STA130H1S - Fall 2022

Problem Set 4

() and STA130 Professors

Instructions

Complete the exercises of **Part 2** in this .Rmd file and submit your .Rmd and .pdf output through Quercus on Oct 6 by 5:00 p.m. ET.

Complete the exercises of $\bf Part~3$ in this .Rmd file and submit your .Rmd and .pdf output through $\bf Quercus$ on Oct 13 by 5:00 p.m. ET.

library(tidyverse)

Part 1: OPTIONAL Warm Up if Needed.

Complete these guided questions if you need some additional help getting started with hypothesis testing before moving on to Part 2, or if you want some additional practice with this hypothesis testing. You are not required to complete these questions as they ARE NOT included as part of your mark.

Question 1: Warm Up with Biased Coin Flipping

Approximately 23% of the general population use the social media platform Twitter. Suppose that the Department of Statistical Sciences (DoSS) is conducting a study to see if this percentage is the same among their undergraduate students (that is, all students in an undergraduate DoSS statistics program). Suppose n=400 students in statistics programs are randomly selected and asked whether or not they use Twitter. Suppose that 103 of these 400 students respond that they use Twitter.

(a) What is the NULL hypotheses H_0 in terms of p? What is H_1 in terms of H_0 ? In a simple sentence without H_0 and p notation, what is the claim of the NULL hypothesis?

REPLACE THIS TEXT WITH YOUR ANSWER

(b) Set set.seed(11) and use the sample() function to simulate the number of students who use Twitter in a random sample of 400 DoSS students under the assumption that the prevalence of Twitter usage is the same among DoSS students as it is in the general population. How many Twitter users did you have in your simulated sample of 400 students?

```
set.seed(11) # REQUIRED so the random sample is reproducible!
# Code your answer here
```

```
sample(c("Head","Tail"), size=10, replace=TRUE)
```

Hints

```
## [1] "Tail" "Tail" "Tail" "Head" "Tail" "Head" "Head" "Tail" "Tail" "Tail"
```

```
# will do the same thing as:
sample(c("Head","Tail"), size=10, prob=c(0.5, 0.5), replace=TRUE)

## [1] "Tail" "Tail" "Head" "Head" "Head" "Tail" "Tail" "Tail" "Tail"

# Even though the exact counts of "Head" and "Tail" differ each time you

# run this code, if you simulate enough coin flips (by increasing

# the value of 'size', you'll get approximately the same proportion

# of "Head" and "Tail" outcomes)

# To modify the code to make Tails much more likely than Heads,

# we could change the probs:
sample(c("Head", "Tail"), size=10, prob=c(0.1, 0.9), replace=TRUE)

## [1] "Tail" "Tail"
```

(c) Use geom_bar() to visualize the number of Twitter users versus non-Twitter users from your simulated sample with a bar plot. How does this simulated proportion compare to the general population rate of 23% and to the 103 of 400 sampled DoSS students?

```
# Code your answer here
```

REPLACE THIS TEXT WITH YOUR ANSWER

```
# You can make a vector a column of a `tibble` like this
tibble(flips = c("Head", "Tail", "Tail"))
```

Hints

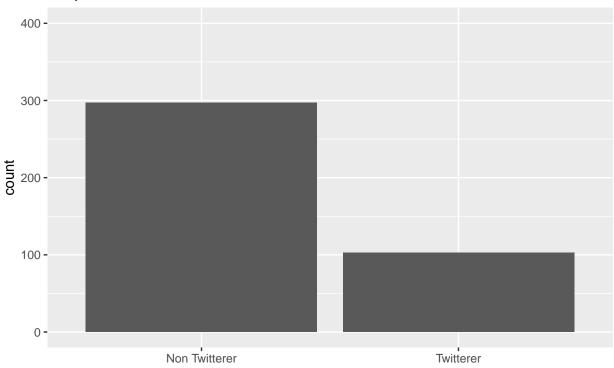
```
## # A tibble: 3 x 1
## flips
## <chr>
## 1 Head
## 2 Tail
## 3 Tail
```

(d) How is the geom_bar() function different than the geom_col() function below?

