Unit-1: Part-1: Control of DC Motors by Single Phase Converters

Introduction to Electrical Drives:

Motion control is required in large number of industrial and domestic applications. Systems employed for getting the required motion and their smooth control are called Drives. Drives require prime movers like Diesel or petrol engines, gas or steam turbines, hydraulic motors or electric motors. These prime movers deliver the required mechanical energy for getting the motion and its control. Drives employing Electric motors as prime movers for motion control are called ***Electric Drives***.

Advantages of Electrical Drives:

- The steady state and dynamic performance can be easily shaped to get the desired load characteristics over a wide range of speeds and torques.
- Efficient Starting /Braking is possible with simple control gear.
- With the rapid development in the field of Power Electronics and availability of high speed/high power devices like SCRs, Power MOSFETs, IGBTs etc., design of Efficient Power Converters to feed power to the electric drives has become simple and easy.
- With the rapid development in the computer's HW & SW, PLCs and Microcontrollers which can easily perform the control unit functions have become easily available.
- Electric motors have high efficiency, low losses, and considerable overloading capability. They have longer life, lower noise and lower maintenance requirements.
- They can operate in all the four quadrants of operation in the Torque/Speed plane. The resulting Electric braking capability gives smooth deceleration and hence gives longer life for the equipment. Similarly Regenerative braking results in considerable energy saving.

- They are powered from electrical energy which can be easily transferred, stored and handled.

Block diagram of an Electrical drive: is shown in the figure below.

Parts of an Electric Drive: The different parts &their functions are explained here.

The load: Can be any one of the systems like pumps, machines etc to carry out a specific task. Usually the load requirements are specified in terms of its speed/torque demands. An electrical motor having the torque speed characteristics compatible to that of the load has to be chosen.

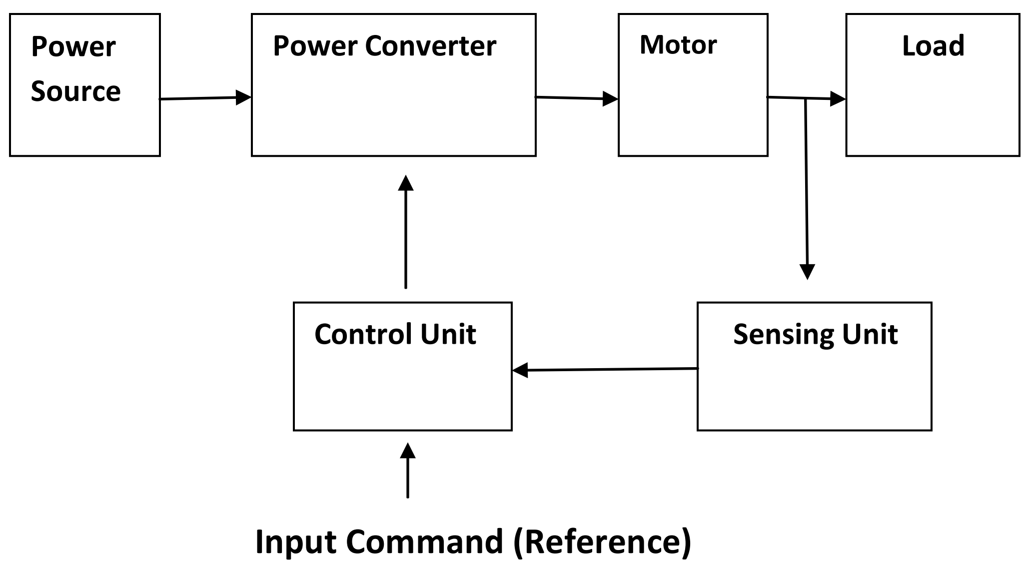


Fig: Block diagram of an Electrical drive

Power Converter: Performs one or more of the following functions.

- Converts Electrical energy from the source into a form suitable to the motor. Say AC to DC for a DC motor and DC to AC for an Induction motor.
- Controls the flow of power to the motor so as to get the Torque Speed characteristics as required by the load.

- During transient operations such as Starting, Braking, Speed reversal etc limits the currents to permissible levels to avoid conditions such as Voltage dips, Overloads etc.
- Selects the mode of operation of the Motor i.e Motoring or Braking

Types of Power Converters:

- There are several types of power converters depending upon the type of motor used in a given drive. A brief outline of a few important types is given below.
- AC to DC converters: They convert single phase/Polyphase AC supply into fixed or variable DC supply using either simple rectifier circuits or controlled rectifiers with devices like thyristors, IGBTs.Power MOSFETs etc. depending upon the application.
- AC voltage controllers or AC regulators: They are employed to get a variable AC voltage of the same frequency from a single phase or three phase supply. Some such controllers are Auto transformers, Transformers with various taps and Converters using Power electronics devices.
- DC to DC converters: They are used to get variable DC voltage from a fixed DC voltage source using Power electronics devices. Smooth step less variable voltage can be obtained with such converters.
- Inverters: They are employed to get variable voltage /variable frequency from DC supply using PWM techniques. Inverters also use the same type of Power electronics devices like MOSFETs,IGBTs,SCRs etc.
- Cycloconverters: They convert fixed voltage fixed frequency AC supply into variable voltage variable frequency supply to control AC drives. They are also built using Power electronic devices and by using controllers at lower power level.They are single stage converter devices .

Control unit/Sensing unit: The control unit controls the operation of the Power converter based on the Input command and the feedback signal continuously obtained from a suitable point (In a closed loop operation) at the load end so as to get the desired load performance. The sensor unit gets the feedback on voltage and current also to operate the motor within its safe operating conditions.

Because of the above advantages, in several applications like Diesel locomotives,Ships etc. the mechanical energy already available from a nonelectrical prime mover is first converted into electrical energy by a generator and then An Electric Drive is used as explained above.

Electrical Motors: most commonly used motors are DC motors – Shunt, Series ,Compound etc., AC motors- Suirrelcage & Slip ring induction motors, Special motors like Brushless DC motors, stepper motors etc.

DC motors have a number of disadvantages compared to Induction motors due to the presence of commutator and brushes. Squirrel cage motors are less costly than DC motors of the same rating, highly rugged and simple. In the earlier days because of easy speed control DC motors were used in certain applications. But with the development in Power electronics and the advantages of AC motors AC drives have become more popular in several applications in present days.

Review of DC Motor Drives:

DC drives are widely used in applications requiring adjustable speed, good speed regulation and frequent starting, braking, and reversing. Although since late sixties, it is being predicted that AC drives would replace DC drives, even today the variable speed applications are dominated by DC drives because of lower cost, reliability and simple control.

DC motors and their performance:

Basic schematic diagrams of DC separately excited, shunt and Series motors are shown in the figure below.

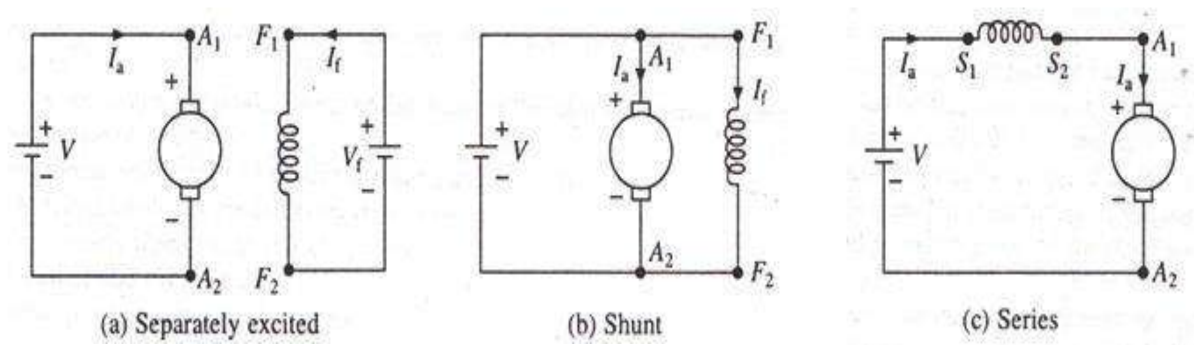


Fig: Basic Schematic digarams of DC motors

a) In a separately excited DC motor the field and armature are connected to separate voltage sources and can be controlled independently.
b) In a shunt motor the field and the armature are connected to the same source and cannot be controlled independently.
c) In a series motor the field current and armature current are same and hence the field flux is dependent on armature current.

The Steady state equivalent circuit of a DC motor Armature is shown in the figure below.

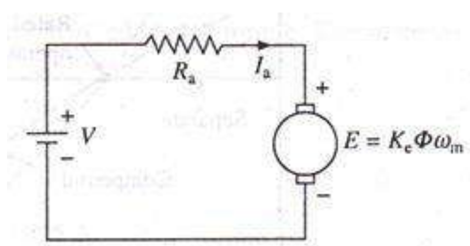


Fig: Steady state equivalent circuit of a DC Motor Armature

Resistance R_a is the resistance of the armature circuit. For separately excited and shunt motors it is resistance of the armature winding and for series motors it is the sum of the field winding and armature winding resistances.

The output characteristics of DC motors (Torque/Speed characteristics): They can be obtained from the Motor's Induced voltage and torque equations plus the Kirchhoff's voltage law around the armature circuit and are given below.

- The internal voltage generated in a DC motor is given by: $\mathbf{E_b = K_a.\ \Phi.\omega}$
- The internal Torque generated in a DC motor is given by: $\mathbf{T = K_a.\ \Phi.I_a}$
- KVL around the armature circuit is given by : $\mathbf{E_a = E_b + I_a.R_a}$

Where	Φ	=	Flux per pole	....	Webers
	I_a	=	Armature current	....	Amperes
	E_a	=	Applied terminal Voltage	....	Volts
	R_a	=	Armature resistance	....	Ohms
	ω	=	Motor speed	....	Radians/sec
	E_b	=	Armature Back EMF	....	Volts
	τ	=	Torque	.....	N-m
	K_a	=	Motor Back EMF/Torque constant	.....	Volt.Sec/Rad.web
				.....	or Nw.m/Amp.web

From the above three equations we get the Basic general relation between Torque and speed as:

$$\boldsymbol{\omega = (E_a / K_a.\ \Phi) -- (R_a / K_a.\ \Phi).\ I_a}$$

$$\boldsymbol{= (E_a / K_a.\ \Phi) -- [R_a / (K_a.\ \Phi)^2]\ \tau.}$$

Shunt and Separately excited motors:

In their case with a constant field current the field flux can be assumed to be constant and then (K_a. Φ) would be another constant K.Then the above Torque speed relations would become :

$$\boldsymbol{\omega = (E_a / K) -- (R_a / K).\ I_a}$$

$$= [E_a / K] -- [R_a / K^2] . \tau$$

The Speed/ Torque Characteristics of a DC Separately Excited Motor for rated terminal voltage and full field current are shown in the figure below. It is a drooping straight line.

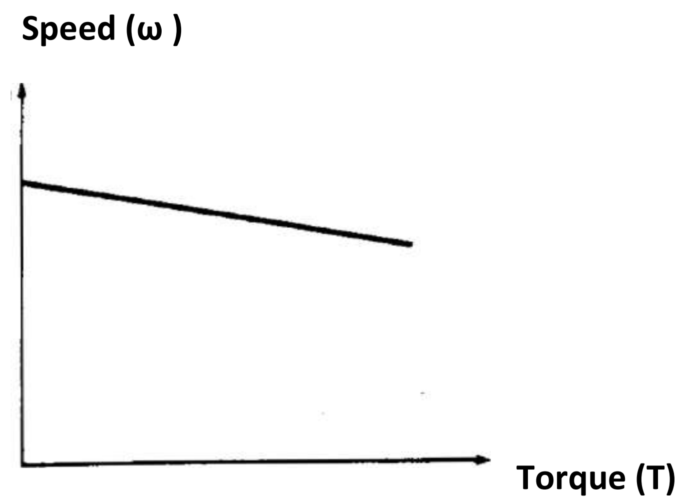


Fig: Speed/ Torque Characteristics of a DC Separately Excited Motor

The no load speed is given by the Applied armature terminal voltage and the field current. Speed falls with increasing load torque. The speed regulation depends on the Armature circuit resistance. The usual drop from no load to full load in the case of a medium sized motor will be around 5%. Separately excited motors are mostly used in applications where good speed regulation and adjustable speed are required.

Series Motor: In series motors the field flux **Φ** is dependent on the armature current $\mathbf{I_a}$ and can be assumed to be proportional to the armature current in the unsaturated region of the magnetization characteristic. Then

$$\Phi = K_f . I_a$$

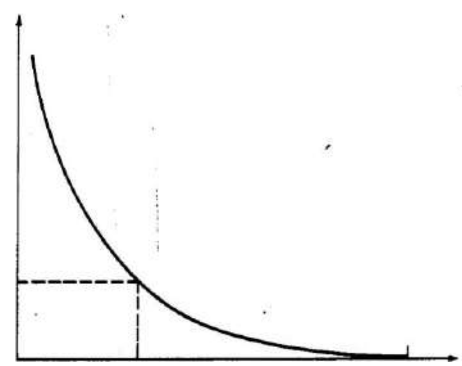
And using this value in the three general motor relations given earlier we get

$$T = K_a.\,\Phi.I_a = K_a.\,K_f.I_a^2 \quad \text{and}$$

$$\omega = E_a/\,K_a.\,K_f.I_a \text{ -- } (R_{af}/\,K_a.\,K_f)$$

$$\omega = {}^*E_a/\surd(K_{af}.\tau)] \text{ -- } [R_a/(K_{af})]$$

Where R_{af} is now the sum of armature and field winding resistances and $K_{af} = K_a.K_f$ is the total motor constant. The Speed-Torque characteristics of a DC series motor are shown in the figure below.



Rated speed and Rated Torque

Fig: Speed-Torque characteristics of a DC series motor

- Series motors are suitable for applications requiring high starting torque and heavy overloads. Since Torque is proportional to square of the armature current, for a given increase in load torque the increase in armature current is less in case of series motor as compared to a separately excited motor where torque is proportional to only armature current. Thus during heavy overloads power overload on the source power and thermal overload on the motor are kept limited to reasonable small values.
- According to the above Speed torque equation, as speed varies inversely to the square root of the Load torque, the motor runs at a large speed at light load. Generally the electrical machines' mechanical strength permits their

operation up to about twice their rated speed. Hence the series motors should not be used in such drives where there is a possibility for the torque to drop down to such an extent that the speed exceeds twice the rated speed.

DC Motor speed control:

There are two basic methods of control

- Armature Voltage Control (AVC) and
- Flux control

Torque speed curves of both SE (separately Excited) motors and series motors using these methods are shown in the figure below.

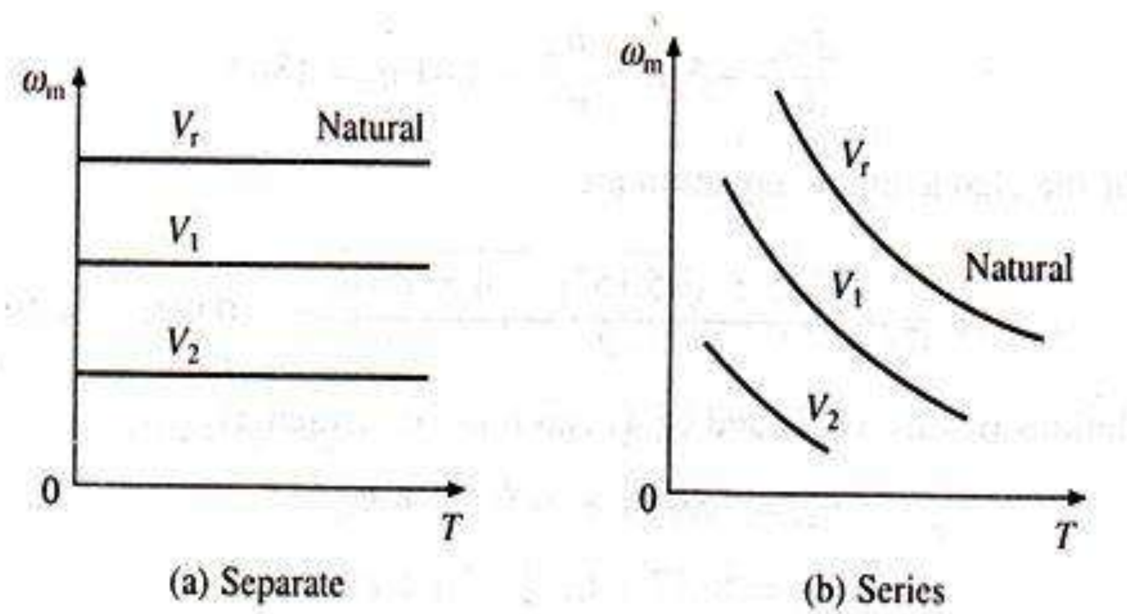


Fig: Torque speed curves with AVC : V_r (V rated) >V_1>V_2)

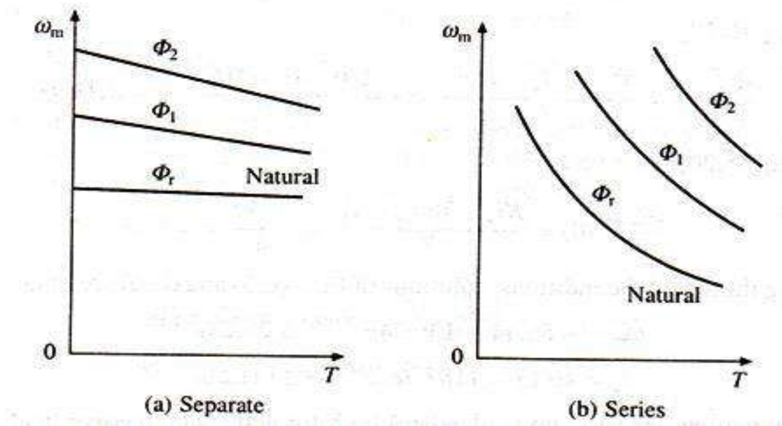


Fig: Torque speed curves with FC : Φ_r (Φ rated) >Φ_1>Φ_2)

Important features of DC Motor speed control:

- AVC is preferred because of high efficiency, good transient response, and good speed regulation. But it can provide speed control below base speed only because armature voltage cannot exceed the rated value.
- For speeds above Base speed Field Flux Control is employed. In a normally designed motor the maximum speed can be twice the rated speed and in specially designed motors it can be up to six times the rated speed.
- AVC is achieved by Single and Three phase Semi & Full converters.
- FC in separately excited motors is obtained by varying the voltage across the field winding and in series motors by varying the number of turns in the field winding or by connecting a diverting resistance across the field winding.
- Due to the maximum torque and power limitations , DC Drives operating
  - With full field, AVC below base speed can deliver a constant maximum torque. This is because in AVC with full field, the Torque is proportional to I_a and consequently the torque that the motor can deliver has a maximum value.
  - With rated Armature Voltage, Flux control above base speed can deliver a constant maximum power. This is because at rated armature voltage, P_m is proportional to I_a and consequently the maximum power that can be developed by the motor has a constant value.

These limitations are shown in the figure below.

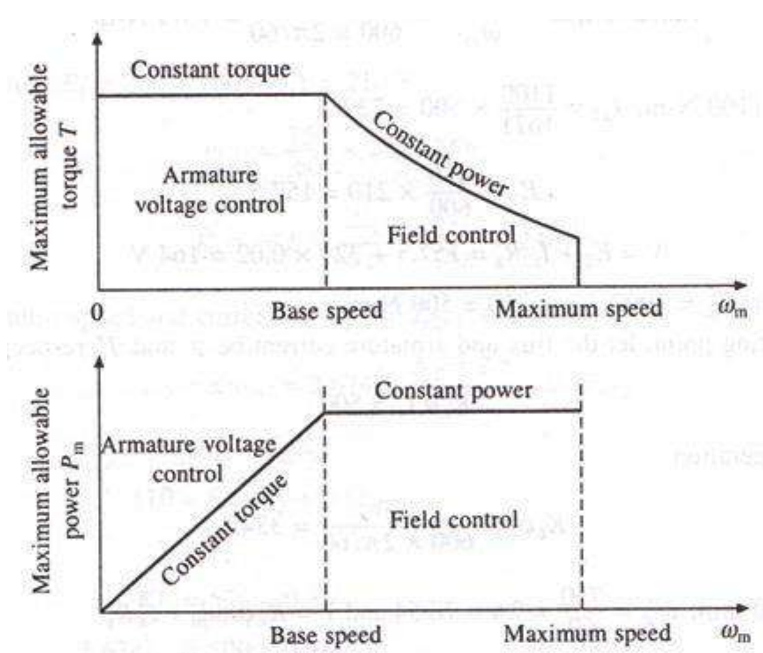


Fig: Torque and Power limitations in Combined Armature Voltage and Flux controls

Single phase Semi converter drives feeding a separately excited DC motor:

Semi converters are one quadrant converters. i.e. they have one polarity of voltage and current at the DC terminals. The circuit diagram of Semi converter feeding a DC separately excited motor is shown in the figure below. It consists of Two controlled rectifiers (Thyristors T1 and T2) in the upper limbs and two Diodes D1 and D2 in the lower limbs in a bridge configuration along with a freewheeling diode as shown in the figure below.The armature voltage is controlled by a 1ϕ semi converter and the field circuit is fed from a separate DC source. The motor current cannot reverse since current cannot flow in the reverse direction in the thyristors. In Semi converters the DC output voltage and current are always positive. Therefore in drive systems using semi converters reverse power flow from motor to AC supply side is not possible. The armature current may be continuous or discontinuous depending on the operating conditions and circuit parameters. The torque speed characteristics would be different in the two modes of conduction. We will limit our study to Continuous conduction mode in this chapter.

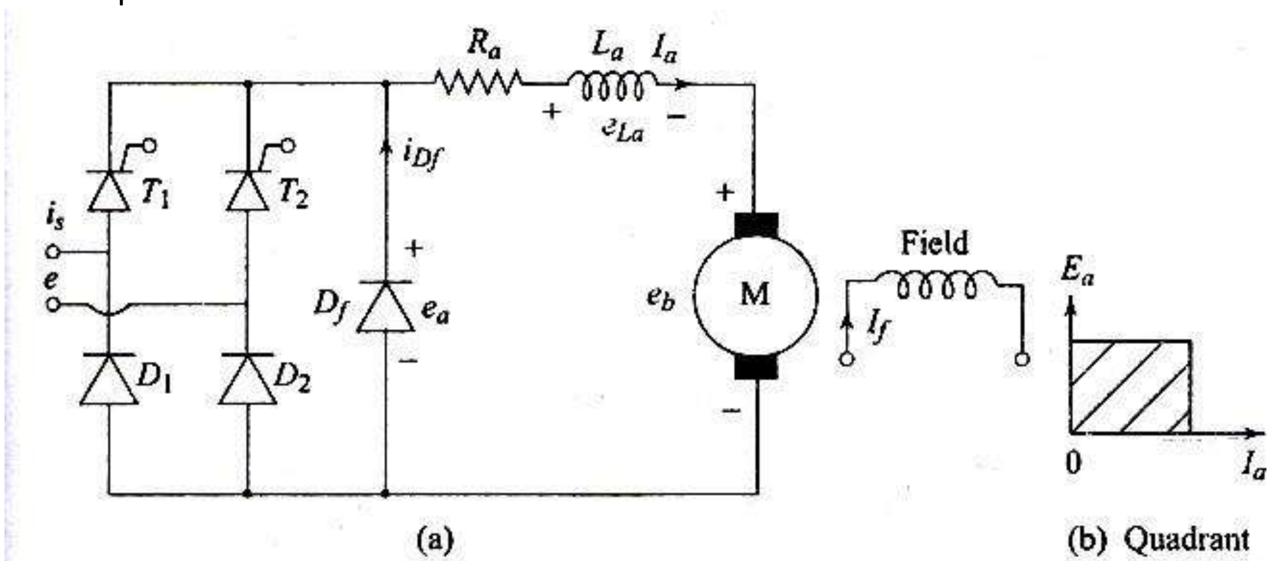


Fig: Single Phase Semi converter feeding a Separately Excited DC Motor

Performance of Semi Converter in Continuous Current Operation:

The voltage and current waveforms are shown in the figure below for operation in continuous current mode over the whole range of operation. SCR T1 is triggered at a firing angle α and T2 at the firing angle (π+α). During the period **$\alpha<\omega t<\pi$** the motor is connected to the input supply through T1 and D2 and the motor terminal voltage **e_a** is the same as the input supply voltage '**e**'. Beyond period **π, e_a** tends to reverse as the input voltage changes polarity. This will forward bias the freewheeling diode D_F and it starts conducting. The motor current i_a which was flowing from the supply through T1 is transferred to D_F (T1 gets commutated).Therefore during the period **$\pi<\omega t<(\pi+\alpha)$** the motor terminals are shorted through D_F making **e_a** zero.

As explained above, when the thyristor conducts during the period **$\alpha<\omega t<\pi$,** energy from the supply is delivered to the armature circuit. This energy is partially stored in the Inductance, partially stored as kinetic energy in the moving system and partially used up in the load. During the freewheeling period **$\pi<\omega t<(\pi+\alpha)$** energy is recovered from the Inductance and is converted to mechanical form to supplement the Kinetic energy required to run the load. The freewheeling armature current continues to produce the torque in the motor. During this period no energy is fedback to the supply.

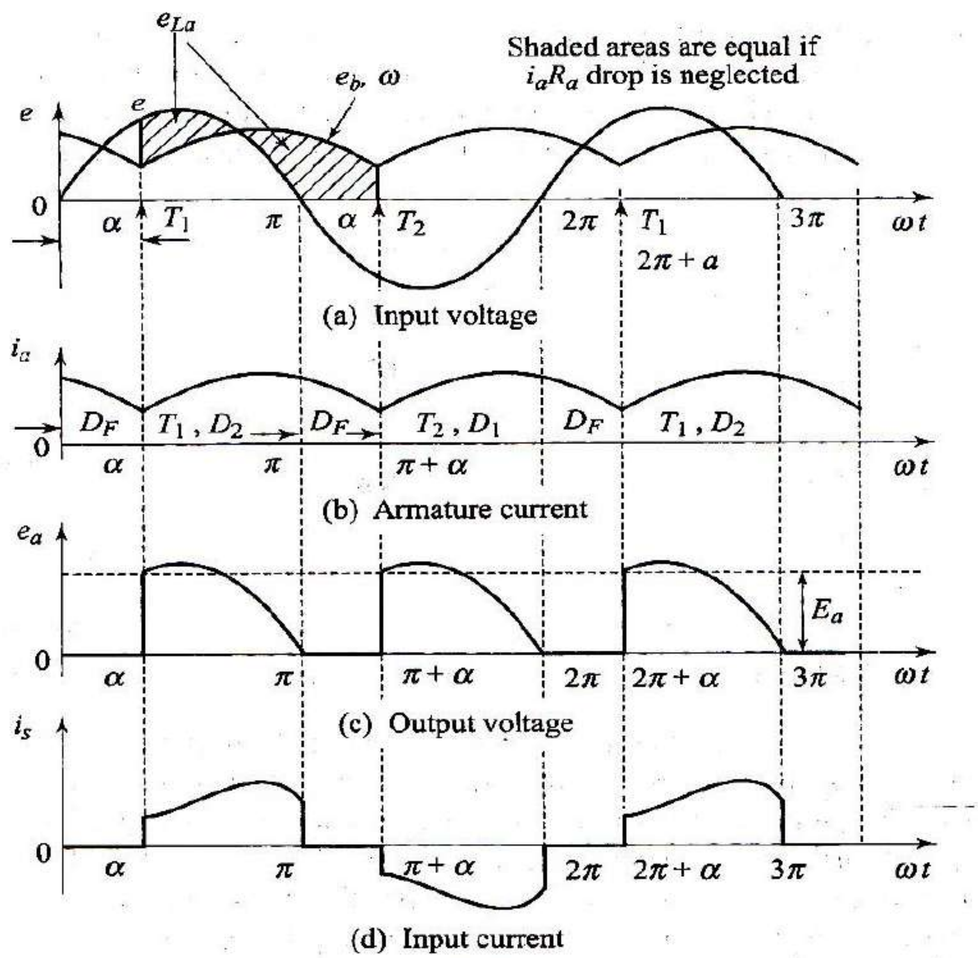


Fig: Voltage and Current waveforms for Continuous current operation in a single Phase semi controlled drive connected to a separately excited DC motor.

Torque Speed Characteristics of a Single phase Semi Converter connected to DC separately excited motor:

In terms of average voltages, KVL around the motor armature gives

$$E_a(\alpha) = E_b + I_aR_a = K_a\phi.\ \omega + I_aR_a$$

Therefore $$\omega = [\ E_a(\alpha) -- I_aR_a\ ]/\ K_a\phi.$$

Assuming motor current to be continuous, the motor armature voltage as derived above for the single phase semi converter is given by:

$$E_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi} E_m \sin \omega t\ \mathrm{d}(\omega t) = \frac{E_m}{\pi}[1 + \cos \alpha]$$

i.e. $$E_a(\alpha) = (\ E_m/\pi)(1+\cos \alpha)$$

Using this in the above expression for speed **ω** we get

$$\omega = [(\ E_m/\pi)(1+\cos \alpha) -- I_aR_a\ ]/\ K_a\phi.$$

$$\omega = [(\ E_m/\pi)(1+\cos \alpha)\ /\ K_a\phi] -- [I_aR_a/\ K_a\phi +$$

$$\omega = [(\ E_m/\pi)(1+\cos \alpha)\ /\ K_a\phi] - [R_a/\ (K_a\phi)^2]\tau$$

The resulting torque speed characteristics are shown in the figure below.

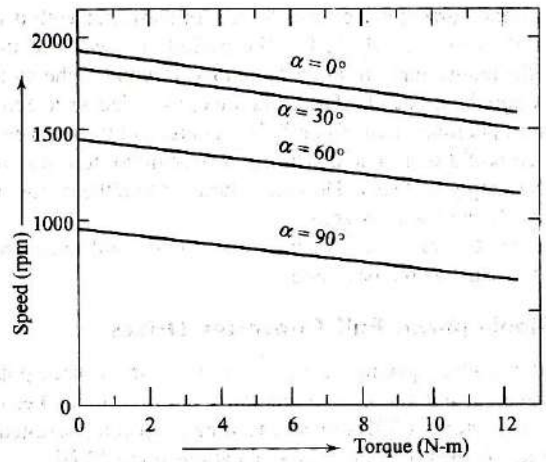


Fig: Torque Speed characteristics of a separately excited DC motor Connected to a single Phase semi controlled drive

Single Phase Full Converter Drive feeding a Separately Excited DC Motor:

A full converter is a two quadrant converter in which the output voltage can be bipolar but the current will be unidirectional since the Thyristors are unidirectional. A full converter feeding a separately excited DC motor is shown in the figure below.

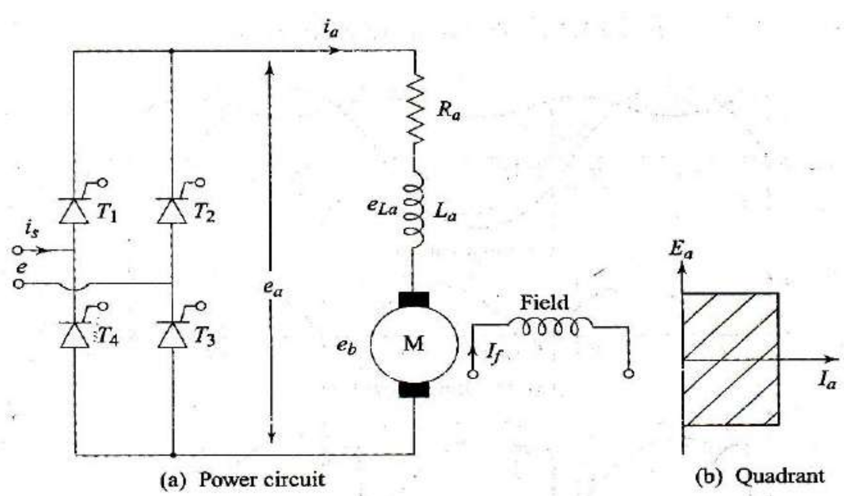


Fig: Single Phase full converter feeding a separately excited DC motor

In this all the four devices are thyristors (T1 to T4) connected in a bridge configuration as shown in the figure.

The operation of the Full converter shown in the figure above is explained with the help of the waveforms shown below.

Thyristors T1 and T3 are simultaneously triggered at a firing angle of **α** and thyristors T2 and T4 are triggered at firing angle **$(\pi+\alpha)$**. The voltage and current waveforms under continuous current mode are shown in the figure below. Figure shows the input voltage **e** and the voltage **e_{La}** across the inductance(shaded area).The triggering points of the thyristors are also shown in the figure.
As can be seen from the waveforms, the motor is always connected through the thyristors to the input supply. Thyristors T1 and T3 conduct during the interval **$\alpha<\omega t<(\pi+\alpha)$** and connect the supply to the motor. From **$(\pi+\alpha)$** to **α** thyristors T2 and T4 conduct and connect the supply to the motor. At **$(\pi+\alpha)$** when the thyristors T2 and T4 are triggered, immediately the supply voltage which is negative appears across the Thyristors T1 and T3 as reverse bias and switches them off.This is called natural or line commutation. The motor current **i_a** which was flowing from the supply through T1 and T3 is now transferred to T2 and T4. During **α to π** energy flows from the input supply to the motor (both **e & i_s** and **e_a & i_a** are positive signifying positive power flow).However during the period **π to $(\pi+ \alpha)$** some of the motor energy is fed back to the input system. (**e & i_s** and similarly **e_a & i_a** have opposite polarities signifying reverse power flow)

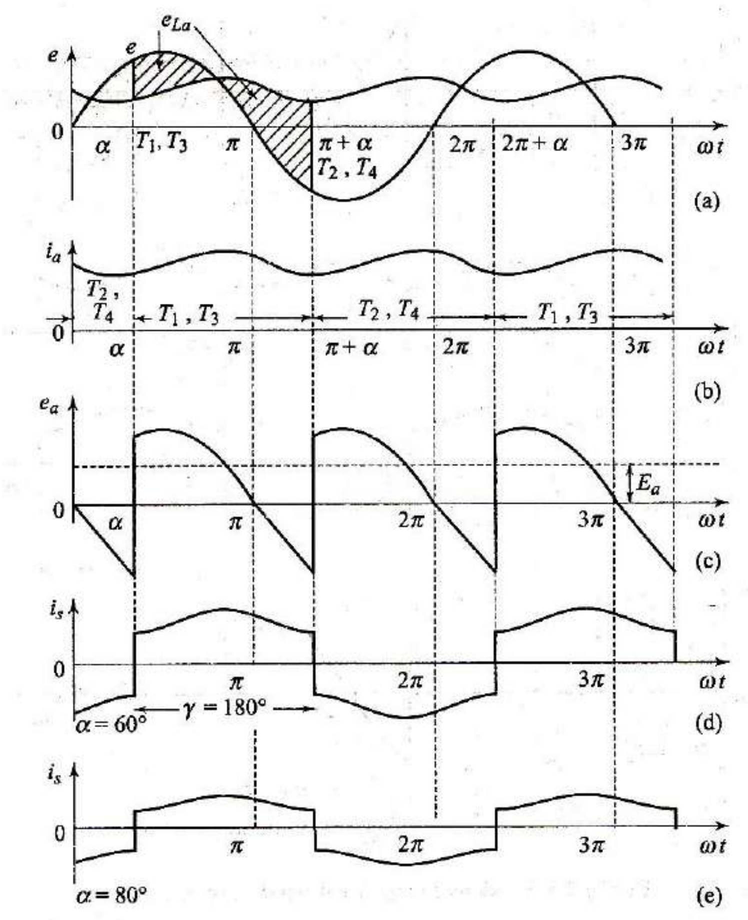


Fig: Voltage and Current waveforms for Continuous current operation in a single Phase fully controlled drive connected to a separately excited DC motor.

Torque Speed Characteristics of a DC separately excited motor connected to a Single phase Full converter:

Assuming motor current to be continuous, the motor armature voltage as derived above for the single phase full converter is given by:

$$E_{dc} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} E_m \sin\omega t \, d(\omega t) = \frac{E_m}{\pi}\left[-\cos\omega t\right]_{\alpha}^{\pi+\alpha}$$

$$= \frac{E_m}{\pi}\left[\cos\alpha - \cos(\pi+\alpha)\right] \quad E_{dc} = \frac{2E_m}{\pi}\cos\alpha$$

$$\text{i.e.} \quad E_a(\alpha) = (2E_m/\pi)(\cos\alpha)$$

In terms of average voltages, KVL around the motor armature gives

$$E_a(\alpha) = E_b + I_aR_a = K_a\phi.N + I_aR_a$$

And therefore the average speed is given by :

$$\omega = [\,E_a(\alpha) -- I_aR_a\,]/\,K_a\phi.$$

In a separately excited DC motor:

$$T = I_a.\,Ka.\phi.$$

And applying this relationship along with the above value of $\mathbf{E_a(\alpha)}$ for the full converter in the above expression for the speed we get :

$$\omega = [(2E_m/\pi)(\cos\alpha) -- I_aR_a\,]/\,K_a\phi.$$

$$\omega = [(2E_m/\pi)(\cos\alpha)\,/\,K_a\phi] -- [I_aR_a/\,K_a\phi+$$

$$\omega = [(2E_m/\pi)(\cos\alpha)\,/\,K_a\phi] - [T.R_a/(\,K_a\phi)^2]$$

The no-load speed of the motor is given by :

$$\omega_{NL} = [(2E_m/\pi)(\cos\alpha)\,/\,K_a\phi+ \quad \text{where the torque } T = 0$$

The resulting torque speed characteristics are shown in the figure below.

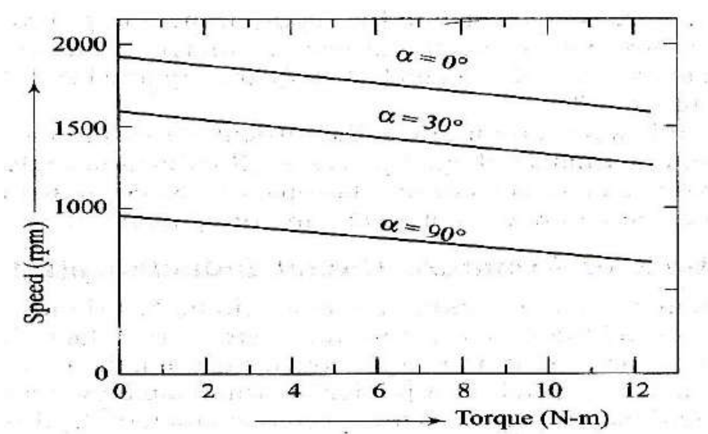


Fig: Torque Speed characteristics of separately excited DC motor Connected to a single Phase fully controlled drive at different firing angles.

Single Phase Converter Drives for DC Series Motors:

Figure below shows the scheme of a basic single phase speed control circuit connected to a DC series motor. As shown the field circuit is connected in series with the armature and the motor terminal voltage is controlled by a semi or a full converter.

- *Series motors are particularly suitable for applications that require a high starting torque such as cranes hoists, elevators, vehicles etc.*
- *Inherently series motors can provide constant power and are therefore particularly suitable for traction drives.*
- *Speed control is very difficult with the series motor because any change in load current will immediately reflect in the speed change and hence for all speed control requirements separately excited motors will be used.*

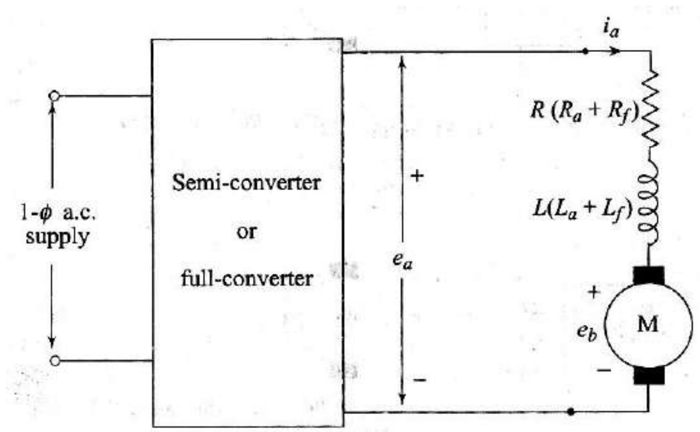


Fig: DC Series motor Power circuit

In the figure the armature resistance Ra and Inductance La are shown along with the field resistance and inductance. The basic DC series motor equations are given below again for ease of reference

- $\mathbf{E_b = K_a.\ \Phi.\omega\ =\ K_a.\ K_f.I_a.\omega}$ **(since** $\mathbf{\Phi = K_f\ .\ I_f\ =\ K_f\ .\ I_a}$ **)**
  $\mathbf{=\ K_{af}.\ I_a.\omega}$ **(where** $\mathbf{K_{af}\ =\ K_a.\ K_f}$ **)**

- $T = K_a . \Phi . I_a = K_a . K_f . I_a^2 = K_{af} . I_a^2$
- $E_a = E_b + I_a . R_a$
- $\omega = E_a / . K_{af} . I_a$ -- (R_a / K_{af})
- $\omega = *E_a / \sqrt{(K_{af} . T)}]$ -- $[R_a / (K_{af})]$

Single Phase Semi Converter Drive connected to DC Series Motors:

The figure below shows the power circuit of a single phase semi converter controlled DC series motor.

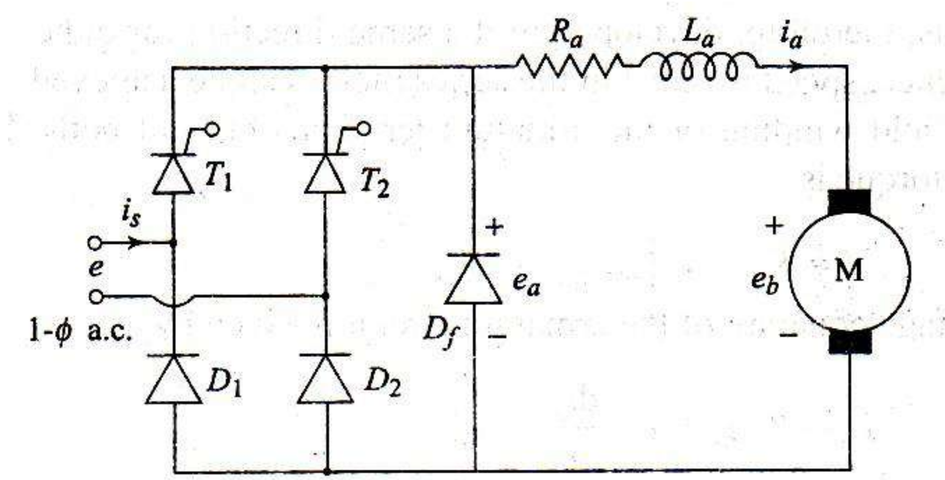


Fig: Power circuit of a Series motor connected to a Semi Controlled converter

Current and voltage waveforms for continuous motor armature current are shown in the figure below. When SCR is triggered at a firing angle **α** the current flows during the period **α to (π + α)** for continuous conduction.

In separately excited motors a large Back EMF is always present even when the armature current is absent. This back EMF E_b tends to oppose the motor current and so the motor current decays rapidly. This leads to discontinuous motor current over a wide range of operations. Whereas in series motors the back EMF is proportional to the armature current and so E_b decreases as I_a decreases. So the motor current tends to be continuous over a wide range of operations. Only at high speed and low current is the motor current is likely to become discontinuous.

Like in earlier semi converters Freewheeling diode is connected across the converter output as shown in the figure above. Freewheeling action takes place during the interval π to (π + α) in continuous current operation.

