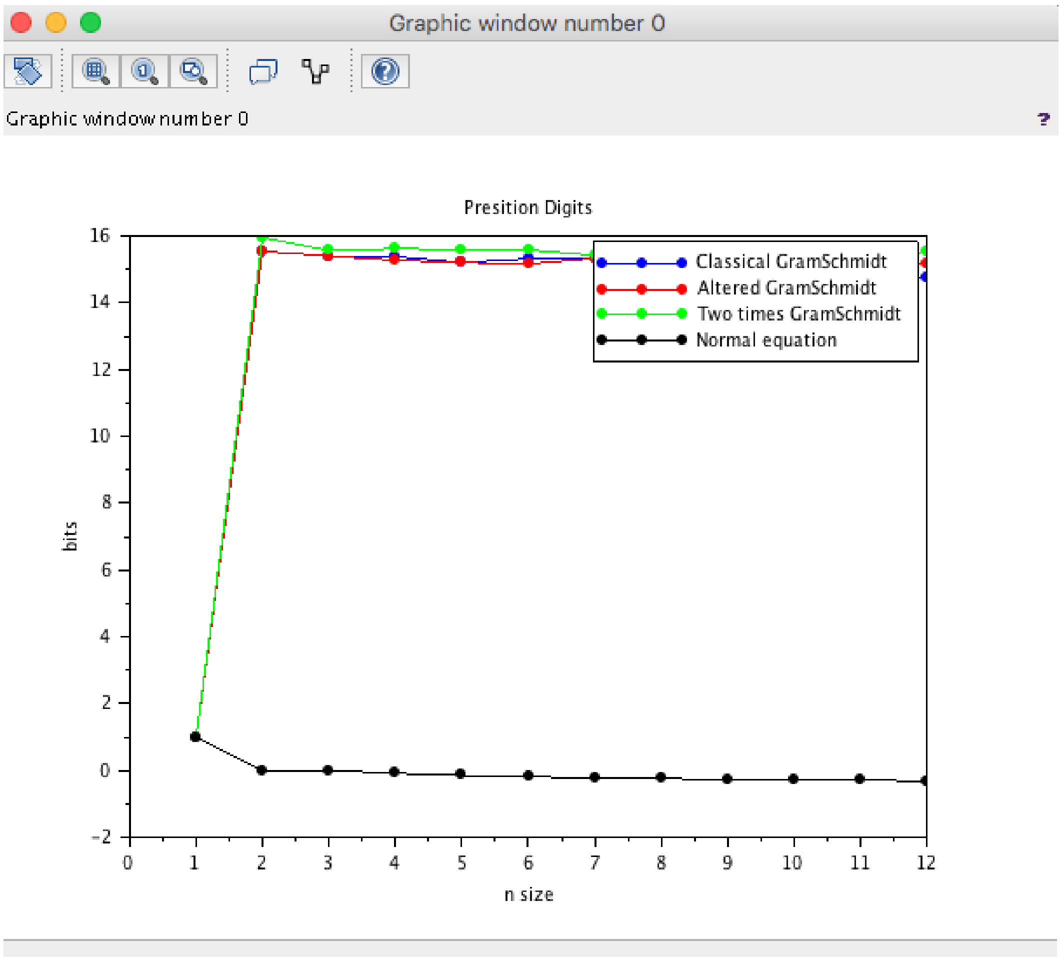
$$H = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{bmatrix}$$

Ahora si, basicamente lo que nos piden es graficar diversos métodos y los digitos de presición conforme se hace crecer esta matriz.

Miremos los resultados y el código necesario para hacerlo funcionar:


Graphic window number 0
Graphic window number 0
Presition Digits
Classical GramSchmidt
Altered GramSchmidt
Two times GramSchmidt
bits
n size
0
2
4
6
8
10
12
14
16
0
1
2
3
4
5
6
7
8
9
10
11
12


Graphic window number 0
Graphic window number 0
Presition Digits
Classical GramSchmidt
Altered GramSchmidt
Two times GramSchmidt
HouseHolder
bits
n size
0
2
4
6
8
10
12
14
16
0
1
2
3
4
5
6
7
8
9
10
11
12