REPLACE THIS TEXT WITH YOUR ANSWER

```
ggplot(data=NULL, aes(x=c("Twitterer","Non Twitterer"), y=c(103,400-103))) +
geom_col() + lims(y=c(0,400)) +
labs(title="Sample of (n=400) DoSS\nDo you Tweet?", y="count", x="")
```

Sample of (n=400) DoSS Do you Tweet?



- (e) Simulate the sampling distribution of the test statistic under the assumption that the prevalence of Twitter usage among DoSS students matches that of the general population. Set the seed to the last 2 digits of your student number, use a simulation of size 1000, make a plot of the simulated sampling distribution, and describe the distribution in a few sentences.
 - If you don't set set.seed() the simulation will be different each time its run.
 - Your knit won't be reproducible and won't align with your interpretations and conclusions.

```
# Clearly label your figure with `labs(x="A primary title\n and a second line")` # Code your answer here
```

(e) What is the definition of a p-value?

REPLACE THIS TEXT WITH YOUR ANSWER

(f) What is the p-value of the hypothesis test based on the sampling distribution above?

```
# Use the `abs()` function to reflect the "as or more extreme" aspect of the p-value # Code your answer here
```

(g) At the alpha=0.05 significance level, what is your conclusion about this hypothesis test based on the p-value computed above?

REPLACE THIS TEXT WITH YOUR ANSWER

- (h) Which of the following statements is correct regarding the p-value above?
- (A) The probability that the proportion of DoSS students who use Twitter matches the general population.
- (B) The probability that the proportion of DoSS students who use Twitter does not match the general population.
- (C) The probability of obtaining a number of students who use Twitter in a sample of 400 students at least as extreme as the result in this study.
- (D) The probability of obtaining a number of students who use Twitter in a sample of 400 students at least as extreme as the result in this study, if the prevalence of Twitter usage among all DoSS students matches the general population.

REPLACE THIS TEXT WITH YOUR ANSWER

(i) What happens to the p-value if you change the seed value in set.seed()? What happens to the p-value if you change the size of the simulation?

REPLACE THIS TEXT WITH YOUR ANSWER

Part 2: One Sample Hypothesis Testing

DUE THURSDAY Oct 6 by 5 p.m. ET

Question 2: Scottish Medicine

A Scottish woman noticed that her husband's scent changed. Six years later he was diagnosed with Parkinson's disease. His wife joined a Parkinson's charity and noticed that odour from other people. She mentioned this to researchers who decided to test her abilities. They recruited 6 people with Parkinson's disease and 6 people without the disease. Each of the recruits wore a t-shirt for a day, and the woman was asked to smell the t-shirts (in random order) and determine which shirts were worn by someone with Parkinson's disease. She was correct for 12 of the 12 t-shirts! You can read about this here.

(a) Without conducting a simulation, describe what you would expect the sampling distribution of the proportion of correct guesses about the 12 shirts to look like if someone was just guessing.

If this was someone who was guessing then the odds of getting a correct result on a shirt would be p=0.5 so you would get ideally 6 correct diagnoses and 6 incorrect diagnoses but since this would be a smaller sample it may not reflect these results accurately you may give or take 1 correct answer

(b) Carry out a simulation and a hypothesis test of the woman being a lucky guesser as opposed to having some ability to identify Parkinson's disease by smell given that she correctly classified 12 of 12 t-shirts. Make a conclusion based on your work.

[1] 7e-04

The p value which in this instance gave us a value of 0.0007 tells us the chances of observing a test statistic at or more than than the extreme value we got. We concluded that the chances of observing this test statistic was 0.7 percent. This tell us our suspicions are true, since the chance is below 5 percent of happening and this proves that this is not a normal circumstance otherwise it would be very unlikely for this to be true. So it must be due to other factor like the claim of the special skill

(c) Actually, initially the woman correctly identified all 6 people who had been diagnosed with Parkinson's but incorrectly identified one of the others as having Parkinson's. It was only eight months later that the final individual was diagnosed with the disease. What is the p-value when only 11 of 12 were known to be correct?

[1] 0.0068

(d) Are you able to get the p-value for the test using the initial data (i.e., 11 correct instead of 12 correct) without running a new simulation?