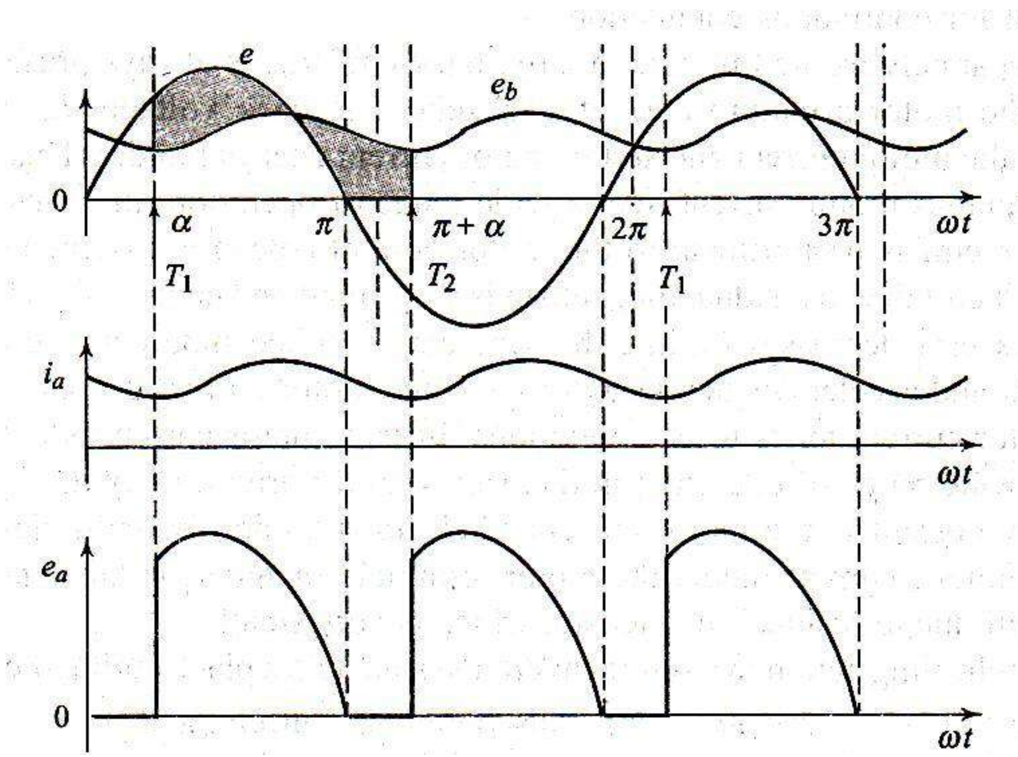


Fig: DC Series motor Semi Converter waveforms in continuous current operation.

In phase controlled converters for Series motors, the current is mostly continuous and the motor terminal voltage can be written as

$$\mathbf{E_a = E_m/\pi\ (1 + \cos\alpha) = I_aR_a + E_b}$$
$$\mathbf{= I_aR_a + K_{af}.\ I_a.\ \omega}$$

Hence from the above equation the average speed can be written as

$$\mathbf{\omega = [(E_m/\pi)(1+\cos\alpha)/(K_{af}.I_a)] -- [(R_a\ .\ I_a/\ K_{af}\ .I_a)]}$$

$$\omega = [(E_m/\pi)(1+\cos\alpha)/\sqrt{(K_{af}.T)}] -- [(R_a/ K_{af})]$$

The torque Speed characteristics under the assumption of continuous and ripple free current flow are shown in the figure below for different firing angles α.

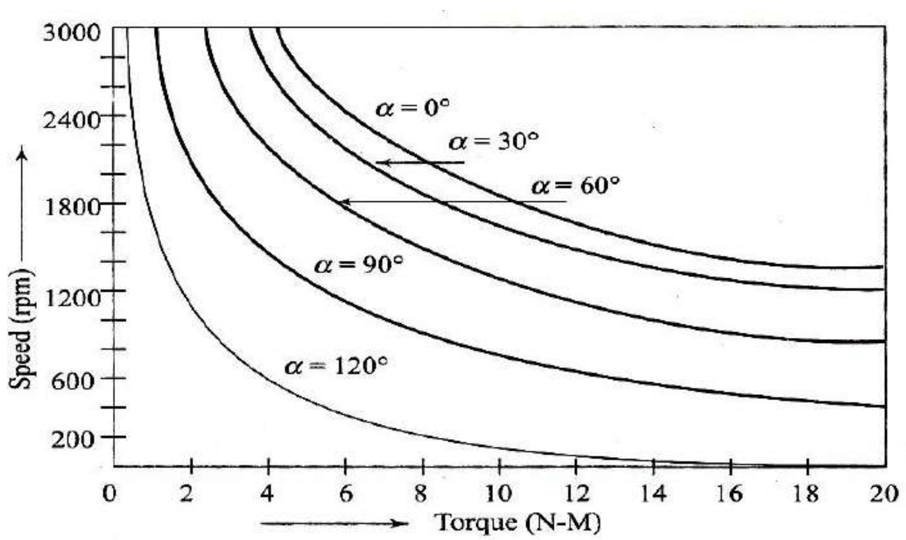


Fig: Torque Speed Characteristics of a DC Series motor controlled by a Single phase Semi converter

Single Phase full converter drive connected to a DC series motor:

The figure below shows the power circuit of a single phase Fully controlled converter connected to a DC series motor.

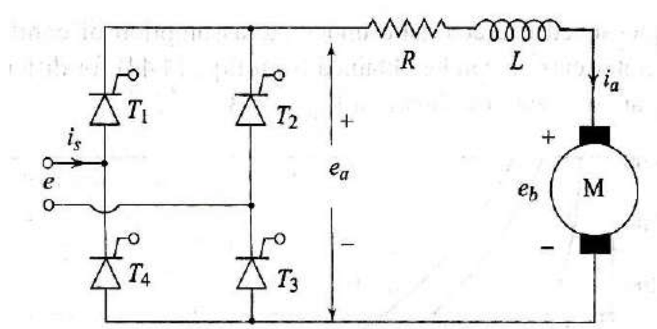


Fig: Power circuit of a Series motor connected to a fully controlled converter

Thyristors T1 & T3 are simultaneously triggered at α and T2 & T4 are simultaneously triggered at (π + α).Current and voltage waveforms for continuous motor armature current are shown in the figure below. When SCR is triggered at a firing angle α the current flows during the period α to (π + α) for continuous conduction.

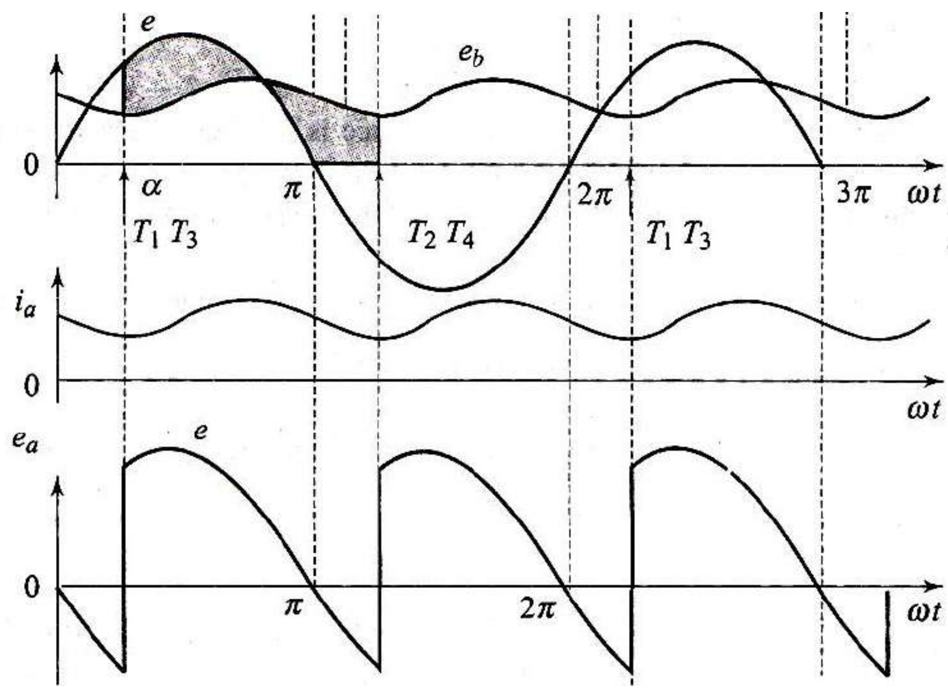


Fig: DC Series motor Full converter waveforms in continuous current operation.

The motor terminal voltage can be written as

$$E_a = 2E_m/\pi\ (\cos\alpha) = I_aR_a + E_b$$
$$= I_aR_a + K_{af}.\ I_a.\ \omega$$

Hence from the above equation the expression for average speed can be written as

$$\omega = *(2E_m/\pi)(\cos\alpha)/(K_{af}.I_a)] -- [(R_a\ .\ I_a/\ K_{af}\ .I_a)]$$

$$\omega = *(2E_m/\pi)(\cos\alpha)/\surd(K_{af}.T)] -- [(R_a/ K_{af.})]$$

The torque Speed characteristics under the assumption of continuous and ripple free current flow are shown in the figure below for different firing angles α.

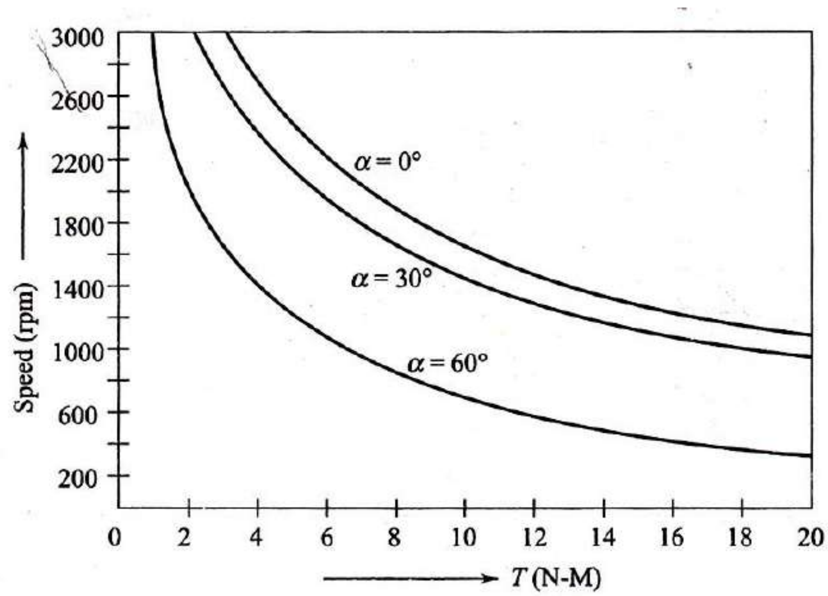


Fig:Torque Speed characteristics of a Series motor connected to a fully controlled converter

Summary:

Important conclusions and concepts:

- In single phase converters output ripple frequency is 100 Hz.(both semi and full)
- Semi converters are one quadrant converters. i.e. they have one polarity of voltage and current at the DC terminals. In this as firing angle varies from 0 to 180^0 DC output varies from maximum ($2E_m/\pi$) to zero.
- A full converter is a two quadrant converter in which the output voltage can be bipolar but the current will be unidirectional since the thyristors are

Unidirectional. In this as firing angle varies from 0 to 180^0 DC output varies from maximum ($2E_m/\pi$) to ($-2E_m/\pi$)

- Separately excited motors are mostly used in applications where good speed regulation and adjustable speed are required.
- Series motors are suitable for applications requiring high starting torque and heavy overloads.
- In case of series motors, Since Torque is proportional to square of the armature current, for a given increase in load torque the increase in armature current is less as compared to separately excited motors where torque is proportional to only armature current.
- There are two basic methods of speed control. Armature Voltage Control and Flux Control.
- AVC is used for speeds below base speeds and FC for speeds above base speed.
- Due to the maximum torque and power limitations DC Drives operating
  - With full field , AVC below base speed can deliver a maximum constant torque and
  - With rated Armature Voltage, Flux control above base speed can deliver a maximum constant power.
- AVC is achieved by Single and three phase Semi & Full converters.
- FC in separately excited motors is obtained by varying the voltage across the field winding and in series motors by varying the number of turns in the field winding or by connecting a diverting resistance across the field winding.

Important formulae and equations:

- *The basic DC motor equations* **:**
  - The internal voltage generated in a DC motor is given by**:** $\mathbf{E_b = K_a.\Phi.\omega}$
  - The internal Torque generated in a DC motor is given by: $\mathbf{T = K_a.\Phi.I_a}$
  - KVL around the armature circuit is given by : $\mathbf{E_a = E + I_a.R_a}$

- *Torque speed relations in semi converter*:

- DC separately excited motor:

$$\omega = [(E_m/\pi)(1+\cos\alpha) / K_a\phi +– [R_a/(K_a\phi)^2]\tau$$

- DC series motor :

$$\omega = [(E_m/\pi)(1+\cos\alpha)/\sqrt{(K_{af}.T)}] -- [(R_a/ K_{af})]$$

- *Torque speed relations in Full converter:*

  - DC separately excited motor:

$$\omega = [(2E_m/\pi)(\cos\alpha) / K_a\phi] – [.R_a/(K_a\phi)^2]\tau$$

  - DC series Motor :

$$\omega = [(2E_m/\pi)(\cos\alpha)/\sqrt{(K_{af}.T)}] -- [(R_a/ K_{af})$$

Illustrative Examples:

Example-1: A separately excited d.c . motor is fed from a 230 V, 50 Hz supply via a single-phase , half –controlled bridge rectifier. Armature parameters are: inductance 0.06 H, resistance 0.3 Ω, the motor voltage constant is K_a = 0.9 V/A rad/s and the field resistance is R_F = 104 Ω. The filed current is also controlled by semi converter and is set to the maximum possible value. The load torque is T_L = 50 N-m at 800 rpm. The inductances of the armature and field circuits are sufficient enough to make the armature and filed current continuous and ripple free.Compute : (i) The field current (ii) The firing angle of the converter in the armature circuit

Solution:

(i) First point to be noted is since the units of K_a are V/A rad/sec the basic governing equations for back emf E_b and Torque T will become : $E_b = K_{af}.\ I_f.\omega$ and $T = K_{af}.\ I_f.I_a$ where K_{af}.is to be taken as the given K_a = 0.9 V/A rad/s

(ii) For single-phase semi converter controlled d.c. drive, we can write the expression for field supply voltage as

$$E_f = \frac{E_m}{\pi}(1+\cos\alpha)$$

So , the maximum field voltage and current are obtained when firing angle α =0.

i.e. $E_f = \frac{2\,E_m}{\pi}$

Hence Field voltage $E_f = \frac{2E_m}{\pi} = \frac{2\times\sqrt{2}\times 230}{\pi}$ = 207.07 V.

And filed current $I_f = \frac{E_f}{R_f} = \frac{207.07}{104} = 1.99\ A$

(iii) Now, we can first find out armature current from the relation

$$I_a = \frac{T}{K_a I_f} = \frac{50}{0.9\times 1.99} = 27.92\ A$$

And then back emf from the relation: $E_b = K_a\,\omega\,I_f = 0.9 \times (800\times\frac{2\pi}{60}) \times 1.99$ = 150.04 V.

Hence finally we can find out armature voltage from the relation : $E_a = E_b + I_a R_a$

$$= 150.04 + 27.92 \times 0.3 = 158.42 \text{ V.}$$

But applied armature voltage from a single phase semi converter is given by the equation $E_a = \frac{E_m}{\pi}(1+\cos\alpha)$ and equating this to the above required armature voltage of 158.42 we get

$\frac{\sqrt{2}\times 230}{\pi}(1+\cos\alpha) = 158.42$ from which we get $\alpha = 58^0$

Note: Sometimes all the data given in the problem may not be required to solve the problem. In this problem there is some such data. Identify….

Example-2: The speed of a 10 HP, 210 V, 1000 rpm separately excited D.C. motor is controlled by a single-phase full-converter. The rated motor armature current is 30 A, and the armature resistance is R_a = 0.25 Ω. The a.c. supply voltage is 230 V. The motor voltage constant is $K_a\Phi$ = 0.172 V/rpm. Assume that sufficient inductance is present in the armature circuit to make the motor current continuous and ripple free. For a firing angle α = 45^0, and rated motor armature current, determine: 1) The motor torque 2) Speed of the motor at Rated armature current.

Solution:

(1) The motor Torque: can be found out directly by using the relation T = K_a ΦI_a. But the constant ***K_a Φ is same in the relations for torque and back emf if it is V/Rad/sec in back emf and N-m/A in torque.*** But it is given in V/RPM . Hence it is first converted to V/Rad/sec and then used in the expression for torque .

The units of the K_a Φ (V/Rad/sec) = K_a Φ (V/RPM) x 60/2π

$$= \frac{0.172 \times 60}{2\pi} \text{ V-s/rad} = 1.64 \text{ V-s/rad.}$$

Rated Motor Torque T_R at rated armature current = K_a ΦI_{aR} = 1.64 × 30 = 49.2 N-m.

(2) Speed of the Motor at Rated armature current: The armature voltage in a fully controlled single phase converter is given by:

$$E_a = \frac{2E_m}{\pi} \cos\alpha = \frac{2\sqrt{2}\times 230}{\pi} \cos 45^0 = 146.42$$

(The given supply voltage of 230 V is RMS value and it is to be converted into E_m by multiplying by $\sqrt{2}$)

$$E_b = E_a - I_{aR} R_a = 146.42 - (30 \times 0.25) = 138.92 \text{ V}.$$

Speed, N = $\frac{E_b}{K_a \emptyset} = \frac{138.92}{0.172} = 807.67\ rpm$

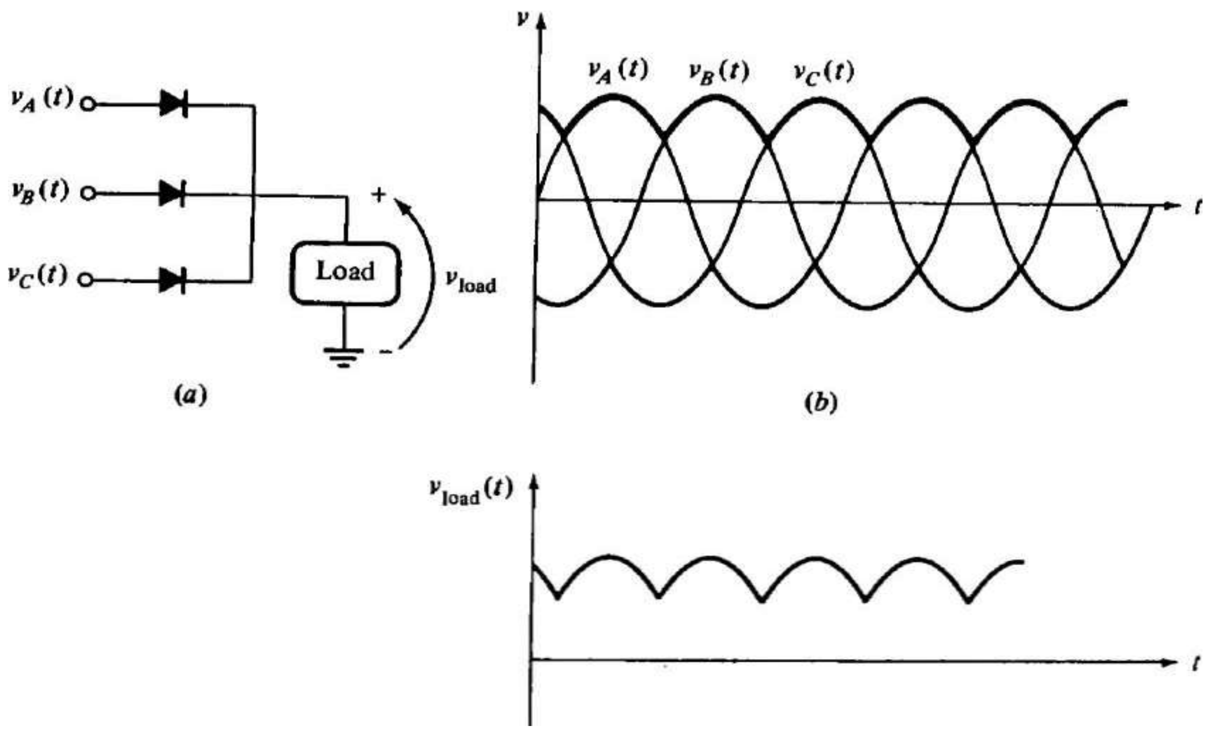
(Here K_a Φ is used directly with the given units of V/RPM so that we can get directly speed N in RPM)

Note: In this problem also all the data given is not used to solve the problem. Identify….

Unit-I: Part 2: Control of DC Motors by Three Phase Converters

Introduction to Three Phase Converters:

Three Phase Half Wave Rectifier:



(c)

Fig: (a) Three Phase half wave rectifier circuit (b) Three Phase input voltages to the circuit
(c) Output Voltage

In the HW circuit shown in the fig (a) above the effect of having all the three diode cathodes connected to a common point and then connecting to the Load is that at any instant of time, the ***Diode with the highest Input voltage applied to it will conduct*** and the other diodes will be reverse biased. The applied three phase voltages are shown in fig (b) and the resulting output voltage across the load is shown in fig (c).It can be seen that the ***OP voltage is just the highest of the three***

input phase voltages at any instant of time. It can be seen that the ripple frequency in this output is 150 Hz. which is larger than the 100 Hz. ripple frequency in a Single Phase FW rectifier.

Three Phase Full Wave Rectifier:

The FW rectifier circuit shown in the fig below consists of basically two parts. One part is just the same as the HW Rectifier and connects the highest of the three input phase voltages to the load. The other part consists of three diodes connected such that their anodes are connected to a common point and then connected to the other end of the load. Their cathodes are connected to the anodes of the first set and to the three phase voltages. This arrangement results in connecting the ***lowest of the input voltages to the other end of the load*** at any instant of time. ***Therefore a Three Phase FWR always connects the highest of the three inputs to one end of the load and the lowest of the three inputs to the other end of the load.***

The OP of a Three Phase FWR is much smoother than a HWR and the ripple frequency is 300 Hz.

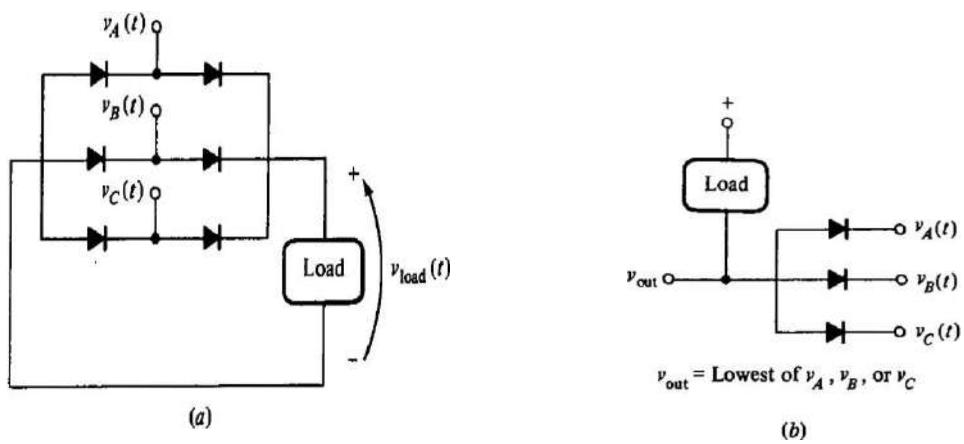


Fig: (a) A TPFWR circuit (b) This circuit places the lowest of the three IPs on the Output

The output from a three phase FWR is shown in the figure below.

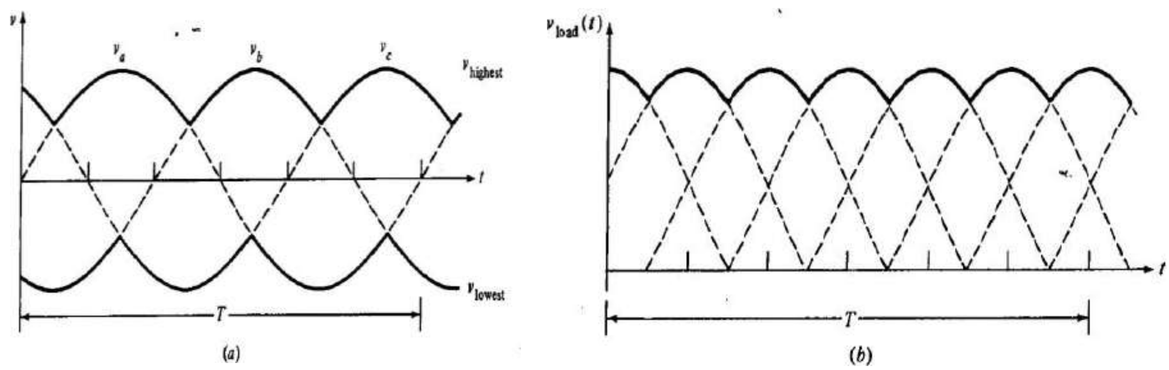


Fig: (a) Highest and lowest voltages in a TPFWR (b) The resulting OP voltage

Three Phase Full Converter drive connected to a DC separately excited motor:

Figure below shows a three phase Full converter drive circuit connected to a DC separately excited DC motor. It is a two quadrant drive without any field reversal. The operation of this circuit is explained with the help of the following important points and with voltage waveforms shown below.

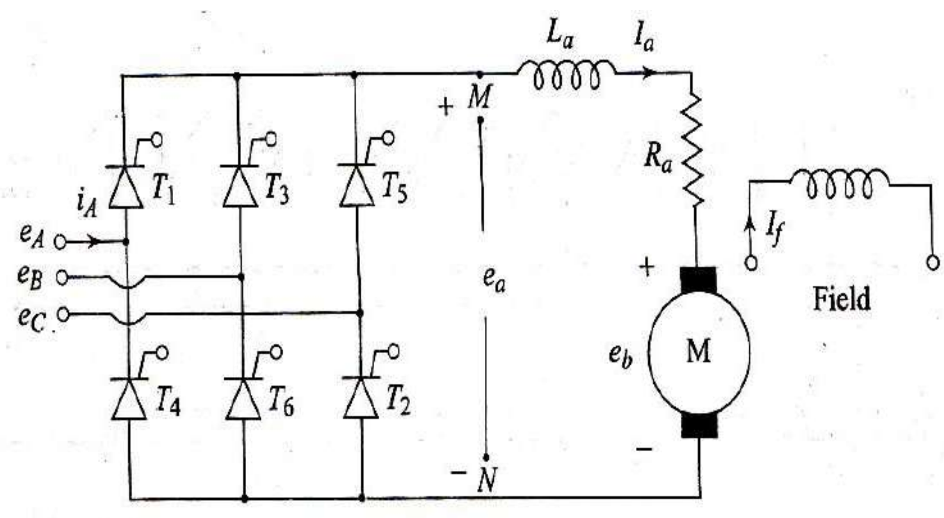


Fig: Three Phase full converter connected to a DC separately excited motor.

Important points:

- Thyristors are fired in the sequence of their numbers T1, T2, T3, T4, T5 and T6 with a phase difference of 60 Degrees.
- Thyristors consist of two groups. Positive (Top) group with odd numbered Thyristors T1, T3 &T5 and Negative (Bottom) group with even numbered Thyristors T2, T4 &T6.
- Each thyristor conducts for a duration of 120 degrees and two thyristors conduct at a time one from the Positive group and the other from the Negative group , applying respective line voltage to the motor.
- At any given instant of time, thyristors conduct in pairs and there are six such pairs. They are :
  (T6, T1), (T1, T2), (T2, T3), (T3, T4), (T4, T5) and (T5, T6).
- Each SCR conducts in two consecutive pairs.
- Transfer of current takes place from an outgoing to an incoming thyristor when the respective line voltage is of such a polarity that it not only forward biases the incoming thyristor but it also leads to reverse biasing of the outgoing thyristor when the incoming thyristor turns on. In other words incoming thyristor commutates the outgoing thyristor. i.e T1 commutates T5,T2 commutates T6,T3 commutates T1 and so on.
- The table below gives the details of conducting thyristor pairs, Incoming and outgoing thyristors and the corresponding Line voltages applied across the load.
- For $\alpha > 60°$ the waveforms are different. The voltages go negative due to the inductive load. The previous SCR pair continues to conduct till the next in the sequence is triggered. For e.g. SCRs T6 and T1 continue to conduct up to $(90+ \alpha)$ when T2 is triggered. When T2 is triggered it commutates T6 and then the pair (T1,T2) will continue.
- For $\alpha = 90°$ the area under the positive & the negative cycles are equal and the average voltage is zero.
- For $\alpha < 90°$ the average voltage is positive and for $\alpha > 90°$ the average voltage is negative.
- The maximum value of α is 180°

- The output voltage is always a six pulse stream with a ripple frequency of 300 Hz irrespective of the firing angle α.

Table: Firing sequence of SCRs in the 3 ϕ full converter

S.No.	ωt	Incoming SCR	Conducting pair	Outgoing SCR	Line voltage across the load
1.	$30° + \alpha$	T_1	(T_6, T_1)	T_5	E_{AB}
2.	$90° + \alpha$	T_2	(T_1, T_2)	T_6	E_{AC}
3.	$150° + \alpha$	T_3	(T_2, T_3)	T_1	E_{BC}
4.	$210° + \alpha$	T_4	(T_3, T_4)	T_2	E_{BA}
5.	$270° + \alpha$	T_5	(T_4, T_5)	T_3	E_{CA}
6.	$330° + \alpha$	T_6	(T_5, T_6)	T_4	E_{CB}

- For a better understanding of this topic the vector diagram of the three Phase voltages (w.r.to Neutral) and the six line to line voltages as obtained from a Star connected source are shown in the figure below.

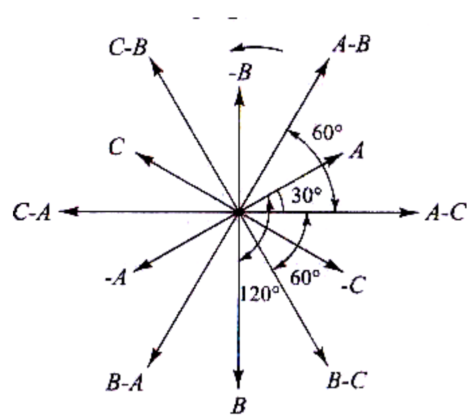


Fig: Phase sequence and Phase relationships between Phase and line voltages

From the above Phasor diagram the Phase and Amplitudes of the three phase voltages and the six line to line voltages can easily be worked out and are given in the table below.

Table: Phase/ Amplitudes of the three phase voltages and the six line voltages

$E_{AN} = E_m \sin(\omega t)$	$E_{BC} = \sqrt{3}\ E_m \sin(\omega t - 90°)$
$E_{BN} = E_m \sin(\omega t - 120°)$	$E_{BA} = \sqrt{3}\ E_m \sin(\omega t - 150°)$
$E_{CN} = E_m \sin(\omega t + 120°)$	$E_{CA} = \sqrt{3}\ E_m \sin(\omega t + 150°)$
$E_{AB} = \sqrt{3}\ E_m \sin(\omega t + 30°)$	$E_{CB} = \sqrt{3}\ E_m \sin(\omega t + 90°)$
$E_{AC} = \sqrt{3}\ E_m \sin(\omega t - 30°)$	

The voltage and current waveforms in this converter for $\alpha = 60°$ are shown in the figure below**.** The instants of firing the thyristors is marked for **$\alpha = 60°$** alone for a clear understanding . The ripple in the output voltage is six pulses per cycle. Since there are six thyristors in the circuit, they are fired at a faster rate (once in 60 °) and the motor current is mostly continuous. Therefore the filtering requirement is less than that in the semi converter system. The operation is explained for the marked firing angle of **$\alpha = 60°$**

Thyristor T1 turns on at $\omega t = (30° + \alpha)$ **.** Prior to this SCR T6 was switched ON. Therefore during the interval $\omega t = (30° + \alpha)$ to $\omega t = (30° + \alpha + 60°)$, thyristors T1 and T6 conduct and the Voltage e_{AB} gets applied to the motor terminals. Thyristor T2 gets triggered at $\omega t = (30° + \alpha + 60°)$ and immediately SCR T6 gets reverse biased and thus gets switched off. The current flow changes from T6 to T2 and so the voltage e_{AC} now gets applied to the motor terminals. This process repeats for every 60° whenever a new thyristor in the sequence gets triggered. The thyristors are numbered in the sequence in which they are triggered.

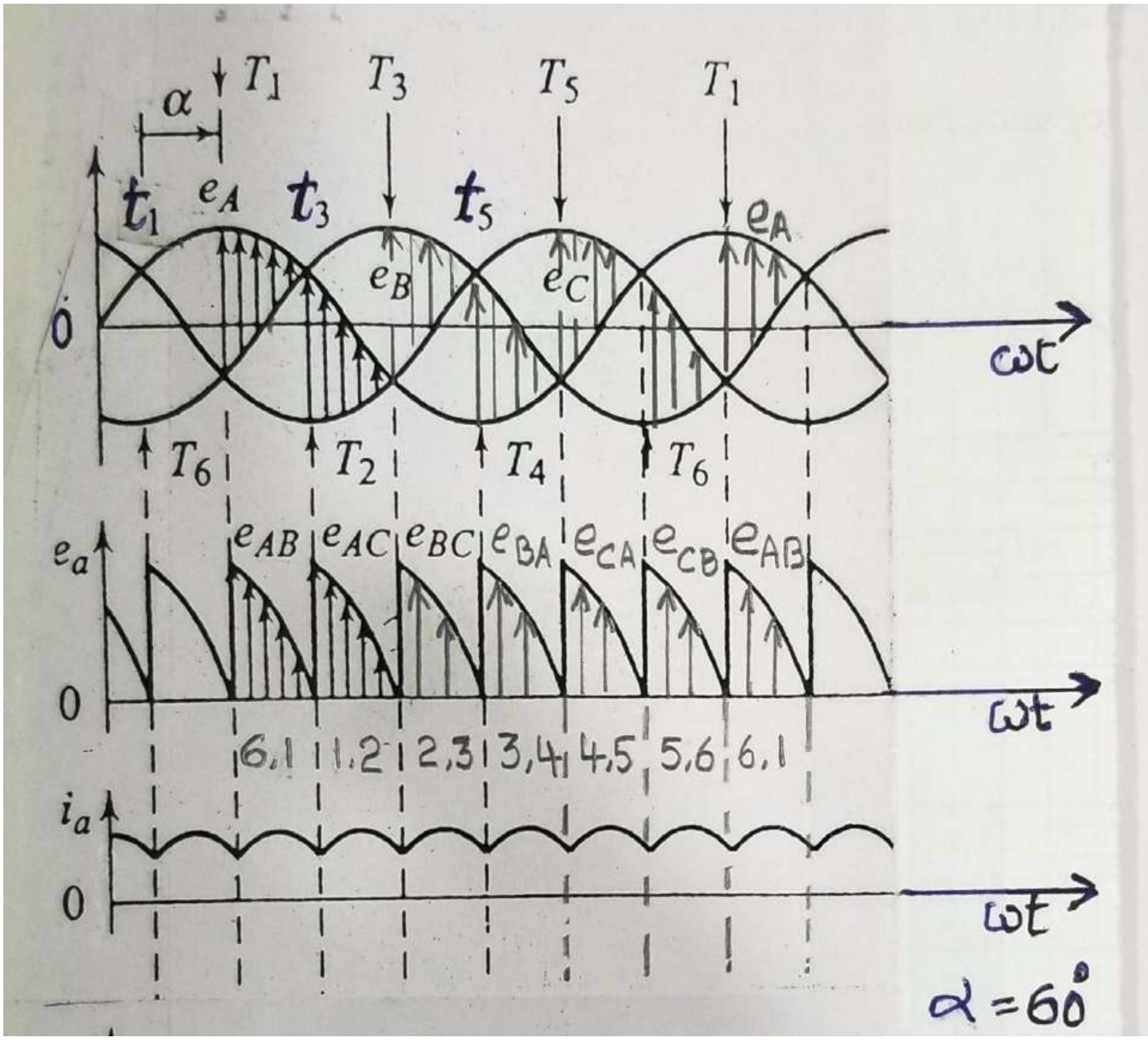


Fig: Three Phase Full Converter Drive Voltage and current waveforms for α = 60°

Applying the same logic the waveform for α = 90^0 and 120^0 is worked out and shown along with that of 60^0 firing angle. It can be seen that the instantaneous voltages that get applied to the motor become negative for half the period and the average value becomes zero for α = 90^0. For firing angle of 120^0 it can be seen that the amplitude of the negative peaks is larger and the average output voltage is negative.

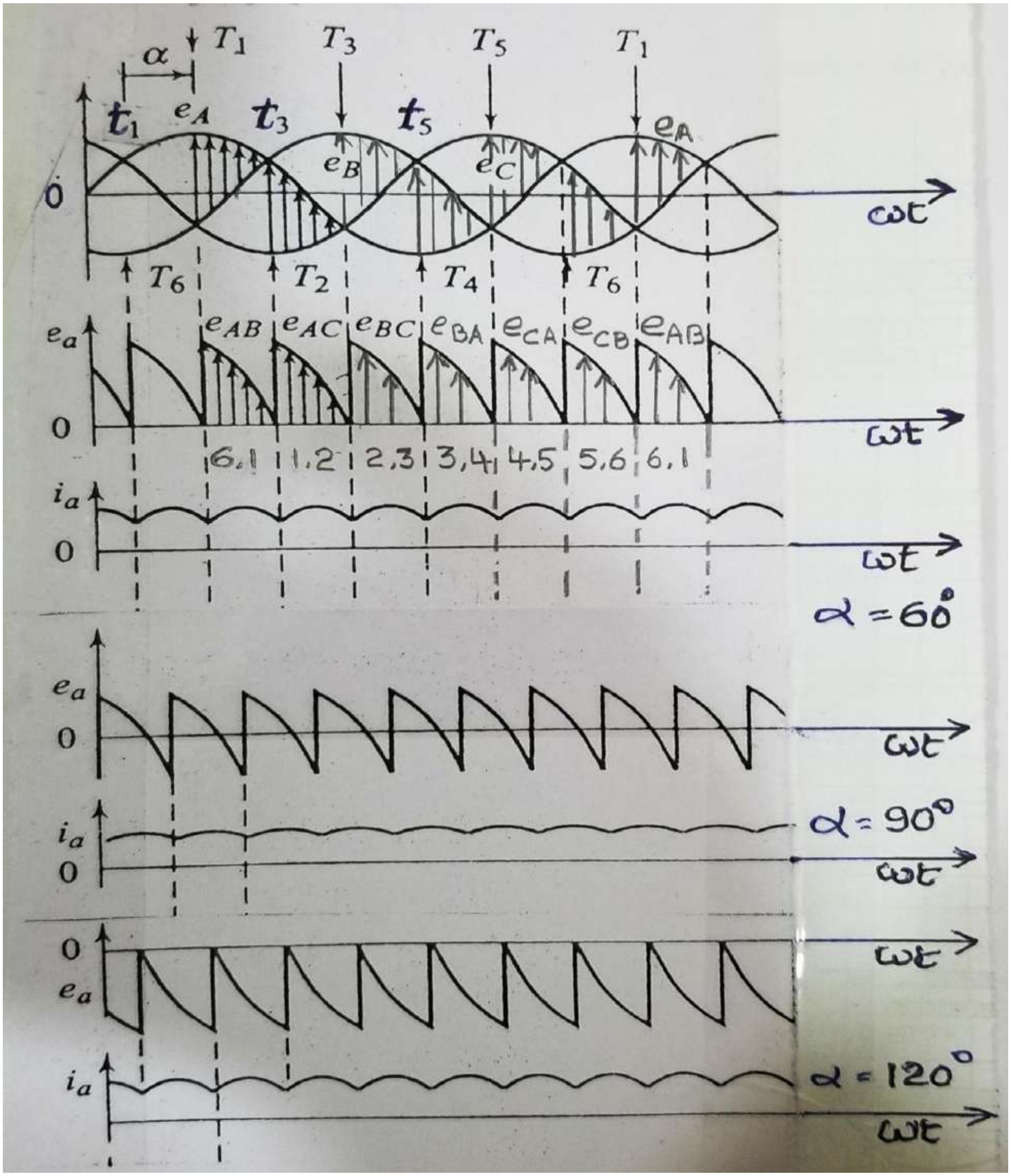


Fig: Three Phase Full Converter Drive Voltage and current waveforms for α = 60°,90°& 120°

Thus it could be summarized as :

- *As the firing angle α changes from 0 to 90° the output load voltage varies from maximum to zero and the converter is working in **Rectifier mode**.*
- *For firing angles of α from 90° to 180° the voltage varies from zero to negative maximum voltage and the converter is working in **Inverter mode**.*

Expression for the Average output voltage:

By observing the waveforms of the output voltage and their symmetry, the average value of output voltage can be obtained by integrating **e_{AB}** ***(E_{RY} in the following equation to be corrected as e_{AB})*** over the time limits ωt = (30°+α) to (90°+α) and averaging over the time period of 60^0 (π/3 Radians)

$$\text{Average output voltage, } E_{dc} = 6 \times \frac{1}{2\pi} \int_{30+\alpha}^{90+\alpha} E_{Ry(\omega t)}\, d\omega t$$

$$= \frac{3}{\pi} \int_{30+\alpha}^{90+\alpha} \sqrt{3}\, E_m \sin(\omega t + 30)\, d\omega t = \frac{3}{\pi} \int_{60+\alpha}^{120+\alpha} \sqrt{3}\, E_m \sin(\omega t)\, d\omega t$$

$$= \frac{3\sqrt{3}\, E_m}{\pi} \left[\cos(\omega t)\right]_{120+\alpha}^{60+\alpha} = \frac{3\sqrt{3}}{\pi} E_m [\cos(60+\alpha) - \cos(120+\alpha)]$$

$$E_{dc} = \frac{3\sqrt{3}\, E_m}{\pi} \cos\alpha \text{ for } 0 \le \alpha \le 180°$$

Where $\mathbf{E_m}$ is the peak value of the phase to neutral voltage.

Torque Speed relationships with Full converter connected to a DC separately excited motor:

Assuming motor current to be continuous, the motor armature voltage as derived above for the full converter is given by:

$$\mathbf{E_a\,(\alpha\,) = (3\sqrt{3}\,E_m/\pi)(\cos\alpha)}$$

Applying this value of **E_a (α)** for the full converter in the general expression for speed **ω** which we already have for a DC separately excited motor

$$\boldsymbol{\omega = [E_a / K_a\phi + - [R_a / (K_a\phi)^2 + \tau}$$

we finally get :

$$\boldsymbol{\omega = *(3\sqrt{3}\,E_m/\pi)(\cos\alpha) / K_a\phi + - [R_a / (K_a\phi)^2]\,\tau}$$

The first term in the above equation for the Speed gives the No-load speed (**τ** = 0) which therefore depends on $E_a(\alpha)$.

*As could be seen, the relationship is identical to that of a single phase full converter connected to a DC separately excited motor we have seen earlier(except that the amplitude of **Ea(α)** is different) and so the torque speed characteristics are identical (Same curves can be redrawn here)*

Three Phase *Semi Converter* drive connected to a DC separately excited motor:

Figure below shows the power circuit of a three Phase Semiconductor drive connected to a DC separately excited motor. It consists of three SCRs, three diodes and an additional freewheeling diode. It is a one quadrant drive with field reversal capability. The field converter can also be a single phase or three phase semi converter.

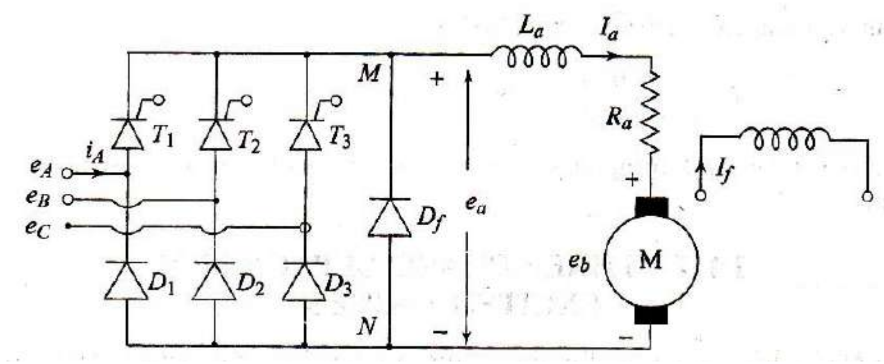


Fig: Three Phase Semiconductor drive connected to a DC separately excited motor

The voltage waveform for this converter is shown in the figure below for a firing angle α = 90° and for continuous current. The operation of this converter is explained with the help of the waveform shown below and the following important points.