Graphic window number 0
Graphic window number 0
Presition Digits
Classical GramSchmidt
Altered GramSchmidt
Two times GramSchmidt
Normal equation
bits
n size
16
14
12
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
9
10
11
12

```
// @Author: Rosas Hernandez Oscar Andres
// @Author: Alarcón Alvarez Aylin Yadira Guadalupe
// @Author: Laurrabaquio Rodríguez Miguel Salvador
// @Author: Pahua Castro Jesús Miguel Ángel

getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms')
clc;


digitsAltered = eye(12, 1)
digitsClassic = eye(12, 1)
digitsTwo = eye(12, 1)
data = eye(12, 1)

for i = (2 : 12)
    H = eye(i, i)
    data(i) = i

    for i = (1 : i)
        for j = (1 : i)
            H(i, j) = 1 / (i + j - 1)
        end
    end


    [Q26Classic, R26] = GramSchmidt(H, 0)
    [Q26Altered, R26] = GramSchmidt(H, 1)
    [Q26Two, R26] = GramSchmidt(Q26Classic, 0)

    digitsClassic(i) = -log10(norm(eye(i, i) - Q26Classic' * Q26Classic))
    digitsAltered(i) = -log10(norm(eye(i, i) - Q26Altered' * Q26Altered))
    digitsTwo(i)     = -log10(norm(eye(i, i) - Q26Two' * Q26Two))

    disp("Q of GramSchmidt Classical:")
    disp(Q26Classic)

    disp("Digits:")
    disp(digitsClassic)

    disp("Q of GramSchmidt Altered:")
    disp(Q26Altered)

    disp("Digits:")
    disp(digitsAltered)
end

plot(data,digitsClassic, '.b-')
plot(data,digitsAltered, '.r-')
plot(data,digitsTwo, '.g-')


hl=legend(['Classical GramSchmidt'; 'Altered GramSchmidt'; 'Two times GramSchmidt']);
xtitle("Presition Digits", "n size", "bits")
```

```
// @Author: Rosas Hernandez Oscar Andres
// @Author: Alarcón Alvarez Aylin Yadira Guadalupe
// @Author: Laurrabaquio Rodríguez Miguel Salvador
// @Author: Pahua Castro Jesús Miguel Ángel

getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms')
clc;


digitsAltered = eye(12, 1)
digitsClassic = eye(12, 1)
digitsTwo = eye(12, 1)
digitsHouse = eye(12, 1)
data = eye(12, 1)

for i = (2 : 12)
    H = eye(i, i)
    data(i) = i

    for i = (1 : i)
        for j = (1 : i)
            H(i, j) = 1 / (i + j - 1)
        end
    end


    [Q26Classic, R26] = GramSchmidt(H, 0)
    [Q26Altered, R26] = GramSchmidt(H, 1)
    [Q26Two, R26] = GramSchmidt(Q26Classic, 0)
    [Q26H, R26] = HouseHolder(H)

```

```
    digitsClassic(i) = -log10(norm(eye(i, i) - Q26Classic' * Q26Classic))
    digitsAltered(i) = -log10(norm(eye(i, i) - Q26Altered' * Q26Altered))
    digitsTwo(i)     = -log10(norm(eye(i, i) - Q26Two' * Q26Two))
    digitsHouse(i)   = -log10(norm(eye(i, i) - Q26H' * Q26H))

    disp("Q of GramSchmidt Classical:")
    disp(Q26Classic)

    disp("Digits:")
    disp(digitsClassic)

    disp("Q of GramSchmidt Altered:")
    disp(Q26Altered)

    disp("Digits:")
    disp(digitsAltered)
end

plot(data,digitsClassic, '.b-')
plot(data,digitsAltered, '.r-')
plot(data,digitsTwo, '.g-')
plot(data,digitsHouse, '.black-')

hl=legend(['Classical GramSchmidt'; 'Altered GramSchmidt'; 'Two times GramSchmidt'; 'HouseHolder']);
xtitle("Presition Digits", "n size", "bits")
```

```
// @Author: Rosas H26ernandez Oscar Andres
// @Author: Alarcón Alvarez Aylin Yadira Guadalupe
// @Author: Laurrabaquio Rodríguez Miguel Salvador
// @Author: Pahua Castro Jesús Miguel Ángel

getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework2/Code/Algorithms')
clc;


digitsAltered = eye(12, 1)
digitsClassic = eye(12, 1)
digitsTwo = eye(12, 1)
digitsNormal = eye(12, 1)
data = eye(12, 1)

for i = (2 : 12)
    H26 = eye(i, i)
    data(i) = i

    for i = (1 : i)
        for j = (1 : i)
            H26(i, j) = 1 / (i + j - 1)
        end
    end


    [Q26Classic, R26] = GramSchmidt(H26, 0)
    [Q26Altered, R26] = GramSchmidt(H26, 1)
    [Q26Two, R26] = GramSchmidt(Q26Classic, 0)
    [L26] = CholeskyGaussian(H26' * H26)
    Q26N = H26 * L26'

    digitsClassic(i) = -log10(norm(eye(i, i) - Q26Classic' * Q26Classic))
    digitsAltered(i) = -log10(norm(eye(i, i) - Q26Altered' * Q26Altered))
    digitsTwo(i)     = -log10(norm(eye(i, i) - Q26Two' * Q26Two))
    digitsNormal(i)  = -log10(norm(eye(i, i) - Q26N' * Q26N))

    disp("Q of GramSchmidt Classical:")
    disp(Q26Classic)

    disp("Q of GramSchmidt Altered:")
    disp(Q26Altered)

end

plot(data,digitsClassic, '.b-')
plot(data,digitsAltered, '.r-')
plot(data,digitsTwo, '.g-')
plot(data,digitsNormal, '.black-')


hl=legend(['Classical GramSchmidt'; 'Altered GramSchmidt'; 'Two times GramSchmidt'; 'Normal
    equation']);
xtitle("Presition Digits", "n size", "bits")
```

