Introduction to Communications and IOT

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Contents

1 Introduction

1.1 What a Signal is

- It's a Quantitative Representation of Information
- The most basic representation of a signal is in the form of a graph (t on X-axis and $f(t)$ on Y-axis)

1.2 Types of Communication

1.2.1 Wired / Wireless

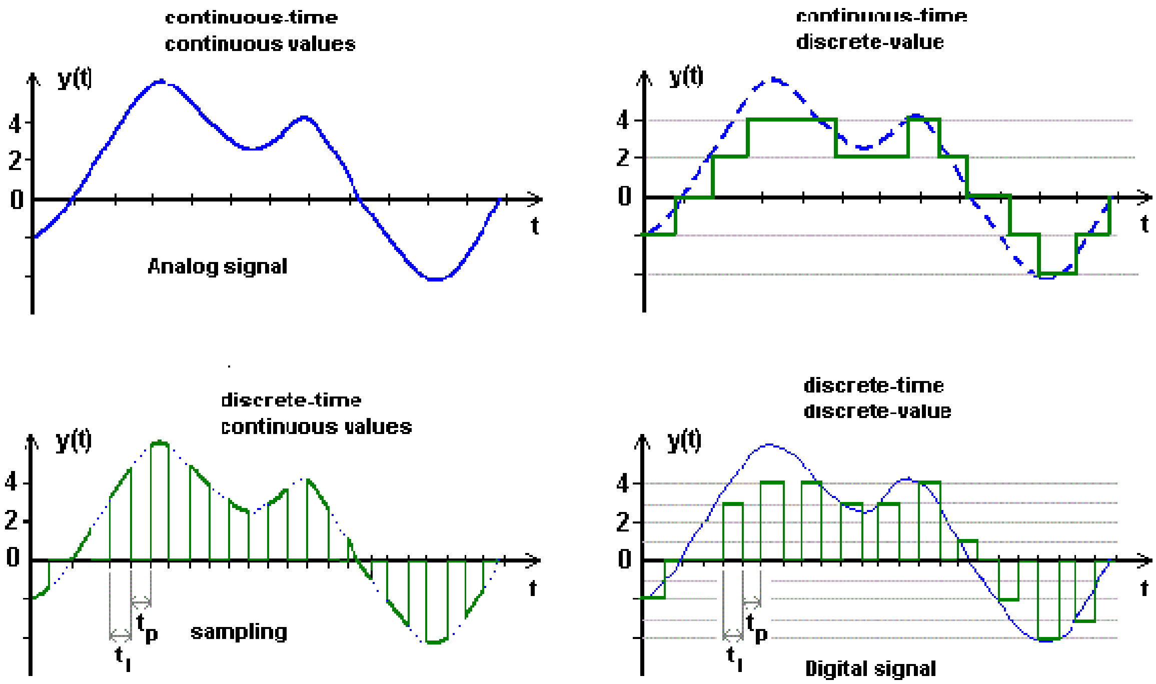
1. **Wired**:
    - Via Coaxial cables or Fibre-Optic Cables
2. **Wireless**:
    - Via Electromagnetic waves or rays

1.2.2 Unidirectional / Bidirectional

1. **Simplex**:
    - One-way
    - Eg. Broadcast, FM
2. **Half-Duplex**:
    - Two-way, but only one direction at a time
    - Eg. walkie-talkie
3. **Duplex**:
    - Two-way, and both directions are simultaneously possible

1.2.3 Analogue / Digital

1. **Analog**:
    - Both t and $f(t)$ are continuous
2. **Digital**:
    - Both t and $f(t)$ are discrete
3. **Continuous-Time**:
    - t is continuous, but $f(t)$ is discrete
4. **Discrete-Time**:
    - t is discrete and $f(t)$ is continuous



1.2.4 Transmission Technique

Before knowing this, you must know what bandwidth is:

Bandwidth:

- Range of frequencies a signal operates.
- In other words:

  Bandwidth = (Highest Frequency of the Wave/Signal)−(Lowest Frequency of the Wave/Signal)

- Fast, irregular variations in frequency $\propto$ Bandwidth

1. **Baseband**:
   - Digital Signals which are sent via TDM (Time Division Multiplexing)
   - One signal uses the entire bandwidth
2. **Broadband**:
   - (I'll add this later)

2 Characteristics of a Signal

2.1 Standard Notation of a Standard Sinusoidal Signal

- For a graph where X-axis = θ and Y-axis = $sin(\theta)$, the measure of input is θ.
- To actually measure a signal against time, X-axis = t (time) and Y-axis = $sin(\theta)$
- Here's what we do for that: $sin(\theta + \phi) = sin(\omega t + \phi)$

2.2 Angular Frequency

- ω = Angular Frequency/Velocity
- $= \frac{Angle}{Time}$
- $= \frac{2\pi}{T}$

2.3 Frequency

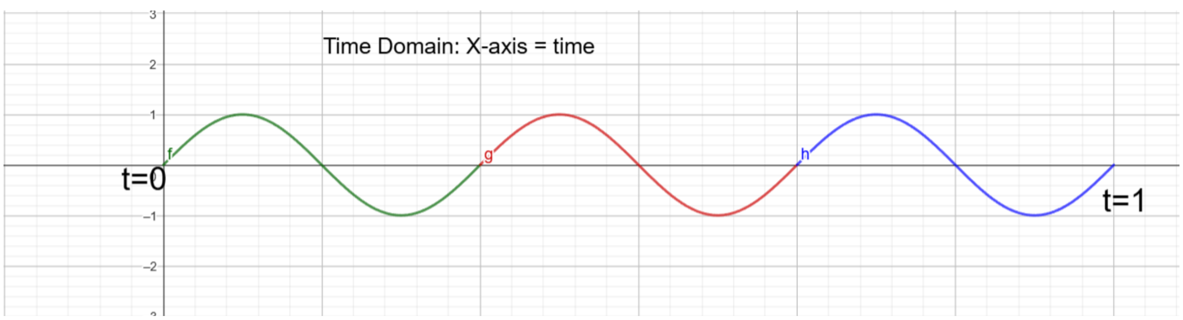
- $f = \frac{1}{T}$
- So, $\omega = \frac{2\pi}{T}$ can also be written as $\omega = 2\pi f$