- Since there are only three SCRs,they fire at 120° interval (It may be recalled that this interval was 180° for single-phase full converters and was 60° for three phase full converters)
- Though SCRs get forward biased when their respective phase voltages are positive maximum, they conduct only when they are fired.
- Diodes start conducting as soon as they are forward biased. And the diodes which get lowest phase voltage get forward biased. Hence It can be seen that only three Line voltages $\mathbf{E_{AC}}$, $\mathbf{E_{BA}}$ and $\mathbf{E_{CB}}$ get applied to the load when the corresponding diodes are forward biased.
- Applying the above basic principles the line voltages that get applied to the load are sketched directly from the three Phase voltages for firing angle α= 90°.
- The conduction periods of the diodes and the thyristors are shown in terms of instants of time t1 to t6. As shown, the diodes D1, D2 and D3 conduct during the intervals t4 to t6,t6 to t8 and t2 to t4 respectively. If thyristors T1,T2 and T3 were also diodes they would have conducted during the periods t1 to t3, t2 to t5 and t5 to t7 respectively. Therefore the references for the triggering angles for T1,T2 and T3 are taken as the instants t1,t3 and t5 respectively. They are the crossing points for the phase voltages $\mathbf{e_A}$, $\mathbf{e_B}$, and $\mathbf{e_C}$

Operation of the converter:

As shown, thyristor T1 gets triggered at ωt = (30°+α) and during the interval ωt = (30°+α) to ωt = ωt4, thyristor T1 & Diode D3 conduct thus applying the voltage e_{AC} to the motor terminals. At ωt4 ,e_A becomes zero and then becomes negative with respect to both e_B and e_C. During this period the freewheeling diode D_f becomes forward biased and the motor current flows through that until the next thyristor T2 is triggered at ωt = (30°+α+ 120°). Then during the interval ωt = (30°+α+ 120°) to ωt = ωt6, thyristor T2 & Diode D1 conduct applying the voltage e_{BA} to the motor terminals. At ωt6 ,e_B becomes zero and then becomes negative with respect to both e_A and e_C. During this period the freewheeling diode D_f again becomes forward biased and the motor current flows through that until the next thyristor T3 is triggered at ωt = (30°+α+ 240°). Then during the interval ωt = (30°+α+ 240°) to ωt = ωt8, thyristors T3 & Diode D2 conduct applying the voltage e_{CB} to the motor terminals.

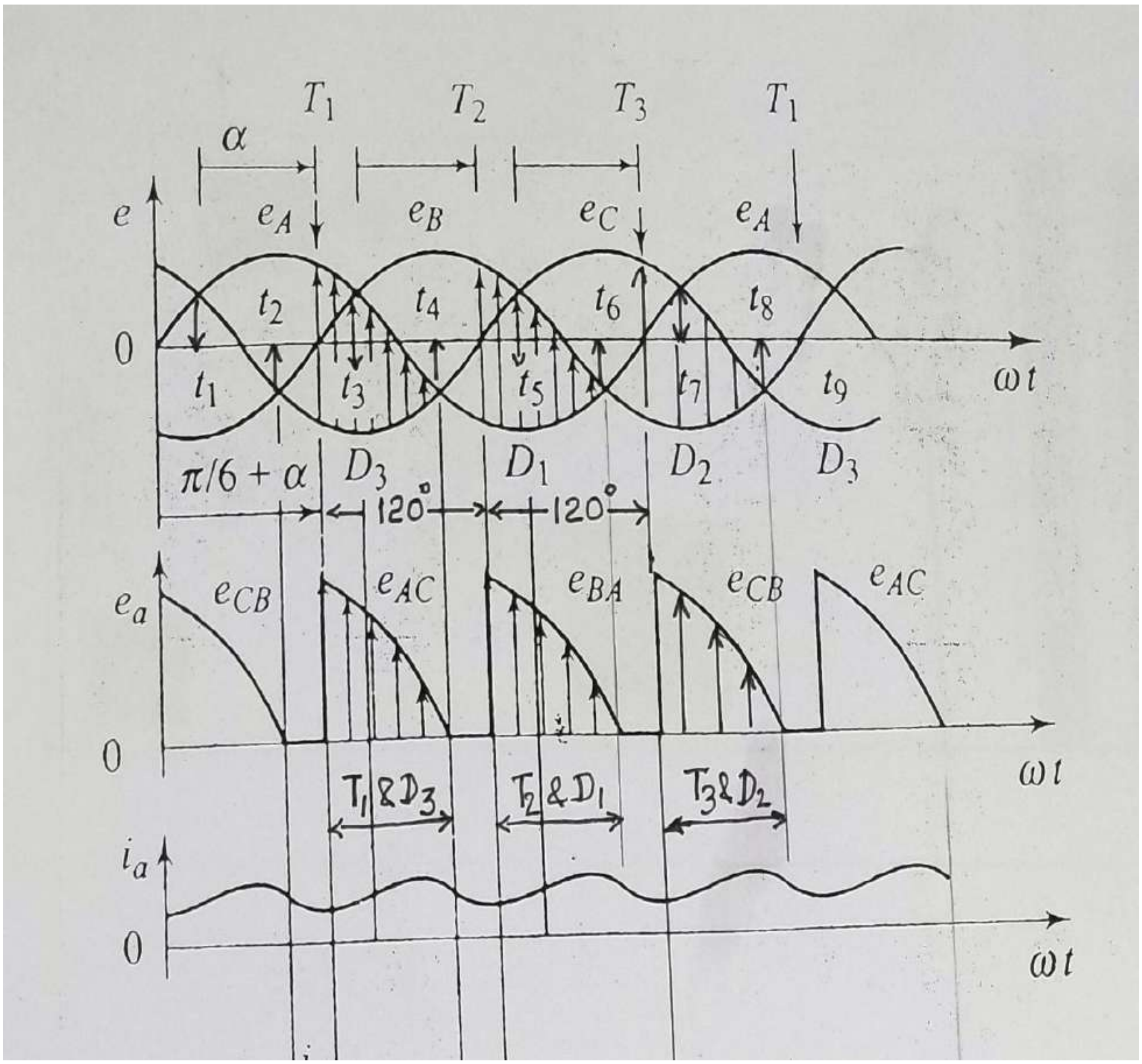


Fig: Waveforms in a three Phase Semi Converter connected to a DC separately excited motor for α = 90°

Expression for the average output voltage:

By observing the waveform of the output voltage and it's symmetry, the average value of output voltage can be obtained by integrating e_{AC} over the time limits ωt = (30°+α) to 210^0 and averaging over the time period of 120^0. It may be noted that the upper limit is taken as 210^0 since it is always fixed and is independent of α for all the three line voltages.

Case II $\alpha \geq 60°, E_{dc} = 3 \times \frac{1}{2\pi}\left[\int_{30+\alpha}^{210} E_{Ac}(\omega t)\, d\,\omega t\right]$

Substitute the value of E_{AC}, we get,

$$E_{dc} = \frac{3}{2\pi}\int_{30+\alpha}^{210} \sqrt{3}\, E_m \sin(\omega t - 30)\, d\omega t = \frac{3\sqrt{3}\, E_m}{2\pi}\left[\cos(\omega t - 30)\right]_{210}^{30+\alpha}$$

$$= \frac{3\sqrt{3}\, E_m}{2\pi}\left[\cos(\alpha) - \cos(180)\right] = \frac{3\sqrt{3}\, E_m}{2\pi}(1 + \cos\alpha) \quad (6.54\ (b))$$

Where E_m is the peak value of the phase to neutral voltage.

Torque Speed relationships with Semi converter connected to a *DC separately excited motor:*

Assuming motor current to be continuous, the motor armature voltage as derived above for the semi converter is given by

$$\mathbf{E_a(\alpha) = (3\sqrt{3}\, E_m/2\pi)(1+\cos\alpha)}$$

Applying this value of $\mathbf{E_a(\alpha)}$ for the full converter in the general expression for speed we already have for a DC separately excited motor

$$\omega = *E_a / K_a\phi + - [R_a/ (K_a\phi)^2 + \tau$$

we finally get :

$$\omega = *(3\surd 3\ E_m/2\pi)(1+\cos\alpha) / K_a\phi + - [R_a/ (K_a\phi)^2 + \tau$$

The first term in the above equation for the Speed gives the No-load speed (**τ** = 0) which therefore depends on $E_a(\alpha)$.

As could be seen the relationship is identical to that of a single phase semi converter connected to a DC separately excited motor we have seen earlier(except that the amplitude of Ea (α) is different) and the torque speed characteristics are identical (Same curves can be redrawn here)

The variation of E_a as a function of α in Semi and Full converters:

The variation of E_a as a function of α for continuous motor current is shown in the figure below for both Semi and Full converters.

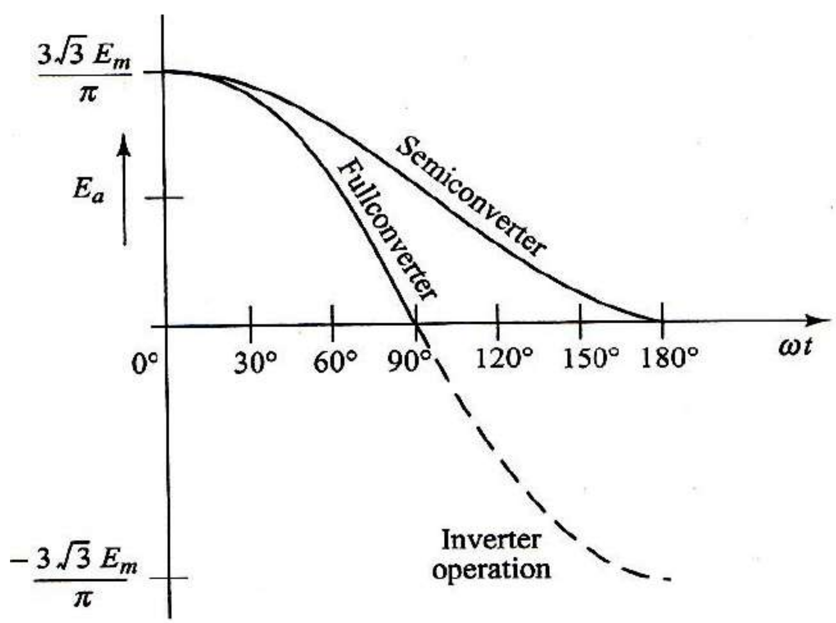


Fig: The variation of E_a as a function of α in Semi and Full converters:

These curves also represent the theoretical no-load speed as a function of firing angle for the separately excited motors. The second term represents the decrease in speed as the motor torque increases. Since the motor armature resistance is small the decrease in speed is small (i.e. good regulation). In large motors, the motor current at no-load is not small and hence if a three phase converter is used, the motor current is more likely to be continuous even at no-load condition. Therefore three phase drives provide better speed regulation and performance compared to single phase drives.

Torque Speed relationships with Full converter connected to *DC series motor :*

In phase controlled converters for Series motors, the current is mostly continuous and the motor terminal voltage as derived earlier for the full converter is:

$$E_a(\alpha) = (3\sqrt{3}\, E_m/\pi)(\cos\alpha)$$

In terms of average voltages, KVL around the motor armature gives:

$$E_a(\alpha) = E_b + I_aR_a = K_a\phi.N + I_aR_a$$

And therefore the average speed is given by :

$$\omega = [\, E_a(\alpha) - I_aR_a \,]/ K_a\phi.$$

In series motors Torque is given by:

$$T = K_a\phi.\, I_a = K_a.\, K_f.I_f.I_a$$
$$= K_{af}.I_a^2$$

Hence from the above equation the average speed can be written as:

$$\omega = [(3\sqrt{3}\, E_m/\pi)(\cos\alpha)/(K_{af}.I_a)] - [(R_a \,.\, I_a/ K_{af}\,.I_a)]$$

$$\omega = [(3\sqrt{3}\, E_m/\pi)(\cos\alpha)/\sqrt{(K_{af}.T)}] - [(R_a/ K_{af})]$$

As could be seen the relationship is identical to that of a single phase semi converter connected to a DC series motor we have seen earlier(except that the amplitude of Ea(α) is different) and the torque speed characteristics are also identical (Same curves can be redrawn here)

Torque Speed relationships with Semi converter connected to *DC series motor:*

In phase controlled converters for Series motors, the current is mostly continuous and the motor terminal voltage from a Semi Converter can be written as

$$\begin{aligned} E_a(\alpha) &= (3\sqrt{3}\, E_m/2\pi)(1+\cos\alpha) \\ &= I_aR_a + E_b \\ &= I_aR_a + K_{af}.\, I_a.\, N \end{aligned}$$

Hence from the above equation the average speed can be written as

$$\omega = [(3\sqrt{3}\, E_m/2\pi)(1+\cos\alpha)/(K_{af}.I_a)] -- [(R_a\, .\, I_a/\, K_{af}\, .I_a)]$$

$$\omega = [(3\sqrt{3}\, E_m/2\pi)(1+\cos\alpha)/\sqrt{(K_{af}.T)}] -- [(R_a/\, K_{af})]$$

As could be seen the relationship is identical to that of a single phase semi converter connected to a DC series motor we have seen earlier(except that the amplitude of Ea(α) is different) and the torque speed characteristics are identical (Same curves can be redrawn here)

Summary:

Important conclusions and concepts:

- In Full converter as the firing angle α changes from 0 to 90° the output voltage varies from *Positive maximum Voltage* to *zero* and the converter works in *Rectifier mode*. And as s the firing angle α changes from 90° to

180° the output voltage varies from *zero* to *negative maximum voltage* and the converter works in *Inverter mode*.ie.

- Where as in a semi converter as the firing angle α changes from 0 to 180° the output voltage varies from *Positive maximum Voltage* to *zero* and the converter works in *Rectifier mode throughout*.
- Hence in both Single Phase and Three phase Full converters work in two quadrants and Semi converters work in single quadrant.
- The ripple frequency of the output of a 3ϕ Half Wave Rectifier is 150 Hz
- The ripple frequency of the output of a 3ϕ Full Wave Rectifier is 300 Hz
- The ripple frequency of the output of a 3ϕ Semi converter is 150 Hz except for α = 0° when it is 300 Hz
- The ripple frequency of the output of a 3ϕ Full converter is 300 Hz
- The motor current in three phase converters may be discontinuous at large firing angles if the current demand is low and the speed is not low.
- In large motors, the motor current at no-load is not small and hence if a three phase converter is used, the motor current is more likely to be continuous even at no-load condition. Therefore three phase drives provide better speed regulation and performance compared to single phase drives.
- The ripple in the output voltage of a three phase Full converter is six pulses per cycle. Since there are six thyristors in the circuit, they are fired at a faster rate (once in 60°) and the motor current is mostly continuous. Therefore the filtering requirement is less than that in the three phase semi converter and single phase converter.

Important formulae and equations:

- **Torque Speed relationships with Full converter connected to *DC Separately excited motor :***
    - **Terminal Voltage $E_a(\alpha) = (3\sqrt{3}\, E_m/\pi)(\cos\alpha)$**
    - **Speed $\omega = *(3\sqrt{3}\, E_m/\pi)(\cos\alpha) / K_a\phi+ - [T.R_a/ (K_a\phi)^2]$**

- **Torque Speed relationships with Semi converter connected to *DC Separately excited motor :***
  - **Terminal voltage E_a (α) = $(3\surd 3\ E_m/2\pi)(1+\cos\alpha)$**
  - **Speed ω = $*(3\surd 3\ E_m/2\pi)(1+\cos\alpha) / K_a\phi + \text{--} [T.R_a/ (K_a\phi)^2]$**
- **Torque Speed relationships with Full converter connected to *DC series motor :***
  - **Terminal Voltage E_a (α) =$(3\surd 3\ E_m/\pi)(\cos\alpha)$**
  - **Speed ω = $[(3\surd 3\ E_m/\pi)(\cos\alpha)/\surd(K_{af}.T)]$ -- $[(R_a/ K_{af})]$**
- **Torque Speed relationships with Semi converter connected to *DC series motor :***
  - **Terminal voltage E_a (α) = $(3\surd 3\ E_m/2\pi)(1+\cos\alpha)$**
  - **Speed ω = $[(3\surd 3\ E_m/2\pi)(1+\cos\alpha)/\surd(K_{af}.T)]$ -- $[(R_a/ K_{af})]$**

Illustrative Examples:

Example-1: A 80 kW, 440V, 800 rpm DC. Motor is operating at 600 rpm and developing 75 % rated torque when controlled by a 3-Ø, six-pulse thyristor converter. If the back emf at rated speed is 410 V, determine the triggering angle of the converter. The input to the converter is 3-Ø, 415 V 50 Hz a.c. supply.

Solution: Given data: E_{bR} = 410 V, N_R = 800 rpm, N_2 = 600 rpm. Rated Power = 80kW, Rated Voltage E_R= 440 V

We know that back e.m.f. is proportional to speed. Hence using the relation:

$$\frac{E_{b_2}}{E_{b_R}} = \frac{N_2}{N_R}$$

We get $E_{b2} = 410 \times \frac{600}{800} = 307.5$ V

We also know that $E_{bR} = E_a - I_{aR} R_a$, $410 = 440 - I_{aR} R_a$. From which we get $I_a R_a$ drop = 30 V at rated conditions.

Rated current = Rated power /Rated Voltage = $I_{aR} = \frac{80 \times 1000}{440} = 181.82$ A.

∴ Armature resistance $R_a = \frac{\text{Ia Ra drop}}{\text{Ia}}$ @rated conditions = $\frac{30}{181.82}$ = 0.165 Ω.

Terminal voltage of DC. motor at 600 rpm and 75 % rated torque,

$$= E_{b2} + 0.75\, I_{aR} R_a = 307.5 + (0.75 \times 181.82 \times 0.165) = 330\text{V}.$$

We know that in the case of a 3Ø, six-pulse thyristor converter the out put voltage is given by: $E_a(\alpha) = \frac{3\sqrt{3}}{\pi} E_m \cos\alpha$

Hence neglecting voltage drop in the converter system, the applied voltage to the motor would be:

$$E_a(\alpha) = \frac{3\sqrt{3}}{\pi} E_m \cos\alpha = 330\text{V}$$

where E_m = maximum value of the phase voltage = √2 Line voltage/ √3 (since input to the converter is normally connected in star,415 V is to be taken as line voltage)

Hence $$330 = \frac{3\sqrt{3}}{\pi} \times \frac{\sqrt{2}}{\sqrt{3}} 415 \cos\alpha$$

$$330 = \frac{3\sqrt{2}}{\pi} 415 \cos\alpha = 1.35 \times 415 \times \cos\alpha$$

From which we have **$\cos\alpha = 0.589$ and $\alpha = 53.91^0$**

Exapmle-2: The speed of a 150 HP, 650 V, 1750 rpm, separately excited d.c motor is controlled by a 3-Ø, 460 V, 50 Hz supply. The rated armature current of the motor is 170 A. The motor parameters are R_a = 0.099 Ω, L_a = 0.73 mH, and $K_a Ø$ = 0.33 V/rpm. Neglecting the losses in converter system, determine:

(a) No-load speeds at firing angles $\alpha = 0^0$ and $\alpha = 30^0$. Assume that at no-load, the armature current is 10% of the rated current and is continuous.
(b) The firing angle to obtain rated speed of 1750 rpm at rated motor current.
(c) The speed regulation for the firing angle obtained in part (b)

Solution: Given data : Rated voltage E_{aR} = 650 V , Rated speedN_R =1750 RPM ,Input Supply 3-Ø, 460 V, rated armature current I_{aR} = 170 A, R_a = 0.099 Ω, L_a = 0.73 mH, rated power = 150 HP and $K_a Ø$ = 0.33 V/rpm

Important Point to be noted:

Since the units of $K_a Ø$ is given as V/rpm the governing equation for back emf has to be taken as $E_b = K_a Ø.N$ where N is speed in rpm. Further since the flux Ø is embedded in the constant we can take the flux and hence the field current as constant. Thus the given $K_a Ø$ constant can be used as it is.

(a) No-load condition speeds for firing angles of $\alpha = 0^0$ and $\alpha = 30^0$:

Though not specifically mentioned, the converter is taken as a fully controlled converter whose output voltage is given by: $E_a(\alpha) = \frac{3\sqrt{3}\,Em}{\pi}\cos\alpha$ where as per our normal convention E_m = maximum Phase voltage.

Input voltage is normally taken as line voltage when not specified. Hence V_L= 460 V

Then supply phase voltage is,

$V_{ph} = \frac{VL}{\sqrt{3}} = \frac{460}{\sqrt{3}} = 265.58$ V and maximum value $V_{ph,max} = \sqrt{2} \times 265.58$ V = E_m as per our convention.

Then, $E_a = \frac{3\sqrt{3}\sqrt{2}\times 265.58}{\pi} \cos\alpha = 621.22 \cos\alpha$

For $\alpha = 0^0$ $E_a = 621.22$ V.

Applying the Armature loop equation for no load condition i.e taking I_a as 10% of rated current of 170 A i.e. 17 A we get

$E_b = E_a - I_a R_a = 621.22 - (17 \times 0.099) = 619.5$ V.

No-load speed with firing angle $\alpha = 0^0$ is

$$N_{NL} = \frac{E_b}{K_a \emptyset} = \frac{619.5}{0.33} = 1877 \text{ rpm.}$$

For $\alpha = 30^0$ $E_a = 621.22 \cos\alpha = 621.22 \cos 30^0 = 537.99$ V

And $E_b = 537.99 - (17\times 0.099) = 536.3$ V

No-load speed $\alpha = 30^0$ is

$$N_{NL} = \frac{536.3}{0.33} = 1625 \text{ rpm.}$$

(b) Firing angle to obtain rated speed of 1750 rpm at rated motor current :

Motor back emf E_b at 1750 rpm is

$$E_{bR} = K_a \emptyset \times 1750 = 0.33 \times 1750 = 577.5 \text{ V}$$

Required converter terminal voltage at rated current and rated speed is

$$E_a = E_{bR} + I_{aR}.R_a = 577.5 + (170 \times 0.099) = 594.33 \text{ V.}$$

Therefore, $594.33 = 621.22 \cos\alpha$

From which $\cos\alpha = 594.33/621.22 = 0.9567$ and $\alpha = 16.92^0$

The firing angle to obtain rated speed of 1750 rpm at rated motor current $\alpha = 16.92^0$

(c). Speed regulation for the firing angle obtained in part (b):

At full-load condition, motor current is 170 A and speed is 1750 rpm. If the load is thrown- off keeping the firing angle same at $\alpha = 16.92^0$, motor current decreases to 10% i.e. 17 A.

To find out the speed regulation we have to find out the speed at no load which can be found out from back emf at no load.

Therefore, E_b with no load and same firing angle of 16.92^0 = Converter out put with firing angle of 16.92^0 – (No load current $\times$ 0.099) i.e.

$E_{bNL} = 594.33 - (17 \times 0.099) = 592.65$ V

Now, no-load speed is

$$N_{NL} = \frac{592.65}{0.33} = 1795.91 \text{ rpm.}$$

The speed regulation $= \frac{Nnl - Nfl}{Nnl} \times 100 = \frac{1795.91 - 1750}{1795.91} \times 100 = 2.56\%$

Note: In this problem also all the data given is not used to solve the problem. Identify….

Example-3: The speed of a 25 Hp, 320 V, 960 rpm separately excited D.C. motor is controlled by a 3-Φ Full Converter. The field current is also controlled by a three-phase Full Converter and is set to the maximum possible value. The A.C. input is a 3-Ø, star-connected 210 V, 50 Hz supply. The armature resistance is $R_a = 0.2\ \Omega$, the field resistance is $R_f = 130\ \Omega$, and the motor voltage constant is $K_a = 1.2$ V/A Rad/s. The armature and field current are continuous and ripple free. Determine:

(a) The firing angle of the armature converter if the field converter is operated at the maximum field current and the developed torque is 110 N-m at 960 rpm.

(b) The speed of the motor if the field circuit converter is set for the maximum field current, the developed torque is 110 N-m and the firing angle of the armature converter is 0^0.

(c) The firing angle of the converter if the speed has to increase to 1750 rpm, for the same load requirement in part (b). Neglect the system losses

Solution:

(a) Since the input is from a 3-Ø, star-connected supply, the given 210 Volts value is r.m.s.value of line voltage. But for calculating the Armature voltage we need maximum value of Phase voltage.

Hence Phase voltage, $E_p = \frac{210}{\sqrt{3}}$ = 121.24 V and $E_m = \sqrt{2} \times 121.24 = 171.46$

In a field controlledfull converter The DC field voltage is given by (same like in a Full converter supplying an Armature)

$$E_f = \frac{3\sqrt{3}E_m}{\pi} \cos \alpha$$

For maximum field current, α = 0,

∴ $$E_f = \frac{3 \times \sqrt{3} \times 171.46 \cos 0}{\pi} = 283.59 \text{ V}$$

And then: $$I_f = \frac{E_f}{R_f} = \frac{283.59}{130} = 2.18 \text{ A.}$$

Before going further we need to observe the units of the armature constant which is given as ***V/A Rad/s*** . instead of the standard ***V/Wb. Rad/s***. i.e The given constant is $\mathbf{K_{af}}$ which is $\mathbf{K_a.K_f}$ combining the field constant also from the relation $\boldsymbol{\phi} = \mathbf{K_f . I_f}$

So we have take the Torque and Speed relations as $\mathbf{T = I_a K_{af} I_f}$ and $\mathbf{E_b = \omega K_{af} I_f}$

Since our objective is to find out firing angle we have to first find out E_a and it can be found out from the equation $E_a = E_b + I_a R_a$.

Now using the equations for torque and back e.m.f E_b . in the above form we can find out first armature current, then E_b and then finally E_a as below.

$$T = I_a K_{af} I_f \;\therefore I_a = \frac{T}{K_a I_f} = \frac{110}{1.2 \times 2.18} = 42.04 \text{ A}$$

$$E_b = \omega K_{af} I_f = 1.2 \times 2.18 \times 960 \times \frac{2\pi}{60} = 262.99 \text{ V.}$$

(Please note that the speed is to be converted from RPM to Rad./Sec since in the above equation for E_b , ω (Rad/sec) is used.

$$E_a = E_b + I_a R_a = 262.99 + (42.04 \times 0.2) = 271.4 \text{ V.}$$

Now, $E_a = 271.4 = \frac{3 \times \sqrt{3} \times 171.46}{\pi} \cos\alpha$.

From which we can get firing angle, $\alpha = 16.86^0$

(b) We have to find out the speed corresponding to α = 0 for armature converter with a load torque of 110 N-m.

Given $\alpha = 0$, $E_a = \frac{3 \times \sqrt{3} \times 171.46}{\pi} = 283.59$.

We have already the value of **I_a** as 42.04 A for a load torque of 110 N-m.Using this in the expression for the back e.m.f. we get

$$E_b = 283.59 - (42.04 \times 0.2) = 275.18 \text{ V}$$

$$\text{Speed } \omega = \frac{E_b}{K_a I_f} = \frac{275.18}{1.2 \times 2.18} = 105.19 \text{ rad/s} = 1004.50 \text{ rpm.}$$

(c) We know that to increase the speed above base speed field is to be weakened.i.e. the field current is to be reduced by reducing the field voltage. We know that to reduce the field voltage we have to increase the firing angle.

So, we can use the same relations as earlier first to get the back e.m.f. corresponding to the speed of 1750 RPM, then I_f then E_f and then finally α_f as below.

$\omega = 1750 \times \frac{2\pi}{60} = 183.26$ rad/s

$E_b = 275.18 = 1.2 \times 183.26 \times If \quad \therefore I_f = 1.25$ A.

$E_f = 1.25 \times 130 = 162.5$ V

$E_f = 162.5 = \frac{3 \times \sqrt{3} \times 171.46}{\pi} \cos \alpha_f, \quad \therefore$ $\boldsymbol{\alpha_f = 55.04^0}$

UNIT – II
FOUR QUADRANT OPERATION OF DC DRIVES

SYLLABUS/CONTENTS:

- Introduction to Four Quadrant Operation
- Motoring operations
- Electric Braking – Plugging, Dynamic and Regenerative Braking operation.
- Four quadrant operation of D.C motors by Dual Converters
- Closed loop operation of DC motor (Block Diagram Only)
- Summary
    - Important concepts and conclusions
- Illustrative Examples

Introduction to Four quadrant operation of electric drives:

An electrical drive has to operate in three modes. i.e. starting, steady state and braking. To achieve this in both directions (forward and reverse) four quadrant operation as shown in the figure below is required which shows the torque and speed coordinates for forward and reverse motions. Power developed by a motor is given by the product of speed and torque.

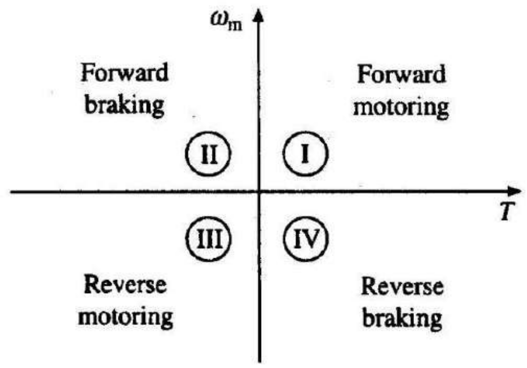


Fig: Four Quadrant operations of Electrical motors

Sign Conventions:

- Positive speed is FORWARD and negative speed is REVERSE.
- Sign of Power is the product of the signs of Torque and Speed. When it is positive it is MOTORING and when it is negative it is BRAKING.

With this convention the four quadrant operation of Motors is explained below. First with reference to the figure above and then with a practical example of Hoist (Lift)

- In Q-1 both *power & speed are positive (forward).* Motor works as a motor delivering mechanical energy to the load. Hence Q-1 operation is designated as *forward Motoring.*
- In Q-2 *power is negative but speed is positive (forward)*. Motor works as a brake opposing the motion. Hence Q-2 operation is designated as *Forward Braking*.
- In Q-3 *power is positive but speed is negative (reverse)*. Motor works as a motor delivering mechanical energy to the load. Hence Q-3 operation is designated as *Reverse Motoring.*

- In Q-4 both *power and speed are negative (reverse)*. Motor works as a brake opposing the motion. Hence Q-4 operation is designated as *Reverse Braking*.

For a better understanding of the four quadrant operation of the drives and the related notations a practical example of a Hoist (Lift) operating in four quadrants is considered here as shown in the figure below. Directions of motor and load torques and direction of speed are marked with arrows.

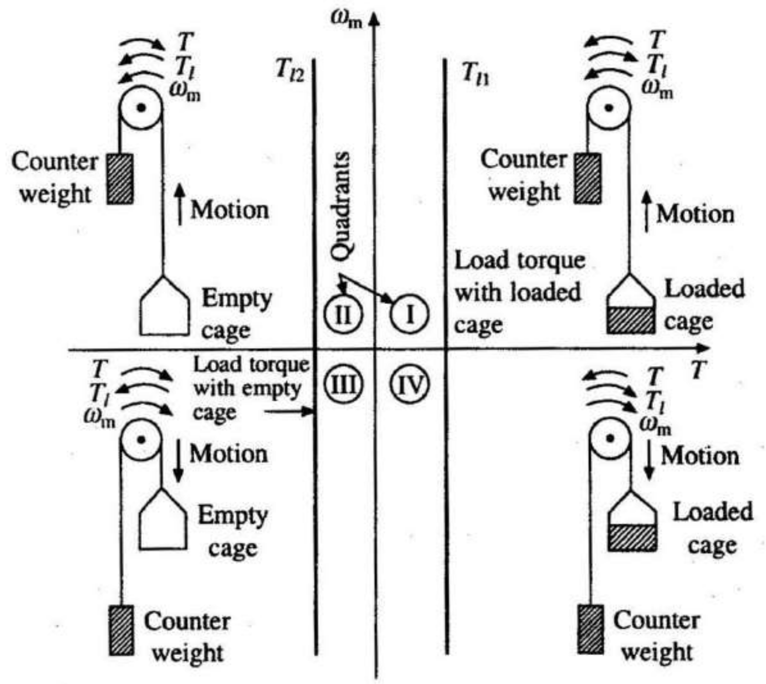


Fig: Typical Example of Four Quadrant operation of a Motor Driving a Hoist (Lift) load.

A hoist consists of a rope wound on a drum coupled to the motor shaft. One end of the rope is connected to the carriage which carries men and/or material from one level to another level. Other end of the rope is connected to a counterweight to balance the carriage so as to distribute the load on the motor in both directions. *Weight of the counterweight is chosen such that it is higher than the empty carriage but lesser than the fully loaded carriage.*

Speed and Torque Sign Conventions: Are explained again with reference to the directions of Speed and Torque shown in the figure above.

- *Forward direction of motion or forward Speed* is considered to be the one which gives *Upward motion* to the carriage which is a result of Anticlockwise movement of the pulley (looking into the page)

- Similarly *Reverse direction of motion or Reverse Speed* is considered to be the one which gives *Downward ward motion* to the carriage which is a result of Clockwise movement of the pulley (looking into the page)

- Similarly the *Torque* is considered to be *Positive* when acting *Anticlockwise* and *Negative* when acting *Clockwise.*

- The sign of the Power becomes the product of the sign of Torque and Speed.

Load torque characteristics are also shown in the diagram and are assumed to be constant. $\mathbf{T_{l1}}$ in quadrants 1 and 4 represents the speed torque characteristic of the loaded carriage. This torque is the difference of torques between loaded hoist and the counter weight and is positive since loaded carriage weight is higher than the counter weight. $\mathbf{T_{l2}}$ in quadrants 2 and 3 represents the speed torque characteristic of the empty carriage. This torque is the difference of torques between empty hoist and the counter weight and is negative since empty carriage weight is lesser than the counter weight.

In Quadrant -1 operation the loaded cage moves upwards corresponding to positive motor speed which in this case is anticlockwise movement of the pulley (looking into the page) This motion will be obtained if the motor produces positive torque in anti clock wise direction equal to the magnitude of the load torque $\mathbf{T_{l1}}$**.** Since both Torque and Speed are Positive Power is also positive and this operation is Forward Motoring.

In Quadrant-4 operation the loaded cage moves downwards corresponding to a negative motor speed which in this case is clock wise movement of the pulley (looking into the page) Since the weight of a loaded cage is higher than the counterweight, it will come down due to the gravity itself. In order to limit the speed of the cage to a safe value, motor must produce a positive torque **T** equal to the load torque $\mathbf{T_{l1}}$ in anticlockwise direction**.** Since Torque is positive and Speed is Negative Power is Negative corresponding to Reverse Braking.

In Quadrant -2 operation the empty cage moves upwards corresponding to a positive motor speed which in this case is anticlockwise movement of the pulley. (Looking into the page) Since the weight of counterweight is higher than the weight of an empty cage, it will automatically move upwards. In order to limit the speed of the cage to a safe value, motor must produce a braking torque **T** equal to the load torque $\mathbf{T_{l2}}$ in clockwise (negative) direction. Since Torque is negative and Speed is positive the Power is Negative corresponding to Forward Braking.

In Quadrant -3 operation, empty cage is lowered. Since an empty cage weight is lesser than the counter weight, the motor must produce a negative torque i.e. in clockwise direction. Since both Speed and Torque both are negative, Power is positive and this operation becomes Reverse Motoring.

Starting:

Maximum current that a DC motor can safely carry is mainly limited by the maximum current that can be commutated without sparking. For normally designed machines twice the rated current can be allowed and in specially designed machines it can be up to 3.5 times the rated current.
During starting when the motor is standstill, the motor back emf will be zero and the only resistance that can limit the current is the armature resistance, which is quite small for almost all DC motors. Hence if a DC motor is started with full rated voltage applied to its terminals then a very large current will flow and damage the motor due to heavy sparking in the commutator and heating of the winding. Hence the current is to be limited to a safe value during starting.
In closed loop speed controllers where *Speed and current controllers* are used the current can be limited to a safe value during starting. But in systems without such controllers a variable resistance controller such as the one shown in figure below is used during starting to limit the current. As the back emf increases with gradual increase in speed, section by section resistances will be removed either manually or remotely with the help of contactors so as to keep the current within the maximum and minimum limits.

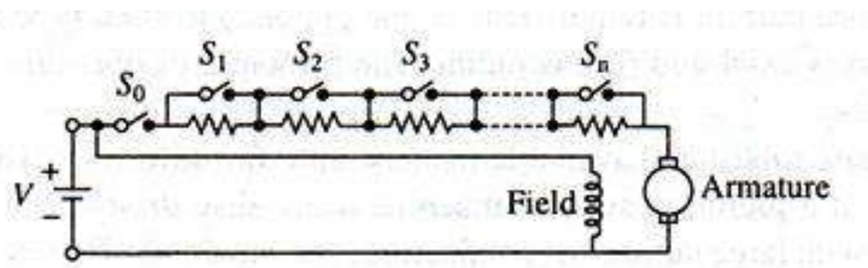


Fig: Starting of a DC Shunt motor

Braking:

An electrical drive has to operate in three modes i.e. steady state, starting and braking during both forward and reverse directions. *Braking* operation is required in two cases.

- For reducing the speed (deceleration) while the drive is operating in Forward (Quadrant-1) or Reverse (Quadrant-3) motoring modes. *Steady state is reached when the motoring torque is equal to the load torque*
- While driving an Active load. That means when the load assists the drive motion [for e.g. moving a loaded hoist in the down ward direction (Reverse braking: quadrant-4) or moving an unloaded hoist in the upward direction (Forward braking: quadrant -2)]. *Steady state is reached when the braking torque is equal to the load torque.*

In both the cases braking can be achieved by mechanical braking. But it has lot of disadvantages: Frequent maintenance like replacement of brake shoes/lining, lower life, wastage of braking power as heat et.. These disadvantages are overcome by Electrical braking but many a times mechanical braking also supplements the electrical braking for reliable and safe operation of the drive.

During electric braking the motor works as a generator developing a torque which opposes the rotational motion. There are three types of electrical braking.

1. Regenerative braking
2. Dynamic or Rheostatic braking and
3. Plugging or Reverse voltage braking.

Regenerative Braking:

In this, the generated energy is supplied to the source. For this to happen, the following conditions should be satisfied:

$|E_b| > |E_a|$ and negative I_a

The concept of regenerative braking can be explained by considering a fully controlled Rectifier connected to a DC separately excited motor as shown in the figure (a) below.

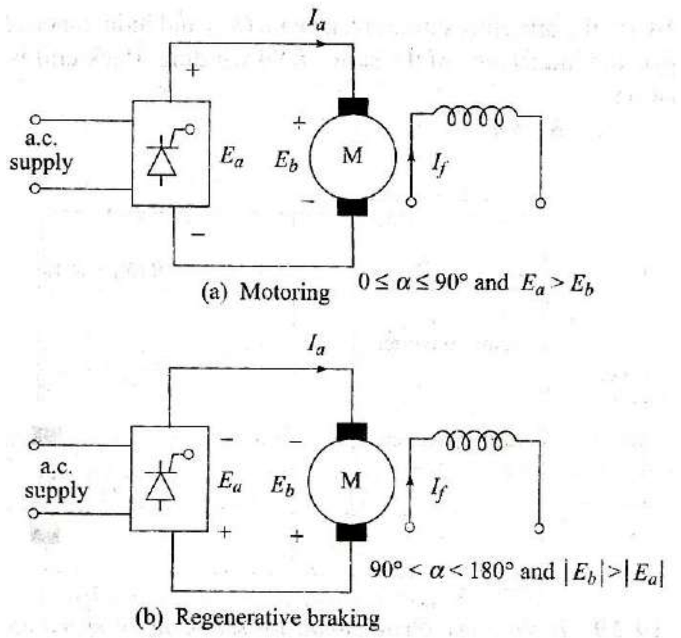


Fig: Two quadrant operation of a Fully Controlled rectifier feeding a DC separately excited motor

The polarities of output voltage, back emf and armature current are shown in the figure (a) above for the motoring operation in the forward direction. The converter output is positive with firing angle in the range $0° \leq \alpha \leq 90°$. With these polarities the converter supplies power to the motor which is converted to mechanical energy. Direction of power flow can be reversed if the direction of current flow is reversed. But this is not possible because the converter can carry current in only one direction. Then the only method available for reversal of power flow is by the following steps.

1. Reverse the Converter output voltage E_a
2. Also reverse the Back emf E_b with respect to the converter terminals
3. And make $|E_b| > |E_a|$

as shown in fig (b). Out of these three steps

- *Step 1*. i.e. the rectifier voltage E_a can be reversed by making $\alpha > 90^0$
- *Step.3.*i.e. the condition $|E_b| > |E_a|$ can be satisfied by choosing a value of α in the range $90^0 \geq \alpha \leq 180^0$
- *Step2*. The reversal of motor emf with respect to rectifier terminals can be done by any of the following changes.
  a. The motor armature terminals can be reversed w.r.to the converter terminals using a reversing switch with the motor still running in the forward direction. (with contactors or thyristors as shown in the figure below) This gives forward regeneration.
  b. The field current may be reversed with the motor running in the forward direction and this also gives forward regeneration without any changes in the armature connections.

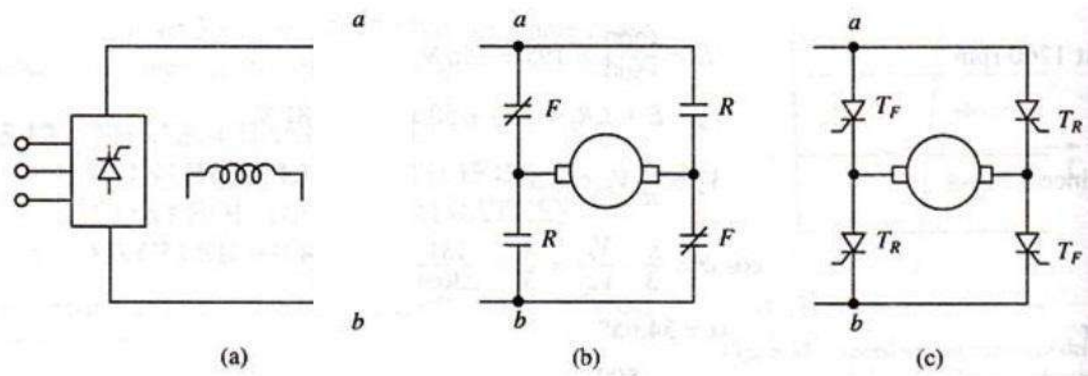


Fig: Four quadrant operation with a single converter and a reversing switch

Regenerative braking cannot be obtained

- If the drive runs in the forward direction only and there is no arrangement for the reversal of either the armature or the field.
- If the converter shown above is a Semi converter.

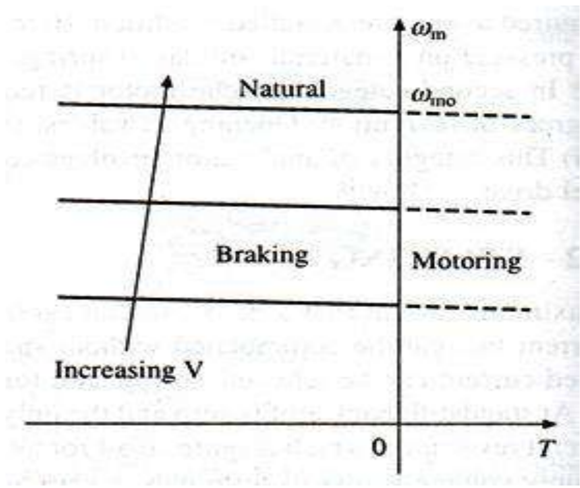


Fig: Regenerative Braking Characteristics of a Separately excited Motor

Dynamic Braking:

In dynamic braking, the motor armature is disconnected from the source voltage and connected across a high wattage resistance R_B .The generated energy is dissipated in the Braking and armature resistances. The braking connections are shown below for DC separately excited motor and DC series motor.

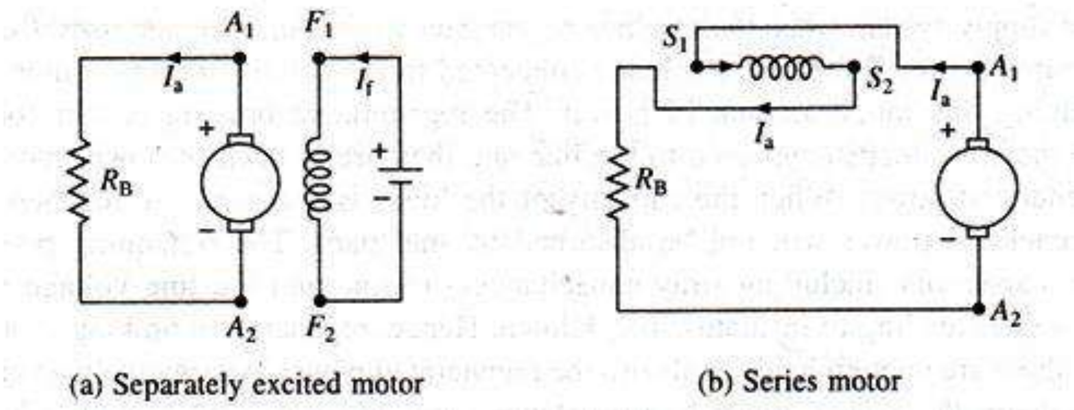


Fig: Connections during Dynamic Braking

In the case of a series motor, it can be seen that the field terminal connections are reversed such that the field current continues to flow in the same direction so that the field assists the residual magnetism. Figure below shows the Speed-Torque curves for both type of motors and the transition from Motoring to Braking.