1.5. 27

Este proceso lo haremos hasta $\epsilon = 2^{-10}$ porque ϵ más pequeñas simplemente ya hacer que se trate a la matriz como singular pues se ve como una matriz con 3 columnas iguales, por lo tanto causa errores al tomarse como una matriz singular.

Lo que podemos ver que mientras epsilon más y más pequeña nos acercamos al claro resultado donde $x_1 = x_2 = x_3 = 0.333$

Algo curioso y diferente de los métodos anteriores, del parcial pasado es que practicamente todos los métodos dan el mismo resultado, curiosamente GramSchmidt es el único diferente en ciertos ϵ.

```
// @Author: Rosas Hernandez Oscar Andres
// @Author: Alarcón Alvarez Aylin Yadira Guadalupe
// @Author: Laurrabaquio Rodríguez Miguel Salvador
// @Author: Pahua Castro Jesús Miguel Ángel

getd('/Users/mac/Documents/Projects/Learning/UNAM/NumericalAnalysis/Homework3/Code/Algorithms')
clc;


values = [
    2;
    1.1;
    1;
    0.9;
    2e-01;
    2e-02;
    2e-05;
    2e-08;
    2e-10;
    2e-11;
]

[numberOfValues, _] = size(values)

for i = (1 : numberOfValues)

    e = values(i)
    disp("============")
    disp("epsilon:")
    disp(e)

    A27 = [
        1 1 1;
        e 0 0;
        0 e 0;
        0 0 e;
    ]

    b27 = [
        1;
        0;
        0;
        0;
    ]

    disp("Ax = b")

    disp("A:")
    disp(A27)

    try
        x27LeastSquares = LeastSquares(A27, b27)

        [Q27HH, R27HH] = HouseHolder(A27)
        x27HouseHolder = QRDecomposition(Q27HH, R27HH, b27)

        [Q27GS, R27GS] = GramSchmidt(A27, 0)
        x27GramSchmidt = QRDecomposition(Q27GS, R27GS, b27)

        [Q27G, R27G] = Givens(A27)
        x27Givens = QRDecomposition(Q27G, R27G, b27)


        disp("x of Least Squares:")
        disp(x27LeastSquares)
```

```
        disp("Ax of Least Squares:")
        disp(A27 * x27LeastSquares)

        disp("x of HouseHolder:")
        disp(x27HouseHolder)

        disp("Ax of HouseHolder:")
        disp(A27 * x27HouseHolder)

        disp("x of GramSchmidt:")
        disp(x27GramSchmidt)

        disp("Ax of GramSchmidt:")
        disp(A27 * x27GramSchmidt)

        disp("x of Givens:")
        disp(x27Givens)

        disp("Ax of Givens:")
        disp(A27 * x27Givens)
    catch
        disp("Error: Singular matrix")
    end




end
```