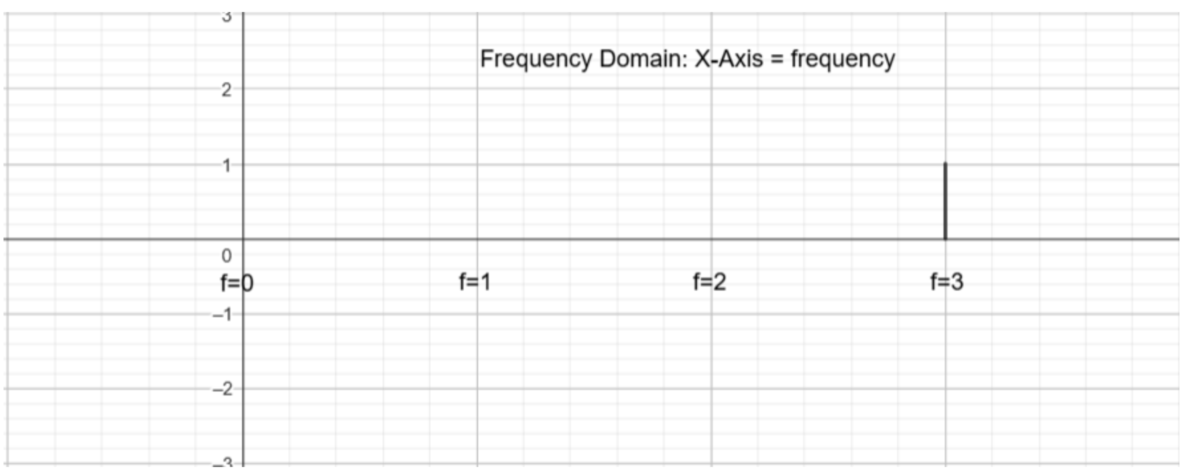
2.4 Phase

- θ or ωt is the X-coordinate.
- Phase ϕ is added to the X-coordinate, so the wave shifts to the left by ϕ
- In a way, it's an offset to a wave. (Check `https://www.geogebra.org/m/rzzqtx6q` for some Visualization)
- For example, if a sine wave is offset by $\frac{1}{6}th$ of a cycle, then the phase would be $\frac{1}{6} * 360^0 \Rightarrow$ Phase $= 60^0$

3 Time Domain vs Frequency Domain

In both cases, Y-Axis = *Amplitude*. Only X-Axis changes





4 Odd Signals vs Even Signals

- **Odd Signals/Functions**: $y(-x) = -y(x)$
- **Even Signals/Functions**: $y(-x) = y(x)$
- Even functions are called even probably because of the exponents they contain (even numbers). Hence even if you input negative numbers, the even exponents turn them into positive numbers. It's probably the same thing with odd functions too (odd exponents preserve sign, so if you input negative numbers, the output is negative too)
- Even functions show reflectional symmetry
  - Symmetrical about the y-axis i.e. $y(-x)$ is just the mirror image of $y(x)$ about y-axis
  - In other words, you only flip the signs of the x-coordinates
- Odd functions show rotational symmetry
  - you rotate the $y(x)$ about the origin to get $y(-x)$.
  - In other words, you flip the signs of both x and y coordinates

5 Some Common Signals

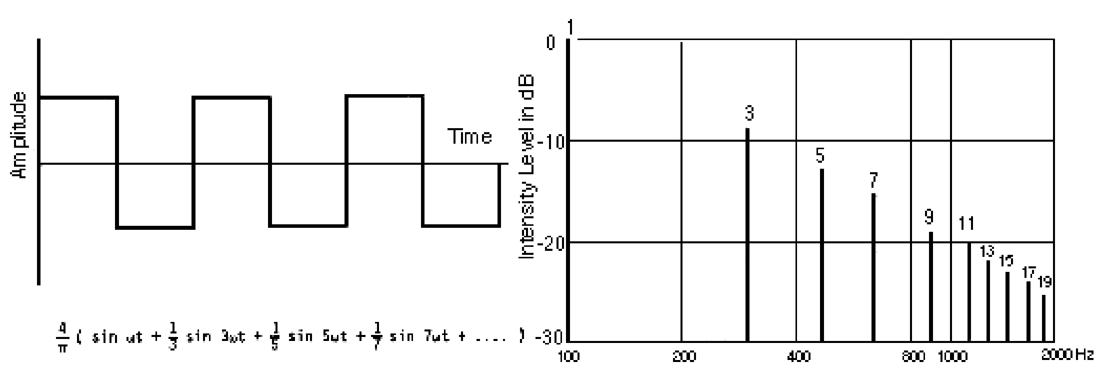
- The GeoGebra plots were made with the help of a simple Python program. I made a string that consists of the first 50 or 100 terms of each of the equations and fed that into GeoGebra.
- For example, the triangle wave sequence was given by:

```
str = f'cos(1)'
for i in range(3,100,2):
    str += f'+cos({i}*x )/{i*i} '
print(str)
```

5.1 Square Wave

- Given by

$$\frac{4}{\pi}(sin(\omega t) + \frac{sin(3 * \omega t)}{3} + \frac{sin(5 * \omega t)}{5} + \frac{sin(7 * \omega t)}{7} + ...)$$