ANSWER: You do not have to run the simulation because of the functionality of set.seed().set.seed(70) allows you to reproduce a particular sequence of numbers made. So to conclude, set.seed() will allow us to redo what the code did and apply different functions to it. The application can be seen in the code above

(e) Is the conclusion of the hypthesis test the same for an observed test statistic of 11/12 as for an observed test statistic of 12/12?

ANSWER: The conclusion states the observed test statistic for the above is not the same for 11/12 and 12/12 because there are different p values involved so probabilty of observing a test statistic as or more extreme is different. So the change is larger for 11/12 vs 12/12

Question 3: Fisher's Tea Experiment

There is an interesting account of the British statistician Ronald Fisher at a tea party in the 1920s. One of the other guests was algae scientist Dr. Muriel Bristol, who refused a cup of tea from Fisher because he put milk in first BEFORE pouring the tea. Bristol was convinced she could taste the difference, and much preferred the taste of tea where the milk was poured in afterwards. Fisher didn't think that there could be a difference and proposed a hypothesis test to examine the situation.

Fisher made 8 cups of tea, 4 with milk in first and 4 with tea in first, and gave them to Dr. Bristol without her seeing how they were made and she would say if she thought the tea or the milk was poured first. As it turned out, Dr. Bristol correctly identified if the tea or milk was poured first for all 8 of the cups. Fisher, being a skeptical statistician wanted to test if this could be happening by chance with Bristol just randomly guessing (or whether it seemed more likely that Bristol was not guessing).

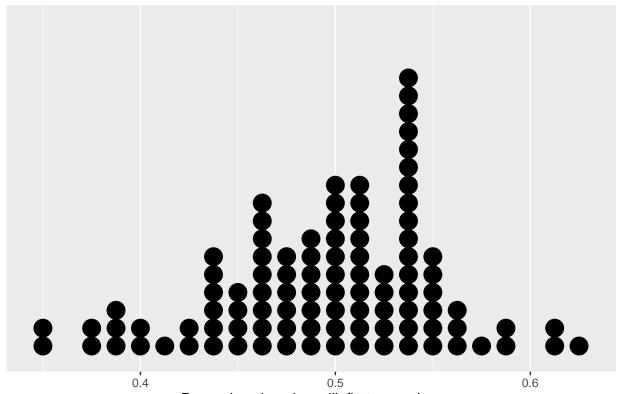
Suppose you run an experiment like this with students in STA130. You get a random sample of 80 STA130 students to each taste one British-style cup of tea and tell you whether they think the milk or tea was poured first. 49 students correctly state which was poured first. Go through the steps to test whether students are just guessing or not.

(a) What is the NULL hypotheses H_0 in terms of p? What is H_1 in terms of H_0 ? In a simple sentence without H_0 and p notation, what is the claim of the NULL hypothesis?

 H_0 p=0.6125 H_0 p=0.6125 H_0 $p \neq 0.6125$ The claim of the null hypothesis is that the probability of 80 STA130 students differenciating and identitying the tea being 61.25%

(b) Conduct a hypothesis test on the basis of the following simulated sampling distribution of the test statistic assuming the NULL hypothesis is true. For simplicity, this distribution shows the results of only 100 simulations, but in practice this likely wouldn't provide very good p-value resolution refinement.

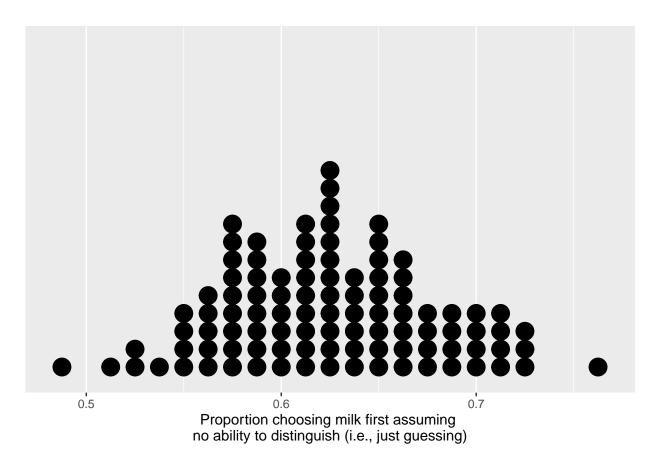
Bin width defaults to 1/30 of the range of the data. Pick better value with `binwidth`.