2. Anexo

2.1. Givens

```
// Get the estimated solution to Ax = b using the HouseHolder transformation
// @param: A a matriz in M_{m x n} where m > n
// @return: Q a matriz in M_{m x m} that is ortogonal
// @return: R a matriz in M_{m x n} that is triangular superior

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function [Q, R] = Givens(A)
    [m, n] = size(A);
    Q = eye(m, m);
    R = A;

    for j = (1 : n)
        for i = (m : -1 : j + 1)
            GivenMatrix = eye(m, m);
            [c, s] = GivensRotations(R(i-1, j), R(i, j));
            GivenMatrix([i-1, i], [i-1, i]) = [c -s; s c];

            R = GivenMatrix' * R;
            Q = Q * GivenMatrix;
        end
    end

end

function [c,s] = GivensRotations(a,b)
    if (b == 0)
        c = 1;
        s = 0;
    else
        if abs(b) > abs(a)
            r = a / b;
            s = 1 / sqrt(1 + r**2);
            c = s * r;
        else
            r = b / a;
            c = 1 / sqrt(1 + r**2);
            s = c * r;
        end
    end

end
```

2.2. GramSchmidt

```
// Get the estimated solution to Ax = b using the Gramm-Schmmidt transformation
// @param: A a matriz in M_{m x n} where m > n
// @param: option if 1 then is the alternate else is the classic algorithm
// @return: Q a matriz in M_{m x m} that is ortogonal
// @return: R a matriz in M_{m x n} that is triangular superior


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function [Q, R] = GramSchmidt(A, option)
    [m, n] = size(A);

    Q = zeros(m, n);
    R = A

    for j = (1:n)

        Q(:,j) = A(:, j)

        for i = (1 : j - 1)
            if (option == 1)
                R(i, j) = Q(:, i)' * Q(:, j);
            else
                R(i, j) = Q(:, i)' * A(:, j);
            end

            Q(:, j) = Q(:, j) - R(i, j) * Q(:, i);
        end

        R(j, j) = norm(Q(:, j));
        Q(:, j) = Q(:, j) / R(j, j);
    end

endfunction
```

2.3. HouseHolder

```
// Get the estimated solution to Ax = b using the HouseHolder transformation
// @param: A a matriz in M_{m x n} where m > n
// @return: Q a matriz in M_{m x m} that is ortogonal
// @return: R a matriz in M_{m x n} that is triangular superior

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function [Q, R] = HouseHolder(A, b)
    [m, n] = size(A);
    Q = eye(m, m);


    for i = (1 : n)
        aei = zeros(m - i + 1, 1);

        alpha = -sign(A(i,i)) * norm(A(i : m, i));
        aei(1) = alpha

        v = A(i : m, i) - aei;
        HouseHolder = eye(m - (i-1), m-(i-1)) -2 * ((v * v') / (v' * v));

        RealHouseHolder = eye(m, m);
        RealHouseHolder(i : m, i : m) = HouseHolder;

        A = RealHouseHolder * A;
        Q = Q * RealHouseHolder;
    end
    R = A;

endfunction
```

2.4. LeastSquares

```
// Get the estimated solution to Ax = b using least squares
// @param: A a matriz in M_{m x n} where m > n
// @param: b a vector of m rows
// @return: x a estimated solution

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function [x] = LeastSquares(A, b)
    [L] = CholeskyBanachiewicz(A' * A, 1);

    y = FowardSubstitution(L, A' * b);
    x = BackwardSubstitution(L', y);
endfunction
```

2.5. BackwardSubstitution

```
// Solve a system Ux = y where U is an upper triangular
// using the famous algorithm backward substitution
// @param: U triangular superior matrix
// @param: b the b in Ux = b
// @return: x the solution vector

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function [x] = BackwardSubstitution(U, b)
    [m, n] = size(U);
    x = zeros(n, 1);

    for i = (n : -1 : 1)
        if (U(i, i) == 0)
            error('Error: Singular matrix');
            return;
        end

        x(i) = b(i) / U(i,i);

        for j = (1 : i - 1)
            b(j) = b(j) - U(j, i) * x(i);
        end
    end
endfunction
```

2.6. CholeskyBanachiewicz

```
// Factor A as A = L * L^T
// using the famous algorithm called Cholesky using this really awesome propierty
// First A = L U then we make U a unit upper triangular matrix so we have L D L' and then
// we do L D2 D2 L' were D2(i, j) = sqrt(D(i, j)) finally we associate and we have A = L2 * L2'
// where L2 = L * D2
// @param: A a positive defined matrix (so A is symetric)
// @param: option if 1 then A = L * L else A = L * D * L
// @return: L lower triangule matrix

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function [L, D] = CholeskyBanachiewicz(A, option)
    [m, n] = size(A);
    D = eye(n, n);
    L = eye(m, n);
    U = A;

    for step = (1 : n - 1)
        if (A(step, step) == 0)
            error('Error: Singular matrix');
            return;
        end

        for row = (step + 1 : n)
            L(row, step) = U(row, step) / U(step, step);
            for column = (1 : n)
                U(row, column) = U(row, column) - L(row, step) * U(step, column);
            end
        end
    end

    if option == 1
        for step = (1 : n)
            for row = (step : n)
                L(row, step) = L(row, step) * sqrt(U(step, step));
            end
        end
    else
        for step = (1 : n)
            D(step, step) = U(step, step);
        end
    end



endfunction
```

