

## CSE13S notes

### Week of 1/30/23

#### Assignment 3: Sortin time

- Sorting is the act of putting things into a defined order
- Dictionaries are sorted in what is called lexicographical order
  - Lexicographical = alphabetical order
- Numbers can be sorted in their natural order, or reverse order (low to high or high to low)
- There are both total and partial orderings, but we will only concern ourselves with total orderings (for now)

#### Why sort?

- Sorting adds information to our data
- For examples, we now can make assertions about before, after, lesser, greater, etc.
- Here are a few examples:
  - We can search most efficiently in sorted data
  - We can merge sorted list efficiently
  - We can detect duplicates efficiently
  - We can find the median efficiently

#### First idea:

- Enumerate all of the possible orderings of  $n$  objects
  - There are  $n!$  such orderings
- Pick the one that is in order
- On second thoughts, that was a bad idea

#### Second idea:

- Pick smallest element from a list of  $n$  elements (you must go through all  $n$  elements)
  - Put it in slot 1, you can ignore that slot now
  - Swap that element in the first slot with the selected element
- This is selection sort

#### Third idea:

- Suppose we have array with 1 element
  - This array is sorted
- Given a second element, it must go either before or after the first element
  - The 2 element array is sorted
- This is slightly better than selection sort
- You can make selection sort better by using binary search
  - Since list being inserted into is already sorted
  - However, this is still somewhat slow and requires a lot of comparisons

#### Fourth idea

- Given unsorted array:
- If element 1 is greater than element 2, swap them.
- Continue doing for each pair of element in the list until sorted
- Bubble sort

Is that the best we can do:

- No, we will learn in later classes where we can prove that we can sort in time proportional to  $n \log n$
- Suppose array is examined in disjoint pairs:  $a[0]$ ,  $a[1]$ , then  $a[2]$ ,  $a[3]$ 
  - We have  $n/2$  such pairs, but we have to look at both so we can call it  $n$
  - We put the elements of every pair in order (length 2), and call it a run
- Now we take a run, each of length 2, and merge them into runs of length 4
  - Again, this will take us time proportional to  $n$
- How many times can we double 1 before we exceed  $n$ ? That's easy,  $\log n$
- Thus, our sort finishes in time  $n \log n$ , this is mergesort

#### Comparative sorting algorithms

- Quicksort
  - Fastest average algorithm for sorting, especially for large number of elements
- Heap sort:
  - A heap is a binary tree
  - A single node is a heap
  - It is a heap if the parent is a heap and the trees rooted at both children are heaps
  - A parent's value is greater than that of either child
  - There is no order among children
- Using comparisons, these are the fastest algorithms
- But, if we make some assumptions about the encoding then you can use a radix sort
  - It runs in time proportional to the number of digits in the key times the number of records
  - It was invented to mechanically sort punched cards

#### Summary

- Sorting is a fundamental operation, so doing it efficiently has a huge impact on computing
- Analysis of algorithm complexity is an important topic, and you will be a lot of it in your advanced classes
  - A better algorithm makes a lot more difference than a faster computer

- You will encounter many instances in your career where you need to employ a sorting algorithm, and the best one will depend on the circumstances and the data structures employed.

#### Computational complexity (simplified)

- BIG O = on the order of
  - $O(\log(n))$  = binary search
  - $O(n)$  = find minimum
  - $O(n \cdot \log(n))$  = merge sort
  - $O(n^2)$  = Bubble sort
  - $O(n^3)$  = Matrix Multiply
  - $O(2^n)$  = Enumerate subsets
  - $O(n!)$  = Enumerate permutations
- When we say  $O(n^2)$  we're not being very precise
- There's some point in time (or number) where  $O(g(n))$  becomes true

#### Estimating complexity

- See slides

#### Constant "c"

- You can think of  $c$  as the overhead an algorithm requires
  - This is why for ~50 items or less, bubble sort is smarter than quicksort
  - Many factors contribute to the overhead, like having to copy the entire array for mergesort, if statements, etc.
- Time space trade of
  - Look up memoizing

#### Summary

- Computer scientists talk about the complexity of algorithms all the time
- Complexity can measure time, space, or both
  - Time is "how many steps it takes" and
  - Space is "how much memory it uses"
- The algorithm that you choose has the largest impact on performance of your program
- For small problems, it may be better to use a worse algorithm if the better algorithm has a larger constant
  - But you need to take the time and examine it carefully

#### Who's Counting?

- We talk about the size of a problem
  - How long is the string
  - How large is the number?

## Bit Vectors and sets

- Units of info
  - Bit = 1 bit, can represent 0 & 1
  - Nibble = 4 bits, can represent hex digits

## Logical shifts

- Left: Same as arithmetic shift left, see below
- Right: zeroes are shifted in on the left

## Arithmetic shift

- Left: Zeroes are shifted in on the right (multiplication by 2 for each shift)
- Right: sign bits are shifted in on the left (division by 2 for each shift)

- Bitwise operations in C
  - & - AND
  - | - OR
  - ~ - NOT
  - ^ - XOR
  - >> - Left shift
  - << - right shift
- In C, we do NOT know if shift is logical or arithmetic, it is implementation defined
  - ONLY shift unsigned integers

## Sets

- Well-defined unordered\* collections that are characterized by the elements they contain
  - Sets can be ordered, says Long (or rather, that we will order them)
- Set intersection
  - AND operator (returns bit only if both sets contain that bit)
- Set Union
  - OR operator
- Set Difference
  - NAND or complement + and
- Set complement
  - NOT or complement
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