0.5
Proportion choosing milk first assuming no ability to distinguish (i.e., just guessing)

[1] 0.3858

Bin width defaults to 1/30 of the range of the data. Pick better value with `binwidth`.



- What does each single dot in the plot represent?

Each single dot in the plot represent a single repetition which basically indicates the number of simulations

• Based on this plot, what is your estimate of the p-value?

So the p value would be values of 0.6125 or less if the null hypothesis is true. So if we count the dots that satisfy this it would be 16.25 and 16.25/100=0.1625 so p=0.1625

• At the alpha = 0.05 significance level, what is your conclusion about this hypothesis test based on the p-value computed above?

Since 0.1625 multipled by 100 is 16.25 and 0.05 is 5% since 16.25>5 we would say that the values were more typical and so the null hypothesis is true.

(c) Suppose the analysis described in (b) is repeated but this time 1000 simulations are used to get a better estimate of the p-value, and the resulting p-value is 0.04. Do not conduct this simulation. At the alpha = 0.05 significance level, what is your conclusion about this hypothesis test for a p-value of 0.04?

If we were to look at this in percentages 0.04 would be 4% and 0.05 would 5% so 4<5 so can be said that the even of your p is so rare that your null hypothesis is false

Question 4: OPTIONAL primer for potential TUT discussion

• You may complete these questions for practice if you wish. You are not required to complete these questions as they ARE NOT included as part of your mark.

A criminal court considers two opposing claims about a defendant: they are either innocent or guilty. In the Canadian legal system, the role of the prosecutor is to present convincing evidence that the defendant is not

innocent. Lawyers for the defendant attempt to argue that the evidence is *not convincing enough* to rule out that the defendant could be innocent. If there is not enough evidence to convict the defendant and they are set free, the judge generally does not deliver a verdict of "innocent", but rather of "not guilty".

(a) If we look at the criminal trial example in the hypothesis test framework, which would be the null hypothesis and which the alternative?

REPLACE THIS TEXT WITH YOUR ANSWER

- (b) In the context of this problem, describe what rejecting the null hypothesis would mean. $REPLACE\ THIS\ TEXT\ WITH\ YOUR\ ANSWER$
- (c) In the context of this problem, describe what failing to reject the null hypothesis would mean.

REPLACE THIS TEXT WITH YOUR ANSWER

- (d) In the context of this problem, describe what a type II error would be. REPLACE THIS TEXT WITH YOUR ANSWER
- (e) In the context of this problem, describe what a type I error would be.

 REPLACE THIS TEXT WITH YOUR ANSWER

Part 3: Two Sample Hypothesis Testing Required Questions

DUE THURSDAY Oct 13 by 5 p.m. ET

Question 5: Social Media and Anxiety

There have been many questions regarding whether or not usage of social media increases anxiety levels. For example, do TikTok and Facebook posts create an unattainable sense of life success and satisfaction? Does procrastinating by watching YouTube videos or reading Twitter posts contribute unnecessary stress from deadline pressure? A study was conducted to examine the relationship between social media usage and student anxiety. Students were asked to categorize their social media usage as "High" if it exceeded more than 2 hours per day, and then student anxiety levels where scored through as series of questions, with higher scores suggesting higher student anxiety.

```
## Rows: 46
## Columns: 2
## $ social_media_usage <chr> "Low", "Low
```

(a) What is the NULL hypotheses H_0 in terms of $Median_{High}$ and $Median_{Low}$? In simple terms, what is the claim of the NULL hypothesis? What is H_1 in terms of H_0 ?

The null hypothesis in this instance is that the group of low social media users have the same median anxiety score adn those with high social media ussage. q $H_0: \mu_{High} = \mu_{Low}$

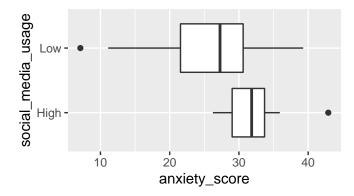
Hint A formal NULL hypotheses that the means of two groups are the same would be $H_0: \mu_{High} = \mu_{Low}$.

(b) Revisit your statements regarding the NULL hypotheses above with confounding in mind; namely, since social media usage is a self selecting process, perhaps social media users are already more anxious people on average regardless of their social media usage. If we make a determination about the NULL hypothesis are we actually addressing the question of "whether or not usage of social media increases anxiety levels"? Or are we just using a hypothesis test to examine if there is an observable difference between the two groups (regardless of its causes)?