2.7. CholeskyGaussian

```
// Factor A as A = L * L^T
// using the famous algorithm called Cholesky using a modification of Gaussian Elimination
// @param: A a positive defined matrix (so A is symetric)
// @return: L lower triangule matrix

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function [L] = CholeskyGaussian(A)

    [m, n] = size(A);
    L = zeros(m, n);

    for step = (1 : n)
        A(step, step) = sqrt( A(step, step) );

        for column = (step + 1 : n)
            A (column, step) = A(column, step) / A(step, step);
        end

        for i = (step + 1 : n)
            for j = (step + 1 : n)
                A(i, j) = A(i, j) - A(i, step) * A(j, step);
            end
        end
    end

    for row = (1 : n)
        for column = (1 : n)
            if (row >= column)
                L(row, column) = A(row, column);
            end
        end
    end

endfunction
```

2.8. CompleteLUDecomposition

```
// Factor A as PAQ = LU
// @param: A a not singular matrix
// @return: L (not sure) lower triangule matrix
// @return: U upper triangule matrix
// @return: P permutation matrix
// @return: Q permutation matrix

// JUST BECAUSE I WRITE IT, IT DOES NOT MEAN IT IS FAST, IT IS NOT FAST!

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function [L, U, P, Q] = CompleteLUDecomposition(A)
    [m, n] = size(A);
    if (m ~= n) == 0 then
        error('Error: Not square matrix');
    end
    P = eye(n, n);
    Q = eye(n, n);
    L = eye(n, n);
    U = A;

    for step = (1 : n - 1)
        Qi = eye(n, n);
        Pi = eye(n, n);

        [maxIndex, index] = max(abs(A(step : n, step : n)));
        index(1) = index(1) + step - 1;
        index(2) = index(2) + step - 1;

        if (maxIndex == 0)
            error('Error: Singular matrix');
        end

        temporal = Pi(step, :);
        Pi(step, :) = Pi(index(1), :);
        Pi(index(1), :) = temporal;

        temporal = Qi(:, step);
        Qi(:, step) = Qi(:, index(2));
        Qi(:, index(2)) = temporal;

        temporal = U(step, :);
        U(step, :) = U(index(1), :);
        U(index(1), :) = temporal;

        temporal = U(:, step);
        U(:, step) = U(:, index(2));
        U(:, index(2)) = temporal;

        for row = (step + 1 : n)
            L(row, step) = U(row, step) / U(step, step);
            for column = (1 : n)
                U(row, column) = U(row, column) - L(row, step) * U(step, column);
            end
        end

        Q = Q * Qi;
        P = Pi * P;
    end

endfunction
```

2.9. FowardSubstitution

```
// Solve a system Ly = b where L is triangular inferior
// using the famous algorithm foward substitution
// @param: L triangular inferior matrix
// @param: b the b in Ly = b
// @return: x the solution vector

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function [y] = FowardSubstitution(L, b)
    [m, n] = size(L);
    y = zeros(n, 1);

    for i = (1 : n)
        if (L(i, i) == 0)
            error('Error: Singular matrix');
            return;
        end

        y(i) = b(i) / L(i, i);

        for j = (i + 1 : n)
            b(j) = b(j) - L(j, i) * y(i);
        end
    end
endfunction
```

2.10. LUDecomposition

```
// Factor A as A = L * U
// @param: A a not singular matrix
// @return: L lower triangule matrix
// @return: U upper triangule matrix

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function [L, U] = LUDecomposition(A)
    [m, n] = size(A);
    L = eye(m, n);
    U = A;

    for step = (1 : n - 1)

        if (A(step, step) == 0)
            error('Error: Singular matrix');
            return;
        end

        for row = (step + 1 : n)
            L(row, step) = U(row, step) / U(step, step);

            for column = (1 : n)
                U(row, column) = U(row, column) - L(row, step) * U(step, column);
            end
        end
    end

endfunction
```