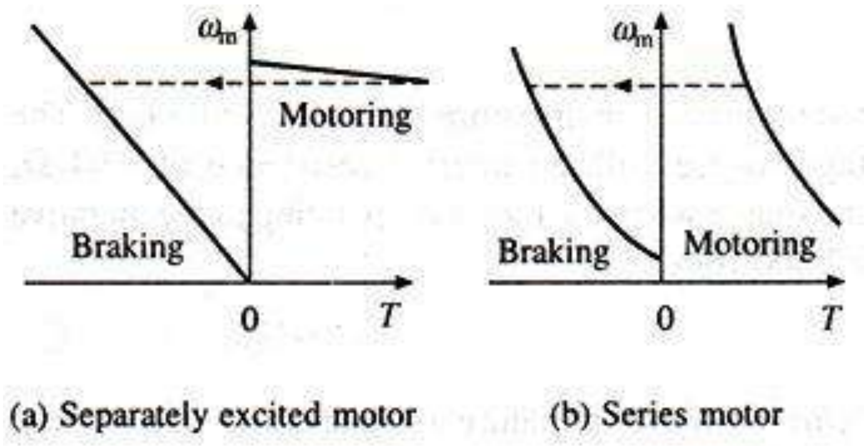


Fig: Speed-Torque curves during Dynamic Braking

Plugging:

In a DC separately excited motor Supply voltage is reversed so that it assists the Back EMF in forcing the Armature current in the reverse direction. In a Series motor Instead of supply voltage, armature alone is reversed so that the field current direction is not changed. In addition, like in dynamic braking, a Braking resistor R_B is also connected in series with the Armature to limit the current as shown in the figure below.

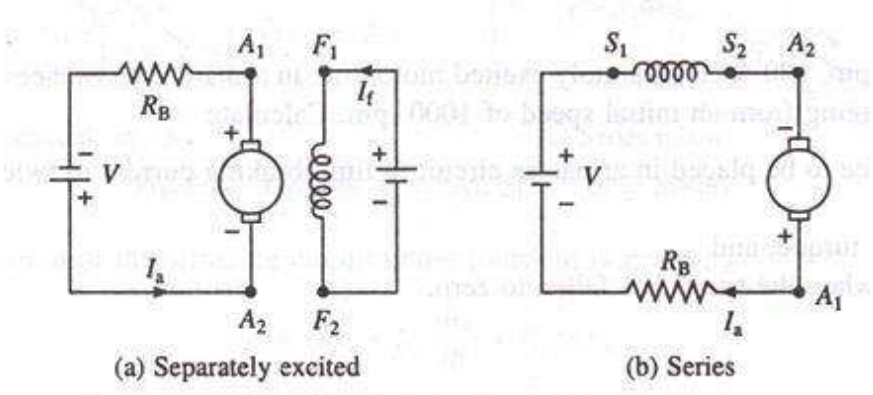


Fig: Plugging operation of DC motors

Speed torque curves can be obtained from the same basic equations by replacing $\mathbf{E_a}$ with $\mathbf{-E_a}$ and are shown in the figure below.

Plugging is highly inefficient because in addition to the generated power additional power from a supply source is also wasted in the Braking resistance.

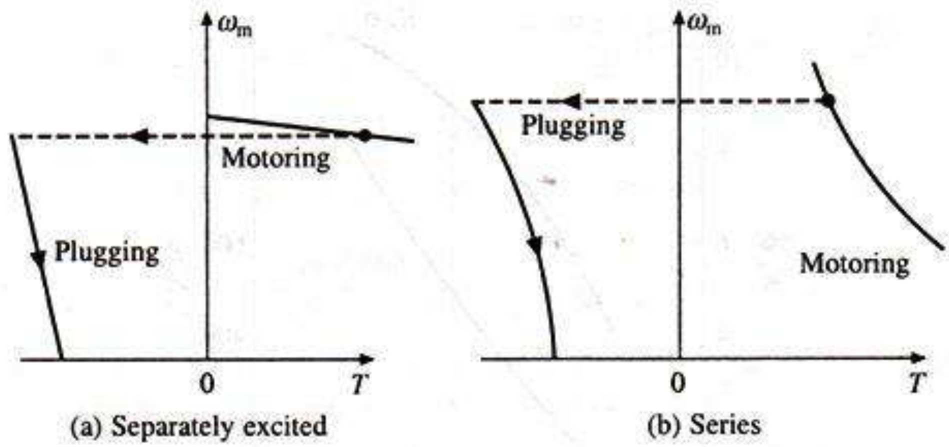


Fig: Torque speed Characteristics of DC motors during Plugging

Four quadrant operation of DC Motors using a Single fully controlled converter:

As studied earlier, a fully controlled converter can provide a reversible output voltage but current in only one direction. In terms of conventional Voltage-Current diagram shown in the figure below it can work in quadrants 1 and 4

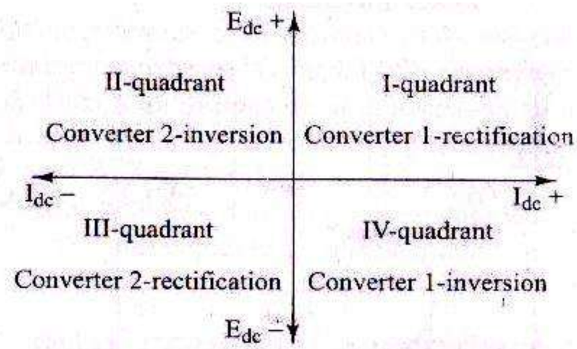
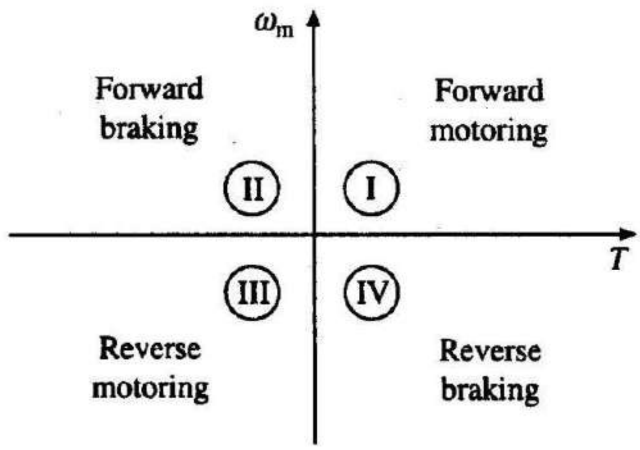


Fig: Voltage-Current Diagram

A converter can be used say in the first quadrant for motoring operation alone in one direction (and in the third quadrant for motoring operation in other direction) during steady state conditions. But during transient requirements such as starting and braking it cannot operate in second (or fourth) quadrant where it is required to extract energy from the load for quick braking. (For faster system response))

If four quadrant operation of a motor is required i.e. reversible rotation and reversible torque in the Torque Speed Plane as shown in the figure below, a single converter along



with changeover contactors to reverse the armature or field connections along with firing angle changeover control [$(0° \leq \alpha \leq 90°)or(90° \leq \alpha \leq 180°)$] can be used so as to change the relationship between the converter voltage and the direction of rotation of the motor.(As explained in the introduction to Regenerative braking). Though they are practicable in suitable circumstances, a better performance can be achieved by going in for a Dual Converter.

Four quadrant operation of DC Motors using Dual Converters:

A dual converter as shown in the figure below consists of two fully controlled converters connected in anti-parallel configuration across the same motor armature terminals. Since both voltage and current of either polarity can be obtained with a dual converter, it can support four quadrant operation of DC motors. Inductors are used to limit the circulating current when both converters are used simultaneously.

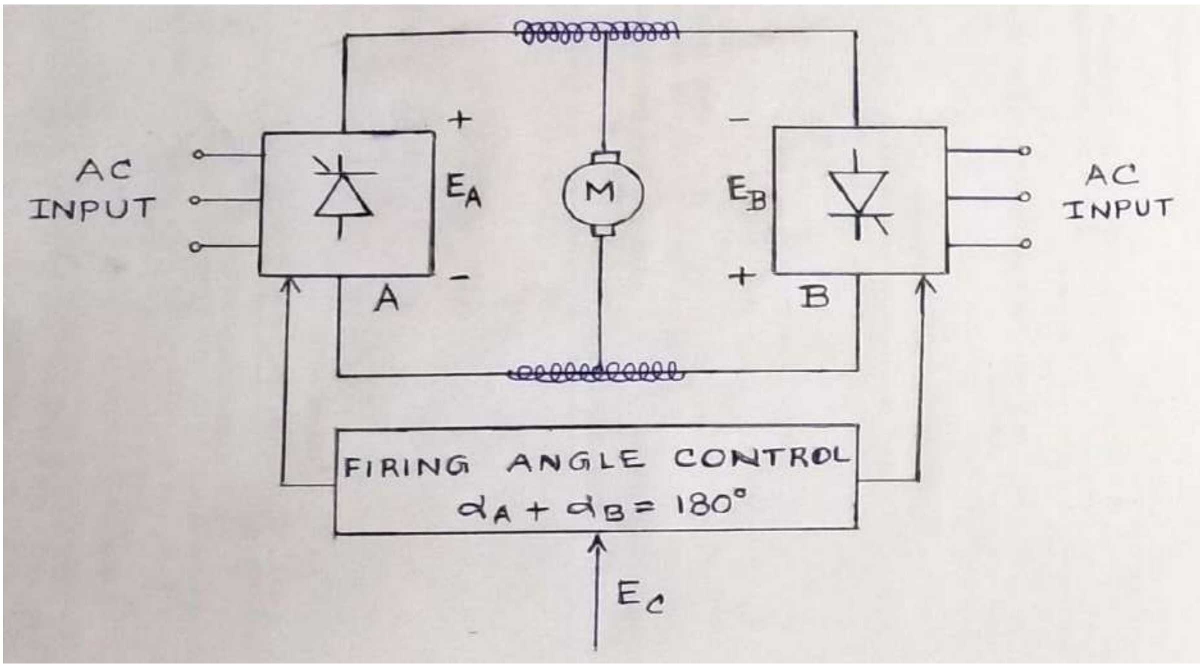


Fig: Dual Converter Control of a DC separately excited motor.
(Inductors are used in only simultaneous or Circulating current mode)

For lower power ratings i.e. up to 10 Kw, single phase Full converters are used and for higher ratings three phase Full converters are used. Typical configuration of both Single phase and three phase Dual converters are shown in the figures below.

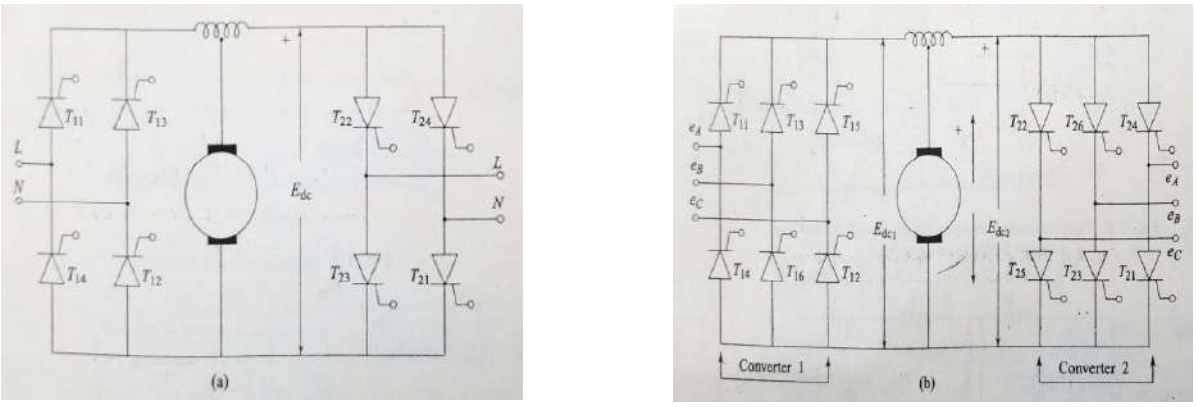
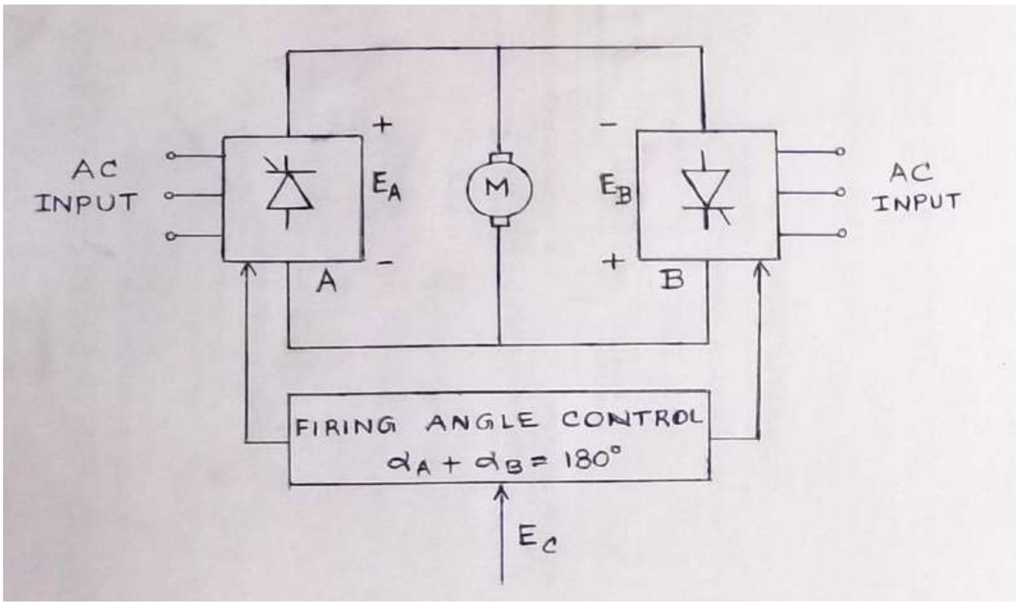


Fig: (a) Single Phase and (b) Three Phase Converters connected as Dual converters.

In a dual converter the converters are configured such that converter-A works in quadrants 1 and 4 and converter-B works in quadrants 2 and 3.

The operation of Dual converter is first explained with the help of an Ideal dual converter (same figure as shown above but without reactors) with the following assumptions:



Simplified diagram of an Ideal Dual Converter

- They produce pure DC output voltage without any ac ripple.
- Each two quadrant converter is a controllable DC voltage source with unidirectional current flow. But the current through the load can flow in either direction.
- The firing angle of the converters is controlled by a control voltage $\mathbf{E_C}$ such that their DC output voltages are equal in magnitude but opposite in polarity. So, they can drive current through the load in opposite directions as per requirement.
- Thus when one converter is operating as a Rectifier and is giving a particular DC output voltage, the other converter operates as an inverter and gives the same voltage at the motor terminals.
- The average DC output voltages are given by:

$$E_{DCA} = E_{max} \cos \alpha_A \text{ and}$$
$$E_{DCB} = E_{max} \cos \alpha_B$$

In an Ideal converter

$$E_{DC} = E_{DCA} = -- E_{DCB}$$

and substituting the above values of $\mathbf{E_{DCA}}$ and $\mathbf{E_{DCB}}$ in this equation we get

$$E_{max} \cos \alpha_A = -- E_{max} \cos \alpha_B$$
$$\text{or } \cos \alpha_A = -- \cos \alpha_B$$
$$= \cos(180° -- \alpha_B)$$
$$\text{or } \quad \alpha_A = 180° -- \alpha_B$$
$$\text{or } (\alpha_A + \alpha_B) = 180°$$

The terminal voltages as a function of the firing angle for the two converters are shown in the figure below. A firing angle control circuit has to see that as the control voltage $\mathbf{E_c}$ changes the firing angles, $\boldsymbol{\alpha_A}$ and $\boldsymbol{\alpha_B}$ are to satisfy the above relation i.e. ***($\alpha_A + \alpha_B$) = 180°***

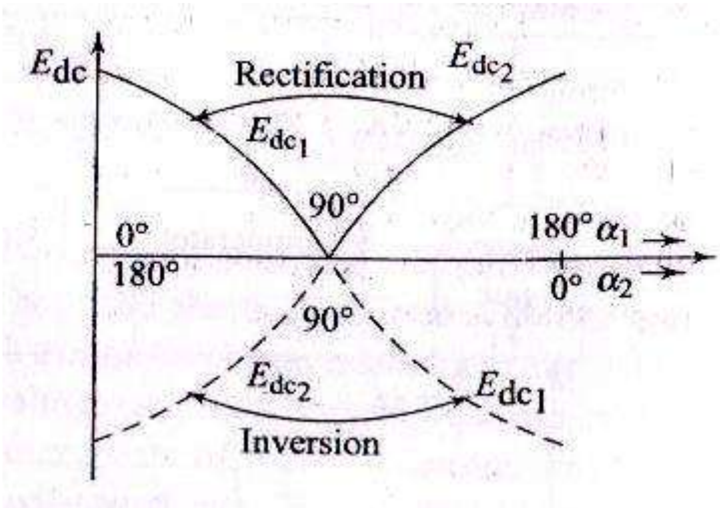


Fig: Firing angle versus Terminal voltage in Dual converter

Practical Dual Converters:

In the above explanation of the Dual Converter it is assumed that when the firing angle is controlled as per the above equation the output voltage is a pure DC voltage without any AC ripple. But in practical dual converters there will be AC ripple and hence the instantaneous voltages from the two converters will be different resulting in circulating current *which will not flow through the load.* If these are not limited they will damage the converters. Hence in order to avoid/limit such circulating currents two methods are adopted.

Method 1: Dual Converter without circulating current (or Non Simultaneous control): The block diagram of a Dual converter operating in this mode is shown in the figure below.

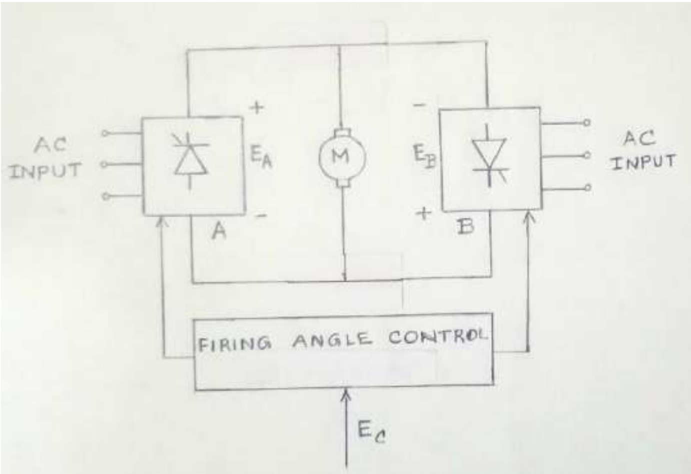


Figure: Block diagram of a Dual Converter without circulating current

In this mode the flow of circulating current is totally inhibited by controlling the firing Pulses such that only one converter which is required to conduct the load current is active at a time. The other converter is kept inactive by blocking its firing pulses. Since only one converter is operating and the other one is in blocking state, no reactor is required.

Suppose converter-A is operating supplying the load current in a given direction and the converter-B is blocked. If now direction is required to be changed, it is done in the following sequence.

1. Load current is first reduced to zero by setting the firing angle of Converter-A to the maximum value (α_A = 90^0.with this firing angle, the converter output voltage and thus output current become zero gradually)
2. The pulses to converter-A are withdrawn after confirming that the current through the load due to converter-A has completely come to zero by continuous current sensing.
3. Now converter-B is made operational by applying the firing pulses and it would build up the current through the load in the other direction. The pulses to Converter-B are applied only after a further gap of a few milli seconds to ensure reliable commutation of converter-A. For converter B also initially the firing angle α_A is set to 90^0 before applying the firing pulses

and then gradually it is brought to 0^0 so that the current build up takes place gradually.

The dead time and hence the reversal time can be reduced by going for accurate zero current sensing methods. When this is done non simultaneous control provides faster response than simultaneous control. Because of these advantages non simultaneous control is more widely used.

In this method at certain load conditions the load current may not be continuous which is not a desirable operating condition. To avoid this second method is used.

Method 2: Dual converter with circulating current:

In this mode Current limiting reactors are introduced between the DC terminals of the two converters as shown in the figure to allow the flow of circulating current due to the AC ripple/unequal voltages.

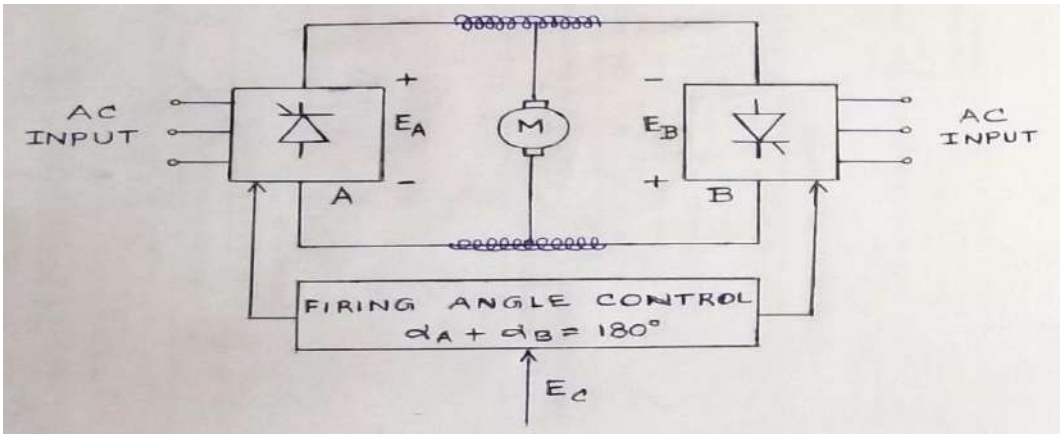


Fig: Dual Converter Control of a DC separately excited motor in simultaneous or Circulating current mode

Just like in an Ideal Dual converter the firing angles are adjusted such that

$$(\alpha_A + \alpha_B) = 180° \text{ -------------- } (1)$$

When operating in quadrant 1 Converter-A will be working as a rectifier ($0^0 \leq \alpha \leq 90^0$) and converter-B will be working as an Inverter. ($90^0 \leq \alpha \leq 180^0$) For e.g. firing angle of converter A is say 60°, then the firing angle of converter B will be 120°. With these firing angles, Converter A will be working as a converter and converter B will be working as an inverter. So, in circulating current mode both converters will be operating. The operation of the converters is to be interchanged for speed reversal i.e. converter1 which was working as a converter should now work as an Inverter and converter 2 which was working as an Inverter should now work as a converter. Two separate firing circuits have to be used for the two converters.

But for speed reversal α_A is to be increased gradually towards 180^0 and α_B is to be decreased gradually towards 0^0 while simultaneously satisfying the above condition (1)

In this process, the armature current reduces to zero, reverses direction , shifts to Converter B and the motor will now operate initially in quadrant 2 during braking and then in quadrant 3 during acceleration and finally at the required steady state speed. The current loop adjusts the firing angle α_B continuously so as to brake the motor at the maximum allowable current from initial speed to zero speed and then accelerates to the desired speed in the opposite direction. As α_B is changed α_A is also changed continuously so as to maintain the above relation-1. During this entire operation, the closed loop control system will ensure the smooth transfer from quadrant 1 to quadrant 2 to quadrant 3.

Advantages and Disadvantages of the Circulating current mode of Dual Converters:

Advantages:

(i) Over the whole control range, the circulating current keeps both converters in virtually continuous conduction, independent of whether the external load current is continuous or discontinuous.

(ii) The reversal of load-current is inherently a natural and smooth procedure due to the natural freedom provided in the power circuit for the load current to flow in either direction at anytime.

(iii) Since the converters are in continuous conduction, the time response of the scheme is very fast.

(iv) The current sensing is not required and the normal delay period of 10 to 20 ms as in the case of a circulating current free operation is eliminated.

(v) Linear transfer characteristics are obtained.

Disadvantages:

The circulating current scheme has the following main disadvantages:

(i) Since the current limiting reactor is required in this scheme, the size and cost of this reactor may be quite significant at high power levels.

(ii) Since the converters have to handle load as well as circulating currents, the thyristors with high current ratings are required for these converters.

(iii) The efficiency and power factor are low because of circulating current which increases losses.

In spite of these drawbacks a dual converter with circulating current mode is preferred if load current is to be reversed quite frequently and a fast response is desired in the four-quadrant operation of the dual converter.

Comparison between Circulating current mode and non circulating current mode Dual converters:

Non Circulating current Mode	Circulating current Mode
1. In this mode of operation, only one converter operates at a time and the second converter remains in a blocking state.	In this mode of operation, one converter operates as a rectifier and the other converter operates as an inverter.
2. Converters may operate in discontinuous current mode.	Converters operates in continuous current mode.
3. Reactors may be needed to make load-current continuous.	Reactors are needed to limit circulating current. These reactors are costly.
4. Since no circulating current flows through the converters, efficiency is higher.	Circulating current flows through the converters and hence increases the losses.
5. Due to discontinuous current, non-linear transfer characteristics are obtained.	Due to continuous current, linear transfer characteristics are obtained.

6.	Due to discontinuous current, response is sluggish.	Due to continuous-current in the converters, response is fast.
7.	Due to spurious firing, faults between converters results in dead short-circuit conditions.	Due to spurious firing, fault currents between converters are restricted by the reactor.
8.	In this mode of operation, the cross-over technique is complex.	In this mode of operation, the crossover technique is simple.
9.	Loss of control for 10 to 20 ms is observed in this mode of operation.	Since converters do not have to pass through blocking unlocking and safety intervals of 10 to 20 ms, hence control is never lost in this mode of operation.
10.	The control scheme needs command module to sense the change in polarity.	As both the converters are operating at the same time, the control scheme does not require command module.
11.	The complete scheme is cheaper compared to circulating current mode.	The complete scheme is expensive.
12.	In this mode of operation, the converter loading is the same as the output load.	In this mode of operation the converter loading is higher than the output load.

Closed loop control of Drives:

Closed loop control in Electrical drives is provided mainly to meet any or all of the following requirements.

- Protection against over current and over voltages
- Enhancement of Speed of response (Transient performance)
- Improve the steady state accuracy

We will study two important schemes of control that are most commonly used in electrical Drive control systems.

Current Limit Control:

Basic block diagram of a typical current limit control employed in electrical drives is shown in the figure below.

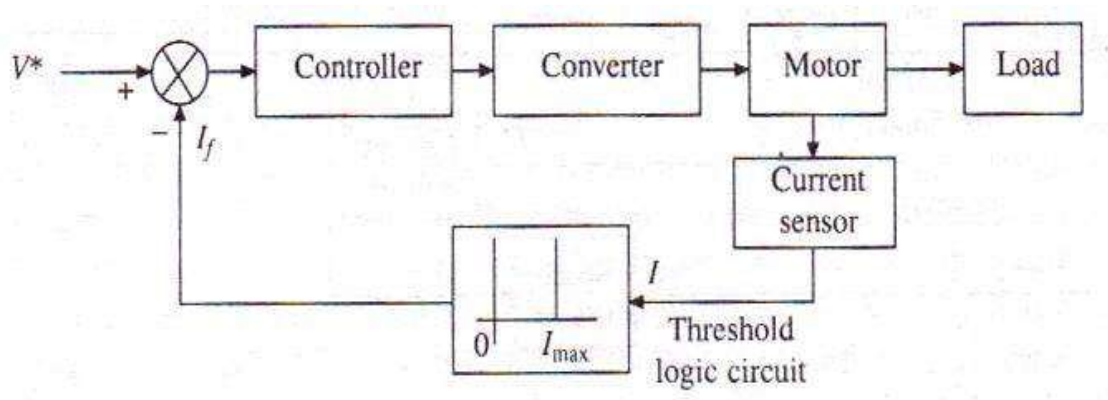


Fig: Current limit Control

This is employed mainly to limit the converter and motor currents to safe values during transient periods like starting and braking. It employs a current feedback loop with a threshold logic circuit. The motor current is sensed using sensors like CTs or Hall Effect sensors and fed to the threshold logic circuit. As long as the motor current is within the set maximum limit, the closed loop control does not come into operation. When the current exceeds the set limit the closed loop control becomes active and the current is brought below the set limit and the control loop becomes again inactive. Whenever current exceeds the limit the control loop becomes active again. Thus the current fluctuates around the maximum limit until the final steady state condition is reached thus ensuring faster response with maximum torque during the transient conditions i.e. starting and braking.

Closed loop Speed control:

The most widely used control loop in electrical drives is the "*closed loop Speed control*" and its Block schematic is shown in the figure below. It employs an inner current control loop within an outer speed control loop. Inner current control loop is provided to limit the converter and motor current (torque) below the safe limits. The speed control loop operates as follows:

$\boldsymbol{\omega^*_m}$ is the input speed reference and when it increases (or decreases) it produces a positive (or negative) speed error (+/-)$\boldsymbol{\Delta\omega_m}$. The speed error is processed through a speed controller and is applied to the current loop as current reference I* through the current limiter. Current limiter works linearly in a small range of error and saturates when the error exceed the set limits.

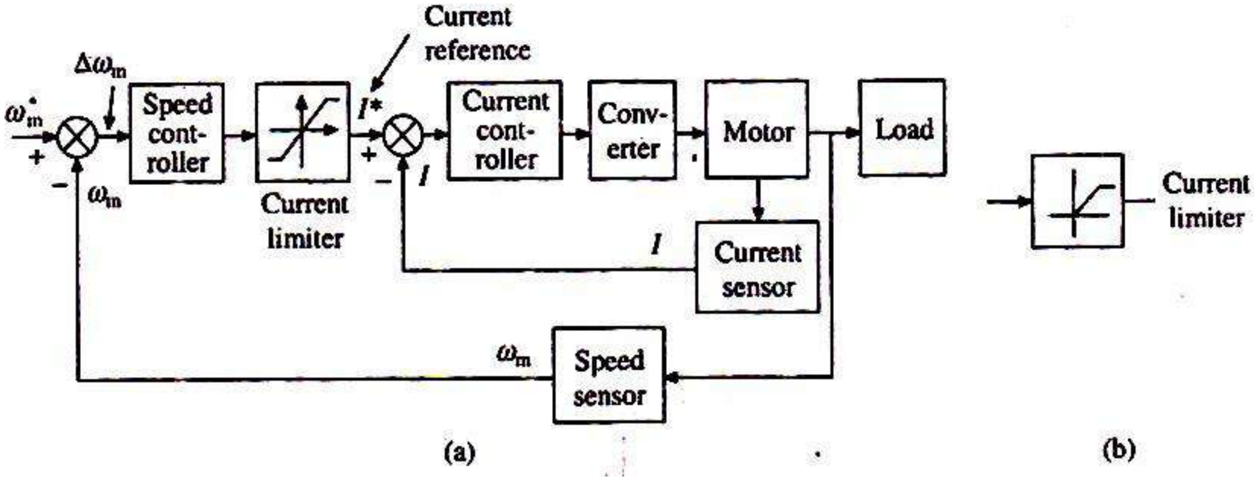


Fig: Closed loop speed control

An increase in the speed reference ω^*_m produces a positive speed error and the current limiter sets a positive maximum allowable input current as reference to the inner current loop. Now the motor accelerates at the maximum current and hence with the maximum torque until it approaches the set speed. When it is close to the set speed, the current limiter desaturates and the speed stabilises at the steady state value of speed with a small steady state error and a current corresponding to the motor torque equal to the load torque.

A decrease in the speed reference ω^*_m produces a negative speed error and the current limiter sets a negative maximum allowable input current as reference to the inner current loop. Now the motor decelerates and operates in braking mode at the maximum current and hence with the maximum torque until it approaches the set speed. When it is close to the set speed, the current limiter desaturates and the speed stabilises at the steady state value of speed with a small steady state error and a current corresponding to the motor torque equal to the load torque.

In drives where there is no provision for current to reverse (single quadrant operation)for braking operation current limiter will have the unipolar I/O characteristics as shown in fig (b).In drive systems where there is enough load torque for braking , electric braking is not required and in such cases also the unipolar current limiter will be used.

Current and speed controllers shown in the speed control loop normally consist of PI (Proportional plus Integral) or PD (Proportional plus Derivative) or PID (Proportional plus Integral plus derivative) controllers depending upon the steady-state accuracy and/or transient response requirements.

Summary:

Important concepts and conclusions:

- An electrical drive operates in three modes. i.e. steady state, starting and braking.
- Steady state operation is also referred to as motoring operation.
- Starting and braking are also referred to as transient operations.
- The three types of electrical braking are :
  - Regenerative braking
  - Dynamic or Rheostatic Braking and
  - Plugging or reverse voltage braking.
- Four quadrant operation can be achieved with a single Full converter along with changeover contactors to reverse the armature or field connections and with firing angle changeover control [($0° \leq \alpha \leq 90°$) or ($90° \leq \alpha \leq 180°$)]. But Dual converters are preferred due to their superior performance.
- In practical Dual converters with circulating current mode, reactors are required to be connected between the two Converter terminals to limit the circulating currents. The firing angles are to be controlled to satisfy the condition ***($\alpha_A + \alpha_B$) = 180°***
- In converters without circulating current only one converter is active at a given time depending on the operation.
- In both modes the closed loop control system takes care of the total control methodology.
- In closed loop speed control systems normally two control loops are used. An inner Current control loop and an outer Speed control loop.
- Current and Speed controllers in a closed loop speed control system normally consist of PI (Proportional plus Integral) or PD (Proportional plus Derivative) or PID (Proportional plus Integral plus derivative) controllers depending upon the steady-state accuracy and/or transient response requirements.

Illustrative Examples:

Example-1: A 210 V**,** 1200 RPM, 10 A separately excited DC motor is controlled by a 1- phase fully controlled converter with an AC source voltage of 230V, 50 Hz. Assume that sufficient inductance is present in the armature circuit to make the motor current continuous and ripple free for any torque greater than 25% of rated torque. R_a = 1.5 Ω

(a) What should be the value of the firing angle to get the rated torque at 800 rpm?
(b) Compute the firing angle for the rated braking torque at -1200 rpm.
(c) Calculate the motor-speed at the rated torque and $\alpha = 165^0$ for the regenerative braking in the second quadrant?

Solution:

This is an example where the constant K_a is not given directly and no mention is made about field current. So we can assume field Ø as constant and determine the motor constant K_aØ from the given rated values of voltage (210 V), speed (1200 rpm), armature current (10 A) and the armature resistance (1.5 Ω)

First we have to find out back emf E_b @rated conditions and from that the constant K_aØ.

$E_b = E_a – I_aR_a = 210 – (10×1.5) = 195$ V

Rated speed in R/s : $\omega = \frac{1200 \times 2\pi}{60}$ = 125.66 rad/s.

We know that $E_b = K_a Ø\omega$ from which we have: $K_a Ø = \frac{E_b}{\omega} = \frac{195}{125.66} = 1.55$ V/Rad/sec

Now we can find out the required quantities one by one.

(a) At rated torque, I_a = 10 A.

Back e.m.f. at 800 rpm = $E_{b1} = \frac{800}{1200} \times 195 = 130$ V

We know that with continuous conduction required armature voltage $E_a(\alpha)$ from the single phase full converter is given by:

$E_a(\alpha) = \frac{2E_m}{\pi}\cos\alpha = I_a R_a + E_b$ ($E_m = 230 \times \sqrt{2} = 325.27$ V)

$\frac{2\times 325.27}{\pi}\cos\alpha = (10\times 1.5) + 130$ From which we get $\alpha = 45.55^0$

(b) For the speed of – 1200 rpm at the rated current E_b = -195 V and the armature loop equation becomes $\frac{2\times 325.27}{\pi}\cos\alpha = (10 \times 1.5) - 195$

from which $\alpha = 150.37^0$

(c) Substituting the value of armature current I_a at rated torque = 10 A in the armature loop equation and equating it to the converter output at a firing angle of $\alpha = 165^0$, we get

$\frac{2\times 325.27}{\pi}\cos 165^0 = 10\times 1.5 + E_b$. or $E_b = -215.02$ V

Regenerative braking (second quadrant operation) is obtained either by the field reversal or the armature reversal, for which,

$K_a\emptyset = -1.55$

Then, $\omega = \frac{E_b}{K_a\emptyset} = \frac{-215.02}{-1.55}$ = 138.72 rad/s and N = 138.72 x 60/2π = 1325 rpm.

Example-2: The speed of a 10 HP, 210 V, 1000 rpm separately excited D.C. motor is controlled by a single-phase, full-converter. The rated motor armature current is 30 A, and the armature resistance is R_a = 0.25 Ω. The a.c. supply voltage is 230 V. The motor voltage constant is $K_a\Phi$ = 0.172 V/rpm. Assume that sufficient inductance is present in the armature circuit to make the motor current continuous and ripple free.

(a) Rectifier operation (motoring action): For a firing angle of $\alpha = 45^0$, and rated motor armature current, determine:

1) Motor torque 2) Motor speed

(b) Inverter operation (regenerative action): The motor back emf polarity is reversed by reversing the field excitation. Determine:

1) Firing angle to keep the motor current at its rated value.

2) Power fed back to the supply

Solution:

(a) Rectifier operation (motoring action):

(1) Motor Torque @rated armature current: can be found out directly by using the relation $T = K_a \Phi I_a$. But the constant ***K_a Φ is same in the relations for torque and back emf if it is V/Rad/sec in back emf and N-m/A in torque.*** But it is given in V/RPM. Hence it is first converted to V/Rad/sec and then used in the expression for torque.

$K_a \Phi$ (V/Rad/sec) = $K_a \Phi$ (V/RPM) x 60/2π

$$= \frac{0.172 \times 60}{2\pi} \text{ V-s/rad} = 1.64 \text{ V-s/rad.}$$

Rated Motor Torque T_R at rated armature current = $K_a \Phi I_{aR} = 1.64 \times 30 = 49.2$ N-m.

(2) Motor Speed at Rated armature current: The armature voltage in a fully controlled single phase converter is given by :

$$E_a = \frac{2E_m}{\pi} \cos\alpha = \frac{2\sqrt{2} \times 230}{\pi} \cos 45^0 = 146.42$$

(The given supply voltage of 230 V is RMS value and it is to be converted into E_m by multiplying by √ 2)

$$E_b = E_a - I_{aR} R_a = 146.42 - (30 \times 0.25) = 138.92 \text{ V}$$

Speed, $N = \frac{E_b}{K_a \phi} = \frac{138.92}{0.172} = 807.67\ rpm$ (here K_a Φ is used directly with the given units of V/RPM so that we can get directly speed N in RPM)

(b) Inverter operation (regenerative action):

(1) Firing angle to keep the motor current at its rated value:

At the time of polarity reversal, the back emf is E_b = 138.92 V(But it is to be taken as negative after polarity reversal for braking)

Then from loop equation $E_a = E_b + I_a R_a = -138.92 + (30 \times 0.25) = -131.42$ V.

But converter out is given by $E_a = \frac{2\sqrt{2}\times 230}{\pi} \cos\alpha$

Equating the converter output voltage to the required motor armature voltage we get: $E_a = \frac{2\sqrt{2}\times 230}{\pi} \cos\alpha = -131.42$ V from which we get $\alpha = 129.39^0$

(2) Power fed back to the supply:

Power from the D.C machine is : $P_g = 138.92 \times 30 = 4167.6$

Power lost in the armature resistance is: $P_L = I_a^2 R_a = 30^2 \times 0.25$

Power fed back to the A.C. supply is: $P_s = 4167.6 - 225 = 3942.6$ W

We can get directly also power fed back as: $P_s = E_a I_a = 131.42 \times 30 = 3942.6$ W.

Example-3: A 220 V, 1500 RPM, 50 A DC separately excited motor with an armature resistance of 0.5 Ω is fed from a Dual converter with 3 **Φ** fully controlled rectifiers. Input supply is a.c. line voltage of 165V. Determine converter firing angles for the following operations.

a) Motoring operation at rated motor torque and 1000 RPM in FW direction.
b) Braking operation at rated motor torque and 1000 RPM in FW direction.
c) Motoring operation at rated motor torque and 1000 RPM in REV direction.
d) Motoring operation at rated motor torque and 1000 RPM in REV direction.

Solution:

Given data: Motor rated Voltage $\mathbf{E_{aR}}$ = 220 V DC**;** Rated speed $\mathbf{N_R}$ = 1500 RPM; Rated Current $\mathbf{I_{aR}}$ = 50 A ; Armature resistance $\mathbf{R_a}$ = 0.5 Ω

Step-1: Find out the back e.m.f. E_b at the rated speed of 1500 RPM using the formula $E_{bR} = E_{aR} - I_{aR} R_a$

$E_{bR} = 220 - 50x0.5 = 195$ V

Step-2: To find out the firing angles corresponding to the required four conditions we have to

(i) First find out the Required armature voltage $\mathbf{E_a}$ = for each of the conditions using the formulae: (1) For Motoring: $\mathbf{E_a = E_b + I_a R_a}$ **(**2) For braking : $\mathbf{E_a = E_b - I_a R_a}$

(ii) And then equating these required armature voltages values to the converter output given by $\mathbf{E_a = [(3\sqrt{3}\, E_m)/\pi] \cos\alpha}$ (where E_m is the maximum value of the input **phase** voltage and $\boldsymbol{\alpha}$ is the firing angle) find out **cos** $\boldsymbol{\alpha}$ and thus finally $\boldsymbol{\alpha}$.

In this problem input voltage is given as 165 V **line** value. It is to be noted that this value is **r.m.s** value. So this to be converted into maximum Phase value.

Thus $\mathbf{E_m = (165/\sqrt{3})\sqrt{2} = 134.7}$

Now we can find out $\boldsymbol{\alpha}$ **for** the four cases:

It is also to be noted here that since in all the four cases we have to find out the firing angle at the rated torque only we need to take the rated current of 50 A only (Torque is proportional to Current)

a)Back e.m.f. $\mathbf{E_b}$ @ 1000 RPM = E_{bR} (1000/1500) = 195x (1000/1500) = 130 V

Then E_a @ 1000 RPM= 130 + 50x0.5 = 155 V

$[(3\sqrt{3} \times 134.7)/\pi] \cos\alpha = 155$ from which we get $\cos\alpha = 0.695$ and $\alpha = 45.9^0$.

And this is for converter A since this is forward motoring operation. Hence

$$\boldsymbol{\alpha_A = 45.9^0}.$$

Since we know that in dual converters while working in simultaneous control mode (Circulating current mode**)** $\boldsymbol{\alpha_A + \alpha_B = 180^0}$ we can get directly get

$$\boldsymbol{\alpha_B = 180^0 - 45.9^0 = 134.1^0}$$

b) For braking $\mathbf{E_b}$ has to be greater than $\mathbf{E_a}$ and hence $\mathbf{E_a = E_b - I_a R_a}$ = 130 – 50x0.5 = 105 V**.** Equating this to the Converter output we get

[(3√3 x 134.7) /π] cos α = 105 from which we get cos α = 0.470 and α = 61.9^0.

And this is for converter A since this is forward braking operation. Hence

$$\boldsymbol{\alpha_A = 61.9^0}$$

And on the same logic as earlier we get

$$\boldsymbol{\alpha_B = 180^0 - 61.9^0 = 118.1^0}$$

For negative speeds the roles of the converters A and B get interchanged between inverter and converter and hence the values of α_A and α_B get interchanged. Hence

c) $\boldsymbol{\alpha_A = 134.1^0}$

$\boldsymbol{\alpha_B = 180^0 - 134.1^0 = 45.9^0}$

d) $\boldsymbol{\alpha_A = 118.1^0}$

$\boldsymbol{\alpha_B = 180^0 - 118.1^0 = 61.9^0}$

Example-4: A 220V, 970 RPM, 100 A DC separately excited motor has an armature resistance of 0.05 Ω. It is braked by plugging from an initial speed of 1000 RPM. Calculate:

a) Resistance to be placed in the armature circuit to limit braking current to twice the full load value.
b) Braking torque
c) Torque when the speed has fallen to zero

Solution:

Given data: Rated armature voltage $\mathbf{E_{aR}}$ = 220 V ; Rated speed $\mathbf{N_R}$= 970 RPM ; Rated armature current $\mathbf{I_{aR}}$ = 100 A ; $\mathbf{R_A}$ = 0.05 Ω

With the given data we can first find out the back e.m.f at the rated speed of 960 RPM using the formula ($E_{bR} = E_{aR} - I_{aR}R_a$) and from that we can find out the back e.m.f at the required speed of 1000 RPM.