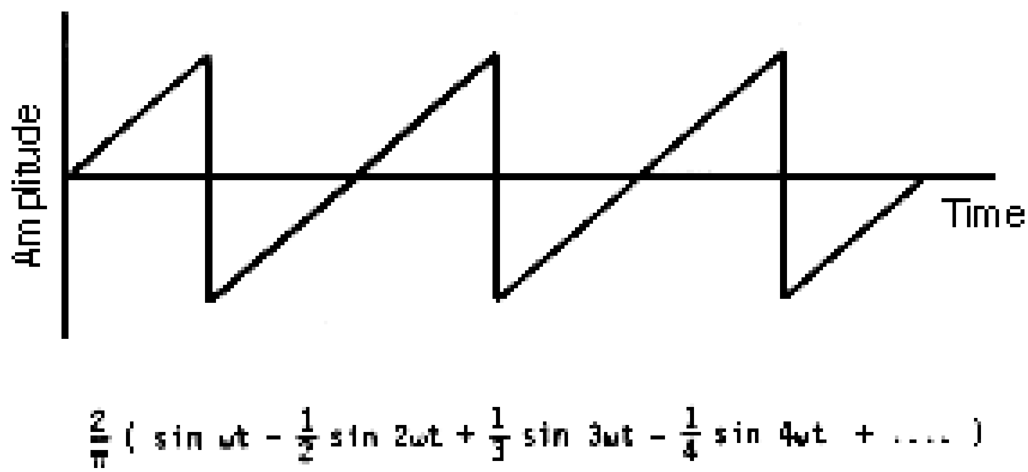
- Click here to view in GeoGebra

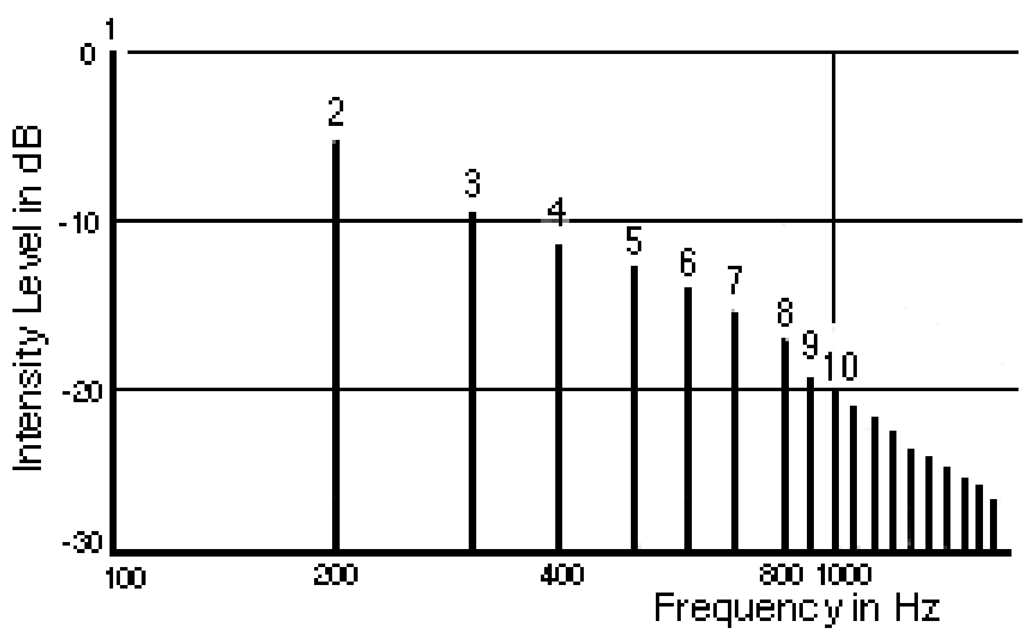
- To make something like this in MATLAB
  - Add as many "Sine Wave" Blocks from Simulink/Sources, and the amplitudes of each of them will be 1, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, etc.
  - Connect them all to an "Add" block, and connect that block to a "Time Scope"

5.2 Sawtooth Wave

- Given by

$$\frac{2}{\pi}(sin(\omega t) - \frac{sin(2*\omega t)}{2} + \frac{sin(3*\omega t)}{3} - \frac{sin(4*\omega t)}{4} + \frac{sin(5*\omega t)}{5} - \frac{sin(6*\omega t)}{6} + \frac{sin(7*\omega t)}{7} + ...)$$