We are testing whether or not there is a difference and not between the two groups because in this form of hypothesis testing we compare mediam values. We seperate the sample into two groups and check the difference. This is why our null hypothesis states a stituation where they are equal and our alternate hypothesis would suggest unequal. To establish causation we would need to prove that anxiety only came after the use of social media which is not a consideration in our test.

(c) Construct boxplots of anxiety_score for the two levels of social media usage, and write 2-3 sentences describing and comparing the distributions of anxiety scores across the social media usage groups.

```
install.packages("tidyverse")
## Installing package into '/opt/r'
## (as 'lib' is unspecified)
library(tidyverse)
install.packages("dplyr")
## Installing package into '/opt/r'
## (as 'lib' is unspecified)
library(dplyr)
install.packages("magrittr")
## Installing package into '/opt/r'
## (as 'lib' is unspecified)
library(ggplot2)
install.packages("ggplot2")
## Installing package into '/opt/r'
## (as 'lib' is unspecified)
anxiety data %>% ggplot(aes(x=anxiety score, y=social media usage)) + geom boxplot()
```



The median or the 50% quartile is higher for those with high social media ussage. The spread of the boxplot with low social media ussage is larger than the one with a high social media ussage which could lead to an incorrect interpretation of data.

(d) What do these data visually suggest regarding the claim that the *median* anxiety level is different for those who use social media in high frequency compared to those who use social media in lower frequency?

The blox-plots suggest that the median anxiety score for those who use social media in a high frequency is higher than those who use it in a low frequency. The boxplot also indicates that the spread of the high frequency users is smaller, so that may lead us to believe that the range was small and more data at different levels may be required to know how true the difference in median is

(e) Look at the code below and write a few sentences explaining what the code inside the for loop is doing and why.

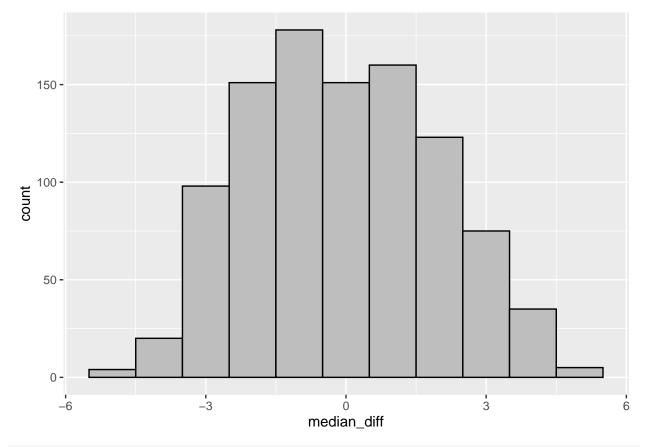
Firstly it puts all the following data into a tibble called sim_value. It takes the tibble simdaata and pipes it into a group_by which groupsthe data by social media ussage and then pipes that information into a summarize function which returns a new column:the list of medians of the anxiety score variable and the it pipes the tibble created by the summary function into another summary function which consists of a varriable which is the difference between medians and adds it to the tibble. It then puts the value into the data set simulated_values[i] at index 1. After a number of repetitions it makes a table of differences

```
test_stat <- anxiety_data %>% group_by(social_media_usage) %>%
  summarise(medians = median(anxiety_score), .groups="drop") %>%
  summarise(value = diff(medians))
test_stat <- as.numeric(test_stat)</pre>
test stat
## [1] -4.57
Х
                  "Correct" "Tails"
                                       "Correct"
                                                 "Tails"
                                                           "Tails"
                                                                      "Correct"
    [8] "Correct" "Correct" "Tails"
set.seed(523)
repetitions <- 1000;
simulated_values <- rep(NA, repetitions)</pre>
for(i in 1:repetitions){
  simdata <- anxiety_data %>% mutate(social_media_usage = sample(social_media_usage))
  sim_value <- simdata %>% group_by(social_media_usage) %>%
```

```
summarise(medians = median(anxiety_score), .groups="drop") %>%
summarise(value = diff(medians)) #finds difference between meedian

simulated_values[i] <- as.numeric(sim_value)
}
sim <- tibble(median_diff = simulated_values)

sim %>% ggplot(aes(x=median_diff)) + geom_histogram(binwidth=1, color="black", fill="gray")
```



```
num_more_extreme <- sim %>% filter(abs(median_diff) >= abs(test_stat)) %>% summarise(n())
p_value <- as.numeric(num_more_extreme / repetitions)
p_value</pre>
```