Back e.m.f at the rated speed of 970 RPM E_{bR} = 220 – 100 x 0.05 = 215 V

Back e.m.f at the required speed of 1000 RPM E_{b2} = (1000 x 215)/ 970 = 221.65 V

a) In plugging operation a braking resistance $\mathbf{R_b}$ is introduced into the armature circuit and the applied voltage is also reversed. Hence In the braked condition total loop voltage would be the sum of the reversed voltage and the motor back e.m.f. Then the braking resistance R_b would be given by:

$R_b + R_a$ = (Ea + Eb)/2I_aR (Since the braking is done at twice the rated current)

= (220 + 221.65) / 200 = 2.21 Ω

Hence additional braking resistance to be introduced R_b = 2.21 – 0.05 = 2.16 Ω

b) We know that mechanical power = τ x ω_m = Electrical power = E_b x I_a = 221.65x200 from which we get $\tau_{@1000\ RPM} = (E_b \times I_a)/\ \omega_m = (E_b \times I_a)/\ (N.2\pi)/60$ = (221.65x200)/(1000x2π/60) = 423.3 N-m

c) At zero speed E_b = 0 V. Therefore only applied reverse voltage will be there in the loop. Hence I_a is given by $I_a = E_a /(R_b+R_a)$ = 220 / 2.21 = 99.55 A

We know that Torque is proportional to armature current. Hence torque at zero speed is given by:

$\tau_{@zero\ speed} = [\tau_{@1000\ RPM} / I_{a@\ 1000\ RPM}\ ]\ I_{a@\ zero\ speed}\ ]$ = 423.3x99.55/200 = 210.7 N-m

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UNIT-III

CONTROL OF DC MOTORS BY CHOPPERS

SYLLABUS/CONTENTS:

- Single quadrant, Two quadrant and Four quadrant chopper fed DC Separately excited and Series excited motors
- Continuous current operation: Output voltage and current wave forms
- Speed torque expressions
- Speed torque characteristics
- Closed Loop operation (Block Diagram Only)
- Summary
    - Important concepts and conclusions
    - Important formulae and equations
- Illustrative Examples

Introduction to Choppers:

Choppers are mainly used to obtain a variable DC output voltage from Fixed DC voltage source. There are two basic types of choppers: AC link choppers and DC choppers.

AC link Choppers: In these, first DC is converted to AC by inverters. Then AC is stepped up or down by transformers to the required level and then it is converted back to DC.

DC choppers: In these a variable DC voltage is obtained from a fixed DC voltage using a static switch.

In this unit we will study the application of DC choppers in the four quadrant operation of DC motors.

Basic DC chopper classification:

1. According to the level of input/output voltages:

    - Step down choppers: Output voltage is lesser than the input voltage
    - Step up choppers: Output voltage is larger than the Input voltage

2. According to the Direction of output voltage and current as shown in the Figure below (as class A to E)

3. According to quadrants of operation: (As shown in the figures above.)

- One quadrant chopper:
    - The output voltage and current are both positive. (class- A)
    - The output voltage is positive but current is negative. (class- B)
- Two quadrant chopper:
    - The output voltage is positive but current can be positive or negative. (Class-C)
    - The output current is positive but voltage can be positive or negative. (Class-D)

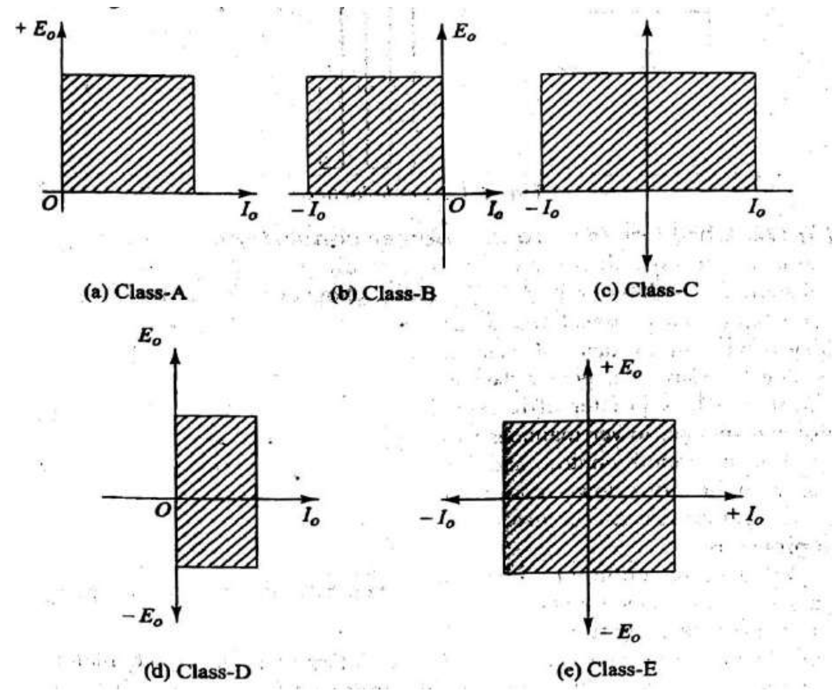


Fig: Classification of DC choppers

- Four quadrant chopper: The output voltage and current both can be positive or negative.

Basic principle of operation of a step down chopper:

A step down chopper consists of a semiconductor device like SCR, BJT, Power MOSFET, IGBT, GTO etc which works like switch along with a DC input source and other components like Inductors, Resistors, Capacitors, Diodes etc. as

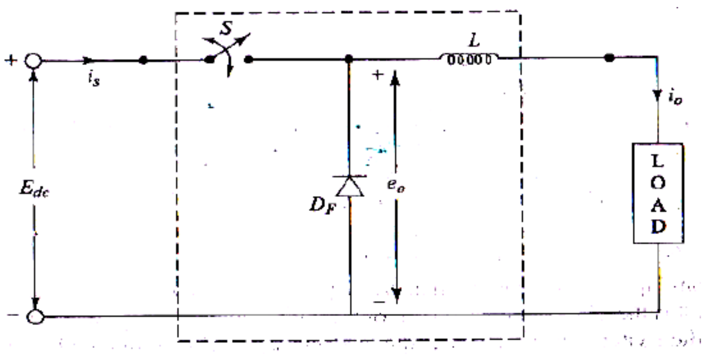


Fig: Basic Chopper circuit

shown in the figure below. The average output voltage across the load is varied by varying the **ON** period (duty cycle) of the chopper with a given Time period.

For SCR based choppers an additional commutation circuit is necessary. Hence in general, gate commutation devices like MOSFETs and IGBTs have replaced the SCRs in Choppers. However for high voltage and high current applications SCRs are still used. The power diode D_F operates in freewheeling mode and provides a path to the load current when the switch is not **ON.** The Inductor works as a filter and smoothes out the switching ripple. The chopped output voltage waveform and the load current are shown in the figure below.

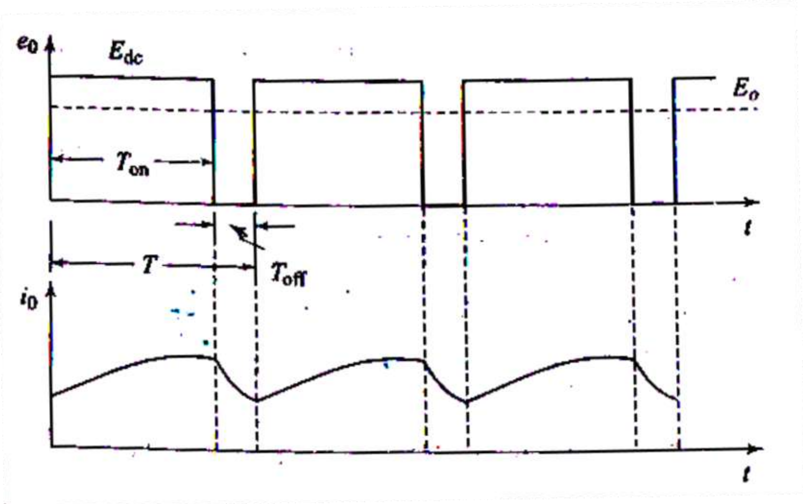


Fig: DC Chopper output voltage and current waveforms

During the **ON** period of the chopper the input voltage gets applied to the load. During the **OFF** period the load gets short circuited by the freewheeling Diode D_F and the load current flows through D_F. Thus a chopped voltage is produced across the load. *This is also called **Time Ratio control.***

The average output voltage is given by :

$$E_O = E_{DC}.T_{ON}/(T_{OFF} + T_{ON}) = E_{DC}.T_{ON}/T$$

Where T_{ON} = **ON** period of the chopper

T_{OFF} = **OFF** period of the chopper and $T = T_{OFF} + T_{ON}$ = Chopping period.

T_{ON}/T is called ***duty ratio*** of the chopper and is represented by the symbol **δ**.

Then the output voltage E_O is given by: $E_O = \delta . E_{DC}$

The output voltage E_O is also given by: $E_O = E_{DC} . T_{ON} . f$ where **f** is the chopping frequency and is equal to **1/T**

The average value of the load current is given by: $I_O = E_O / R = \delta . E_{DC}/R$

Types of chopper control:

- If the chopper is Transistor based , the base current will switch ON and OFF the transistor.
- If it is GTO thyristor based then a positive gate pulse will switch it ON and a negative gate pulse will switch it OFF
- If it is SCR based a commutation circuit is required to turn it OFF.

Class-A Chopper (First Quadrant operation):

The basic power circuit of a Class-A chopper connected to a separately excited motor operating in the first quadrant is shown in the figure below.

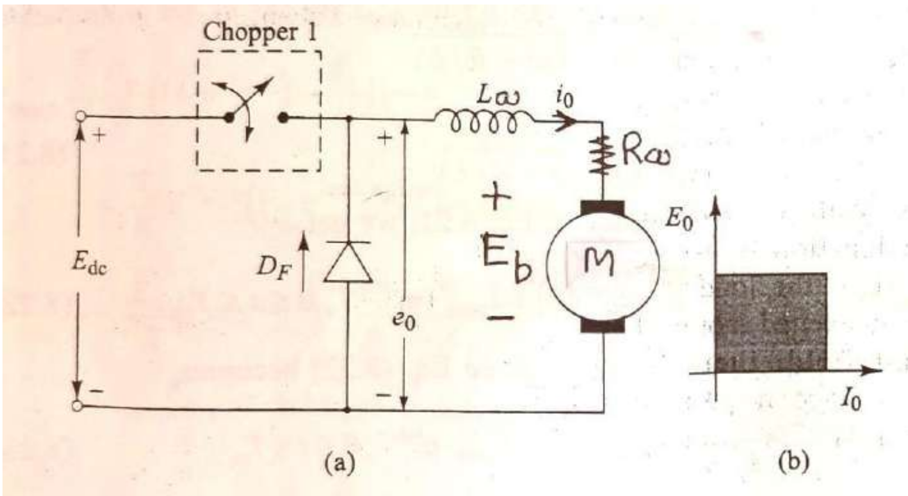
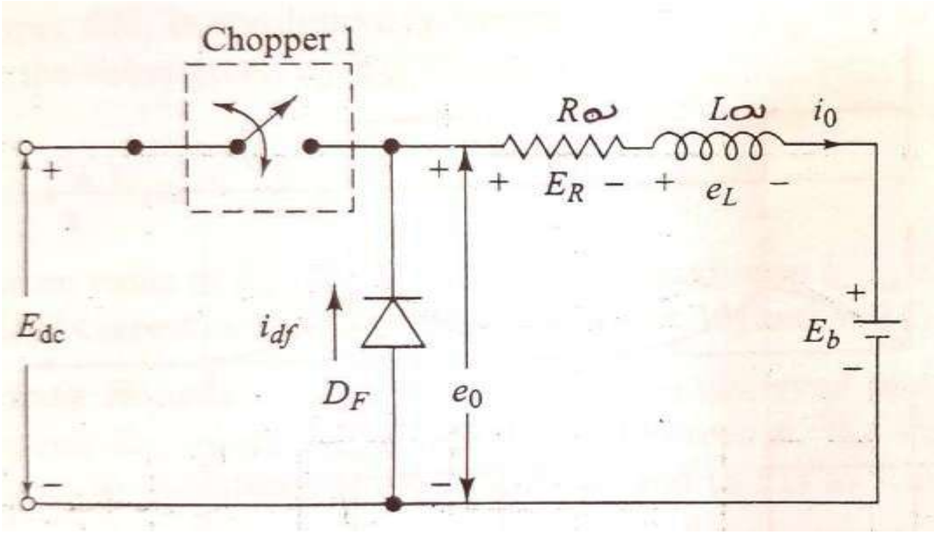


Fig: First quadrant operation of a Class-A chopper connected to a DC separately Excited motor

The term first quadrant refers to the operation with both voltage E_0 and current I_0 polarities confined to the directions as shown. When the chopper is ON the output voltage $E_O = E_{DC}$ and when the chopper is OFF $E_O = 0$ volts but the current I_O flows in the load in the same direction through the freewheeling diode D_F.



Electrical equivalent circuit of Class-A chopper (polarities indicated are when Chopper is ON)

Both average load voltage and load current are positive and hence power flows from source to load. ***Hence this is Motoring operation***. The output voltage and current waveforms are shown in the figure below.

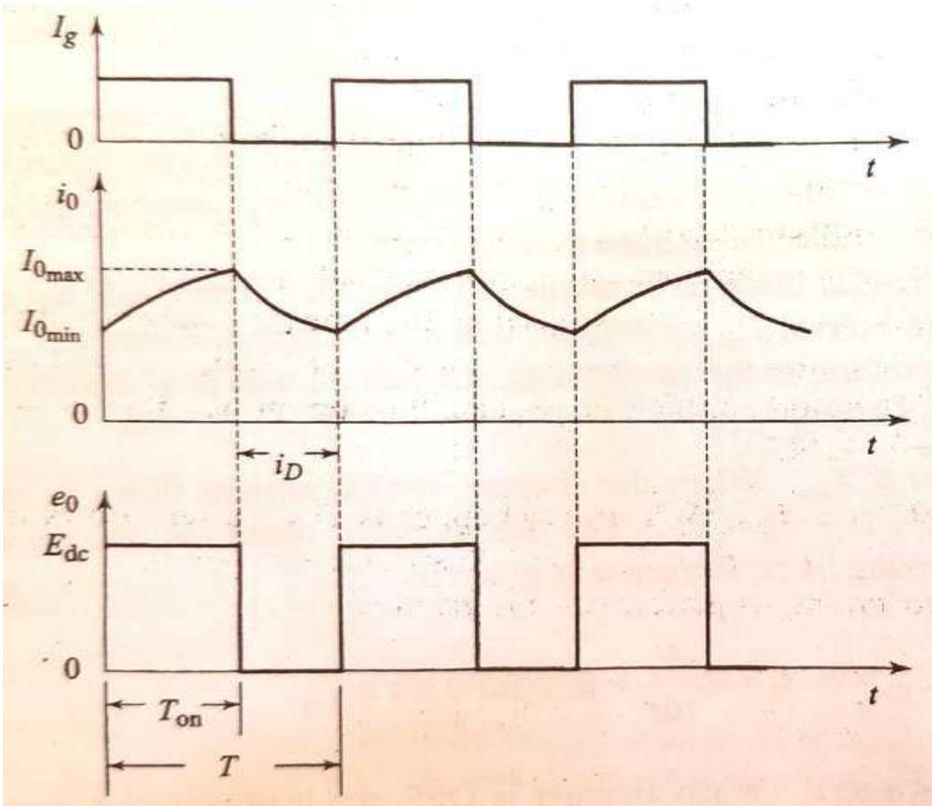


Fig: Class-A Chopper gate current, armature Voltage and current waveforms with continuous load current

During the ON period the rate of rise of current is positive and hence the voltage across the Inductance will be positive and the governing relation will be:

$$\mathbf{E_{DC} = R_a.i_0 + L.di_0/dt + E_b \qquad \text{for } 0< t < T_{on}}$$

During the OFF period rate of rise of current is negative and hence the voltage across the Inductance will be negative and the governing relation will be

$$\mathbf{0 = R_a.i_0 + L.di_0/dt + E_b \qquad \text{for } T_{on} < t < T}$$

The average output voltage $\mathbf{E_O}$ is given by $\mathbf{E_O = E_{DC}.\delta}$ where **δ = duty ratio = T_{on}/T**. The torque speed relation is identical to those we have seen earlier with single/three phase converters connected to DC SE motors and it is given by:

$$\mathbf{\omega_m = (E_{DC}.\delta/K_a.\phi) \text{ -- } (R_a.T)/(K_a.\Phi)^2}$$

Class-B Chopper (Second Quadrant operation):

The basic power circuit of a chopper connected to a DC separately excited motor operating in the second quadrant is shown in the figure below. The term second quadrant refers to the operation with both voltage E_0 and current I_0 polarities confined to the directions as shown.

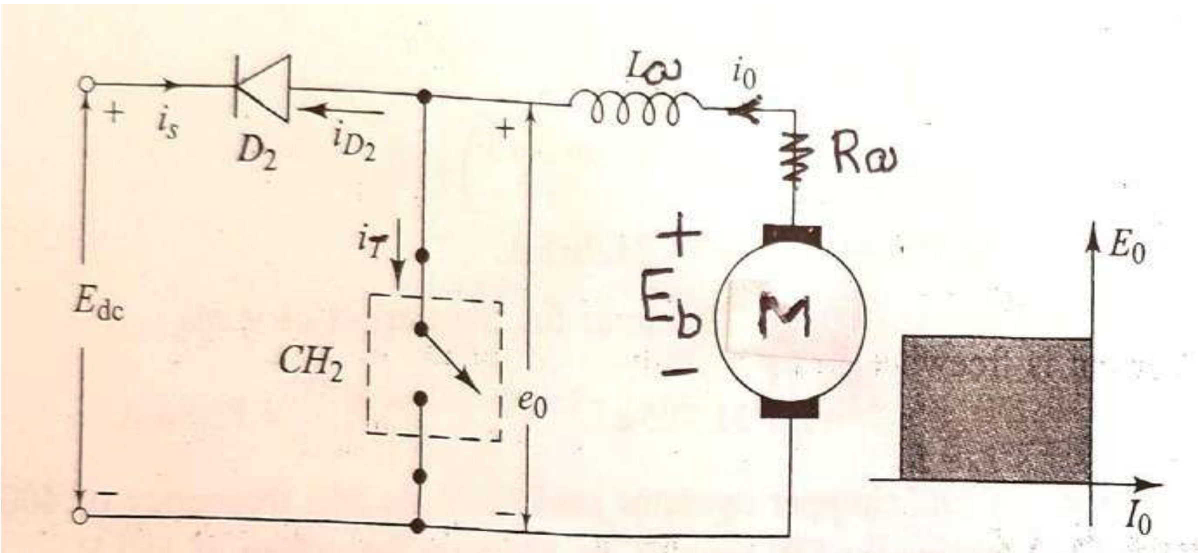


Fig: Second quadrant operation of a Class-A chopper connected to a DC separately Excited motor

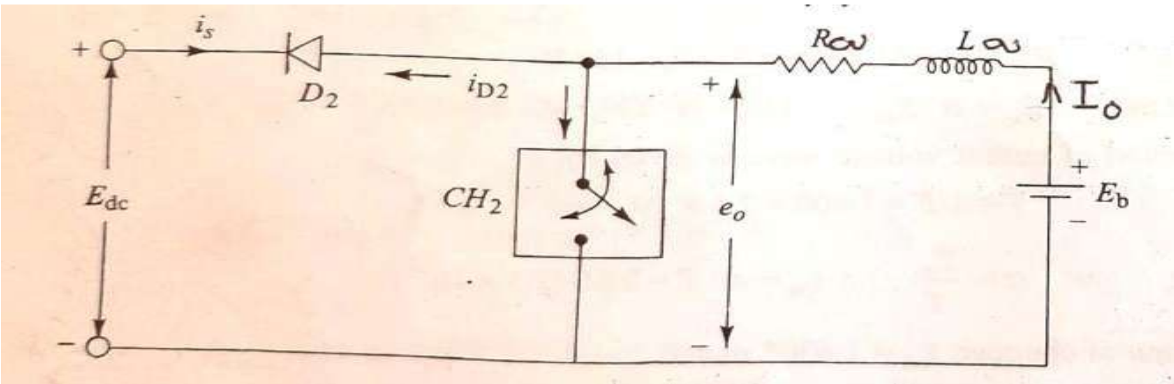


Fig: Second quadrant operation of a Class-A chopper connected to a DC separately Excited motor along with its equivalent circuit.

Chopper is turned ON and OFF at regular intervals of period T. The back emf E_b stores energy in the inductance L whenever the chopper is ON and this stored energy is delivered to the source E_{DC} by flow of current through the diode D_2 and in the same direction through the motor as it was flowing when the chopper was ON. In this, the average load voltage is positive and load current is negative. Hence power flows from load to source. Hence this is ***regenerative braking operation.*** The output voltage and current waveforms are shown in the figure below.

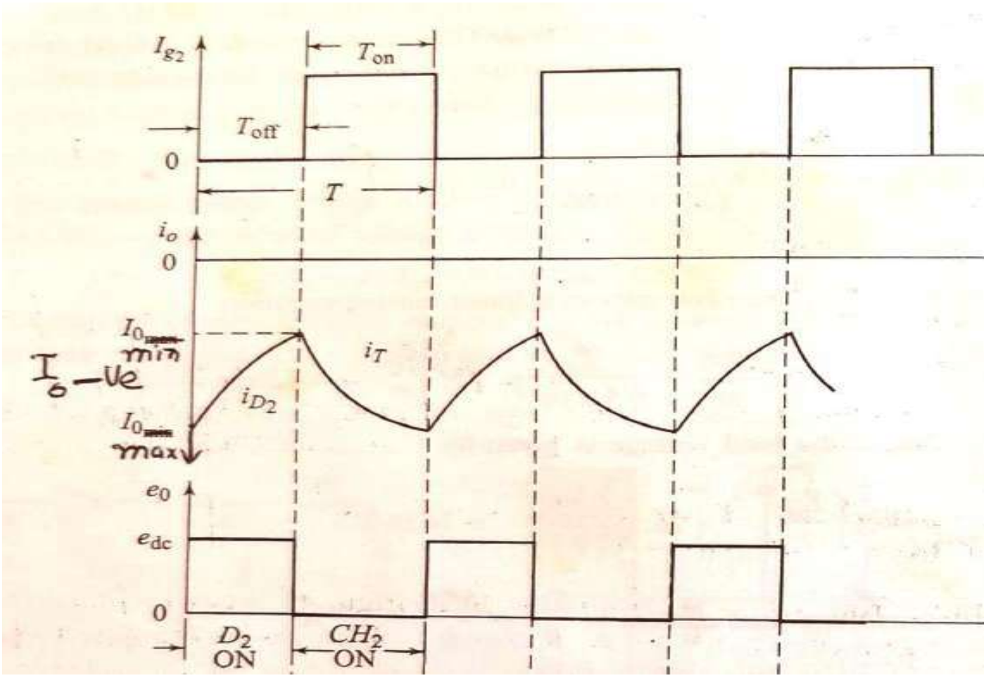


Fig: Class-B Chopper Voltage and current waveforms with continuous load current

Chopper control of series motor:

Motoring:

Chopper circuit and the waveforms are same as those of a Class - A chopper connected to a DC separately excited motor. Here also $\mathbf{E_O = E_{DC}.\delta}$ but $\mathbf{E_b}$ will not be constant and varies with i_o. Due to saturation of the field magnetic circuit, relationship between E_b and I_o is non linear. However the basic motor relations we have derived earlier for the series motor are still applicable and are given here again for quick reference.

Since $\mathbf{\Phi = K_f.I_a}$

$$E_b = K_a.\ \Phi.\ \omega = K_a.\ K_f.I_a.\ \omega$$

$$T = K_a.\ \Phi.I_a = K_a.\ K_f.I_a^2$$

$$E_a = E_b + I_a.R_a \quad \text{and}$$

$$\omega = E_o / K_a.\ K_f.I_a - (R_a / K_a.\ K_f)$$

$$\omega = [E_o/\sqrt{(K_{af}.T)}] - [R_a/(K_{af})]$$

Where R_a is now the sum of armature and field winding resistances and $\mathbf{K_{af} = K_a.K_f}$ is the total motor constant. Using these equations the torque speed relation for a choppers controlled DC series motor would become

$$\omega = [E_{DC}.\delta / \sqrt{(K_{af}.T)}] - [R_a/(K_{af})]$$

Regenerative braking:

For series motor also for regenerative braking the same Class-B chopper that was used for a DC separately excited motor is used. During regenerative braking, series motor works like a self excited series generator. Bur for self excitation, the current flowing through the field winding should assist the residual magnetism (as already explained during the braking of series motor).Therefore, when changing from motoring to braking connection, while direction of armature current should reverse, field current should flow in the same direction. This is achieved by reversing the field with respect to armature when changing from motoring to braking operation. Voltage and current waveforms will be same as those shown for regenerative braking of a DC separately excited motor.

The governing equations during braking are:

$$\mathbf{E_0= E_{DC}.\delta}$$

$$\mathbf{E_b = E_o+ I_a.R_a}$$

$$\mathbf{\omega = E_o/ K_a. K_f.I_a + (R_a/ K_a. K_f)}$$

$$\mathbf{\omega \ = *E_{DC}.\delta /\surd(K_{af}.T)]+ [R_a/(K_{af})]}$$

For a chosen value of I_a. K_f is obtained from the magnetisation characteristic. Then, **ω** and T are obtained from the above equations. The nature of torque speed characteristics is shown in the figure below.

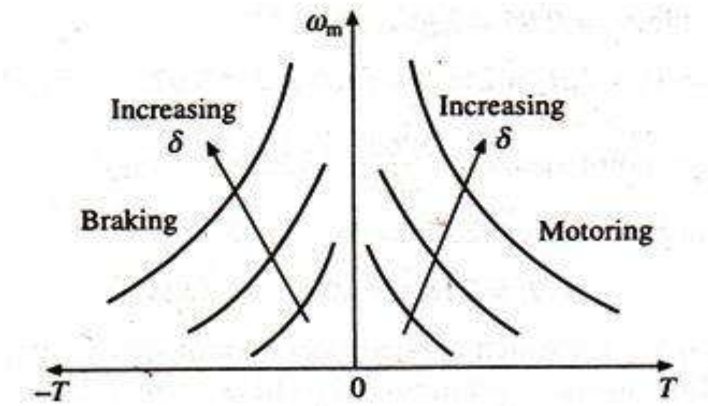
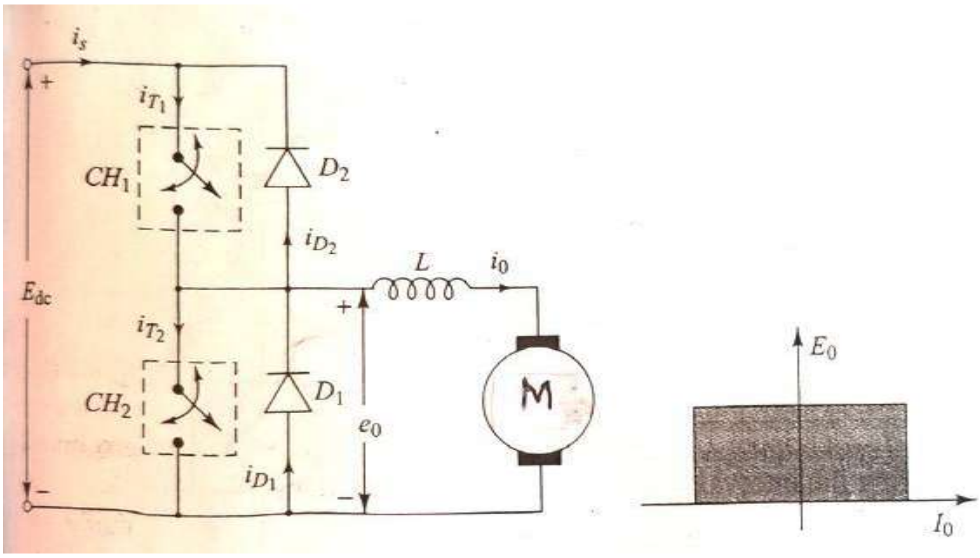


Fig: Motoring and Regenerative Braking characteristics of a Chopper controlled DC series motor.

Two quadrant (type –A) or class-C chopper:

Class-C chopper can be realised by combining the class-A and class-B choppers together as shown in the figure below. This combined circuit provides both forward motoring and forward regenerative braking. CH1 along with diode D1 performs forward motoring operation while CH2 along with diode D2 performs the function of forward regenerative braking. Thus for motoring operation CH1 is controlled and for braking operation CH2 is controlled. Shifting of control from CH1 to CH2 shifts operation from motoring to braking and vice versa.



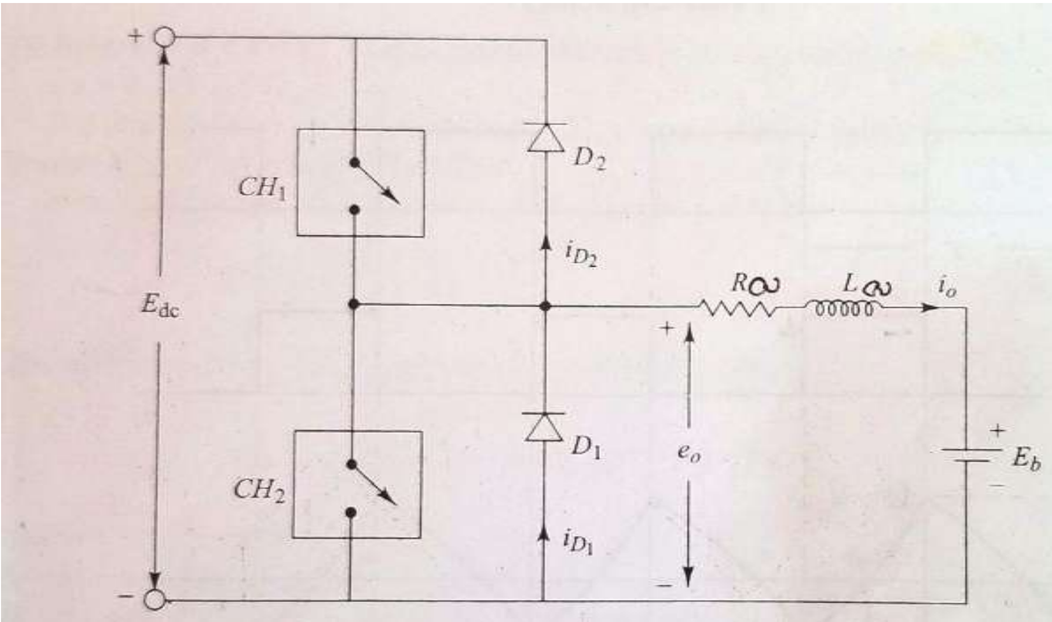


Fig: Two quadrant Type-A (class- C) Chopper, the permissible E-I coordinates and Electrical equivalent circuit.

But in many applications a smooth and fast changeover from motoring to braking and vice versa is required and in such cases Ch1 and Ch2 are controlled simultaneously as explained below with the help of the Motor terminal voltage and the current waveforms shown in the figure below.

Important points to be noted/conventions followed in this explanation are:

- With the given polarity of E_{DC}, the motor current is positive when flowing down wards (during motoring) and negative when flowing upwards (during braking).
- Since we are considering two quadrant operation with forward motoring & braking, the polarity of E_b is considered positive as shown.
- The choppers conduct in the direction as shown by the arrow in the respective chopper when triggered and only when forward biased.
- The voltage across the inductance is positive (terminal **Ra** side of **L_a** is positive as shown in eq.circuit) and adds up to the motor back emf E_b when the rate of rise of current is positive. And this happens when Ch-1 is ON or when diode D2 is conducting.
- The voltage across the inductance is negative (terminal **Ra** side of **L_a** is negative as shown in eq.circuit) and opposes the motor back emf E_b when the rate of rise of current is negative. And this happens when Ch-2 is ON or when diode D1 is conducting.

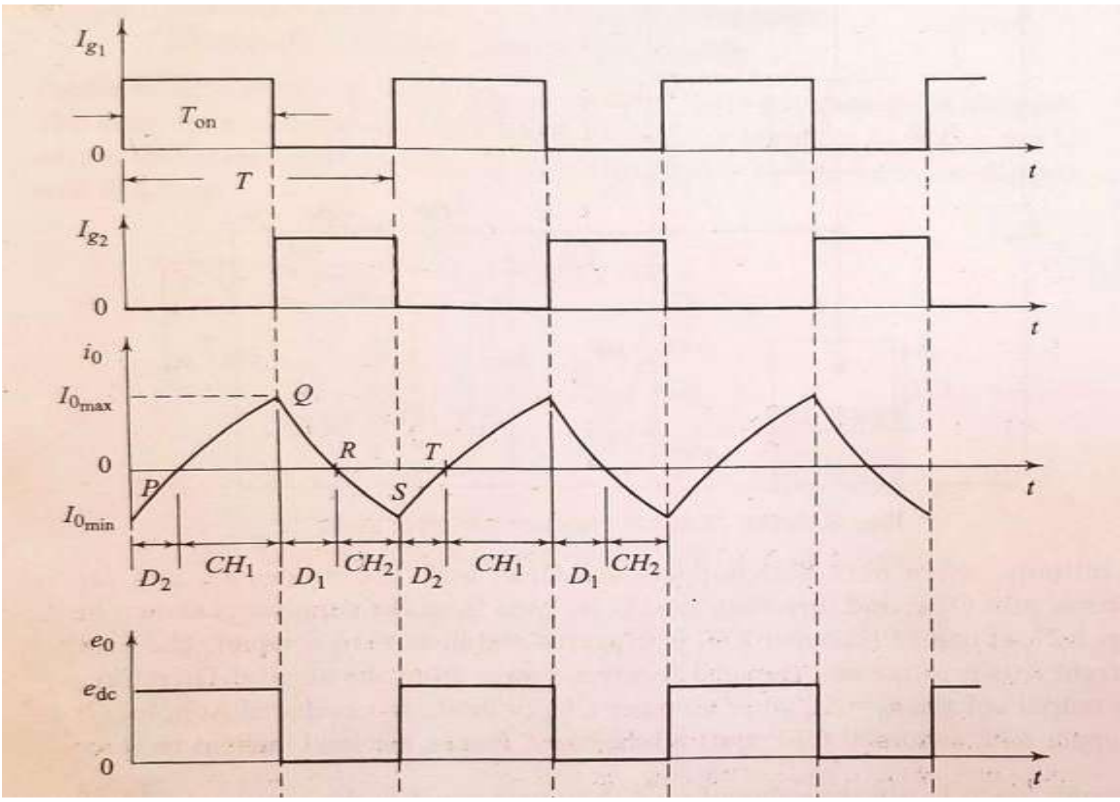


Fig: Voltage and current waveforms in a Class-C chopper

Operation:

- Initially when both choppers are OFF, both diodes also are not conducting and hence the load is isolated from the source. As shown in the waveforms above, say initially at point P chopperCh1 is triggered and it starts conducting. The load current is positive and the load receives power from the source. So the output voltage $e_o = E_{DC}$ whenever chopper Ch1 conducts.
- At point Q chopper Ch1 is turned off, polarity of voltage across inductance L_a changes (becomes negative) and the energy in the inductance forces load current to flow through the diode D1 (in the same direction through the motor i.e. positive) till the voltage across the inductance L.di/dt becomes equal to the back emf E_b and the load current becomes zero i.e. up to point R.
- At this point R, the motor back emf E_b is greater than the voltage across the inductance and since the gate signal for Ch2 is present, now E_b forces a current in the opposite direction (negative current) through La and Ch2.This continues up to point S i.e. until Ch2 is turned off and Ch1 is turned on.
- Now at point S when Ch2 is turned off, polarity of voltage across inductance L_a changes (becomes positive) and the energy in the inductance forces same negative current through the diode D2 into the source until point T when the input current reduces to zero. In this period the current is negative and hence Ch1 cannot conduct though it is triggered.
- At this point T since gate signal is available to Ch1 load current becomes positive, conducts through Ch-1 and the sequence repeats.

Summary observations:

- In a period **T,** Ch1 is switched on from **0 to δ .T** and Ch2 is switched on from **δ.T to T** where **δ** is the duty ratio of Ch1.Therefore during the period **0 to δ .T** motor is connected to the source through Ch1 or D2 depending upon whether the motor current is positive (Ch1) or negative (D2).

- Similarly during the period **δ.T to T** motor armature is shorted through Ch2 or D1 depending upon whether the motor current is negative (Ch2) or positive (D1). And during this period the rate of change of current is always negative.
- For first quadrant operation i.e. motoring, torque has to be positive, so motor current has to be positive and thus Ch1 and D1 perform the motoring.
- For the second quadrant operation i.e. braking , torque has to be negative, so motor current has to be negative and thus Ch2 and D2 perform the braking.
- Load voltage is zero if either Ch2 or diode D1 conducts and equal to E_{DC} if Ch1 or D2 conducts. So average output voltage is always positive.
- Load current is positive whenever Ch1 or diode D1 conduct and negative when Ch2 or diode D2 conducts.
- Load voltage is positive but current is reversible and hence power flow is also reversible.
- Both Ch1 and Ch2 should not be switched on simultaneously as it would short circuit the source voltage E_{DC}. They are turned on alternatively as shown by the gate signals I_{g1} and I_{g2}. i.e I_{g1} and I_{g2} are complementary.

Torque- Speed Characteristics:

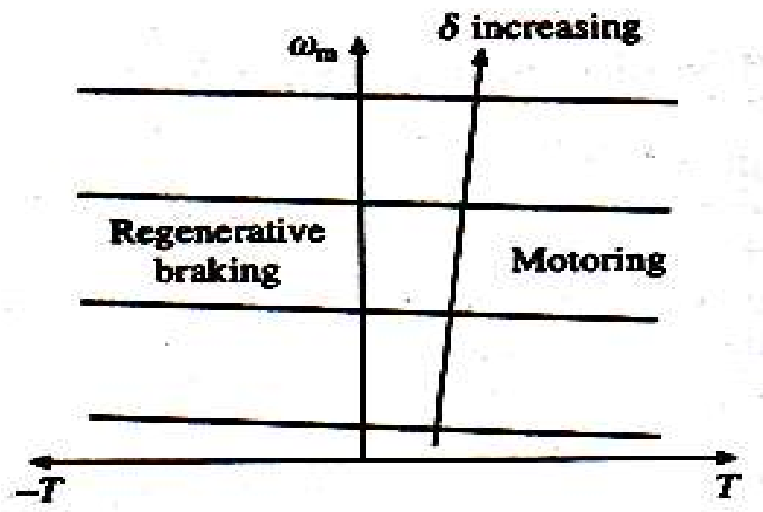


Fig: Torque speed characteristics of a Class-C chopper controlled DC separately excited motor.

Four Quadrant or Class-E Chopper:

The circuit diagram of a four quadrant or class-E chopper is shown in the figure below. It can be considered to be consisting of either two Class-C or Class-D choppers together as shown. With this type of chopper, motor direction of rotation can be changed without changing the field excitation direction and both motoring and braking operations in both directions can also be obtained by controlling the choppers 1 to 4 as explained below.

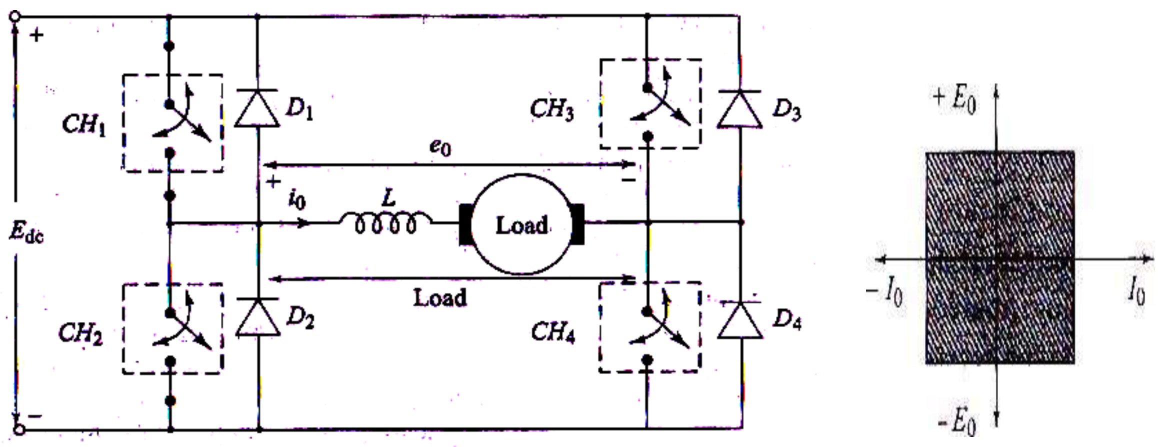


Fig: Four quadrant or class-E chopper circuit Diagram and characteristic.

With Ch-4 continuously ON and Ch-3 continuously OFF the chopper can be considered to be a Class-C chopper. Controlling choppers 1&2 will make E_o positive and motor current reversible thus operating in first and second quadrants. Similarly with Ch-2 continuously ON and Ch-1 continuously OFF, controlling Ch-3 and Ch-4 will make E_o negative and motor current reversible thus operating in third and fourth quadrants.

The operation of a Four Quadrant chopper is explained below:

When choppers Ch1 and CH4 are turned ON, current flows through the path: E_{DC+}, Ch1, load, Ch4, E_{DC--}. Since both E_O and I_O are positive we get First Quadrant operation. When both the choppers Ch1 and Ch4 are turned OFF, load dissipates its' energy through the path: Load, D3, E_{DC+}, E_{DC--}D2, Load. In this case E_O is negative while I_O is positive and we get fourth Quadrant operation. When choppers Ch2 and Ch3 are turned ON current flows through the path: E_{DC+}, Ch3, load, Ch2, E_{DC--}.Since both E_O and I_O are negative we get

Third Quadrant operation. When both the choppers Ch2 and Ch3 are turned OFF, load dissipates its' energy through the path: Load, D1, E_{DC+}, E_{DC--}, D4, Load. In this case E_O is positive while I_O is negative and we get Second Quadrant operation.

Four quadrant chopper circuit consists of two bridges, Forward Bridge and Reverse Bridge. Chopper Bridge Ch1 to Ch4 is the forward bridge which permits flow of energy from source to load. Diode Bridge D1 to D4 is the Reverse Bridge which permits flow of energy from load to source. This four-Quadrant Chopper configuration can be used for a reversible regenerative DC drive.

Summary:

Important concepts and conclusions:

- Choppers are classified as single quadrant (Class-A&B), two quadrant (Class-C&D) and four quadrant (Class-E) depending on the quadrants of operation.
- They are also classified as step-down and step-up choppers defending on whether the output voltage is lesser than or greater than the input voltage.
- The duty ratio of a chopper is given by **δ = duty ratio = T_{on}/T** where **T_{on}** is the **ON** time and **T** is total time period.
- Choppers conduct in only one direction i.e. when they are forward biased and also when they are triggered to start.
- The voltage across the Armature inductance is positive and adds up to the motor back emf **E_b** when the rate of rise of current is positive.
- The voltage across the Armature inductance is negative and opposes the motor back emf **E_b** when the rate of rise of current is negative

Important formulae and equations:

- The output voltage E_O of a chopper is given by: **$E_O = \delta.E_{DC}$**
- The output voltage E_O is also given by: **$E_O = E_{DC}.T_{ON}.f$** where **f** is the chopping frequency and is equal to **1/T.**
- The average value of the load current is given by: **$I_O = E_O / R = \delta.E_{DC}/R$**

Illustrative Examples:

Example-1: A 220 V, 24A, 1000 RPM, DC separately excited motor has an armature resistance of 2 Ω. The motor is controlled by a Chopper with a frequency of 500 Hz from a supply of 230 V. Calculate the duty ratio **δ** for 1.2 times the rated Torque and 500 RPM.