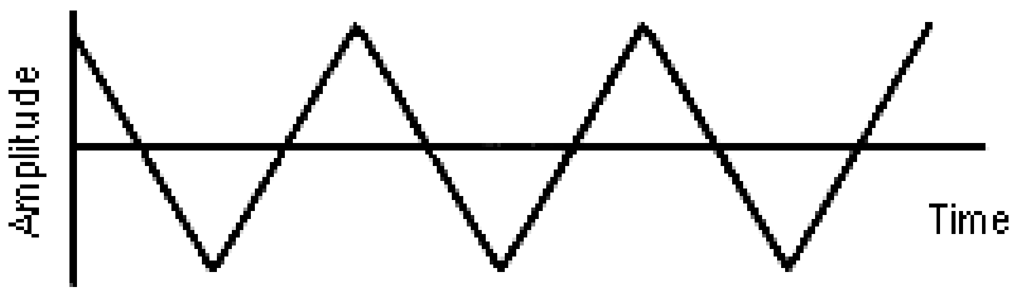


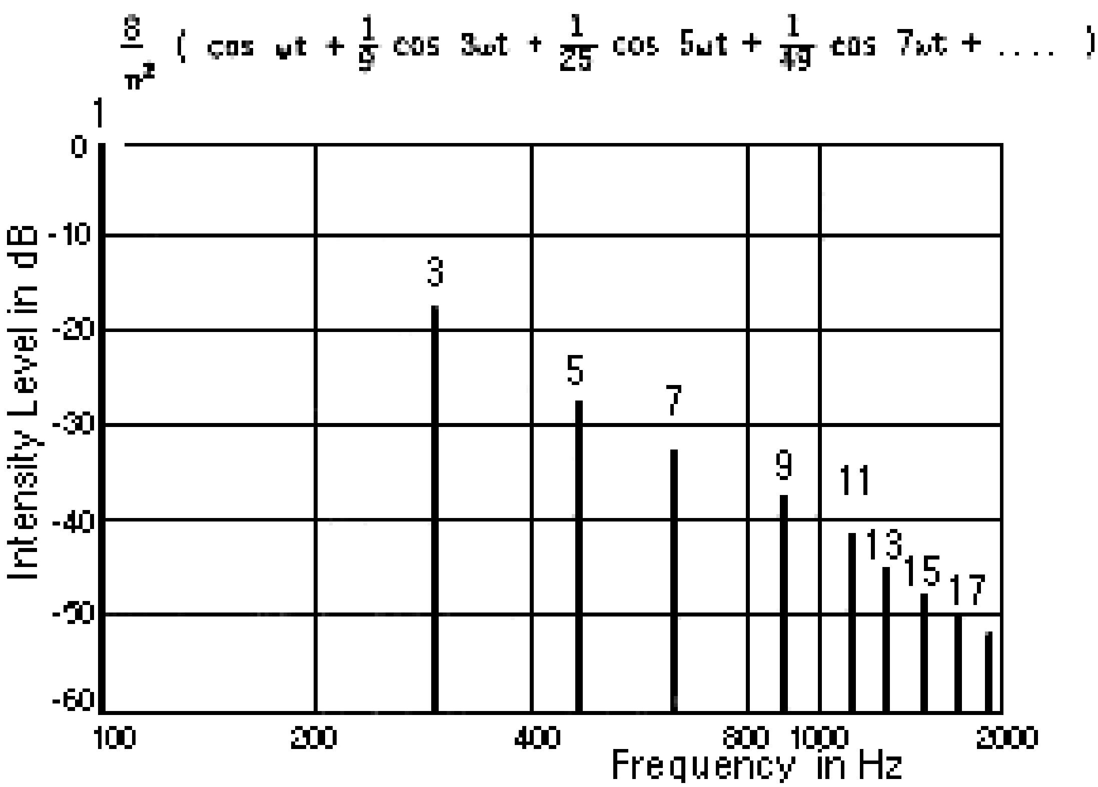
- Click here to view in GeoGebra

5.3 Triangle Wave

- Given by

$$\frac{8}{\pi^2}(cos(\omega t) + \frac{cos(3 * \omega t)}{3^2} + \frac{cos(5 * \omega t)}{5^2} + \frac{cos(7 * \omega t)}{7^2} + ...)$$





- Click here to view in GeoGebra

6 Energy and Power of a Signal

6.1 Prerequisite knowledge

- Let's assume we have a sinusoidal voltage and current
- $P = \frac{V^2}{R} = I^2R$
- This means that the power of a signal is some **constant** times **voltage squared** or **current squared**

- Let us have a general signal $x(t)$ which can either be sinusoidal voltage or sinusoidal current

$$x(t) = V \textbf{ or } x(t) = I$$

- So Instantaneous Power $= P = (x(t))^2$

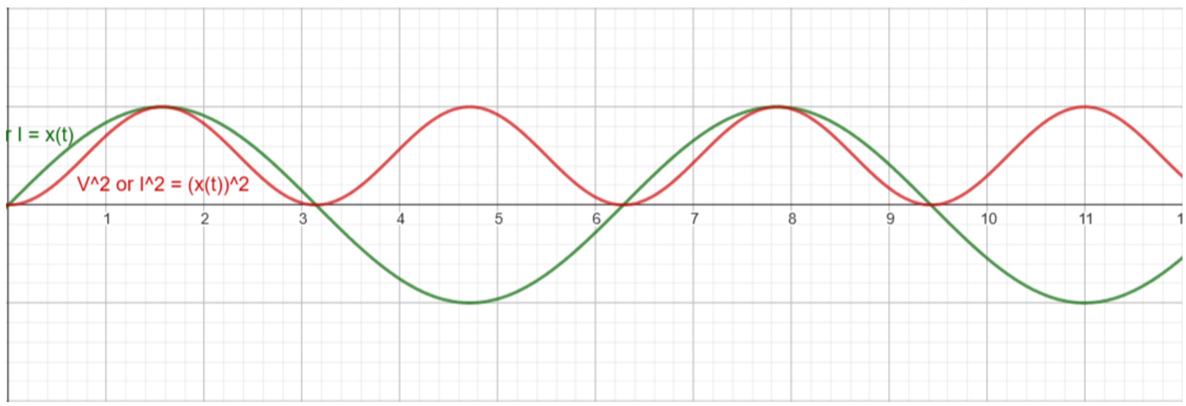


Figure 1: Green Curve showing V or I and the Red Curve showing P

6.2 Energy

- Energy $=$ Power $*$ time
- But the above formula is only applicable for discrete values.
- So the energy of a signal would be the area of the Power-Time Graph