[1] 0.009

(f) Summarize the NULL hypothesis and then, at the alpha=0.05 significance level, state your conclusion about the hypothesis test of the NULL hypothesis based on the p-value computed above.

The null hypothesis stated that median1 = median2. Since 0.009 is less than 0.05, we reject the null hypothesis

(g) Do these data support the claim that the *median* anxiety level is different for those who use social media in high frequency compared to those who use social media in lower frequency? How about the claim that "usage of social media increases anxiety levels"?

Yes, this calim supports the fact that the median anxiety level for people who use social media is different from those who don't use social media. In our hypothesis we have only proven that the values are different however we have not proven that the cause of increased anxiety levels is social media.

Question 6: Airbags

The table below is adapted from "Biostatistics for the Biological and Health Sciences" and presents data from a random sample of passengers sitting in the front seat of cars involved in car crashes. Based on this data we'd like to make a determination as to whether or not death rates differ for passengers in cars with airbags and passengers in cars without airbags.

	Airbag available	No airbag available
Passenger Fatalities	45	62
Total number of Passengers	10,541	9,867

The code below creates a tidy data frame for this problem using the rep() function.

```
## Rows: 20,408
## Columns: 2
## $ group <chr> "airbag", "airbag", "airbag", "airbag", "airbag", "airbag", "airbag", "airbag", "dead", "d
```

(a) What is the NULL hypotheses H_0 in terms of p_{airbag} and $p_{no-airbag}$? What is H_1 in terms of H_0 ? In a simple sentence without H_0 and p_{airbag} and $p_{no-airbag}$ notation, what is the claim of the NULL hypothesis?

What is the NULL hypotheses H_0 : of p_{airbag} - $p_{no-airbag}$ =0; H_1 divided by H_0 /neq 1. The calim of the null hypothesis is that the chances of a fatality with an airbag and without an airbag are equal *

(b) Simulate the sampling distribution of the test statistic under the assumption that the NULL hypothesis state above is TRUE.

```
#don't want to see that warning.

Mean_tester_table <- data %>%
    mutate(outcome=case_when(outcome=="alive"~0,outcome=="dead"~1))%>%
    group_by(group) %>%
    summarise(mean_value_table = mean(outcome), .groups="drop") %>%
    summarise(value_differences = diff(mean_value_table))
test_stats <- as.numeric(Mean_tester_table)
test_stats</pre>
```

```
## [1] 0.002014527

set.seed(770)
repetitions <- 1000;
```

```
simulated_values <- rep(NA, repetitions)

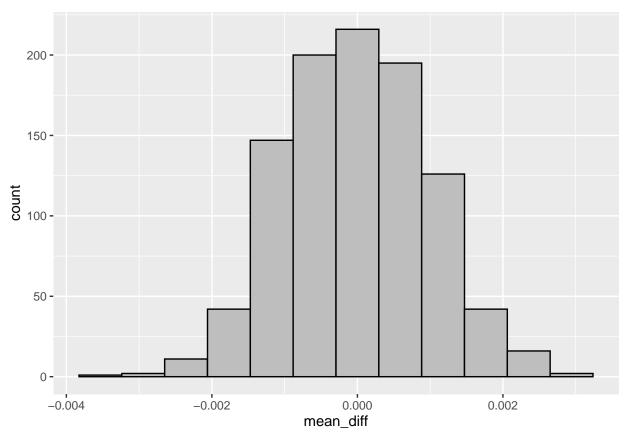
for(i in 1:repetitions){
    Simulated_data_set <- data %>% mutate(group = sample(group))

    simulated_Value <- Simulated_data_set %>% mutate(outcome=case_when(outcome=="alive"~0,outcome=="dead"
    group_by(group) %>%
    summarise(means = mean(outcome), .groups="drop") %>%
    summarise(value = diff(means)) #finds difference between meedian

    simulated_values[i] <- as.numeric(simulated_Value)
}

sim <- tibble(mean_diff = simulated_values)

sim %>% ggplot(aes(x=mean_diff)) + geom_histogram(bins=12, color="black", fill="gray")
```



```
num_more_extreme <- sim %>% filter(abs(mean_diff) >= abs(test_stats)) %>% summarise(n())
p_value <- as.numeric(num_more_extreme/repetitions)
p_value</pre>
```

