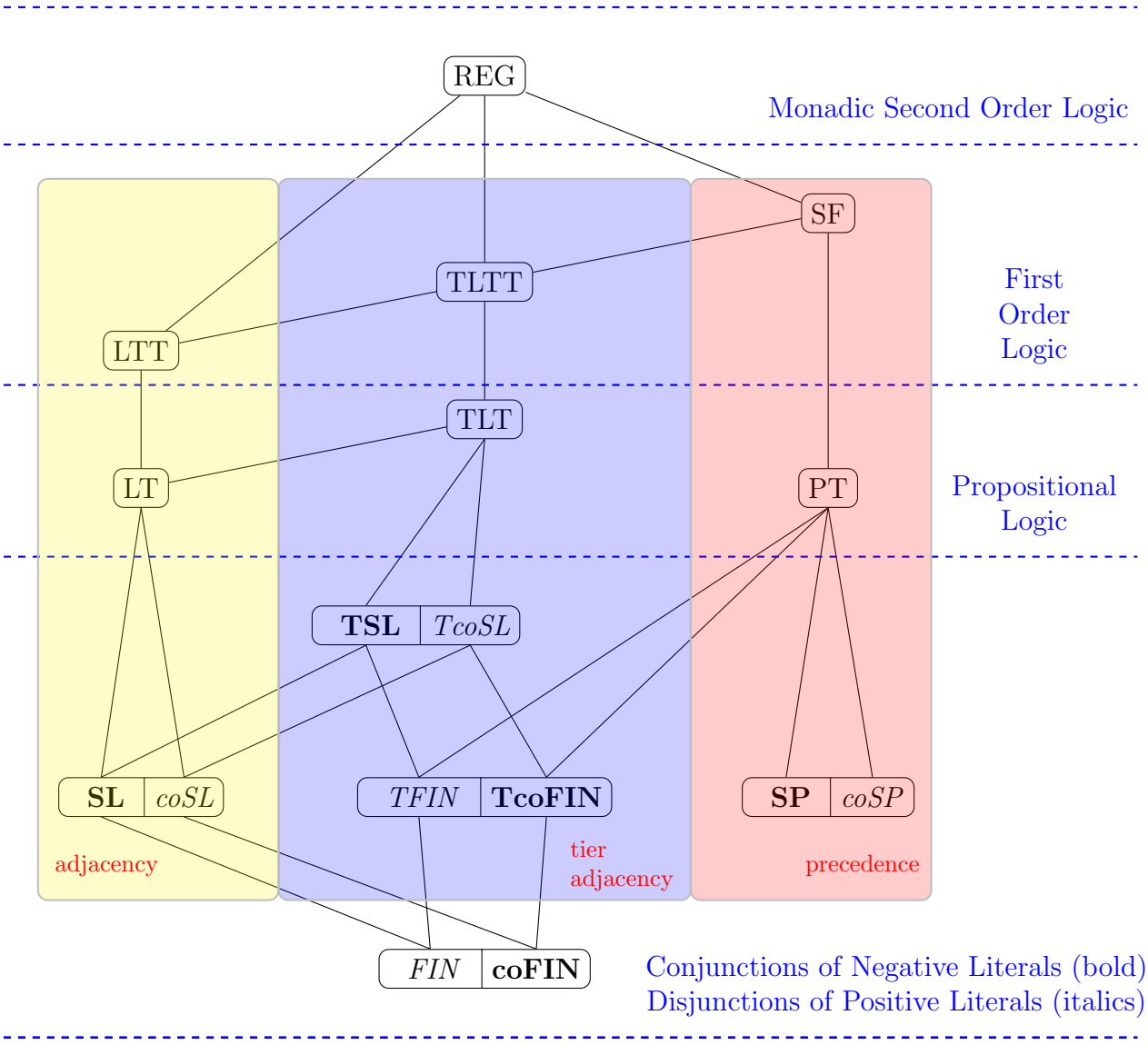
REG
Monadic Second Order Logic
SF
TLTT
First
Order
Logic
LTT
TLT
LT
PT
Propositional
Logic
TSL
TcoSL
SL
coSL
TFIN
TcoFIN
SP
coSP
adjacency
tier
adjacency
precedence
FIN
coFIN
Conjunctions of Negative Literals (bold)
Disjunctions of Positive Literals (italics)

## Grammar-Independent Characterizations

SL **Suffix Substitution Closure:**
A language $L$ is Strictly Local iff there is a $k$ such that for all strings $u_1, v_1, u_2, v_2 \in \Sigma^*$ and for all strings $x$ of length $k$ whenever $u_1xv_1, u_2xv_2 \in L$ then $u_1xv_2 \in L$.

LT **Local Testability:**
A language $L$ is Locally Testable iff there exists $k$ such that for all $u, v \in \Sigma^*$, if $u$ and $v$ have the same $k-1$ prefix, $k$-long substrings, and $k-1$ suffix then either $u, v \in L$ or $u, v \notin L$.

LTT **Local Theshold Testabilty:**
A language $L$ is Locally Testable iff there exists $k$ such that for all $u, v \in \Sigma^*$, if $u$ and $v$ have the same $k-1$ prefix, $k-1$ suffix, and the same number of occurrences of the same $k$-long substrings, counting up to some threshold $t$, then either $u, v \in L$ or $u, v \notin L$.

SP **Subsequence Closure:**
A language $L$ is Strictly Piecewise iff whenever $w \in L$ every subsequence of $w$ also belongs to $L$.

PT **Piecewise Testability:**
A language $L$ is Piecewise Testable iff there exists $k$ such that for all $u, v \in \Sigma^*$, if $u$ and $v$ have the same $k$-long subsequences then either $u, v \in L$ or $u, v \notin L$.

SF **Aperiodicity:**
A language $L$ is Star-Free iff there exists $n$ such that for all $x, y, z \in \Sigma^*$ and $m > n$ if $xy^nz \in L$ then $xy^mz \in L$.

Reg **Nerode** Theorem:
$L$ is regular iff $\left|\{T_L(u) \mid u \in \Sigma^*\}\right|$ is finite, where for all $L \subseteq \Sigma^*, u \in \Sigma^*$, $T_L(u) = \{v \mid uv \in L\}$.

(See also the Myhill theorem.)

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