Homework Complexity IBC028

To be handed in on March 16, 2018, in the exercise group.

This is the second and last set of homework exercises.

By handing in these homework exercises an extra bonus can be obtained for the examination: one full point if both sets of homeworks exercise are done perfectly, and otherwise a corresponding part of one point.

Exercise 1.

Assume access to an algorithm for matrix multiplication of two $n \times n$ matrices of time complexity $\Theta(n^2\sqrt{n})$. In the lecture a sketch of an approach was given for a matrix inversion algorithm of the same complexity, based on a way to describe matrix inversion by two recursive calls of matrix inversion of $n/2 \times n/2$ matrices, plus a finite number of matrix multiplications and matrix additions. Establish whether the same conclusion can be drawn for an alternative approach using k recursive calls of matrix inversion of $n/2 \times n/2$ matrices, plus a finite number of matrix multiplications and matrix additions, for $k = 3, 4, 5, 6$.

Exercise 2.

Let $L_1, L_2 \in P$. Prove that

$$\{u \in L_1 \mid u \notin L_2\} \in P.$$

Exercise 3.

Prove or disprove: if $L_1 \leq_P L_2$ and $L_2 \subseteq L_3$, then $L_1 \leq_P L_3$.

Exercise 4.

A CNF ϕ is satisfiable if an assignment exists for which every clause in ϕ is true. A CNF ϕ is called pre-satisfiable if an assignment exists for which every clause in ϕ is true, except for at most one. We want to prove that pre-satisfiability of CNFs is NP-complete.

(a) Describe what has to be proven for this.

(b) Give the proof.