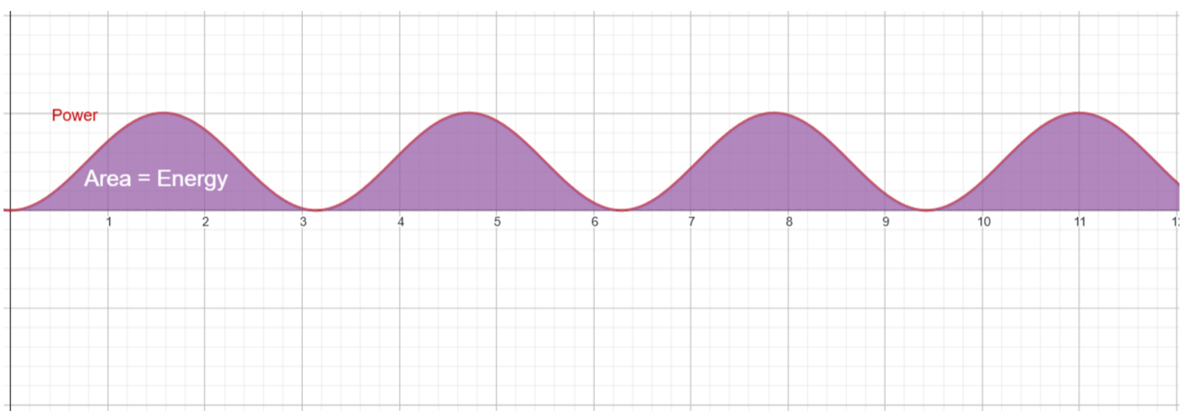


Figure 2: Area under the Red Curve

$$\text{Energy} = \int \mathrm{P}dt = \int_{\frac{-T}{2}}^{\frac{T}{2}} (x(t))^2 dt$$

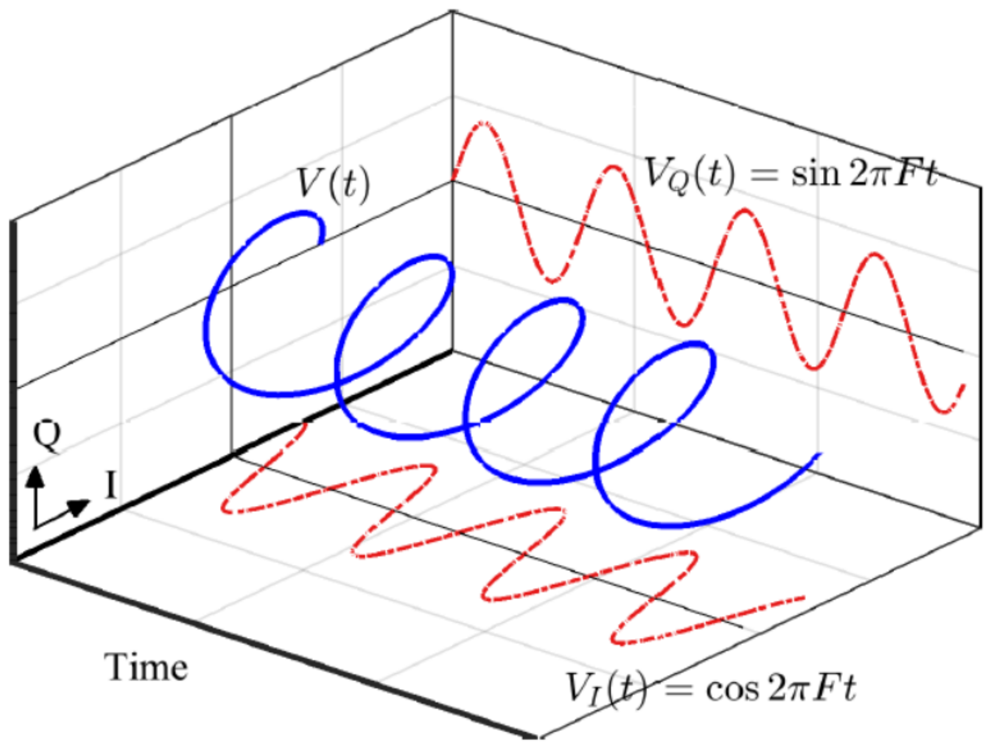
- The limits are actually from 0 to T, but having them from $\frac{-T}{2}$ to $\frac{T}{2}$ simplifies calculations.

6.3 Power

- Power is just $\frac{\text{Energy}}{\text{Time}}$.
- Power $= \frac{\int_{\frac{-T}{2}}^{\frac{T}{2}} (x(t))^2 dt}{T}$

7 Complex Sinusoids

- **In phase**: Two signals are said to be in phase if they have phase difference 0
- **Quadrature**: Two signals are said to be in quadrature if they have a phase difference 90
- A complex sinusoid is given as $cos(\theta) + jsin(\theta)$
- Now $cos(\theta)$ is taken to be on the $Inphase - Time$ plane, and $sin(\theta)$ is taken to be on the $Quadrature - Time$ plane
- This results in a helical structure.



- Number of rotations about the *time* axis, per unit time, is the frequency of the complex sinusoid.
- Anti-Clockwise rotation means Positive frequency, so clockwise rotation means negative frequency

8 Sampling

8.1 What it is

- Converting a continuous time signal into a discrete time signal by taking samples of the signals at discrete time intervals
- Say we have a continuous sinusoidal signal:

$$s(t) = Acos(2\pi Ft + \phi)$$

- In its discrete form, instead of a parameter t, you'd have parameters n and T_s:

$$s[n] = Acos(2\pi FnT_s + \phi)$$

or

$$s[n] = Acos(2\pi F\frac{n}{F_s} + \phi)$$

Here, T_s = Sampling Time Period and F_s = Sampling Frequency

8.2 Sampling Theorem or Nyquist Theorem

- F_s is the number of samples taken per second i.e. the **sampling rate**. Likewise, T_s is the time taken to record one sample
- If F_s is too less, you won't be able to capture the wave correctly. You'll end up over-simplifying the wave.
- This is called **aliasing**, and it's where high-frequency components appear as low-frequency components because of insufficient sampling rate.
- Nyquist Theorem states that:

$$F_s \geq 2B$$

  where B is the highest bandwidth present in the signal

  Another way of saying this would be:

$$B \leq \frac{F_s}{2}$$

9 Filters

9.1 Analog Filters

1. **Low Pass Filters**:
    - Keeps frequencies below a cutoff, and cuts off everything after it
    - It's used for smoothening images
    - Helps removing aliasing effect, as instead of increasing sampling frequency (it has to be at least double the highest frequency), you could cut off all the higher frequencies and then sample.
2. **High Pass Filters**:
    - Keeps frequencies after a cutoff, and cuts off everything below it
    - It's used for sharpening images
    - Helps in removing noise (blurriness, caused due to low frequency signals)
3. **Band Pass Filters**: Keeps frequencies inside a range (above a lower cutoff, and below a higher cutoff), and cuts off everything outside
4. **Band Reject/Stop Fitlers**: Keeps frequencies outside a range, and cuts off everything inside

9.2 Digital Filters

- They're mathematical algorithms used on discrete time signals
- Before knowing this, you must know about impulse signals:
  - **Impulse Signal** $\partial[n]$:
    - $\partial[n] = 1$ if $n = 0$, and $\partial[n] = 0$ for any other value of n
    - This function is used for representing frequencies in digital signals
  - **Impulse Response** $h[n]$: Output of a system, if the input is an impulse signal.

9.2.1 Finite Impulse Response (FIR)

1. What it is
   - Output depends only on current and past input
   - Output does NOT depend on past output

$$y[n] = \sum_{i=0}^{M} b_i x[n-i]$$

where $y[n]$ = output of filter, $x[n]$ = input signal, b_i = filter coefficients, M = filter order = number of taps

- Eg. $y[n] = 0.25x[n] + 0.5x[n-1] + 0.25x[n-2]$
  - This is called a 3-tap FIR Filter
  - The impulse response for this filter would be
    * $h[n] = [y[0], y[1], y[2], y[3], ..., y[n]]$
    * $h[n] = [0.25, 0.5, 0.25, 0, 0, 0, ...]$

2. Characteristics
   - Stable (Phase response is linear, and can be good for image/audio processing)
   - Phase Accurate
   - Computationally expensive
   - Impulse response decays to 0

9.2.2 Infinite Impulse Response (IIR)

1. What is is
   - Output depends on past input AND past output i.e. it uses **feedback**.

$$y[n] = \sum_{i=0}^{N} a_i y[n-i] + \sum_{i=0}^{M} b_i x[n-i]$$

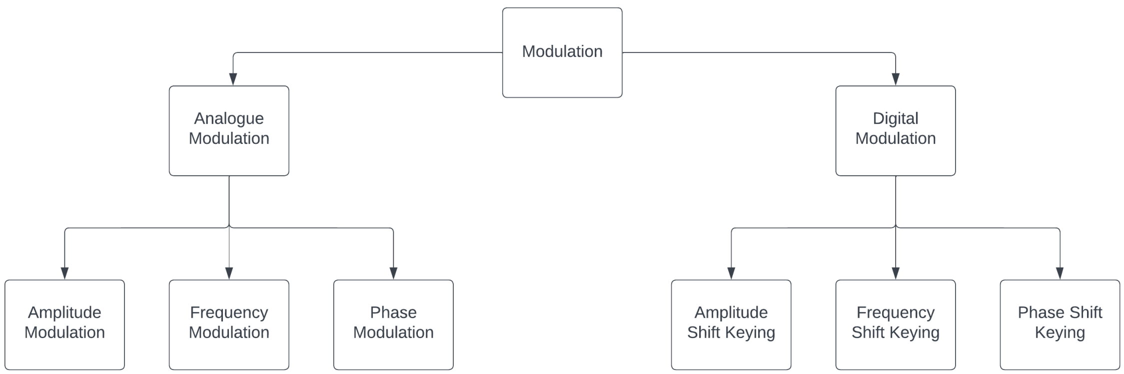
Here, it's just whatever FIR was, but you're also doing the same thing for the previous **outputs** too.

2. Characteristics
   - If order is low, frequency cutoffs will be sharp (Phase response is non-linear) (Can be good for real-time communication)

- Computationally efficient (you'll need less parameters)
- Can cause phase distortion
- Impulse response never decays to 0 (hence, it's infinite)

10 Modulation

$$\text{Base Signal/ Message} + \text{Carrier Signal} = \text{Modulated Signal}$$



10.1 Analogue Modulation

This is when the Message Signal is analog.

10.1.1 Amplitude Modulation (AM)

- Amplitude of carrier signal is changed
- Say the message signal is:
$$m(t) = A_m sin(2\pi f_m t)$$
And the original carrier signal is:
$$c(t) = A_c sin(2\pi f_c t)$$
- The modulated signal i.e. the new carrier signal is
$$s(t) = (A_c + A_m sin(2\pi f_m t))sin(2\pi f_c t)$$
- Another standard notation for this is obtained by taking A_c out common
$$s(t) = A_c(1 + \frac{A_m}{A_c} sin(2\pi f_m t))sin(2\pi f_c)$$
$$s(t) = (1 + \frac{A_m}{A_c} sin(2\pi f_m t)) * A_c sin(2\pi f_c)$$
$$s(t) = (1 + \mu sin(2\pi f_m t)) * A_c sin(2\pi f_c)$$
Where μ is the modulation index (described in the next section)
- In some cases, instead of adding 1 to $\mu sin(2\pi f_m t)$, a larger number is added. This is allowed as all you're doing is shifting the entire function up or down ($y(x) +$ some constant)

10.1.2 Modulation Index (m)

$$m = \mu = \frac{\text{Amplitude of Message Signal}}{\text{Amplitude of Carrier Signal}} = \frac{A_m}{A_c}$$