Solution: Given Data: E_a = 220 V, I_a = 24 A, N = 1000 RPM, R_a = 2 Ω E_s = 230 V

*(Note: Since the rated voltage of the motor is 220 V which is less than the supply voltage, even for normal operation we have to work with a duty ratio **δ** of 220 / 230 = 0.956 to ensure that never the applied voltage to the motor exceeds 220 V)*

First let us find out E_b for 1000 RPM using the relation $E_a = E_b + I_a R_a$ i.e. $220 = E_b + 24 \times 2$

From which we get E_b = 220 – 48 = 172 V for a speed of 1000 RPM

Hence for a speed of 500 RPM E_b = (500/1000) x 172 = 86 V

Then the required voltage to be applied to the armature is given by:

$E_a = (E_{b\ @\ 500RPM} + I_{a\ @\ 1.2\ \text{times rated torque}} \times 2) = (86 + 24 \times 1.2 \times 2) = 143.6$ V

[Here it is to be noted that the rated current of 24 A is to be multiplied by 1.2 since the motor is now working with a load torque which is 1.2 times the rated torque]

Required $\mathbf{E_a}$ for 500 RPM @ 1.2 times rated Torque = 143.6 V

*Required duty ratio **δ** = 143.6/230 = 0.624*

Example-2: A DC separately excited motor with an armature resistance of 0.08 Ω is powered by a chopper from a power source of 450V DC. The armature and field currents are 275 A and 3 A respectively. The armature current is continuous and ripple free. Back EMF constant of the motor is K_t = 1.527 V/A.rad/sec. If the duty ratio of the converter is 65% determine:

(i) Input power from the DC source (ii) Speed of the motor and Torque

Solution: Given Data**:** E_s = 450 V, I_a = 275A, I_f = 3A, R_a = 0.08 Ω, δ = 0.65, K_t= 1.527 V/A.rad/sec

(i) Input power is given by the product of the duty ratio, supply voltage armature current.

Thus IP Power = δ x E_s x I_a = 0.65 x 450 x 275 = 80.43 kW

(ii) To find out the speed and the torque we must first be clear of the units of the given motor constant.

We know that motor back EMF and Torque are given by the formulae:

(1) $E_b = K_a\varphi\omega$ and (2) $T= K_a\varphi I_a$ where the units of K_a are (1) Vots/Web.Rad/sec or (2) N-m/ Web.Amp.

*But here the units are given as V/**A**.rad/sec. i.e. Assuming the field magnetization to be linear, constant K_f in the relation $\varphi = K_f.I_f$ is combined with K_a in the above relations for Back EMF and Torque thus making them: $E_b = K_a K_f I_f \omega$ and $T= K_a K_f I_f I_a$ or $E_b = K_{af} I_f \omega$ and $T= K_{af} I_f I_a$ where $\boldsymbol{K_{af}}$ is the combined constant of motor including field magnetization and its units are given by:*

In case of Back EMF: V/A.rad/sec where A (Amperes) refers to the field current and

In case of Torque : V/A.A where first 'A' refers to the field current and second 'A' refers to the armature current.

In this problem from the units of the given constant Kt we have to take it as the combined constant K_{af} of the motor including field magnetization.

Now we can directly find out the torque by using the equation: $T= K_t I_f I_a$ where $\mathbf{K_t}$ is nothing but $\mathbf{K_{af}}$ as explained above and **T** = 1.527 x 3 x 275 = **1259.7 N-m**

To find out the speed we have to find out the back EMF corresponding to the applied armature voltage with a duty ratio of 0.65 using the standard relation $E_a = \delta E_s = E_b + I_a R_a$ form which we get : $\mathbf{E_b} = \delta E_s - I_a R_a$ = 0.65 x 450 – 275 x 0.08 = 292.5 – 22 = **270.5 V**

Now from the relation $E_b = K_{af} I_f \omega$ we get $\omega = E_b / K_{af} I_f$ = 270.5/ (1.527 x 3) = 59.04 Rad/sec and **Speed in RPM** = (59.04 /2π) 60 = **564 RPM**

Example-3: A DC separately excited motor with an armature resistance of 2 Ω is powered by a chopper from a power source of 220V DC. The chopper is working with an ON time of 15 msec and OFF time of 10 msec. The motor constant K_m = 0.4 V/Rad/sec. Assuming continuous current conduction calculate the average motor current for a speed of 1400 RPM.

Solution: Given Data**:** E_s = 220 V, R_a = 2 Ω , K_m= 0.4 V/Rad/sec, t_{ON} = 15 mses , t_{OFF} = 10 mses

From the given units of the motor constant and the standard back EMF relation $E_b = K_a.\Phi.\omega$ we can easily see that it is normal motor constant K_a combined with a constant flux φ resulting in a simpler relation $E_b = K_m\ \omega$ where $K_m = K_a.\Phi$ = 0.4 V/Rad/sec

To calculate I_a we have to use the standard DC motor relation: $\mathbf{E_a = E_b + I_a R_a}$

We know $\mathbf{R_a}$ **.** We have to find out $\mathbf{E_a}$ **&** $\mathbf{E_b}$ then we can find out $\mathbf{I_a}$

$\mathbf{E_a}$ = δ. $\mathbf{E_s}$ where $\mathbf{E_s}$ = 220 V and δ = t_{ON} /Total Period T = t_{ON} / (t_{ON} + t_{OFF}) = 15/(15+10)= 15/25=0.6

Thus E_a = 0.6 x 220 = 132 V

$\mathbf{E_b}$ = Speed in Rad/sec (Speed in RPM x 2π/60) x K_m = 146.5 x 0.4 = 58.6 V

From the basic DC motor relation we get $\mathbf{I_a}$ **=** (E_a- E_b)/ R_a = **(**132 – 58.6)/2 = 73.4/2 **= 36.7 A**

Example-4: A separately excited DC motor with an armature resistance of 0.01 Ω works on a DC supply of 220 V. It draws an armature current of 100A and its rated speed is 1000 RPM. It is fed from a chopper controller for its motoring and braking operations. Assuming continuous conduction

(i) Calculate the duty ratio of chopper at rated torque with a speed of 500RPM during motoring (ii) Calculate the duty ratio of chopper at rated torque with a speed of 500RPM during braking

Solution: Given Data**:** E_a = 220 V, I_a = 100A, N = 1000 RPM, R_a = 0.01 Ω

Motoring operation Governing equation is: $E_a = E_b + I_a \times R_a$ i.e. 220 = E_b + 100 x 0.01

From which we get E_b at rated 1000 RPM = 220 -100 x 0.01 = 220-1 = 219 V

E_b at 500 RPM = (500/1000) x 219 = 109.5 V

(i) Required terminal voltage during braking is given by Ea = $E_{b@500RPM} + I_a \times R_a$

= 109.5 + 100 x0.01 = 110.5 V

From which we get; *Required duty ratio **δ** = 110.5/220 = **0.5***

(i) Required terminal voltage Ea during braking is given by Ea = $E_{b@500RPM} - I_a \times R_a$

= 109.5 -100 x0.01 = 108.5 V

From which we get : *Required duty ratio **δ** = 108.5/220 = **0.493***

Example-5: A separately excited DC motor is controlled by an ideal step down chopper with an ideal voltage source of 230 V. Motor armature resistance R_a = 1.5 Ω, L_a = 1mH, motor back emf constant = 0.05 volts/rpm. The motor drives a load with constant Torque drawing an average current of I_a = 15 A. Obtain (i)The range of speed control (ii) Corresponding range of duty ratio.

Solution:

Minimum speed /corresponding duty ratio of motor: Here we have to start with *minimum speed as zero* and find out the corresponding **δ**. To find out the required **δ** we have to find out the required $\mathbf{E_a}$ taking $\mathbf{E_b}$ as zero corresponding to zero speed.

i.e. $E_a = E_b + I_a R_a$ = 0 + 15x 1.5 (Since Torque is constant and corresponding I_a = 15) = 22.5 V = δ x 230 from which we have **δ** = 22.5 /230 = **0.097**

For finding out maximum speed and the corresponding maximum duty ratio δ: the procedure is different. Here we have to start with *maximum δ as 1* and then find out the maximum speed.

With δ as 1 we have Es = 1 x 230 = 230 V = $E_b + I_a R_a = E_b$ + 15 x 1.5 from which we get : E_b = 230 – 22.5 = 207.5 V

Now maximum speed can be obtained using the relation E_b = N x K (where N is in RPM and motor back emf constant K is = 0.05 Volts /RPM and E_b = 207.5 Volts) from which we get

207.5 = N_{max} x 0.05 and thus $\mathbf{N_{max}}$ = 207.5/ 0.05 = **4150 RPM**.

Thus, to get a **Speed range of 0 to a maximum of 4150 RPM** the required **δ range is: 0.097 to 1.0**

Example-6: A DC shunt motor draws a current of 50 A on a DC supply of 440 V and runs at 1000 RPM. It has an armature resistance of 0.5 Ω & field resistance of 100 Ω and is connected to a load having a constant torque. Its armature is controlled by a Chopper with an ON period of 2ms in the speed range of 400-800 RPM. The field current is held constant from a separate DC supply of 440V. Determine the range of frequencies of the chopper to get the required speed range.

Solution: Given Data: E_S = 440 V, I_A = 50 A, Speed N = 1000 RPM, R_A = 0.5 Ω R_F = 100 Ω

Range of speed required = 400 to 800 RPM t_{ON} = 2 ms

It is to be noted here that since the chopper is operated at a constant ON period of 2 ms, to get a variable duty ratio δ, we have to vary the chopper frequency f.

The first step is to find out the back emf E_b of the motor at the rated speed of 1000 RPM when it is running with full supply voltage of 440 V

Back EMF $\mathbf{E_b}$ at rated speed of 1000 RPM = $E_S - I_A R_A$ = 440 – 50 x 0.5 = 415 V

The next step is from this value of E_b we can find out the back emfs corresponding to the two speeds and from them, the required armature *voltages, then the required duty ratios and then finally the required chopper frequencies.*

To find out chopping frequency for lower end of speed i.e. 400 RPM:

Back EMF E_b at lower end speed of 400 RPM = 415 x 400 /1000 = 166 V

The required armature supply voltage for this speed $E_S = E_b + I_A R_A$ = 166 + 50 x 0.5 =191 V

(Current is taken here as the rated current of 50 A since the load torque is given to be constant)

Hence δ x 440 = 191 V from which we get δ = 191 /440 = 0.434

But δ = t_{ON} / Chopping period = 2 x 10^{-3} x Chopping frequency 'f '

(Since Chopping frequency = 1/Chopping period T)

Hence chopping frequency 'f' required to get 400 RPM = δ / (2 x 10^{-3})

= 0.434/ (2 x 10^{-3}) = 0.117 x 10^{3} = **117 Hz**

To find out chopping frequency for upper end of speed i.e. 800 RPM:

Back EMF E_b at upper end speed of 800 RPM = 415 x 800 /1000 = 332 V

The required armature supply voltage for this speed $E_S = E_b + I_A R_A$ = 332+ 50 x 0.5 =357 V

Hence δ x 440 = 357V from which we get δ = 357 /440 = 0.811

But δ = t_{ON} / Chopping period = 2 x 10^{-3} x Chopping frequency 'f '

Hence we get chopping frequency 'f' required to get 800 RPM = δ / 2 x 10^{-3}

= 0.811/ (2 x 10^{-3}) = 0.4055 x 10^{3} = **405.5 Hz**

Hence the Range of chopping frequencies is 117 Hz to 405.5 Hz to get a range of speeds from 400 to 800 RPM

(It is to be noted that sometimes all the given data in the problem may not be required to get the required solution. For e.g.in this problem field voltage and field resistance are not required to get the required solution.)

Example-7: A 230 V, 500 RPM, 4.1A, DC 1 HP motor has an armature resistance of 7.56 Ω and inductance of 55.0 mH. Its armature is controlled by a class A Chopper with a 240 V DC source. The field current is held constant at a value that gives rated operation on 230V DC at a chopping frequency of 50 Hz. The minimum load torque is 5 N-m. Determine the value of $\mathbf{t_{ON}}$ for the minimum load Torque of 5 N-m at rated speed of 500 RPM

Solution: Given Data: E_s = 240 V, E_a = 230, I_a = 4.1 A, R_a = 7.56 Ω , Output Power = 1 HP = 746 watts, Rated speed = 500 RPM

From Rated speed in RPM we get rated speed in Rad/sec as:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi.500}{60} = 52.36 \text{ Rad/sec}$$

Then let us get the back e.m.f at the rated speed as: $E_b = E_a - I_aR_a = 230 - 4.1 \times 7.56 = 199$ Volts

We know that $E_b = K_a\phi\omega$ from which we get the motor constant $K_a\phi$ as:

$K_a\phi = E_b / \omega = 199/52.36 = 3.801$ Volts /Rad/sec or N-m/Amp

(Here we have considered the motor constant as $K_a\phi$ instead of just K_a i.e. including ϕ since we are have data as constant field current and constant flux ϕ)

To find out $\mathbf{t_{ON}}$ we have to follow the following sequence:

1.) From the given data first find out the mechanical torque losses as below:

Armature Input Power = E_aI_a = 230 x 4.1 = 943 watts

Actual output power is given as (or shaft power) = 1 HP = 746 watts

Armature Copper losses = $I_a^2R_a = (4.1)^2 \times 7.56$ = 127 watts

Mechanical rotational power losses = Armature Input Power – (Actual Motor output power + Armature Copper losses)

= 943 – (746 + 127) = 70 Watts

Hence we have @500 RPM rotational torque loss = τ_{loss} = (Rotational power loss @500RPM / ω) = 70 /52.36 = 1.337 N-m

2.) Now we can find out $\mathbf{t_{ON}}$ as below :

For minimum load torque @500 RPM the average internal torque developed by the motor: $\tau_d = \tau_{min}$ + rotational torque loss = 5+1.337 = 6.337 N-m

We know that the torque developed by the motor is given by $\tau_d = K_a\phi.\ I_a$ from which we have : $I_a = \tau_d / K_a\phi$ = 6.337/ 3.801 = 1.667 Amp (corresponding to the minimum torque)

Hence the required armature voltage $E_a = E_{b\ @500rpm} + I_a R_a$ = 199 + 1.667 x 7.56 = 211.6 V

We know that on time t_{ON} is given by $t_{ON} = (E_a / E_s) \times T$

Where

E_a = Required armature voltage = 211.6 V

E_s = Supply DC voltage = 240 V

T = Time period (corresponding to a chopping frequency of 50 Hz) = 1/50 sec

From which we have $\mathbf{t_{ON}}$ = (211.6/240) x (1/50) = **1.763 x10^{-3} Sec** = **1.763 m Sec**

(It is to be noted that sometimes all the given data in the problem may not be required to get the required solution. For e.g. in this problem armature inductance is not required to get the required solution.)

UNIT-IV

SYLLABUS/CONTENTS:

Part -1: CONTROL OF INDUCTION MOTOR THROUGH STATOR VOLTAGE:

- Review of Basic Induction Motor Concepts
  - Development of Induced Torque, concept of Rotor slip, Electrical Frequency on the Rotor
  - Power and Torque in Induction Motor
  - Losses and Power flow diagram
  - Derivation of Expressions for Developed Torque, Slip at maximum Torque ,Maximum Developed Torque, and Starting Torque
- Variable voltage characteristics
- Speed-Torque characteristics
- Control of Induction Motors by AC Voltage Controllers
- Waveforms
- Summary
  - Important concepts and conclusions
  - Important formulae and equations
- Illustrative Examples

Part-2: CONTROL OF INDUCTION MOTOR THROUGH STATOR FREQUENCY:

- Variable frequency characteristics
- Variable frequency control of Induction Motors by voltage & current source inverters and Cycloconverters
- PWM control
- Comparison of VSI and CSI operations
- Speed- Torque characteristics
- Numerical problems on Induction Motor drives
- Closed loop operation of Induction Motor drives(block diagrams only)
- Summary:
  - Important concepts and conclusions
- Illustrative Examples

Review of Basic Induction Motor Concepts:

Principle of operation:

The Development of Induced Torque in an Induction Motor:

- When current flows in the stator, it will produce a magnetic field in stator such that **Bs** (stator magnetic field) will rotate at a speed:

$$n_S = 120.f_S/P$$

- Where ***fs*** is the system frequency in hertz and **P** is the number of poles in the machine. This rotating magnetic field **Bs** passes over the rotor bars and induces a voltage in them. The voltage induced in the rotor is given by:

$$e_{ind} = (v \times B)\, l$$

- Where **v** = velocity of the Rotor bars relative to the Stator magnetic field, **B =** magnetic flux density vector and **l =** length of the rotor bar in the magnetic field.
- Hence there will be a rotor current flow which would be lagging due to the fact that the rotor is Inductive. And this rotor current will produce a magnetic field at the rotor, **Br**. The Interaction between these two magnetic fields would give rise to an induced torque:

$$T_{ind} = k.B_R \times B_S$$

- The torque induced would accelerate the rotor and hence the rotor will rotate.
- However, there is a finite upper limit to the motor's speed due to the following interactive phenomenon:
  If the induction motor's speed increases and reaches synchronous speed then the rotor bars would be stationary relative to the magnetic field

↓

No induced voltage

↓

No rotor current

↓

No rotor magnetic field

↓

Induced torque = 0

↓

Rotor will slow down due to friction

Conclusion: An induction motor can thus speed up to such a near synchronous speed where the induced torque is just able to overcome the load torque but it can never reach synchronous speed.

The Concept of Rotor Slip:

The induced voltage in the rotor bar is dependent upon the *relative speed between the stator Magnetic field and the rotor*. This is termed as slip speed and is given by:

$$n_{slip} = n_{sync} - n_m$$

Where n_{slip} = slip speed of the machine

n_{sync} = speed of the magnetic field (also motor's synchronous speed)and

n_m = mechanical shaft speed of the motor.

Apart from this we can describe this relative motion by using the concept of ***slip*** which is the relative speed expressed on a per-unit or percentage basis. ***Slip s*** is defined as

$$s = \frac{n_{slip}}{n_{sync}} (\times\ 100\%)$$

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}} (\times\ 100\%)}$$

On percentage basis and is defined as

$$S = (N_{sync} - N_m) / N_{sync}$$

On per unit basis.

Slip S is also expressed in terms of angular velocity ***ω*** (Rad/Sec) as given below:

$$\boxed{s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} (\times\ 100\%)}$$

It can be noted that if the motor runs at synchronous speed the slip ***S*** =0 and if the rotor is standstill then the slip ***S*** =1. It is possible to express the mechanical speed of the Rotor in terms of Slip ***S*** and synchronous speed n_{sync} as given below:

$$\boxed{n_m = (1 - s)n_{sync}}$$

$$\boxed{\omega_m = (1 - s)\omega_{sync}}$$

Rotor e.m.f., Rotor frequency, Rotor reactance, rotor current and Power factor at standstill and during operation: (Effect of Slip on Rotor parameters)

In case of a transformer, the frequency '**f**' of the induced e.m.f. in the secondary is same as that of the voltage applied to the primary. But in the case of an Induction motor it is not same as that of the applied voltage to the stator and depends on the slip. At start, the speed N = 0, the slip's'= 1 and the frequency of the induced voltage in the rotor is same as that of the voltage applied to the stator. As the motor picks up speed, the slip becomes smaller and hence the frequency of the induced e.m.f. in the rotor also becomes lesser. Due to this, some of the Rotor parameters also get affected. Let us study the effect of slip on the following parameters. 1. Rotor frequency 2.Magnitude of induced e.m.f. 3. Rotor reactance 4. Rotor power factor and 5. Rotor current.

Rotor frequency:

The speed of the Stator rotating magnetic field is given by $\mathbf{N_S = 120.f_S/P}$ **(1)** where $\mathbf{f_s}$ is the system frequency in hertz and **P** is the number of poles in the machine. At start, the speed N = 0, the slip's'**= 1** and the rotor which is stationary has maximum relative motion i.e. same as that of the R.M.F. Hence the frequency of the induced voltage in the rotor is same as that of the voltage applied to the stator. As the motor picks up speed the relative speed of the Rotor with respect to the Stator RMF decreases and becomes equal to slip speed (N_S – N). As we know, the frequency and Magnitude of induced e.m.f in the rotor depends on the rate of change of cutting flux i.e. relative speed (N_S- N). Hence in running condition the magnitude and frequency of induced voltage decreases. The rotor is wound for the same number of poles as that of the Stator i.e. **P**. If $\mathbf{f_r}$ is frequency of the Rotor induced e.m.f. in running condition at slip speed of (N_S - N) (when the motor is running at a speed of N)

then there exists a fixed relation between slip speed (N_S - N), $\mathbf{f_r}$ and **P** just as in the case of stator. So for Rotor we can write: $\mathbf{N_s - N = 120f_r/P}$ ---------- **(2)**
Dividing equation (2) by (1) we get:

$$(N_s-N)/N_s = (120f_r/P) / (120.f_S/P)$$

But $\mathbf{(N_S - N) / N_s}$ **= Slip'****s'**

$$\textbf{Hence } s = f_r/f_s \textbf{ or } \qquad f_r = sf_s$$

Thus we can say that the frequency of the Rotor induced e.m.f $\mathbf{f_r}$ is slip '**s**' times the supply frequency $\mathbf{f_s}$.
As slip of an induction motor is normally in the range of 0.01 to 0.05 the Rotor frequency is very small in the running condition.

Rotor Induced e.m.f:

We know that just like the induced frequency, the induced e.m.f is also is proportional to the relative speed between the Rotor and the stator.
Let $\mathbf{E_2}$ = Rotor induced e.m.f when it is standstill i.e. relative speed is N_s
And $\mathbf{E_{2r}}$ = Rotor induced e.m.f when it is running i.e. relative speed is N_s – N
So we have $\mathbf{E_2}$ a $\mathbf{N_s}$ **i.e** $\mathbf{E_2 = k\,N_s}$ ----- **(1)**
And $\mathbf{E_{2r}}$ a $\mathbf{N_s - N}$ **i.e** $\mathbf{E_{2r} = k\,(N_s - N)}$ ---- **(2)**

Dividing the second equation by first equation we get : $\mathbf{E_{2r} / E_2 = (N_s - N) / N_s}$
.
But $\mathbf{(N_s - N) / N_s}$ **= slip 's' .** Hence we get finally **:**

$$E_{2r} = s\,E_2$$

i.e. The magnitude of the *Rotor e.m.f. in running condition* also gets reduced to ***slip*** times the magnitude of the *e.m.f. in standstill condition*.

Rotor Resistance and Reactance:

Just like the stator, Rotor winding also has its own Resistance and Reactance and let them be $\mathbf{R_2\ \Omega/Ph}$ and $\mathbf{X_2\ \Omega/Ph}$ respectively.
We know that Resistance of a coil is independent of frequency while its Reactance is given by $\mathbf{X = 2\pi fL}$ where **L** is the Inductance of the coil. Thus

$$X_2 \text{ (@ standstill)} = 2\pi f_s L$$

and since $\mathbf{f_r = sf_s}$

$$X_{2r} \text{ (@ running condition)} = 2\pi f_r L = 2\pi s f_s L = s\,X_2$$

$$\text{i.e.} \quad X_{2r} = s\,X_2$$

Thus we can conclude that the Resistance of the Rotor which is independent of frequency remains the same at both standstill and in running condition while the reactance which is dependent on the frequency gets reduced to slip times the Reactance in standstill condition.

Then we have Rotor impedance $\mathbf{Z_2}$ per phase as**:**

$$Z_2 = R_2 + j\,X_2 \quad = \quad \sqrt{R_2^2 + X_2^2} \qquad \Omega\,/Ph \qquad (\text{@standstill})$$

And $$Z_{2r} = R_2 + j\,X_{2r} \quad = \quad \sqrt{R_2^2 + (sX_2)^2} \qquad \Omega\,/Ph \qquad (\text{@ Running condition})$$

Rotor power factor:

We know that the power factor of any inductive circuit is given by:

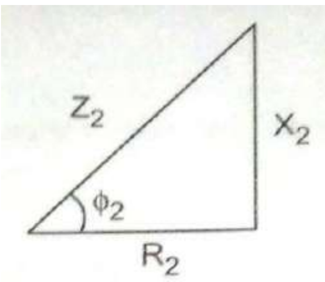
$$\cos\theta = R/Z$$

Using the above values of Resistance and impedance of the Rotor in both standstill and running conditions in this relation for p.f we get:

$$\cos\theta = R_2/Z_2 = R_2/\sqrt{R_2^2 + X_2^2} \quad \Omega\,/Ph \qquad (\text{@standstill}) \text{ and}$$

$$\cos\theta_r = R_2/Z_{2r} = R_2/\sqrt{R_2^2 + (sX_2)^2} \qquad (\text{@ Running condition})$$

The corresponding impedance triangles for both standstill and running conditions are shown in the figures (a) and (b) below.



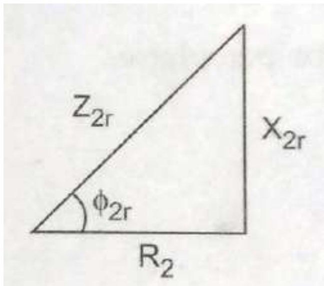


Fig: Impedance triangles (a) at standstill (b) while running

Note: As Rotor circuit is inductive the p.f is always lagging.

Rotor current:

The rotor currents (per phase) in both cases are given by (using the basic relation **I = E/Z**) :

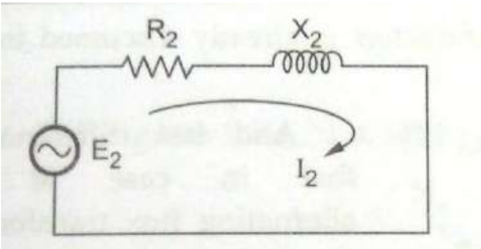
$I_2 = E_2 / Z_2 = E_2 / \sqrt{R_2^2 + j X_2^2}$ (@standstill)

and

$I_{2r} = E_{2r} / Z_{2r} = E_{2r} / \sqrt{R_2^2 + j X_{2r}^2} = s E_2 / \sqrt{R_2^2 + (sX_2)^2}$ (@ Running condition)

Note: (θ_{2r} is the phase angle between the Rotor voltage E_{2r} and Rotor current I_{2r} which decides the power factor while the motor is running)

The corresponding Rotor equivalent circuits for both standstill and running conditions are shown in the figures (a) and (b) below.



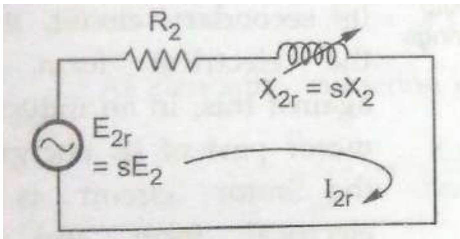


Fig: Rotor equivalent circuit (a) At standstill (b) while running

Rotor power input, Rotor copper loss and mechanical power developed and their interrelation:

An induction motor can be basically described as a rotating transformer. Its input is a 3 phase system of voltages and currents. For an ordinary transformer, the output is electric power from the secondary windings. The secondary windings in an induction motor (the rotor) are shorted and so no electrical output exists from normal induction motors. Instead, the output power is mechanical. The power flow diagram given below shows how the Input Electrical power given to the Induction Motor stator gets converted into Mechanical power at the Rotor end and what are the losses taking place in between.

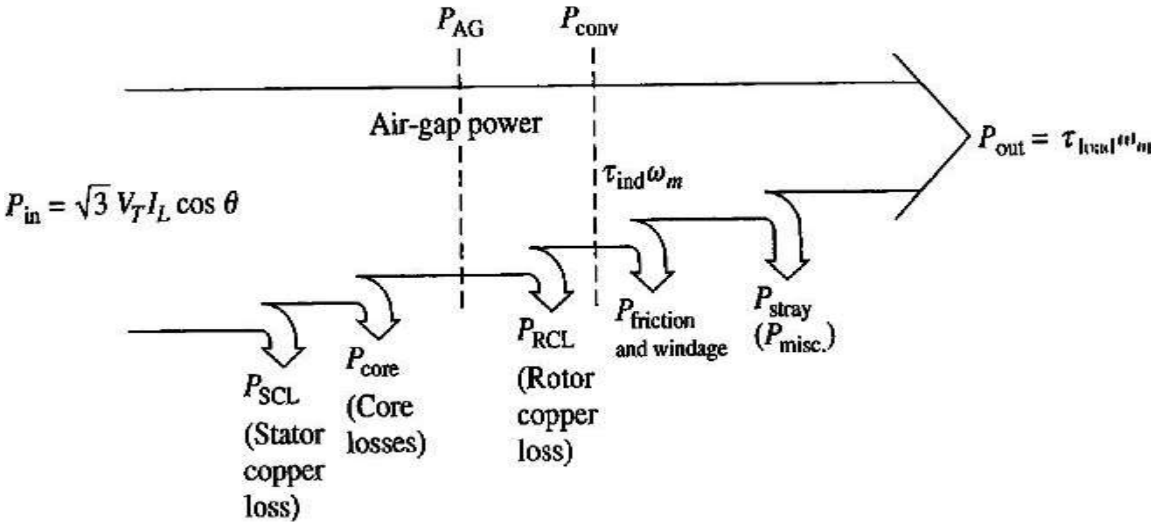


Fig: Power flow diagram of an Induction motor.

The input power to an induction motor $\mathbf{P_{in}}$ is in the form of 3-phase electric voltages and currents and is given by:

$$\mathbf{P_{IN} = \sqrt{3}\ V_L I_L \cos\theta}$$

where $\mathbf{V_L}$, $\mathbf{I_L}$ are line values of voltage & current and **cos θ** is motor power factor.

The first losses encountered in the machine are $\mathbf{I^2R}$ losses in the stator windings (the stator copper loss $\mathbf{P_{SCL}}$). Then, some amount of power is lost as hysteresis and eddy currents in the stator ($\mathbf{P_{core}}$). The power remaining at this point is transferred to the rotor of the machine across the air gap between the stator and rotor. This power is called the air gap power $\mathbf{P_{AG}}$ of the machine.i.e.

$$\mathbf{P_{AG} = P_{IN} - (P_{SCL} + P_{core}) = T_{ind}.\omega_s}$$

After the power is transferred to the rotor, some of it is lost as $\mathbf{I^2R}$ losses (the rotor copper loss $\mathbf{P_{RCL}}$), and the rest is converted from electrical to mechanical form ($\mathbf{P_{CONV}}$).i.e.

$$\mathbf{P_{CONV} = P_{AG} - P_{RCL} = T_{ind}.\omega_m}$$

When this mechanical power is delivered to the load through the rotor shaft again some more power is lost as mechanical losses known as friction and windage losses $\mathbf{P_{F\&W}}$ and then again some unaccounted losses known as stray losses $\mathbf{P_{MISC}}$. Finally the remaining power is the net output power delivered by the Motor to the load as $\mathbf{P_{OUT}}$. i.e.

$$\mathbf{P_{OUT} = P_M - (P_{F\&W} + P_{MISC}) = T_{load}\,.\omega_m}$$

This total power flow along with the losses in between is shown the diagram above.

The core losses do not occur in the stator side alone as shown in the figure above. The core losses of an induction motor come partially from the stator circuit and partially from the rotor circuit. Since an induction motor normally operates at a speed near synchronous speed, the relative motion of the magnetic fields over the rotor surface is quite slow, and the rotor core losses are very tiny compared to the stator core losses. Since the largest fraction of the core losses come from the stator circuit, all the core losses are lumped together and shown as if they are occurring at the stator end. The *higher* the speed of an induction motor, the *higher* the friction, windage, and stray losses. On the other hand, the *higher* the speed of the motor (up to n_{sync}), the *lower* its core losses. Therefore, these three categories of losses are sometimes lumped together and called as *rotational losses*. The total rotational losses of a motor are often considered to be constant with changing speed, since the component losses change in opposite directions with a change in speed as explained.

Torque equation – expressions for maximum torque and starting torque:

Torque equation: The torque developed in an Induction motor depends on the following factors.

1. The stator magnetic field ϕ which induces e.m.f. in the rotor.

2. The magnitude of the Rotor current I_{2r} in running condition.

3. The power factor '$\cos \theta_{2r}$'of the Rotor circuit in running condition.

Thus the expression for Torque can be given as: $T \propto \phi . I_{2r} . \cos \theta_{2r}$ --------- **(1)**

We know that the flux ϕ produced by the stator is proportional to the voltage applied to the stator E_1. And similarly the Stator and Rotor voltages E_1 and E_2 are related to each other by a ratio of their effective number of turns **'K '**.

i.e. $\phi \propto E_1$ and $E_1 / E_2 = K$ and so effectively $\phi \propto E_2$ ----- **(2)**

We have earlier obtained expressions for the Rotor current and Rotor power factor as:

$\mathbf{I_{2R} = E_{2r} / Z_{2r} = s\,E_2 / \sqrt{R_2^{\,2} + (sX_2)^2}}$ (@ Running condition) ----- **(3)**

$\mathbf{Cos\ \Theta_{2R} = R_2 / Z_{2r} = R_2 / \sqrt{R_2^{\,2} + (sX_2)^2}}$ (@ Running condition) ----- **(4)**

Using the above equations at (2),(3) and (4) in equation (1) we get :

$\mathbf{T \propto [s\,E_2^{\,2} R_2 / R_2^{\,2} + (sX_2)^2]}$

$\mathbf{T = k\,[s\,E_2^{\,2} R_2 / R_2^{\,2} + (sX_2)^2]}$

Where ‘k’ is the constant of proportionality and can be shown that k $\mathbf{= 3/2\pi n_s}$ where $\mathbf{n_s}$ **=** synchronous speed in r.p.s. = $N_s/60$ (N_s= Synchronous speed in RPM). Substituting this value of the constant ‘k’ in the above expression for Torque we get finally

$$\mathbf{T = (3/2\pi n_s)\,[s\,E_2^{\,2} R_2 / R_2^{\,2} + (sX_2)^2]} \quad \textbf{N-m}$$

So, ***Torques*** at any load condition can be obtained if ***Slip‘s’*** at that load and ***Standstill Motor parameters*** are known**.**

Starting Torque: Is the torque at the time of start in an induction motor and can be obtained by substituting the corresponding value of slip’s’. At the time of starting the speed N= 0 and hence the slip‘s’ = 1. Using this value of‘s’ in the above equation for Torque we get the starting torque as:

$$\mathbf{T_{st} = (3/2\pi n_s)\,[E_2^{\,2} R_2 / R_2^{\,2} + X_2^{\,2}]} \quad \textbf{N-m}$$

Maximum torque: Condition for maximum Torque:

As can be seen from the above Torque equation, the torque depends only on the slip with which the motor is running since all the other parameters are constant. Supply voltage to the stator is usually rated and hence constant and the turn’s ratio between Stator and Rotor is also constant. Hence E_2 is constant. Similarly R_2, X_2 and n_s are constants in an Induction motor. So to find out the maximum torque we have to find out at what slip maximum torque occurs. Hence, mathematically we can write the condition for maximum Torque as

$$\mathbf{dT/ds = 0}$$

where $\mathbf{T} = k\ [\mathbf{s\ E_2^{\,2}\ R_2 / R_2^2 + (sX_2)^2}]$ **.** While evaluating the above differential it is to be noted that in the above expression for Torque all the parameters like $\mathbf{E_2, R_2}$ and $\mathbf{X_2}$ are also constants apart from the constant of proportionality **'k'** and the only variable is **'s'** and this term is present in both numerator and denominator. Hence we can differentiate the expression for torque using the formula for differential of a quotient (u/v) after taking out all the constant terms out of the differential as shown below.

$$\mathbf{T = (k\ E_2^{\,2}\ R_2)\ [s / R_2^2 + s^2X_2^2]}$$

Now differentiating the term within the square brackets and equating the numerator alone to zero we get:

$$\mathbf{s.d/ds\ [R_2^{\,2} + s^2X_2^{\,2}] - [R_2^{\,2} + s^2X_2^{\,2}]d/ds\ (s) = 0}$$

i.e. $\mathbf{s.\ [2\ sX_2^{\,2}] - [R_2^{\,2} + s^2X_2^{\,2}].1 = 0}$

i.e. $\mathbf{2\ s^2X_2^{\,2} - s^2X_2^{\,2} - R_2^{\,2} = 0}$

i.e. $\mathbf{s^2 = R_2^{\,2} / X_2^{\,2}}$

Or finally $\mathbf{s = R_2 / X_2}$

So we conclude that the torque is maximum at a slip's' = $\mathbf{R_2 / X_2}$ or in other words the slip at maximum torque is given by:

$$\mathbf{'s_m' = R_2 / X_2}$$

Maximum Torque:

Now we can obtain the magnitude of maximum torque $\mathbf{T_{max}}$ by substituting the value of $\mathbf{'s_m' = R_2 / X_2}$ in place of **'s'** in the general expression for Torque.

$$\mathbf{T_{max}} = k[\mathbf{s_m\ E_2^{\,2}\ R_2 / \{R_2^{\,2} + (s_mX_2)^2\}}]$$

$$\mathbf{T_{max}} = k[\mathbf{(R_2 / X_2)\ E_2^{\,2}\ R_2 / \{R_2^{\,2} + \{(R_2 / X_2)X_2)^2\}}]$$

Or finally $\mathbf{T_{max}} = k\mathbf{E_2^2/2\ X_2}$ **N-m**

From the above expression for ***Maximum Torque*** we can observe the following important points:

- It is directly proportional to the **Square of the induced e.m.f. $\mathbf{E_2}$** in the rotor at stand still.

- It is inversely proportional to the **Rotor Reactance X_2 at stand still**
- The most interesting fact is: ***It is not dependent on the Rotor resistance*** **R_2.** But the slip or speed at which such a maximum Torque occurs depends on the value of ***Rotor resistance*** **R_2**

Torque slip characteristic:

When an Induction motor is loaded from no load to full load its speed decreases and slip increases. Due to increased load, motor has to produce higher torque to satisfy higher load torque demand. The torque ultimately depends on the slip as we have seen earlier. The behaviour of the motor can be easily analyzed by looking at the Torque versus slip curve from s=0 to 1. (Instead of Torque versus Speed Characteristics because we have readily available equations for Torque in terms of **slip's'**. The Torque vs. Slip Characteristics can then be easily translated to Torque vs. Speed Characteristics since they are complementary to each other.)

We have already seen that for a constant supply voltage, E_2 is also constant. So we can rewrite the basic Torque equation $T \propto [s E_2^2 R_2 / R_2^2 + (sX_2)^2]$

as: $T \propto [s R_2 / R_2^2 + (sX_2)^2]$.

To study the Torque versus Slip characteristics let us divide the slip range **(s = 0 to 1)** into **three** parts and analyze.

The Torque speed characteristic can be divided into three important regions:

1. **Low Slip Region**:

   In this region's' is very small. So, the term $(sX_2)^2$ would be **small** compared to R_2^2 and hence can be neglected. Thus $T \propto s R_2 / R_2^2$. i.e. Torque becomes directly proportional to slip's'. Thus torque increases linearly with increase in **slip's'** and satisfies the load demand. Thus we can conclude that in this region.

   - The mechanical speed decreases approximately linearly with increased load
   - The motor slip increases approximately linearly with increased load.

- Induced Torque increases linearly with slip thus satisfying the load demand.
- Rotor reactance is negligible. So Rotor Power factor is almost unity.
- Rotor current increases linearly with slip.

The entire normal steady state operating range of an Induction motor lies in this linear low slip region. Thus in normal operation, an induction motor has a linear speed drooping characteristic.

2. Moderate slip region: In this region:

- Rotor frequency is higher than earlier and hence the Rotor reactance is of the same order of magnitude as the rotor resistance.
- Rotor current no longer increases as rapidly as earlier and the Power factor starts dropping.
- The peak torque (Pull out or Break down Torque) occurs at a point where for an incremental increase in load the increase in the current is exactly balanced by the decrease in rotor power factor.

3. High slip region: In this region:

Slip is high i.e approaching the value 1. Here it can be assumed that the term $\mathbf{R_2}^2$ is very small compared to $(\mathbf{sX_2})^2$. Hence the expression for Torque becomes **T** a **s** $\mathbf{R_2 / (sX_2)^2}$ **i.e T** a **1/ s.** So in high slip region Torque is inversely proportional to **slip's'.** Hence the induced Torque decreases with increase in load torque since the increase in Rotor current is dominated by the decrease in Rotor power factor where as it should increase to meet the increase in Load demand. So speed further comes down and Induced Torque still reduces further. So in this process the motor comes to standstill. i.e. the motor cannot run at any point in the high slip region. Hence this region is called ***unstable region*** .On the other hand the low slip region where the characteristic is linear is called the ***stable region***.

The maximum Torque which the motor can produce before going into unstable region occurs at **s' = '$\mathbf{s_m}$'.** Since beyond this torque the motor gets into unstable region, this maximum Torque is also called as ***Break down Torque*** or ***pullout Torque***. The entire Torque slip characteristics are shown in the figure below.

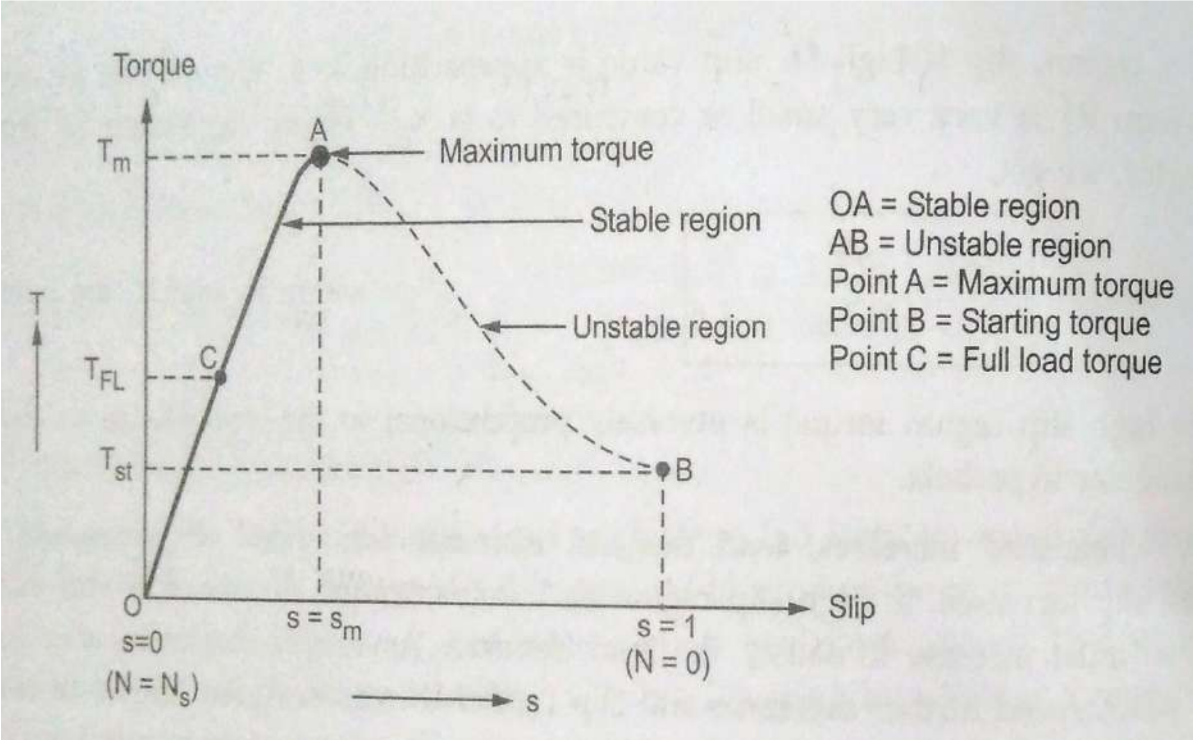


Fig: Torque-slip Characteristics (Pl change Point 'C' as Starting Torque and Point 'B' as Full Load Torque in the above figure)

Torque vs. Speed Characteristics: Are just complimentary to the Torque-slip Characteristics. The detailed Torque speed characteristics of an a Induction Motor Showing the Starting, Pull-out and Full-load torques are shown in the figure below.

Important characteristics of the Induction Motor Torque Speed Curve:

- Induced Torque is zero at synchronous speed.
- The graph is nearly linear between no load and full load (at near synchronous speeds).In this region the Rotor resistance is much larger than the Rotor reactance ,and hence the Rotor Current, magnetic field and the induced torque increases linearly with increasing slip.
- There is a Max. Possible torque that cannot be exceeded which is known as pull out torque or breakdown torque. This is normally about two to three times the full load torque.
- The Starting torque is higher than the full load torque and is about 1.5 times. Hence this motor can start with any load that it can handle at full power.