[1] 0.032

(c) At the alpha = 0.10 significance level, what is your conclusion about this hypothesis test based on the p-value computed above?

Since our p value is less than our alpha we reject the null hypothesis.

(d) Based on your conclusion above, what kind of error could you have made?

This could have potentially been a type 1 error because we have rejected the null hypothesis eventhough it may have been true. One potential reason for this may have been an incorrect measure placed upon alpha

(e) Does your conclusion support the claim that "airbags save lives"? Or does it seem reasonable to believe that there could be some sort of confounding (like in Question 5) by which people who choose to drive in cars without airbags are just more likely on average do die if they're in a car crash irrespective of any safety benefit of airbags?

This calim does not state that airbags save lives, however it does conclude that there is a difference between the amount of people that use airbags and do not use airbags. It would be accurate to say that the people who choose to drive in cars without airbags are more likely however, this does not promote airbags, or bring to light any particular causation.

Question 7: OPTIONAL practice specifying NULL Hypotheses

• You may complete these questions for practice if you wish. You are not required to complete these questions as they ARE NOT included as part of your mark.

We've covered two kinds of hypothesis tests.

- In the first version we hypothesize a proportion in a population (the parameter p) and test that value using the proportion observed in the sample (the sample average \bar{x} , also sometimes notated as \hat{p}). These ONE sample hypotheses tests have a NULL hypothesis of the form $H_0: p = p_0$ (and of course the ALTERNATIVE hypothesis then takes the form $H_1: H_0$ is FALSE). Since this one-sample framework works for any average (not just proportions), another form of this test that is often encountered is $H_0: \mu = \mu_0$ where μ is the mean of the population (corresponding to the sample average \bar{x}).
- In the second version we hypothesize a relationship between two populations, such as that both populations have the same mean or median (or proportion or standard deviation, etc.). These TWO sample hypotheses tests have a NULL hypothesis of the form $H_0: \mu_1 = \mu_2$ or $H_0: p_1 = p_2$ (or $H_0: Median_1 = Median_2$ or $H_0: \sigma_1 = \sigma_2$, etc.) and of course the ALTERNATIVE hypothesis is still $H_1: H_0$ is FALSE.

For each of the following scenarios, state appropriate hypotheses H_0 and H_1 . For each scenario, also state in simple terms what the claim of the NULL hypothesis is.

Be sure to carefully define any parameters you refer to.

(a) A health survey asked individuals to report the number of times they exercised each week. Researchers were interested in determining if the proportion of individuals who exercised at least 100 minutes per week differed between people who live in the condos vs people who do not live in condos.

REPLACE THIS TEXT WITH YOUR ANSWER

(b) A study was conducted to examine whether a baby is born prematurely/early (i.e., before their due date) to whether or not the baby's mother smoked while she was pregnant.

REPLACE THIS TEXT WITH YOUR ANSWER

(c) Nintendo is interested in whether or not their online advertisements are working. They record whether or not a user had seen an ad on a given day and their amount of spending on Nintendo products in the next 48 hours. They are interested in determining if there is an association between whether or not the user say an ad and their expenditures.

REPLACE THIS TEXT WITH YOUR ANSWER

(d) Based on results from a survey of graduates from the University of Toronto, we would like to compare the median salaries of graduates from the statistics and graduates of mathematics programs.

REPLACE THIS TEXT WITH YOUR ANSWER

Part 4: OPTIONAL but STRONGLY Recommended for Practice

You may complete these questions for practice if you wish. You are not required to complete these questions as they ARE NOT included as part of your mark.

Question 8

Complete this One Sample Hypothesis Testing Practice Quiz using this Rmd file

Question 9

Complete this Two Sample Hypothesis Testing Practice Quiz using this Rmd file