- If $m < 1 \Rightarrow$ UnderModulated (Not Using full bandwidth)
- If $m = 1 \Rightarrow$ Uses full bandwidth and is 100% Modulated
- If $m > 1 \Rightarrow$ OverModulated, causes distortion

10.1.3 Side Band

- We already know that $s(t) = (A_c + A_m cos(2\pi f_m t))cos(2\pi f_c t)$
- $s(t) = A_c cos(2\pi f_c t) + A_m cos(2\pi f_m t)cos(2\pi f_c t)$ ($\because$ Multiply the $cos(2\pi f_c t)$ term inside)
- $s(t) = A_c \cos 2\pi f_c t + \frac{A_m}{2} \cos 2\pi t(f_c - f_m) + \frac{A_m}{2} \cos 2\pi t(f_c + f_m)$ ($\because \cos A \cos B = \frac{\cos(A-B)}{2} - \frac{\cos(A+B)}{2}$)
- $f_c + f_m =$ upper band
- $f_c - f_m =$ lower band

10.1.4 Bandwidth

Band width = (upper band) - (lower band) = $2f_m$

10.1.5 Power of AM

- We know that

$$s(t) = A_c cos(2\pi f) + \frac{A_m}{2} cos 2\pi t(f_c - f_m) + \frac{A_m}{2} cos 2\pi t(f_c + f_m)$$

- Power of a sinusoidal signal is $\frac{A^2}{2}$, and here there are 3 sinusoidal signals.
- Total Power = $\frac{A_c^2}{2} + \frac{(\frac{A_m}{2})^2}{2} + \frac{(\frac{A_m}{2})^2}{2}$

$$\begin{aligned} Power &= \frac{A_c^2}{2} + 2\frac{(\frac{A_m}{2})^2}{2} \\ &= \frac{A_c^2}{2} + (\frac{A_m}{2})^2 \\ &= \frac{A_c^2}{2} + \frac{A_m^2}{4} \\ &= \frac{A_c^2}{2} + \frac{mA_c^2}{4} \end{aligned}$$

$\because m = \frac{A_m}{A_c} \Rightarrow A_m = mA_c$

- Taking $\frac{A_c^2}{2}$ common,

$$= \frac{A_c^2}{2}(1 + \frac{m^2}{2})$$

$$Power_{AM} = Power_{Carrier}(1 + \frac{m^2}{2}) = Power_{Carrier} + 2 * Power_{1Sideband}$$

10.1.6 AM Demodulation

1. Synchronous/ Coherent demodulation
   - Multiply recieved signal with a sine wave of same frequency and phase
2. Asynchronous/ Non-Coherent demodulation
   - Doesn't need frequency or phase matching
   - Uses diode, resistor, capacitor

10.1.7 Example

A carrier signal with a frequency of 1 MHz and amplitude of 10V is amplitude modulated by a sinusoidal message signal with frequency 10 kHz and amplitude 5V. Calculate:

1. The modulation index m.
$$m = \frac{5}{10} = 0.5$$
2. The carrier power Pc.
$$P_c = \frac{A_c^2}{2} = \frac{10^2}{2} = 50W$$
3. The sideband power P_{SB}.
$$P_{SB} = (\frac{A_m}{2})^2 = (\frac{5}{2})^2 = 6.25$$
4. The total transmitted power P_t.
$$P_t = P_c(1 + \frac{m^2}{2}) = 50(1 + \frac{0.5^2}{2}) = 56.25$$
5. The bandwidth of AM.
$$\text{Bandwidth} = 2f_m = 2 * 10 = 20kHz$$

10.2 Frequency Modulation (FM)

- We change frequency of carrier signal
$$s(t) = A_c cos(2\pi f_c t + \beta sin(2\pi f_m t))$$
where $\beta = \frac{\Delta f}{f_m}$, Δf = Frequency Deviation = (some constant) $* A_m$
- The more the value of Δf is, the better it will be noise-resistant

10.2.1 Bandwidth

- Bandwidth of FM is given by Carson's rule:
$$\text{Bandwidth} = 2(\Delta f + f_m)$$
$$\text{Bandwidth} = f_{max} - f_{min} + 2f_m$$

10.2.2 Classification based on modulation index

1. Narrow Band Frequency Modulation

$$\beta < 1$$

   - Frequency deviation is small
   - Used in voice communication, walkie-talkies

2. Wide Band Frequency Modulation

$$\beta > 1$$

   - Frequency deviation is large
   - Used in FM Broadcasting

10.2.3 Power in FM

- This is just the power of a regular sinusoidal signal

$$P = \frac{A_c^2}{2}$$

10.2.4 AM vs FM

AM	FM
Larger Range	Shorter Range
Lower Sound Quality	Better Sound Quality
Prone to distortion from electrical equipment	Not really

$$SNR_{FM} = (1 + \beta^2) SNR_{AM}$$

where SNR stands for Signal-to-noise

$$SNR = \frac{P_{\text{Signal}}}{P_{\text{Noise}}}$$

10.2.5 Example

A 100 MHz carrier is frequency modulated by a 5 kHz signal. The maximum frequency deviation is 75 kHz. Calculate:

1. The modulation index β.

$$\beta = \frac{\Delta f}{f_m} = \frac{75}{5} = 15$$

2. The bandwidth of FM using Carson's Rule.

$$2(\Delta f + f_m) = 2(75 + 5) = 160kHz$$

10.3 Phase Modulation

$$s(t) = A_c cos(2\pi f_c + k_p A_m cos(2\pi f_m))$$

where k_p = Phase Sensitivity Constant = $\frac{\text{Number of phase changes}}{\text{Amplitude}}$

10.3.1 Modulation Index

$$\beta = k_p A_m$$

- In FM, β depends on both frequency and amplitude
- But in PM, β depends on only amplitude.