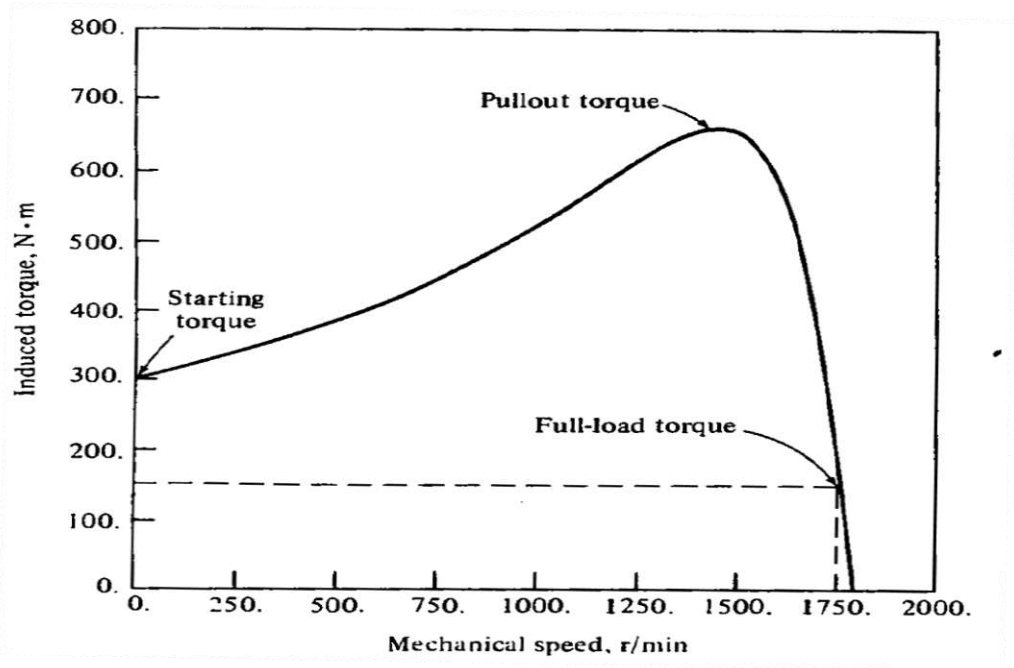


Fig: Torque speed characteristics of an a Induction Motor Showing the Starting, Pull-out and Full-load torques

- Torque for a given slip varies as the square of the applied voltage. This fact is useful in the motor speed control with variation of Stator Voltage.
- If the rotor were driven faster than synchronous speed, then the direction of the Induced torque would reverse and the motor would work like a generator converting mechanical power to Electrical power.
- If we reverse the direction of the stator magnetic field, the direction of the induced torque in the Rotor with respect to the direction of motor rotation would reverse, would stop the motor rapidly and will try to rotate the motor in the other direction. Reversing the direction of rotation of the magnetic field is just phase reversal and this method of Braking is known Plugging.

Full load Torque: When the load on the motor Torque increases, the slip increases and thus the Induced torque also increases. The increase in induced Torque is produced by a corresponding increase in the current drawn from the supply.

The load which the motor can drive safely depends on the current which the motor can draw safely. When the current rises, the temperature rises. Hence the safe limit on the current is dictated by permissible temperature rise. ***The safe limit of current is that which when drawn for continuous operation of the motor produces a temperature rise which is well within the limits.*** Such a full load point is shown as point **'C'** on the plot and the corresponding torque is called the Full load Torques $\mathbf{T_{FL}}$. If the motor is operated beyond this full load continuously the windings' insulation is likely to be damaged. But for short durations of time the motor can be operated beyond the Full load Torque but up to the limit of ***Breakdown Torque/Pull out Torque***

Speed control of Induction motors - Basic Methods:
Stator side:

1. Stator Voltage control
2. Stator variable frequency control

Rotor side:

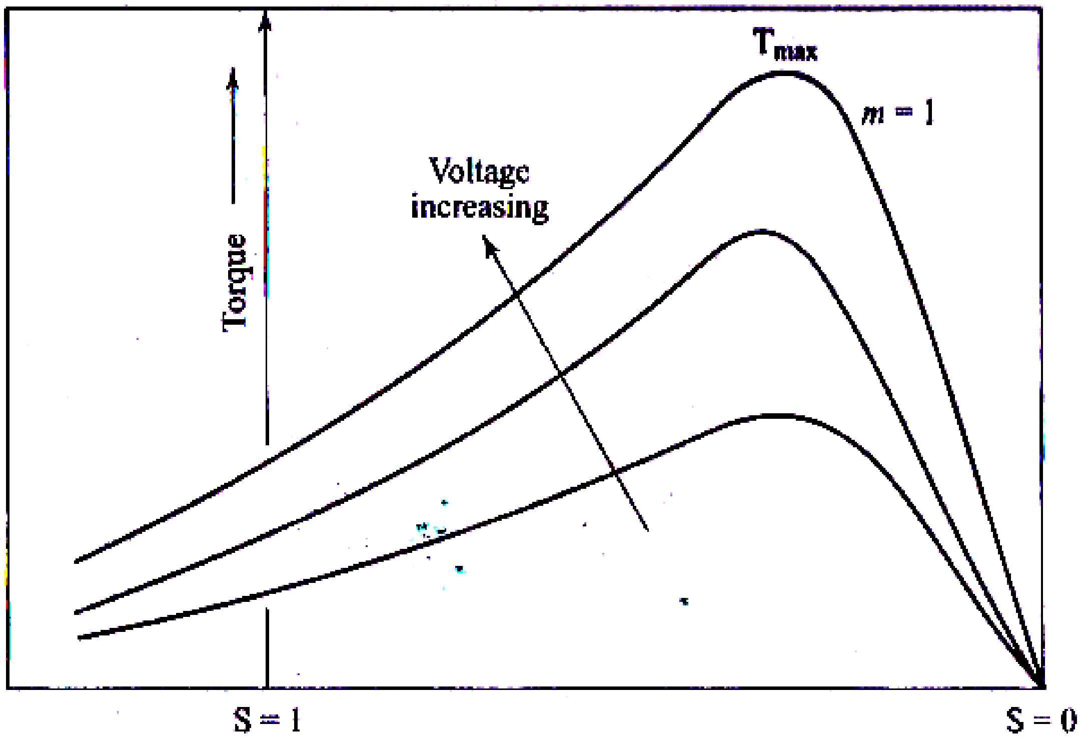
- Rotor resistance control
- Slip-energy recovery

Stator voltage control:

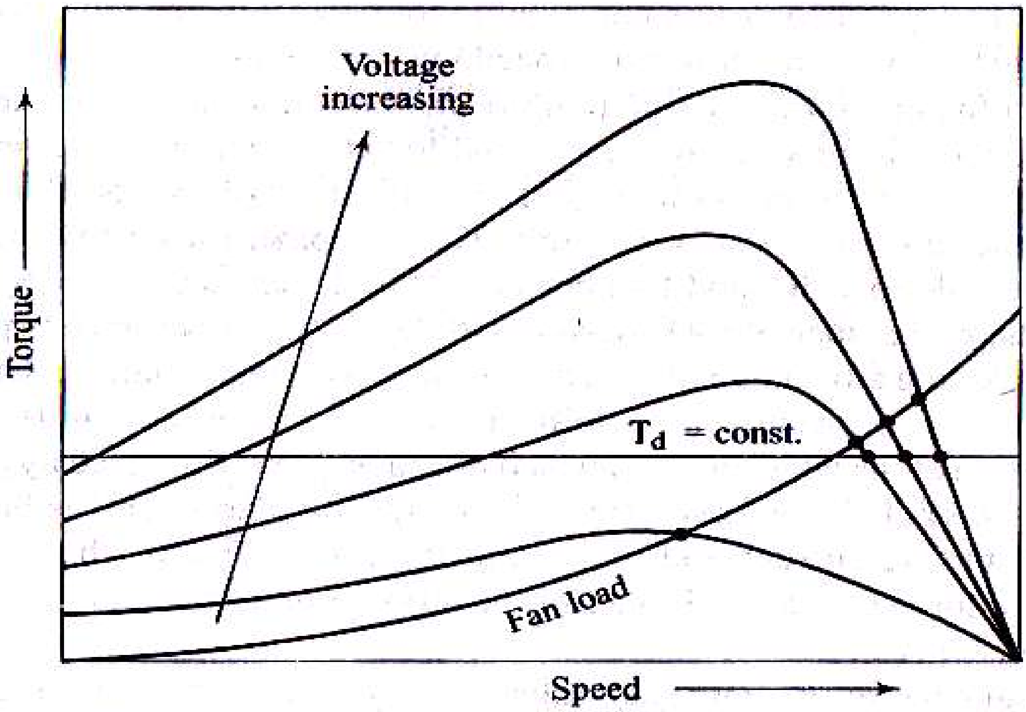
- From the expression for the torque developed by an induction motor, we can see that it is directly proportional to the square of the applied terminal voltage at a constant value of supply frequency and slip. By varying the applied voltage, a set of torque-speed curves as shown below can be obtained. When the applied voltage changes by n times the resulting torque changes by n^2 times.

- If constant torque is required at different voltages, the slip increases with decreasing voltage to accommodate the required rotor current. But the power factor deteriorates at low voltages.

- Fig(b) shows the torque- speed curves along with a constant load and varying load (with speed).From this it can be seen that speed control is possible only in a limited range



(a) Typical speed-torque curves for variation in stator voltage (low-resistance rotor)



(b) Operating points and speed range for constant torque and fan type load (rotor resistance low)

Limitations of Stator voltage control:

- The portion of the speed control beyond the maximum torque is unstable and is not suitable for speed control.
- Normal squirrel cage motors will have low rotor resistance and therefore will have a large unstable region. Hence speed control is possible only in a limited band.
- The starting current is also very high for these motors (because of low rotor resistance). Hence the equipment used for control of these motors must be able to handle/withstand such large starting currents.
- The power factor also will be poor at large slips.
- Therefore special rotor design with high resistance is required to be able to take advantage of speed control with stator voltage variation. This shifts the point of slip for maximum torque to the left and decreases the unstable region.
- The unstable region can be reduced or even completely eliminated by properly designing the rotor. This increases the range of speed control substantially, reduces the starting current and improves the power factor.
- However motors designed with high rotor resistance to achieve higher speed control range will have higher rotor losses at large slips and will have to dissipate the resulting large heat in the Rotor itself.
- But slip ring motors allow the insertion of the high resistance externally. Hence the losses will be dissipated in the external resistors only and Rotor heating will be avoided.

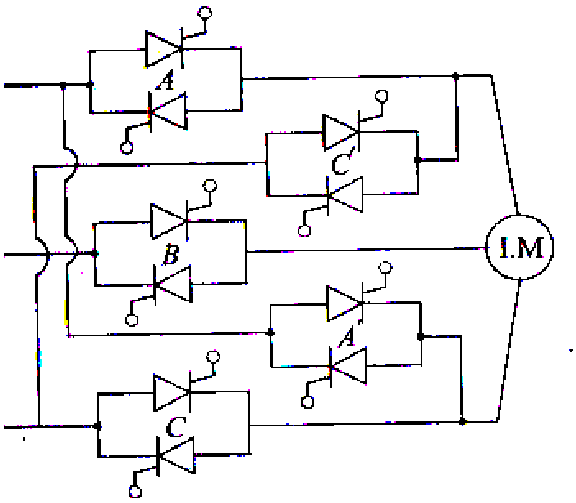
Method of stator voltage control:

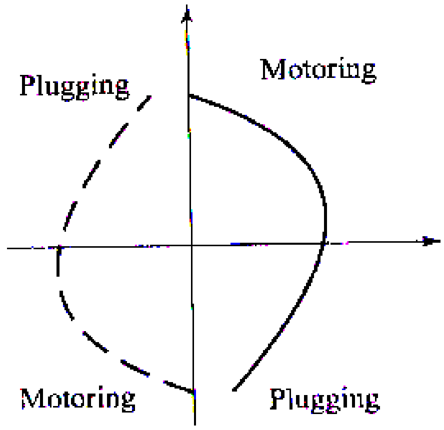
AC voltage controllers can be used for varying the applied input stator voltage. By controlling the firing angle of the thyristors connected in anti parallel in each phase the RMS value of the stator voltage applied to each phase can be varied to get the desired speed control.

Four quadrant operation with plugging is obtained by the use of the circuit shown in the figure below. Thyristor pairs A,B and C provide operation in quadrants 1 &4 (as shown by the solid line) . Thyristor pairs A',B and C' changes the phase sequence and thus provide operation in quadrants 2&3(as shown by the dotted line).

Precaution:

While changing from one set to another set of thyristor pairs, i.e from ABC to A'BC' or *vice versa, care* should be taken to ensure that the incoming pair is activated only after the outgoing pair is fully turned off. This is to avoid short circuiting of the supply by the conducting thyristor pairs. Protection against such faults can be provided only by the fuse links and not by the current control.





Limitations:

A review of the AC controllers reveals that:

- The output voltage from an AC controller is dependent not only on the delay angle of the gate firing pulses but also on the periods of current flow which in turn are dependent on the load power factor. An induction motor will draw a varying power factor current and this will influence the voltage being applied to it. Whenever the load current is continuous, the controller will not have any influence on the circuit conditions at all.

- Control is achieved by distortion of the voltage waveforms and by the reduction of the current flow periods. Significant amounts of stator and rotor harmonic currents will flow and eddy currents will be induced in the iron core. These will cause additional motor heating and alter the motor performance compared with sinusoidal operation.

The practical results of these limitations are:

- The motor performance can be predicted only after a full understanding of the motor, thyristor converter and the load.
- A closed loop speed control based on a tachogenerator speed feedback is essential to ensure stable performance.
- The system gains most practical application when the load is predictable and the load torque required at low speeds is relatively low.

Important formulae and equations:

- Synchronous speed of rotating magnetic field : $\mathbf{n_S = 120.f_S/P}$
- Voltage induced in the rotor : $\mathbf{e_{ind} = (v \times B)\, l}$
- Torque induced in the rotor : $\mathbf{T_{ind} = k.B_R \times B_S}$
- slip **s** on percentage basis:

$$s = \frac{n_{slip}}{n_{sync}} (\times\ 100\%)$$

$$\boxed{s = \frac{n_{sync} - n_m}{n_{sync}} (\times\ 100\%)}$$

- Slip **s** on per unit basis: $\mathbf{S = (N_{sync} - N_m) / N_{sync}}$
- The magnitude of the rotor induced voltage $\mathbf{E_R}$ in terms of the rotor induced voltage at rotor locked condition $\mathbf{E_{R0}}$: $\mathbf{E_R = s.E_{R0}}$
- The magnitude of the rotor Reactance $\mathbf{X_R}$ in terms of the rotor Reactance at rotor locked condition $\mathbf{X_{R0}}$: $\mathbf{X_R = s.X_{R0}}$ (since $f_r = s.f_s$ and $X_R = s.2\pi f_s L_R$)
- The rotor frequency can be expressed as :

$$\mathbf{f_r = (P/120).\,(n_{sync} - n_m.)}$$

- Important relationships between Air gap power $\mathbf{P_{AG}}$, converted power $\mathbf{P_{conv}}$, Rotor induced Torque $\mathbf{T_{ind}}$,Rotor copper losses $\mathbf{P_{rcl}}$ and the slip **s :**

$$\mathbf{T_{ind} = P_{conv} / \omega_m}$$
$$\mathbf{T_{ind} = P_{AG} / \omega_s}$$
$$\mathbf{P_{rcl} = s.P_{AG}}$$
$$\mathbf{P_{conv} = (1-s)\, P_{AG}}$$

- **Expressions for Torque considering Rotor circuit only :**

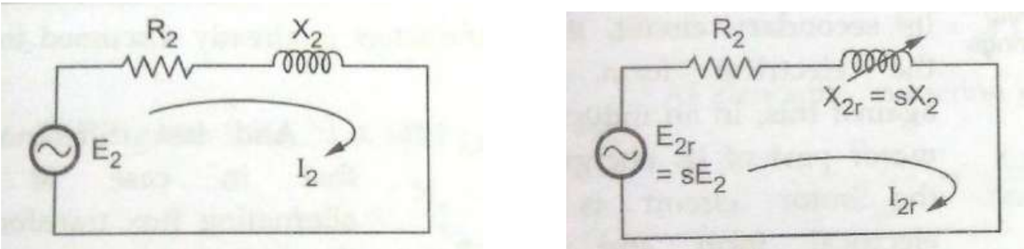


Fig: Rotor equivalent circuit (a) At standstill (b) while running

- Torque developed by the motor T_d: $T_d = k\,[s\,E_2^2\,R_2 / R_2^2 + (sX_2)^2]$

  (Where constant $K = 3/2\pi N_s = 3/\omega_s$)
- Slip at maximum Torque S_m: **'s_m' = R_2 / X_2**
- Maximum developed torque T_{max}: $T_{max} = kE_2^2/2X_2$
- Starting Torque T_{st} : $T_{st} = k\,[E_2^2\,R_2 / R_2^2 + X_2^2]$

- **Torque–Speed relations using an Equivalent circuit with both Stator and Rotor circuit parameters.**

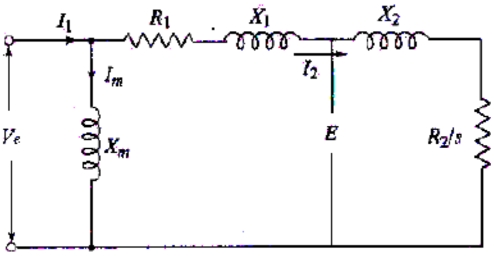


Fig: Simplified Per-phase equivalent circuit of an Induction Motor with both Stator and Rotor circuit parameters

- Torque developed by the motor T_d:

$$T_d = \frac{P_{gross}}{\omega_r} = \frac{P_{gross}}{\omega_s(1-s)} = \frac{3V_1^2 R_2/s}{\omega_s[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]}$$

or $$T_d = \frac{3}{\omega_s} I_2^2 \frac{R_2}{s} \text{ N-m}$$

- Slip at maximum Torque S_{maxT}:

$$S_{max\,T} = \pm \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$

- Maximum developed torque **T_{max}:**

$$T_{max} = \frac{3V_{1ph}^2}{2\omega_s\left[R_1 \pm \sqrt{R_1^2 + (X_1 + X_2)^2}\right]}$$

- Starting orque **T_{st} :**

$$T_{start} = \frac{3V_1^2 R_2}{\omega_s\left[(R_1 + R_2)^2 + (X_1 + X_2)^2\right]}$$

Illustrative Examples:

Example-1: A 3 kW, 400 V, 50 Hz, 4 pole, 1400 RPM, delta connected induction motor has the following parameters referred to stator. R_1 = 2.5 Ω; R_2 = 4.5 Ω; X_1 = X_2 = 6 Ω. Speed control is achieved by Stator Voltage Control. When driving a *fan load*, the motor runs at rated speed and rated voltage. Calculate the voltage to be applied to the motor to run at 1300 RPM.

Solution: Given data: V_L = 400 V, f = 50 Hz,P =4, N_r = 1400 RPM

R_1 = 2.5 Ω; R_2 = 4.5 Ω; X_1 = X_2 = 6 Ω

Since the motor is delta connected V_L = 400 V = V_{ph}

We know that the motor synchronous speed is given by N_s = 120 f/P

= 120 x 50 /4 = 1500RPM

Hence ω_s = 1500 x2π /60 = 157 Rad/sec

Slip at the rated speed of 1400 RPM is given by s = (1500-1400)/1500 = 0.0667

Induction motor's simplified equivalent circuit taking into account both stator and rotor circuit parameters (referred to stator) is shown below.

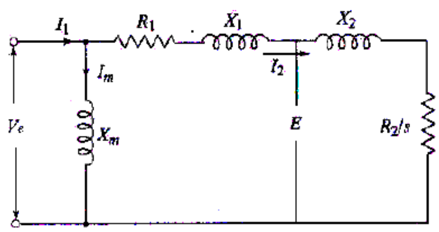


Fig: Induction motor Equivalent circuit with both Stator and Rotor circuits

From the equivalent circuit, the rotor current I_2 can be shown to be:

$$I_2 = V_{ph} / [\ (R_1+R_2/s)^2 + (X_1+X_2)^2\ ]^{1/2}$$

Substituting the values from the given data we get:

$I_2 = 400 / [\ (2.5+4.5/0.0667)^2 + (6+6)^2\ ]^{1/2} = 400/70.98 = 5.634\ A$

We know that the torque developed T_d by an Induction motor is given by:

$$T_d = (3/\omega_s).\ I_2^2.\ (R_2/s)\ N\text{-}m$$

$= (3/157).\ (5.634)^2.\ (4.5/0.0667) = 40.92\ N\text{-}m$

[*Here it may be noted that the torque developed at the rated speed was found out by first calculating the motor current using two separate formulae for current and torque. It could have been found out directly also by using the formula*

$$T_{d@slip\ 's'} = (3/\ \omega_s).[V^2_{ph} / \{(R_1+R_2/s)^2 + (X_1+X_2)^2\}].(R_2/s)$$

We know that with a fan type of load, the load torque T_L a $\omega_r{}^2$ and $T_d = T_L$ at steady state.

From which we have T_d a $\omega_r{}^2$ or T_d a $*\omega_s (1\text{-}s)]^2$ i.e $T_d = K_s.\omega_s{}^2 (1\text{-}s)^2 = K (1\text{-}s)^2$

(Where K is the final constant including $\omega_s{}^2$ which is also a constant since ω_s, the synchronous speed is constant)

We have the value of T_d at the rated speed of 1400 RPM (i.e. @a slip of 0.0667) and using that in the above relation $T_d = K\ (1\text{-}s)^2$ we can find the value of the constant K. Then using that value of K we can use the same relation and find out the developed Torque at the required speed of 1300 RPM [i.e. @a slip of (1500-1300/1500)= 0.133]

Thus: $40.92 = K(1\text{-}0.0667)^2$ from which we get K = 46.97

And $T_{L\ @1300RPM} = K\,(1\text{-}s_{@\ 1300\ RPM})^2 = 46.97 \times (1\text{-}0.133)^2 = 35.28$ N-m

And we know that this is the steady state torque developed by the motor @1300 RPM which is also given by:

$$T_{d@1300\,RPM} = (3/\omega_s).[V^2_{ph} / \{(R_1+R_2/s)^2 + (X_1+X_2)^2\}].(R_2/s)$$

Where V_{ph} is the required phase voltage for running the motor at 1300 RPM,'s' is the corresponding slip at 1300 RPM and all other parameters are already known. Substituting these values we get:

$35.28 = (3/157).[V^2_{ph} / \{(2.5 + 4.5/0.133)^2 + (6 + 6)^2\}].(4.5/0.133)$

$35.28 = 0.0191 \times [V^2_{ph} / 1464.2] \times 33.834$ from which we get

$V^2_{ph} = (35.28 \times 1464.2)/(0.0191 \times 33.834) = 79{,}936$ and $V^2_{ph} = 282.72$ V

Thus finally V_{ph} = 282.72 V is the voltage/phase to be applied to the stator windings to get a speed of 1300 RPM.

Example-2: A 440 V, 3ϕ, 50 Hz, 6 pole, 945 RPM, delta connected induction motor has the following parameters referred to stator side.R_1 = 2.0 Ω; R_2 = 2.0 Ω; X_1 = 3 Ω, X_2 = 4 Ω. Motor speed is controlled by stator Voltage Control. When driving a fan load, the motor runs at rated speed with rated voltage. To run the motor at 800 RPM calculate (a) torque developed by the motor (b) the voltage to be applied to the motor and (c) the corresponding current drawn.

Solution: Given data: V_L = 440 V [, f = 50 Hz,P =6, N_r = 945RPM

R_1 = 2.0 Ω; R_2 = 2.0 Ω; X_1 = 3 Ω, X_2 = 4 Ω.

Since the motor is delta connected $V_L = 400\ V = V_{ph}$

We know that the motor synchronous speed is given by N_s = 120 f/P = 120 x 50 /6 = 1000RPM

Hence ω_s = 1000 x2π /60 = 104.67 Rad/sec

Slip at the rated speed of 945 RPM is given by s = (1000-945)/1000 = 0.055

(Refer the previous Simplified equivalent circuit of an Induction motor considering both Stator and Rotor circuits)

(a) Torque developed to run the motor at 800 RPM:

To find the torque developed to run the motor at 800 RPM first we have to find the torque developed at the rated speed.

From the above equivalent circuit, we know that the torque developed by the motor at rated speed is given by : (here we are finding directly by using the formula for $T_{d@rated\ speed)}$

$$T_{d@rated\ speed} = (3/\ \omega_s).[V^2_{ph} / \{(R_1+R_2/s)^2 + (X_1+X_2)^2\}].(R_2/s_{\ rated\ speed})$$

Substituting the values from the given data we get:

$$T_{d@\ 945\ RPM} = (3/\ 104.67)[440^2 / \{(2 + 2/0.055)^2 + (3 + 4)^2\}](2/0.055)$$

$$= (3/\ 104.67)[440^2 / \{(2 + 2/0.055)^2 + (3 + 4)^2\}](2/0.055)$$

$$= (3\ x440^2\ x\ 2)/\ [104.67\ x\ \{(2 + 36.36)^2 + (3 + 4)^2\}\ x0.055)]$$

$$= (3\ x440^2\ x\ 2)/\ [104.67\ x\ 1520.48\ x\ 0.055)] = 132.7\ N\text{-}m$$

$$T_{d@\ 945\ RPM} = 132.7\ N\text{-}m$$

We know that with a fan type of load, the load torque T_L a ω_r^2 and $T_d = T_L$ at steady state.

From which we have T_d a ω_r^2 or T_d a $*\omega_s (1\text{-}s)]^2$ i.e $T_d = K_s.\omega_s^2 (1\text{-}s)^2 = K (1\text{-}s)^2$

(Where K is the final constant including ω_s^2 which is also a constant since ω_s ,the synchronous speed is constant)

We have the value of T_d at the rated speed of 945 RPM (i.e. @a slip of 0.055) and using that in the above relation $T_d = K (1\text{-}s)^2$ we can find out the value of the constant K. Then using that value of K we can use the same relation and find out the developed Torque at the required speed of 800 RPM [i.e. @a slip of (1000-800/1000)= 0.2]

Thus: $132.7 = K (1\text{-}0.055)^2$ from which we get K = 148.6

And $T_{L\ @800RPM} = K (1\text{-}s_{@\ 800\ RPM})^2 = 148.6\ x\ (1\text{-}0.2)^2 = 95.1\ N\text{-}m$

And we know that this is the steady state torque developed by the motor @800 RPM and hence:

Torque developed by the motor @800 RPM = 95.1 N-m

(b) Voltage to be applied to the stator to run the motor at 800 RPM:

From the above equivalent circuit we know that the per phase voltage V_{ph} to be applied to the stator in terms of the steady state torque developed by the motor @800 RPM is given by:

$$\mathbf{T_{d@800\,RPM} = (3/\omega_s).[V^2_{ph} / \{(R_1+R_2/s)^2 + (X_1+X_2)^2\}].(R_2/s_{@800\,RPM})}$$

where 's' is the corresponding slip at 800 RPM and all other parameters are already known. Substituting these values we get:

$$95.1 = \mathbf{(3/\ 104.67)[\ V^2_{ph} / \{(2 + 2/0.2)^2 + (3 + 4)^2\}](2/0.2)}$$

$$95.1 = \mathbf{(3/\ 104.67)[\ 10V^2_{ph} / (144 + 49)]}$$

$$\textbf{From which } \mathbf{V^2_{ph} = 95.1 \times 104.67 \times 193/30 = 64056.5}$$

And thus finally $V_{ph} = \sqrt{\mathbf{64056.5}}$ **=** 253.09 V is the voltage/phase to be applied to the stator windings to get a speed of 800 RPM.

(c) Current drawn by the motor to run the motor at 800 RPM:

From the above equivalent circuit we also know that the Torque developed, current drawn and the slip at any speed are related by:

$$\mathbf{T_d = (3/\ \omega_s).I_2^{\,2}.\ (R_2/s)\ \text{i.e.}\ I_2^{\,2} = T_d\ .\omega_s.\ s\ /3R_2}$$

Substituting the values we have for the RHS expression we get

$I_2^2 = 95.1 \times 104.67 \times 0.2 / 3 \times 2 = 331.8039$ and $I_2 = \sqrt{331.8039} = 18.21$ A

But since the motor is delta wound the input current is to be taken as line current and hence:

$I_{2L} = \sqrt{3} \times I_2 = \sqrt{3} \times 18.21 = 31.54$ A

Current drawn by the motor to run at 800 RPM = 31.54 A

UNIT- IV Part 2

CONTROL OF INDUCTION MOTOR THROUGH STATOR FREQUENCY:

Variable frequency control:

Speed control By Change of frequency:

The synchronous speed is given by N_S = 120 f /P. Thus by controlling the supply frequency smoothly, the synchronous speed can be controlled over a wide speed range. But form the basic transformer voltage equation we have the expression for the air gap flux:

$V = [4.44\ K_1\ \Phi\ T_{ph} f]$ from which $\Phi = [1/\ 4.44\ K_1\ T_{ph}]\ (V/f)$

Where K_1 = Stator winding constant , T_{ph1} = Stator turns /phase,

V =Supply voltage and f = Supply frequency

From the above expression it can be seen that if the frequency is reduced the flux will increase which results in saturation of the stator and rotor magnetic cores. This saturation in turn results in increase in magnetization current (no load current) which is undesirable. Hence it is required to maintain the air gap flux constant when supply frequency is changed. From the above expression for flux Φ we can see that this can be achieved by changing the Voltage also correspondingly so as to maintain a constant V/f ratio. Hence with V/f control method which ensures constant flux Φ, we can get smooth speed control.

Such a constant V/f with both variable voltage and frequency can be obtained using a electronic converter and an inverter as shown in the figure below.

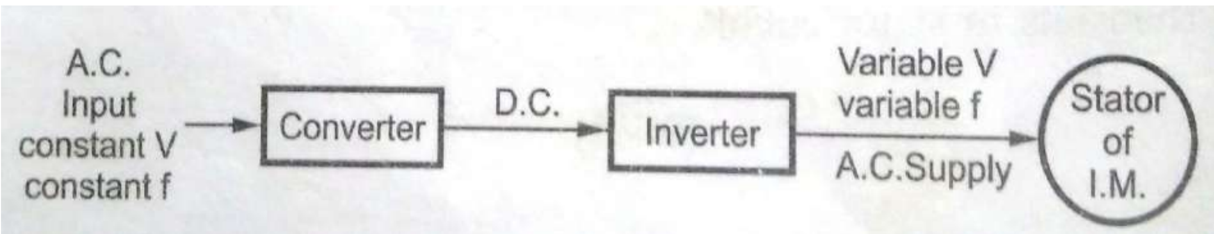


Figure: Electronic V/f control scheme

The converter converts the normal input power supply into DC. The inverter then converts the DC supply into a variable frequency supply as per the speed required but maintaining a constant V/f. If f_1 is the nominal frequency, then the figure below shows the Torque – slip characteristics with frequency $f_5 < f_4 < f_3 < f_2 < f_1$

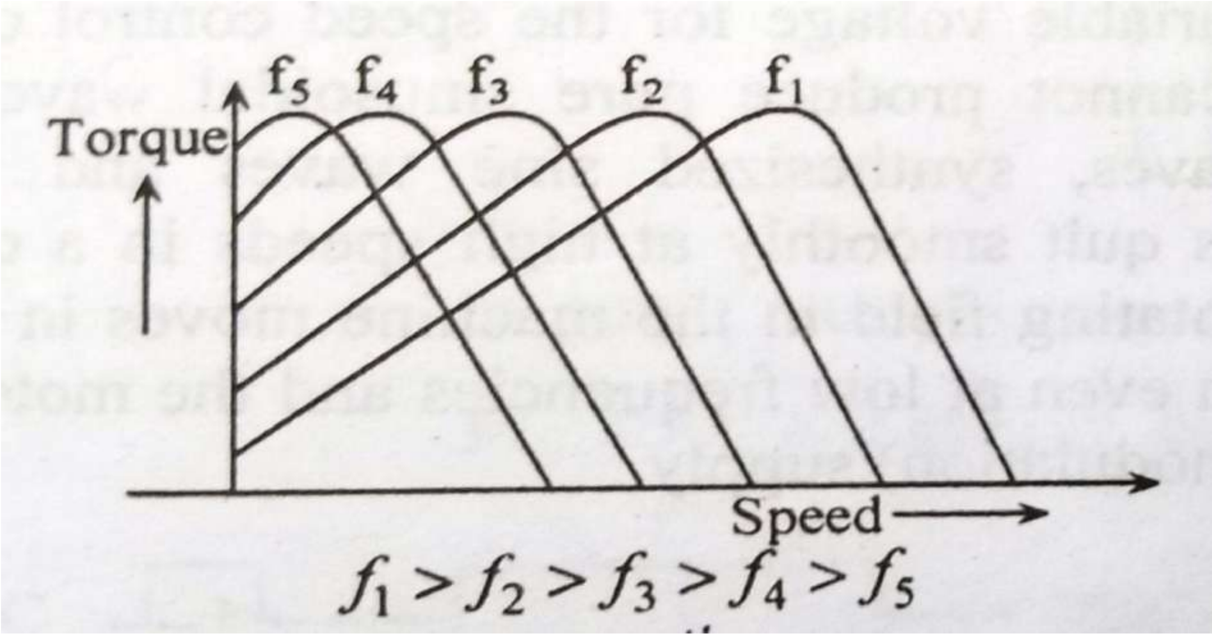


Figure: Torque – Speed Characteristics with variable f and constant V/f

Control of Induction Motors by Voltage Source Inverters:

An Inverter belongs to the VSI category if looking from the load side the AC terminals of the Inverter function as a Voltage Source. A voltage source has very ***low*** internal Impedance and the terminal voltage remains substantially constant with variations in load. Hence it is suitable for both single motor and multi motor drives. Any short circuit across its terminals causes current to rise very fast due to low internal impedance. The fault current cannot be regulated by current control and must be cleared by fast acting fuse links.

In a Voltage source Inverter the DC source is connected to the Inverter through a series Inductor $\mathbf{L_S}$ and a parallel capacitor **C**. The capacitance of **C** is sufficiently large that the Voltage would almost be constant. The output voltage waveform would be roughly a square wave since voltage is constant and the output current waveform would be approximately triangular. Voltage variations will be small but current can vary widely with variations in load.

The figure below shows the circuit diagram of a VSI employing transistors. Any other self commutating device can also be used instead of transistors. Generally MOSFETs are used in low voltage and low power inverters. IGBTs and power transistors are used up to medium power levels. GTOs and IGCTs (**I**nsulated **G**ate **C**ommutated **T**hyristors) are used for high power levels.

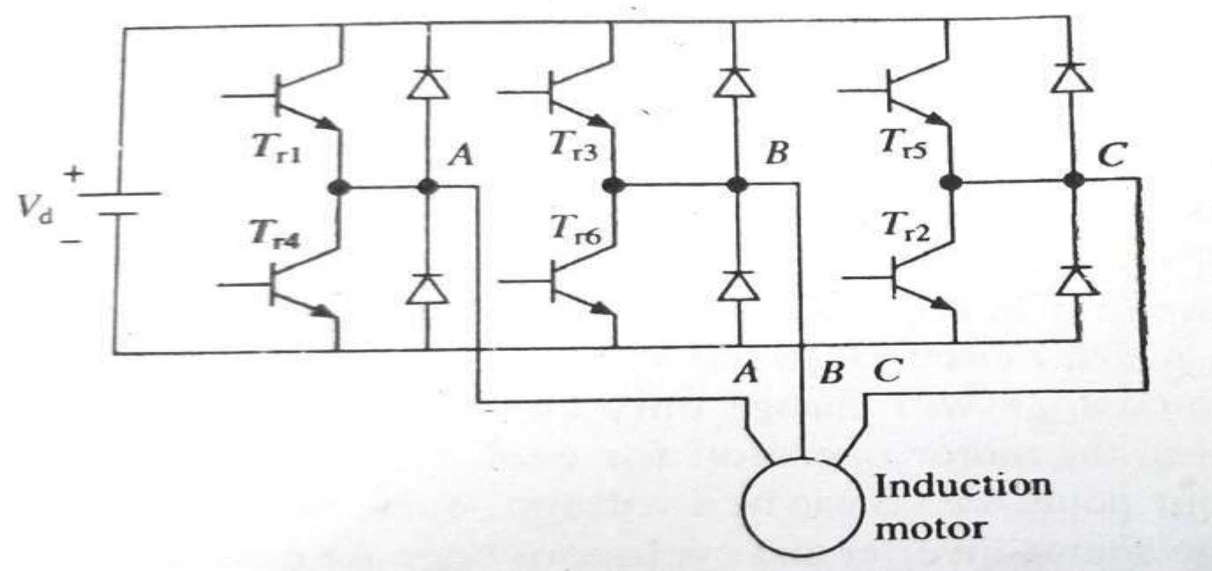


Fig: Circuit Diagram of a Transistor based Three Phase Voltage Source Inverter

VSI can be operated as a stepped wave Inverter or a PWM Inverter. When operated as a stepped wave Inverter, transistors are switched in the sequence of their numbers with a time difference of T/6 and each transistor is kept ON for a period of T/2.The resultant line voltage is shown in the figure (b) below. Frequency of operation is varied by varying the time period T and the output voltage of the inverter is varied by varying the DC input voltage.

The limitations of low frequency operation in Stepped wave Inverter can be eliminated in a PWM inverter by obtaining voltage control in the inverter itself. The inverter is supplied with a constant DC voltage and the inverter is controlled so that the average voltage is variable. In this method the operation of the inverter can be extended up to zero frequency as the commutation is effective at all frequencies.

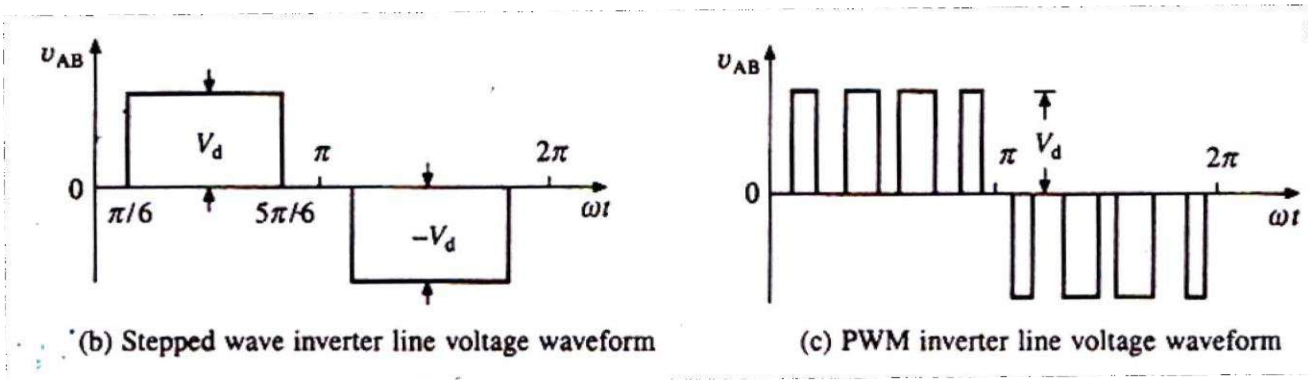


Fig: Stepped wave and PWM Inverter waveforms

In PWM the output voltage is no longer a square wave but a pulsed wave. This method results in a pure sinusoidal output if sinusoidal modulation is used. The output voltage waveform is shown in the figure (c) below.

The speed of an induction motor can be controlled using a DC or an AC source and four typical schemes of VSIs are shown and explained below with the figure shown below.

(a) The controlled rectifier varies the DC voltage to the inverter at the same time as the inverter output frequency is varied. The section between the DC source and the Inverter is known as the DC link and it includes a series Inductance and large capacitance which smoothes the DC voltage to an almost constant value, $\mathbf{E_{DC}}$. In this if the inverter is a six step Inverter the motor voltage is controlled by adjusting the DC link voltage.

(b) The above system cannot regenerate since current flow cannot be reversed. If regeneration is required it can be obtained by replacing the phase controlled rectifier with a Dual Converter as shown in figure **(b).**

(c) A system in which the DC link voltage is constant is shown figure **(c).** In this scheme the Inverter is a PWM based system and it varies both the voltage and the frequency.

(d) In the fourth scheme the variation of voltage is obtained by a chopper. Due to the chopper the harmonic injection into the AC supply is reduced. This scheme is a combination that is used when a high frequency output is required and hence a PWM inverter is not possible.

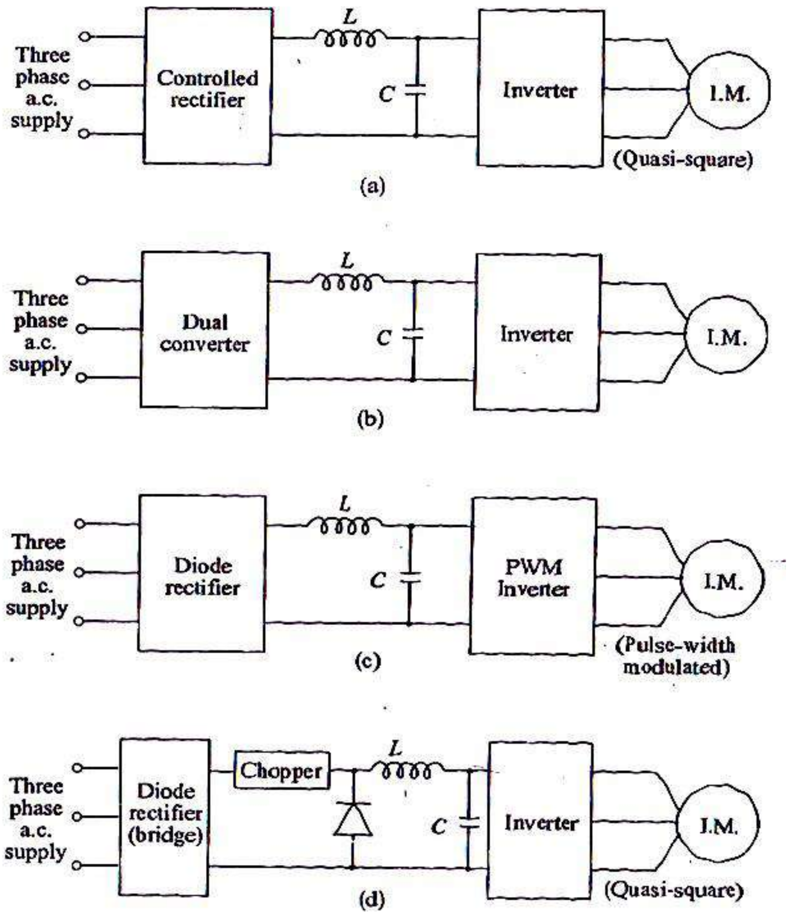


Fig: Schemes for Induction Motor speed control by VSIs

Control of Induction Motors by Current Source Inverter:

An Inverter belongs to the CSI category if looking from the load side the AC terminals of the Inverter function as a Current Source. A current source has ***large*** internal Impedance and hence the terminal voltage of a CSI changes substantially with change in load. If used in a multi motor drive a change in load would affect the other motor drives and hence a CSI is not suitable for multi motor drives. But since the inverter current is independent of load impedance it has inherent protection against short circuits across its terminals.

In a Current Source Inverter the DC source is connected to the Inverter through a large series Inductor $\mathbf{L_S}$ which would limit the current to be almost constant. The output current waveform would roughly be a square wave since current is constant and the output voltage would be approximately triangular. It is easy to limit the over current conditions in this system but the output voltage can swing widely in response to changes in load conditions.

A thyristor based **C**urrent **S**ource **I**nverter (CSI) is shown in the figure (a) below. This is a stepped wave inverter whose operation is already explained. Diodes D1- D6 and capacitors C1-C6 provide commutation of thyristors T1-T6 which are fired with a phase difference of 60 ° in sequence of their numbers. Figure (b) below shows the nature of output current waveforms. The inverter behaves as a current source inverter due to the presence of the large Inductor in the DC link.