10.3.2 Bandwidth

$$\text{Bandwidth} = 2(\beta + f_m)$$

10.3.3 FM vs PM

FM	PM
Needs wider bandwidth	Needs narrower bandwidth
depends on frequency deviation	depends on rate of phase shift
needs complex circuitry	easy to implement, but you need good timing

10.3.4 Example

A phase modulator has a phase sensitivity constant of 5 radians/volt. The modulating signal has a maximum amplitude of 3V and a maximum frequency of 4 kHz. Calculate:

1. The modulation index β.
$$\beta = k_p A_m = 5 * 3 = 15$$
2. The bandwidth of PM using Carson's Rule.
$$2(\beta + f_m) = 2(15 + 4) = 38kHz$$

10.4 Digital Modulation

10.4.1 Basics

- This is when the message signal is digital.

1. Bit Rate
$$\text{Bit Rate} = \text{Number of Bits per second} = \frac{\text{Number of bits}}{Time}$$
2. Baud Rate
$$\text{Baud Rate} = \text{Number of Symbols per Second} = \frac{\text{Number of Symbols}}{Time}$$
    - A symbol is a combination of bits, which represents 1 change of state.

$$\text{Bit Rate} = f_b = \text{Baud Rate} * \text{Number of Bits Per Symbol}$$

10.4.2 Amplitude Shift Keying

- You're still multiplying two waves: message and carrier.
- But there's a catch: The carrier signal is still analogue (some sort of sinusoidal wave), but the message signal is digital. At a given point of time, it's either 0 or 1.
- So the modulated signal becomes:

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t), & \text{if bit} = 1 \\ 0, & \text{if bit} = 0 \end{cases}$$

1. Bandwidth

$$\text{Bandwidth} = 2f_b$$

   where f_b is the bit rate

   - In AM, it was $2f_m$, where f_m was the message frequency.
   - Digital message frequency is what we call bit rate.

2. Power

$$\text{Power} = \frac{A_c^2}{2}$$

3. Properties

   - Easy to implement
   - Works with existing AM systems
   - Highly sensitive to noise
     - 0 bit means no power transmission, and so at low amplitudes, noise makes it very uncertain whether it's a 0, or it's a 1.
   - Used in Optical Fiber Communication and RFID (Radio Frequency Identification)

10.4.3 Frequency Shift Keying

$$\text{Carrier Frequency} = \begin{cases} f_1, & \text{if Message bit} = 1 \\ f_0, & \text{if Message bit} = 0 \end{cases}$$

Hence,

$$s(t) = \begin{cases} A_c \cos(2\pi f_1 t), & \text{if bit} = 1 \\ A_c \cos(2\pi f_0 t), & \text{if bit} = 0 \end{cases}$$

1. Bandwidth

$$f_1 - f_0 + 2f_b$$

2. Properties

   - More noise-resistant than ASK
   - Used in low-power applications
   - Requires larger bandwidth
   - Complex to implement
   - Used in Bluetooth and RFID

10.4.4 Phase Shift Keying

- Every binary number represents one phase i.e. an angle

o

$$\text{Number of Phases} = 2^{\text{Number of bits}}$$

or

$$\text{Number of bits} = log_2(\text{Number of Phases})$$

1. Binary Phase Shift Keying (BPSK) There is only 1 bit, hence there are 2^1 phases: $0^0 or 180^0$

$$s(t) = A_c \cos(2\pi f_c + \theta)$$

where:

$$\theta = \begin{cases} 0, & \text{for bit 0} \\ 180, & \text{for bit 1} \end{cases}$$

   (a) Power and Bandwidth

$$\text{Power} = \frac{A_c^2}{2}$$
$$\text{Bandwidth} = 2f_b$$

   (b) Properties
      - Immune to noise, as phase changes are very distinct
      - Used in deep-space communication due to its robustness
      - Very slow, as the number of bits per second is only 1.
      - It needs coherent demodulation

2. Quadrature Phase Shift Keying (QPSK)
   - There are 2 bits, hence there are $2^2 = 4$ phases: $0^0, 90^0, 180^0, 270^0$

$$s(t) = A_c \cos(2\pi f_c + \theta)$$

where:

$$\theta = \begin{cases} 0, & \text{for bit 00} \\ 90, & \text{for bit 01} \\ 180, & \text{for bit 10} \\ 270, & \text{for bit 11} \end{cases}$$

   (a) Power and Bandwidth

$$\text{Power} = \frac{A_c^2}{2}$$
$$\text{Bandwidth} = f_b$$

   (b) Properties
      - QPSK is twice as fast as BPSK, because the data rate is two times larger (2 bits as opposed to 1)
      - QPSK has higher noise sensitivity, as the phases aren't as distinct

3. Higher Order PSK

(a) 8-PSK:
   - 8 Phases
   - $log_2(8) = 3$ bits

(b) 16-PSK:
   - 16 Phases
   - $log_2(16) = 4$ bits

4. Quadrature Amplitude Modulation aka. IQ Modulation aka. Quadrature Carrier Multiplexing
   - It's a combination of ASK and PSK i.e. both amplitude and phase is modulated.
   - Consider two independent carrier waves $I(t)$ (inphase) and $Q(t)$ (quadrature). The transmitted signal is:
   $$s(t) = I(t)cos(2\pi f_c t) + Q(t)sin(2\pi f_c t)$$
   - 16-QAM basically means there are 16 waveforms/symbols. So the number of bits per symbol is $log_2(16) = 4$.
   - Wi-Fi 5 is basically 256-QAM, which means there are 256 symbols, and the number of bits per symbol is $log_2(256) = 8$
   - Wi-Fi 6 is 1024-QAM