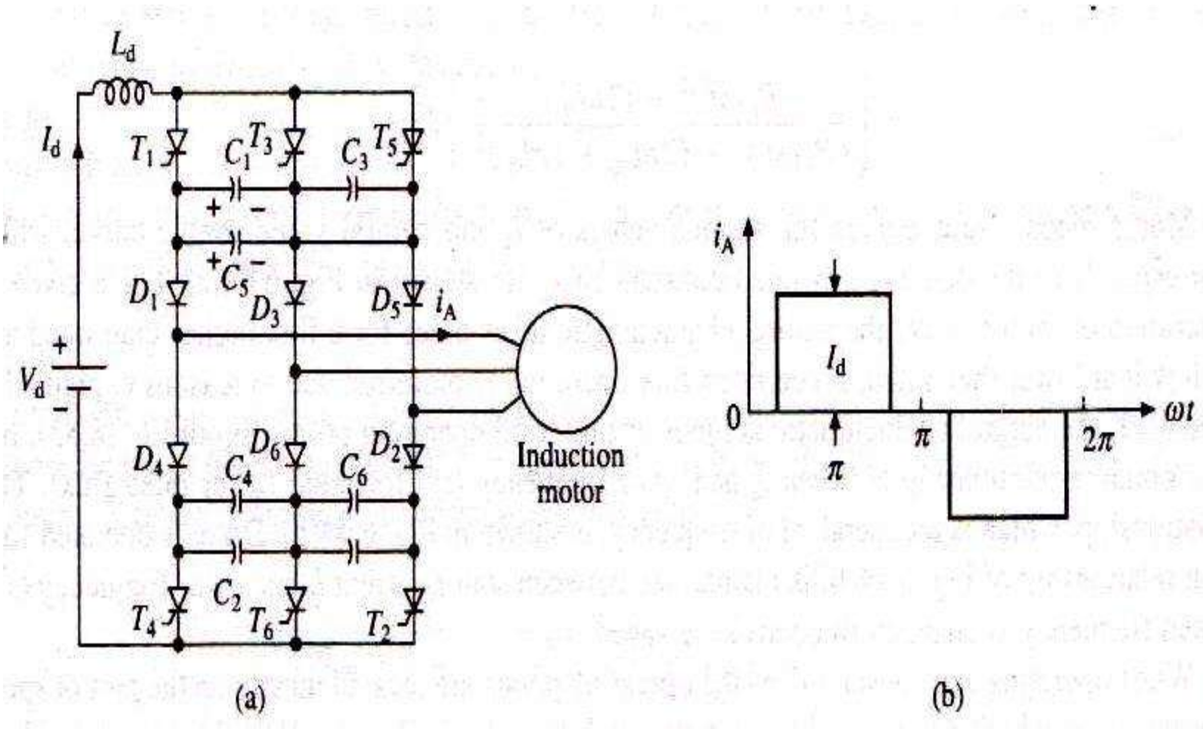


Fig: (a) Circuit diagram of a Current Source Inverter (b) Current waveform

The fundamental component of motor phase current from figure (b) is given by

$$\mathbf{I_s = (\sqrt{6}/\pi).I_d}$$

For a given speed, torque is controlled by varying the DC link current $\mathbf{I_d}$ by changing the value of $\mathbf{V_d}$. Hence when supply is AC, a controlled rectifier is connected between the supply and Inverter. When the supply is DC a chopper is connected between the supply and Inverter as shown in the figure (b) below. The maximum value of DC output voltage of the fully controlled rectifier and chopper are chosen such that the motor terminal voltage saturates at rated value.

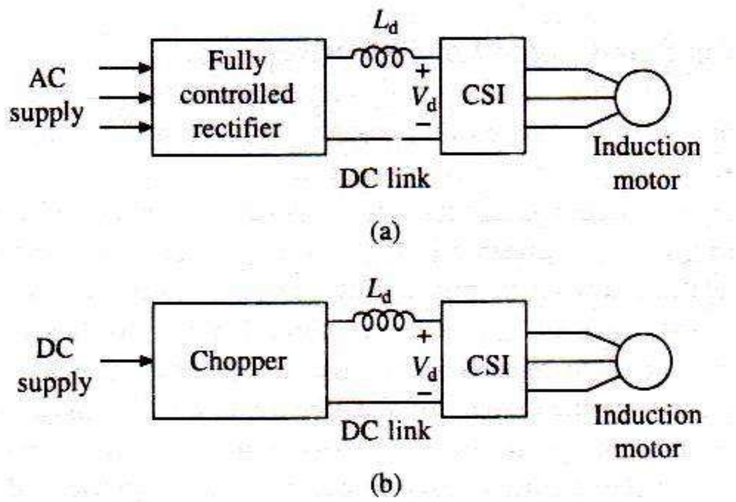
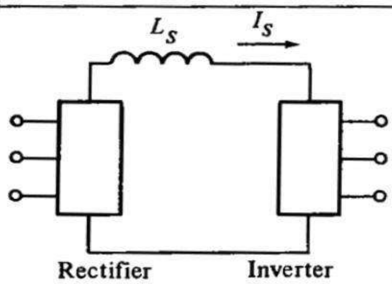
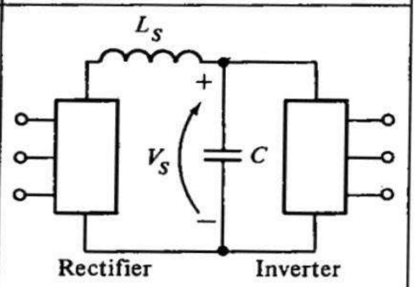
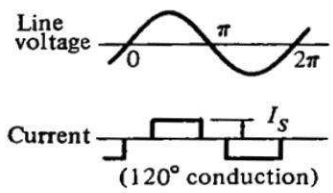
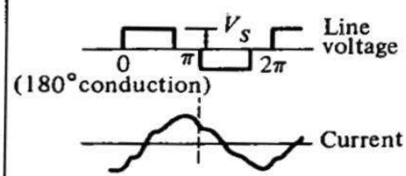


Fig: Different configurations of CSI Induction motor drives.

Comparison of VSIs with CSIs:

	Current source inverter	Voltage source inverter
Main circuit configuration	 L_S, I_S, Rectifier, Inverter	 L_S, V_S, C, Rectifier, Inverter
Type of source	Current source $-I_s$ almost constant	Voltage source$-$ V_S almost constant
Output impedance	High	Low
Output waveform	 Line voltage, 0, π, 2π, Current, I_S, (120° conduction)	 V_S, Line voltage, 0, π, 2π, (180°conduction), Current
Characteristics	1. Easy to control overcurrent conditions with this design 2. Output voltage varies widely with changes in load	1. Difficult to limit current due to capacitor 2. Output voltage variations small due to capacitor

- The major advantage of CSI is its reliability. In case of VSIs a commutation failure would cause the switching devices in the same leg to conduct simultaneously. This causes a shorting of the source voltage and hence the current through the devices would rise to very high levels. Expensive high speed semiconductor fuses are required to be used to protect the devices.
- In case of CSIs simultaneous conduction of two devices in the same leg will not lead to sudden rise of current due to the presence of the large Inductance. This allows time for commutation to take place and normal operation will get restored in the subsequent cycles. Further less expensive HRC fuses are good enough for protection of thyristors.
- As seen in the CSI current waveforms, the motor current rise and fall are very fast. Such a fast rise and fall of current through the motor leakage Inductance of the motor produces large voltage spikes. Therefore a motor with low leakage reactance is used. Even then voltage spikes could be large. The commutation capacitors C1-C6 reduce the voltage spikes to some extent by limiting the rise and fall of current. But large values of capacitors are required to substantially reduce the voltage spikes. Large values of commutation capacitors have the advantage that cheap converter grade thyristors can be used but then they reduce the frequency range of the inverter and hence the speed range of the drive.
- Further, due to large values of Inductors and capacitors, the CSI drive is expensive *and will have more weight and volume.*

Cycloconverter:

Cycloconverter is a device for directly converting AC power at one frequency to AC power at another frequency. The input to cycloconverter is a three phase source which consists of three AC voltages equal in magnitude and phase shifted from each other by 120°. The output is the desired frequency at the required voltage and power level.

As we know, in a three phase full converter the mean output DC voltage is maximum with a firing angle of 0° and is zero with a firing angle of 90° and is negative maximum with a firing angle of 180°. In between it varies from

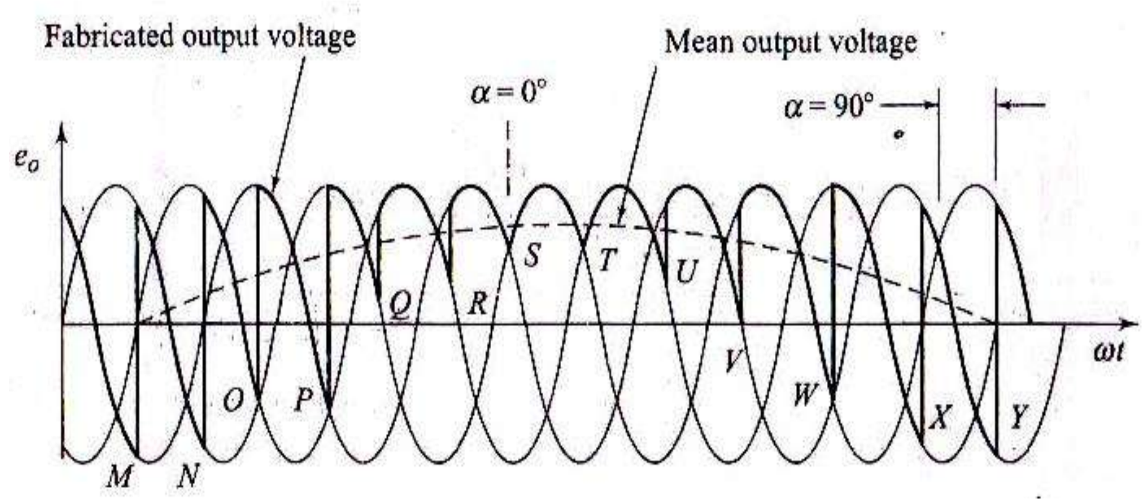


Fig: Fabricated and mean output voltage waveform for a single phase Cycloconverter (half cycle)

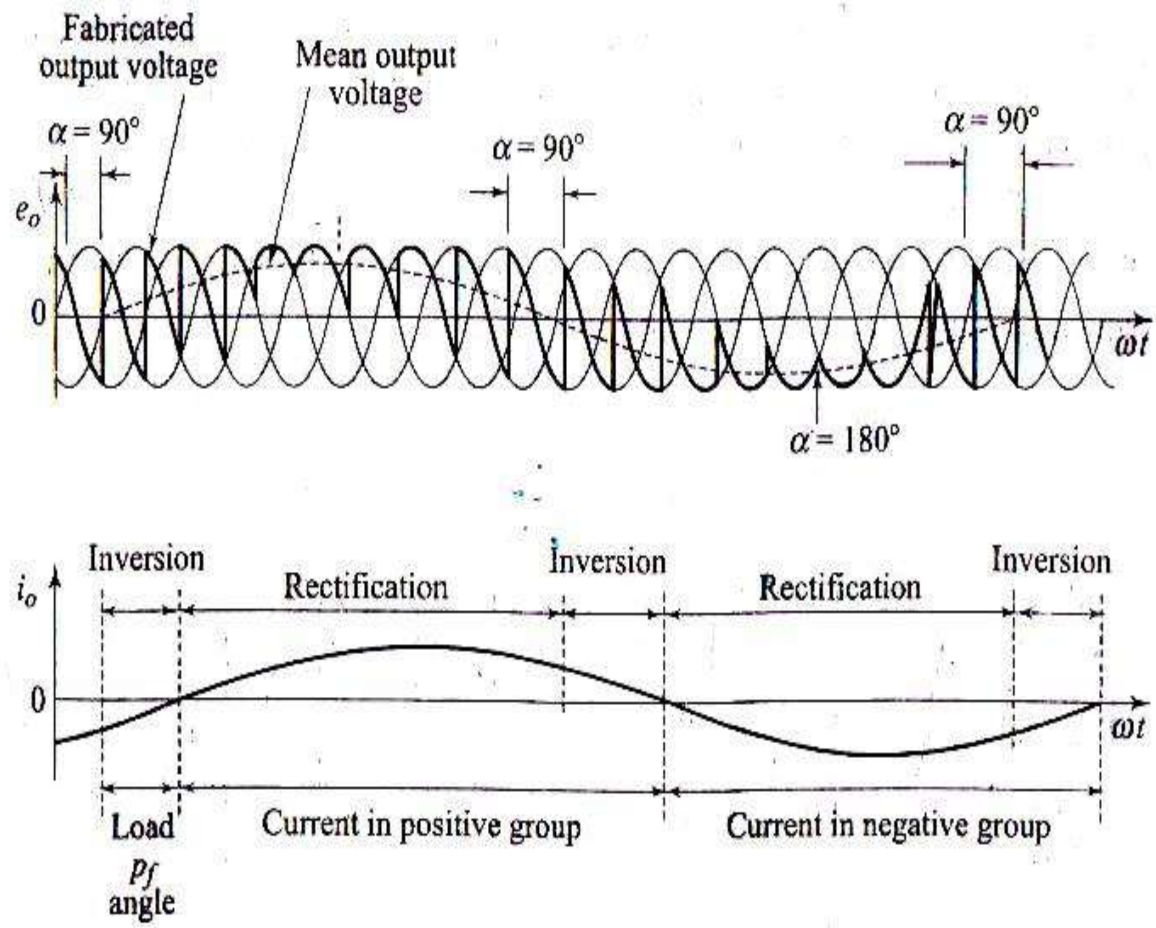


Fig: Fabricated and mean output voltage waveform for a single phase Cycloconverter (full cycle)

positive maximum to negative maximum with corresponding firing angle variation. Cycloconverter makes use of this basic principle and generates its output voltage by selecting the combination of the three phases which are made to closely approximate the desired single phase output by varying the firing angle continuously in accordance with a control signal. The control signal is the low level frequency of the desired output.

The synthesized (fabricated) output voltage from the three phases along with the corresponding desired mean output voltage for half cycle and full cycle for one phase are shown in the figure below.

A full three phase Cycloconverter is made up of three such cycloconverters connected together as shown in the figure below utilising half wave converters connected in anti parallel in a circulating current mode as shown in the subsequent figures below.

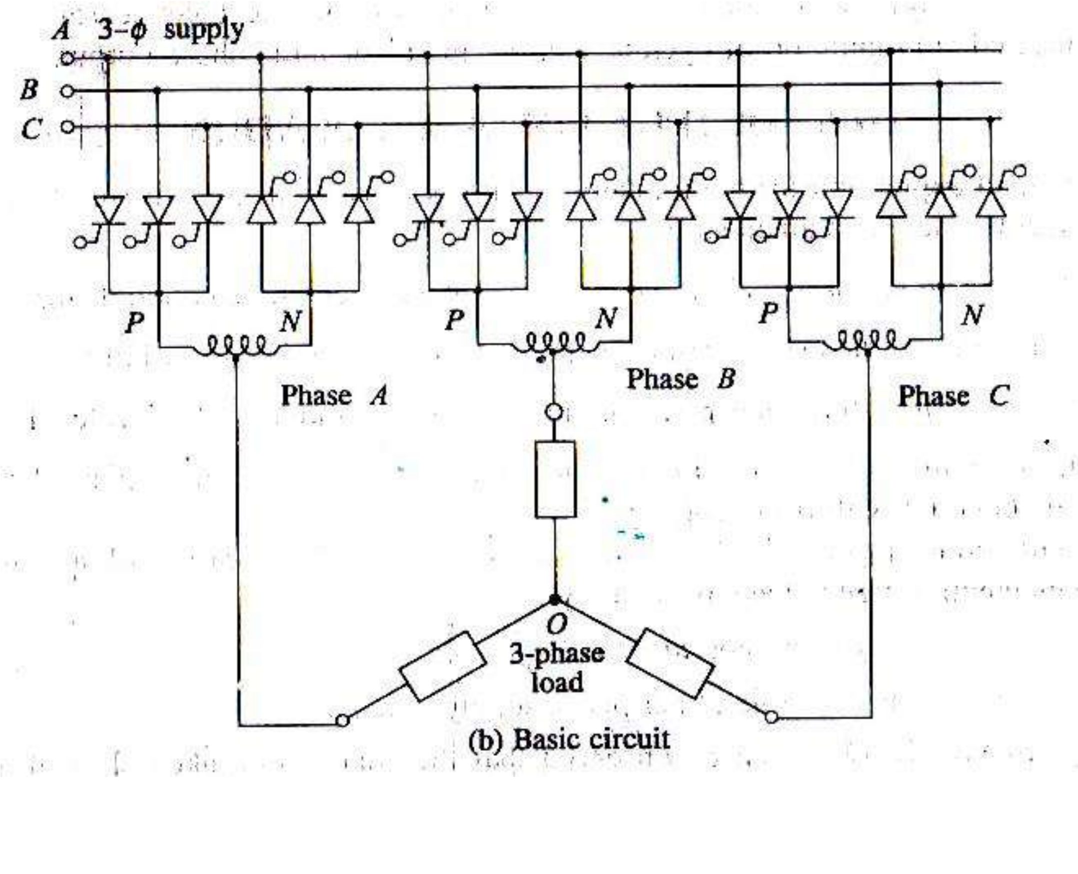


Fig: Three phase to Three phase Cycloconverter basic circuit diagram

Closed loop speed control with VSI/Cycloconverter based Induction Motor drives:

A closed loop speed control system is shown in the figure below. It employs a slip speed inner loop and an outer speed loop. Since for a given slip speed, current and Torque are constant, slip speed inner loop is used in place of inner current loop. Further it ensures that speed of operation is always on that portion of the Speed Torque curve between synchronous speed and the speed at maximum Torque for all frequencies. This ensures high Torque to current ratio. The drive shown here uses a PWM inverter fed from a DC source which has capability for regenerative braking and four quadrant operation. This scheme is applicable to any of the VSI or Cycloconverter drives as well which has Regenerative or dynamic braking capability. The closed loop operation is explained below.

The speed error is processed through a PI controller and a slip regulator. PI controller is used to get good steady state accuracy. The slip regulator sets the slip speed command ω^*_{sl} whose maximum value is limited to limit the inverter current to a permissible value. The synchronous speed obtained by adding actual speed ω_m and slip speed ω^*_{sl} determines the inverter frequency. The reference signal for the closed loop control of the machine terminal voltage V* is generated from frequency f using a function generator which ensures a constant flux operation up to base speed and operation at constant terminal voltage above base speed.

A step increase in speed command ω^*_m produces a positive speed error. The slip speed command ω^*_{sl} is set to the maximum positive value. The drive accelerates at the maximum permissible inverter current producing maximum available torque until the speed error is reduced to a small value. The drive finally settles at a slip speed for which the motor torque balances the load torque.

A step decrease in speed command ω^*_m produces a negative speed error. The slip speed command ω^*_{sl} is set to the maximum negative value. The drive decelerates under regenerative braking at the maximum permissible inverter current producing maximum available braking torque until the speed error is reduced to a small value. The drive finally settles at a slip speed for which the motor torque balances the load torque.

With this scheme the drive has fast response because the speed error is corrected at the maximum available torque. Direct control of slip assures stable operation under all operating conditions.

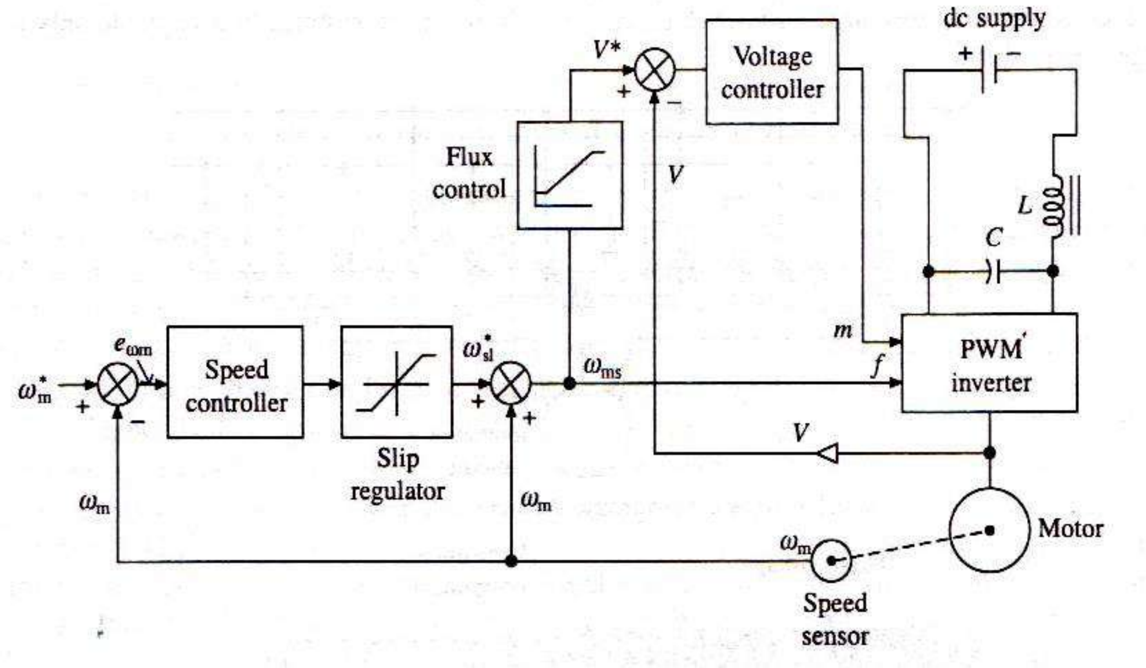


Fig.: Closed loop slip controlled VSI Induction motor drive with PWM inverter.

Summary:

Important concepts and conclusions:

- Synchronous speed of an induction motor is directly proportional to the supply frequency. Hence by changing the supply frequency the synchronous speed and hence the motor speed can be varied.
- The motor terminal voltage is proportional to the product of the frequency and the flux neglecting the stator voltage drop as given by the relation: **v(t) α ω.ϕ.** Hence any reduction in the supply frequency without a corresponding reduction in the Stator voltage would cause an

increase in the air gap flux and a corresponding increase in the magnetisation current which is not desirable.

- Hence to avoid excessive magnetisation currents and also to maintain the torque constant, variable frequency control below the base speed is normally carried out by reducing the stator voltage along with frequency in such a manner that magnetic flux is maintained constant. This method is called constant V/f control. But above the base speed, the stator voltage is maintained constant because of the limit imposed by the stator insulation or by supply voltage limitations and hence the developed torque would come down.
- The two important systems of Induction motor speed control using variable frequency are **V**oltage **S**ource **I**nverters (VSI) and **C**urrent **S**ource **I**nverters (CSI).
- The important type of Inverters used in these systems are Quasi Square Wave Inverters (QSW), Pulse Width Modulated Inverters (PWM) and Cycloconverters.

Illustrative examples:

Example-1: A 3ϕ, 415 V, 50 Hz, 4 pole, 1460 RPM, star connected induction motor has the following parameters. $R_1 = 0.65\ \Omega$; $R_2 = 0.35\Omega$; $X_1 = 0.95\ \Omega$, $X_2 = 1.43\ \Omega$, $X_m = 28\ \Omega$. Motor speed is controlled by varying stator Voltage and frequency keeping the V/f ratio constant at the rated condition. Determine the maximum Torque and speed at which it occurs for stator frequencies (a) 50 Hz (b) 35 Hz (c) 10 Hz.

Solution: Given data: $V_L = 415$ V, f = 50 Hz, P =4, $N_r = 1460$ RPM, Stator STAR connected and $R_1 = 0.65\ \Omega$; $R_2 = 0.35\Omega$; $X_1 = 0.95\ \Omega$, $X_2 = 1.43\ \Omega$, $X_m = 28\ \Omega$

To understand how to work out the problem, the following points are to be noted first:

- It is not mentioned whether the rotor parameters R and X are referred to stator. But we know that in the equivalent circuit and in the corresponding formulae they are normally considered to be referred to stator. Since the stator/rotor turns ratio is required for calculating stator referred parameters and it is not given we can assume that the given rotor parameters are referred to stator.

- Since the stator is connected in STAR, the given stator voltage of 415 V is line voltage. Hence : $V_{ph} = 415/\sqrt{3} = 239.6$ V
- The given reactance parameter values X_1 and X_2, (which are frequency dependent) though not mentioned specifically, we can always take them to be at the rated frequency of 50 Hz. Their values at the other required frequencies of 35 Hz and 10 Hz are to be scaled down correspondingly.
- It is important to note that for the two lower frequencies the applied voltage V_{ph} is to be scaled down correspondingly so as to maintain constant V/f as specified in the problem.
- The synchronous speed for the two lower frequencies is also to be scaled down.
- Then, using these values appropriately, the maximum Torque and the speed at which it occurs for stator frequencies (a) 50 Hz (b) 35 Hz (c) 10 Hz. can be found one by one using the following formulae. (the formula for 'Slip @ maximum Torque' is also given below since it is required to find out the 'Speed @maximum Torque'

The slip @maximum torque, the speed @maximum torque and the maximum torque are given by:

- Slip @ maximum Torque: $s_m = R_2 / [R_1^2 + (X_1+X_2)^2]^{1/2}$
- Speed @maximum Torque: $N_{r@max\,\tau} = N_s(1 - s_m)$
- Maximum Torque: $T_{d\,max} = 3V_{ph}^2 / 2\omega_s[R_1 + \{R_1^2 + (X_1+X_2)^2\}^{1/2}]$

Now, substituting the corresponding values from the above data we can find out the above three for the three frequencies.

(a) 50 Hz:

Synchronous speed (RPM) : $N_s = 120f/P = 120 \times 50/4 = 1500$ RPM
Synchronous speed (Rad/sec): $\omega_s = 1500 \times 2\pi/60 = 157$ Rad/sec

i. Slip @ maximum Torque: $s_m = 0.35/[0.65^2 + (0.95+1.43)^2]^{1/2} = 0.142$

ii. Speed @max Torque: $N_r = N_s (1- s_m) = 1500 (1- 0.142) = 1287$ RPM

iii. Maximum torque : $T_{d\,max} = 3 \times 239.6^2 / 2\times157 [0.65 + \{0.65^2+ (0.95+1.43)^2\}^{1/2}]$

$= 175.96$ N-m

(b) 35 Hz:

Synchronous speed (RPM): $N_s = 120f/P = 120 \times 35 /4 = 1050$ RPM

Synchronous speed (Rad/sec): $\omega_s = 1050 \times 2\pi /60 = 109.95$ Rad/sec

R_1 and R_1 remain same.

But $X_1 = 0.95 \times 35/50 = 0.665\ \Omega$ and $X_2 = 1.43 \times 35/50 = 1.0\ \Omega$

$V_{ph} = 239.6 \times 35/50 = 167.72$ V

i. Slip @ maximum Torque: $s_m = 0.35/ [0.65^2+ (0.665+1.0)^2]^{1/2} = 0.1957$

ii. Speed @max Torque: $N_r = N_s (1- s_m) = 1050 (1- 0.1957) = 844.515$ RPM

iii. Maximum torque : $T_{d\,max} = 3 \times 167.72^2 / 2\times109.95 [0.65 + \{0.65^2+ (0.665+1)^2\}^{1/2}]$

$= 84389.99/219.9 (0.65+1.7883) = 157.38$ N-m

(a) 10 Hz:

Synchronous speed (RPM) : $N_s = 120f/P = 120 \times 10/4 = 300$ RPM

Synchronous speed (Rad/sec): $\omega_s = 300 \times 2\pi /60 = 31.4$ Rad/sec

R_1 and R_1 remain same.

But $X_1 = 0.95 \times 10/50 = 0.19\ \Omega$ and $X_2 = 1.43 \times 10/50 = 0.286\ \Omega$

$V_{ph} = 239.6 \times 10/50 = 47.92$ V

i. Slip @ maximum Torque: $s_m = 0.35/ [0.65^2+ (0.19+0.286)^2]^{1/2}$

$= 0.35/ 0.8056 = 0.434$

ii. Speed @max Torque: $N_r = N_s (1- s_m) = 300 (1- 0.434) = 169.8$ RPM

iii. Maximum torque : $T_{d\,max} = 3 \times 47.92^2 / 2 \times 31.4\ [0.65 + \{0.65^2 + (0.19+0.286)^2\}^{1/2}]$

$= 6888.979/62.8\ (0.65+0.805) = 75.37$N-m

Example-2: For a 3-phase delta connected 6-pole 50 Hz 400 V, 925 rpm squirrel cage induction motor is having R_s= 0.2 Ω, R_r= 0.3 Ω , X_s= 0.5, and X_r= 1.1 Ω . The motor is operated from a voltage source inverter with constant V/f ratio form 0 to 50 Hz and having the constant voltage of 400 V above 50 Hz frequency. Calculate (i) speed for a frequency of 35 Hz with half full load torque (ii) torque for a frequency of 35 Hz for a speed of 650 rpm. **(April-2018 JNTU)**

Solution: Given data: V_L = 400 V, f = 50 Hz, P =6, N_r = 925 RPM, Stator DELTA connected and R_1 = 0.2 Ω; R_2 = 0.3Ω; X_1 = 0. 5 Ω, X_2 = 1.1 Ω

The following points are to be noted first:

- It is not mentioned whether the rotor parameters R and X are referred to stator. But we know that in the equivalent circuit and in the corresponding formulae they are considered to be referred to stator. Since the stator/rotor turns ratio is required for calculating stator referred parameters and it is not given we can assume that *the given rotor parameters are referred to stator.*
- Since the stator is connected in DELTA, the given stator voltage of 400 V is line voltage as well as Phase voltage . Hence : V_{ph} = 400 V
- The given reactance parameter values X_1 and X_2, (which are frequency dependent) though not mentioned specifically, we can always take them to be at the rated frequency of 50 Hz. Their values at the other required frequency of 35 Hz are to be scaled down correspondingly.
- It is important to note that for the lower frequency of 35 Hz the applied voltage V_{ph} is also to be scaled down correspondingly so as to maintain constant V/f as specified in the problem.

(i) Calculation of speed for a frequency of 35 Hz with half full load torque: We have the standard relation for the Torque developed in an Induction motor as:

$$\mathbf{T_d = (3V^2_{ph} \cdot R_2 /s)/ \omega_s\ [(R_1 + R_2 /s)^2 + (X_1 + X_2)^2]}$$

A close observation of this equation indicates that:

- If we can first find out 'half full load torque' then we can find out the slip @35 Hz and then we can get the speed corresponding to a frequency of 35 Hz with half full load torque.
- And full load torque can be found out by using the same relation with data corresponding to the full load condition. (Same as rated values)

Calculation of Full load (rated) Torque:

Synchronous speed (RPM): $N_s = 120f/P = 120 \times 50/6 = 1000$ RPM

Synchronous speed (Rad/sec): $\omega_s = 1000 \times 2\pi/60 = 104.7$ Rad/sec

Slip 's' @ full load = (1000 – 925)/1000 = 0.075

Now using these values along with the given data in the above equation for the developed torque we can find out the Full load Torque. $R_1 = 0.2\ \Omega$; $R_2 = 0.3\Omega$; $X_1 = 0.5\ \Omega$, $X_2 = 1.1\ \Omega$

$$T_d = (3V_{ph}^2 . R_2/s)/\ \omega_s [(R_1 + R_2/s)^2 + (X_1 + X_2)^2]$$

$$T_{FL} = (3 \times 400^2 \times 0.3/0.075)/\ 104.7\ [(0.2 + 0.3/0.075)^2 + (0.5 + 1.1)^2]$$

$$= (480000 \times 4)/\ 104.7\ [(0.2 + 4)^2 + (1.6)^2] = 1920000 / 104.7 \times 20.2 = 907.8 \text{ N-m}$$

Full load (rated) Torque = 907.8 N-m

Half full load torque = 907.8/2 = 453.9 N-m

Before we use this value in the equation for T_d, we have to find out the other required parameters which are frequency dependent i.e. N_s, ω_s, X_1 and X_2 @35 Hz and also $V_{ph@}$ 35 Hz

$N_{s\ @35\ Hz} = \omega_{s\ @50\ Hz} \times 35/50 = 1000 \times 35/50 = 700$ RPM

$\omega_{s\ @35\ Hz} = \omega_{s\ @50\ Hz} \times 35/50 = 104.7 \times 35/50 = 73.29$ Rad/sec

$X_{1\ @35\ Hz} = X_{1@50\ Hz} \times 35/50 = 0.5 \times 35/50 = 0.35\ \Omega$

$X_{2\ @35\ Hz} = X_{2@50\ Hz} \times 35/50 = 1.1 \times 35/50 = 0.77\ \Omega$

$V_{ph@\,35\,Hz} = V_{ph@\,50\,Hz}$ x 35 /50 = 400 x 35/50 = 280 V

Now we can use these values in the equation for T_d and find out the slip at 35 Hz

$T_d = (3V^2_{ph}\, R_2/s)/\ \omega_s[(R_1 + R_2/s)^2+(X_1+X_2)^2]$

Let K = R_2 /s.Then

453.9 = 3 x 280 ^{2} x K / 73.29 [(0.2+ K)^2 + (0.35+0.77)^2]

453.9 = 235200 K / 73.29 [(0.04+ K^2+ 0.4K) + 1.2544] = 235200 K / 73.29 [(1.2944+ K^2 + 0.4K)]

453.9 = 235200 K / [(73.29 x 1.2944 + 73.29 K^2 + 29.32 K)]

From which we get: 43060 + 33266K^2 - 221892K = 0

K^2 – 6.67K + 1.2944 = 0

K = [6.67 +/_ (6.67^2 – 5.18)$^{1/2}$] /2 = [6.67 +/- 6.27]/2 = 12.94/2 or 0.4/2 i.e. 6.47 or 0.2

But K = R_2 /s and hence R_2 /s = 6.47 or 0.2 or s = 0.3 /6.47 or 0.3/0.2 = 0.046 or 1.5

But slip cannot be larger than 1 and hence slip's' at half full load torque = 0.046

Speed with 35 Hz supply frequency and half full load torque = Synchronous speed $_{@35Hz}$x (1-s)

= 700 x (1-0.046) = 700 x 0.954 = 668 RPM

Speed for a frequency of 35 Hz with half full load torque = 668 RPM

(ii) Torque for a frequency of 35 Hz for a speed of 650 rpm:
This can be found out directly by using the formula for torque developed i.e.

$$\mathbf{T_d = (3V^2_{ph}\ .\ R_2/s)/\ \omega_s\ [(R_1+ R_2/s)^2+(X_1+X_2)^2]}$$

and using the same parameters which we have already obtained for 35 Hz and given below again. But slip alone is required to be calculated taking $N_{s@35\,Hz}$ =700 RPM and N = 650 RPM

$\omega_{s\ @35\ Hz} = \omega_{s\ @50\ Hz}$ x 35/50 = 104.7 x35/50 = 73.29 Rad/sec

$X_{1\ @35\ Hz} = X_{1@50\ Hz}$ x 35/50 = 0.5 x 35/50 = 0.35 Ω

$X_{2\ @35\ Hz} = X_{2@50\ Hz}$ x 35/50 = 1.1 x35/50 = 0.77 Ω

$V_{ph@\ 35\ Hz} = V_{ph@\ 50Hz}$ x 35 /50 = 400 x 35/50 = 280 V

$s_{@35\ Hz} = (N_{s@35\ Hz} – N)/N_{s@35\ Hz}$ = (700-650)/700 = 50/700 = 0.071

Thus: $T_d = (3V^2_{ph} \cdot R_2/s)/\ \omega_s[(R_1 + R_2/s)^2+(X_1 +X_2)^2]$

$= (3 \times 280^2 \times 0.3 /0.071)/\ 73.29[(0.2 + 0.3 /0.071)^2+(0.35+0.77)^2]$

$= (235200 \times 4.225)/\ 73.29[(0.2+ 4.225)^2 + (1.12)^2]$

= 993720 / 73.29(19.58+1.25) = 993720 / (73.29 x 20.83) **= 650.92 N-m**

Torque for a frequency of 35 Hz for a speed of 650 rpm = 650.92 N-m

xxxxx

UNIT-V

SYLLABUS/CONTENTS:

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- Static Rotor Resistance Control
- Slip Power Recovery
- Static Scherbius Drive
- Static Kramer drive
  - Their Performance
  - Speed -Torque Characteristics
  - Advantages
  - Applications
  - Problems
- Summary
  - Important concepts and conclusions
- Illustrative Examples

PART-2: CONTROL OF SYNCHRONOUS MOTORS :

- Introduction
- Separate control and self control of Synchronous Motors
- Operation of Self controlled Synchronous Motors by VSI and CSI Cycloconverters
- Load commutated CSI fed Synchronous Motor:
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- Closed loop operation of Synchronous motor drives (Block Diagram only)
- Variable frequency control, Cycloconverter, PWM, VSI, CSI

PART-1: CONTROL OF INDUCTION MOTORS FROM ROTOR SIDE:

Static Rotor resistance control:

Introduction to Rotor Resistance Control:

Before explaining the static Rotor resistance control a brief introduction to the basic method of ***Rotor resistance control*** is given here. The speed of an Induction motor can be controlled by the introduction of an external resistance in the Rotor circuit as shown in the figure below.

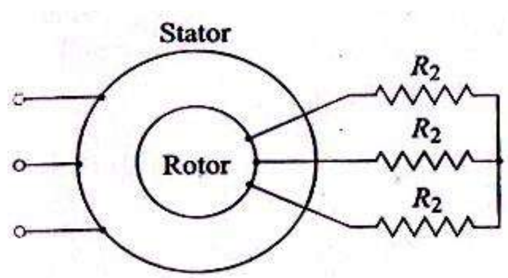


Fig: External Rotor resistances connected in a Slip Ring Induction Motor

The speed-Torque characteristics of an Induction motor with such a control are shown in the figure below.

Before studying /analyzing these characteristics, the basic Torque speed relations in an induction motor (***considering only the Rotor circuit parameters***) what we have learnt earlier are given here for a quick reference. These relations are the basis for the nature of the characteristics shown in the figure below .

- *Torque developed by the motor* T_d: $T_d = k\,[s E_2^2 R_2 / R_2^2 + (sX_2)^2]$
- *Slip at maximum Torque* S_m: 's_m' $= R_2 / X_2$
- *Maximum developed torque* T_{max}: $T_{max} = kE_2^2 / 2X_2$
- *Starting Torque* T_{st} : $T_{st} = k\,[E_2^2 R_2 / R_2^2 + X_2^2]$

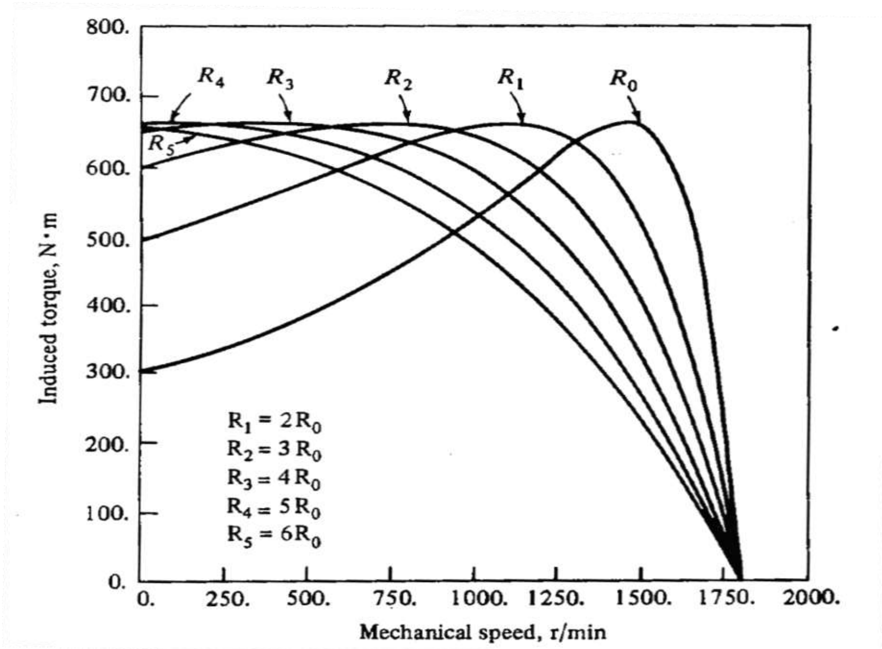


Fig: Induction Motor Torque-Speed Characteristics with variation of Rotor Resistance.

A study of the above relations along with the characteristics shows that:

- For a given Load torque, the motor speed is reduced (since slip **s** Increases) as the Rotor resistance is increased. However the no load speed remains the same with the variation of the Rotor Resistance.
- The increase in rotor resistance does not affect the value of the maximum Torque but increases the value of Slip at which it occurs.
- With increase in Rotor resistance the starting torque increases and the starting current reduces. Hence the ***Torque to current ratio*** improves.

Advantages and disadvantages of Rotor resistance control:

- External resistors can be added only during the accelerating period to increase the starting torque and can be removed later during the steady state. This minimises the losses with dissipation in external resistors.
- The rotor temperature rise is substantially lower than it would have been if the higher resistance were incorporated in the rotor winding as in the case

of squirrel cage motors. This allows the optimum utilisation of the motor torque capabilities.

- It provides a constant torque operation with high Torque to current ratio.
- Though Rotor copper losses increase with decrease in speed most of it is dissipated in the external resistors. The copper losses inside the motor remains constant for a given fixed torque. Because of this,a motor of smaller size can be employed.
- **Motor efficiency decreases and the rotor copper losses increase with the decrease in speed**.

  This is the main disadvantage and hence to overcome this, static Rotor resistance control is adopted.

Static Rotor Resistance control with a Chopper:

Instead of mechanically varying the Rotor Resistance or electrically by using contactors it can be varied electronically by using a chopper as shown in the figure below. This gives a stepless and smooth variation of Resistance and hence the Speed of the motor. In this system the external resistor is introduced in the rotor circuit after converting the slip power into DC using a three phase bridge rectifier instead of directly connecting in the rotor circuit. Along with the resistor a chopper is also connected in parallel. By switching the chopper ON and OFF at a high frequency the effective value of the Resistance is controlled smoothly. As $\mathbf{T_{on}}$ is changed from 0 to its full time period of **T t**he resistance changes from **R** to **0**.In terms of the duty ratio **δ** of the Chopper the effective value of the resistance $\mathbf{R_E}$ introduced into the Rotor is given by :

$$\mathbf{R_E = (1 - \delta).R}$$

A filter inductor $\mathbf{L_d}$ is provided in series between the rectifier and the external resistor to smoothen the current $\mathbf{I_d}$**.** A higher ripple in I_d produces higher harmonics in the rotor current and hence the rotor copper losses will increase. ***The diode bridge is the main contributor for the ripple and not the Chopper Switch since it operates at a relatively higher frequency.***

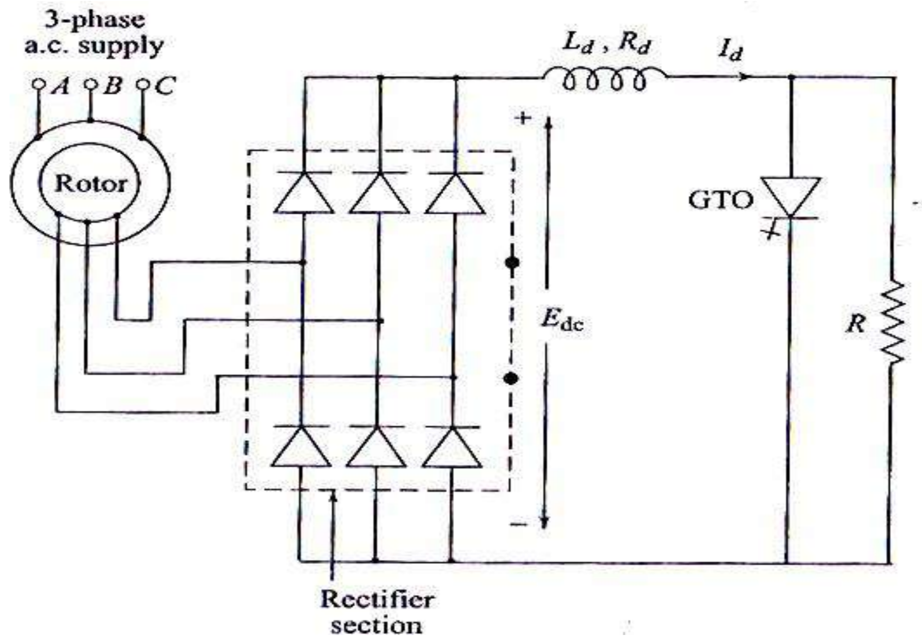


Fig: Induction Motor Speed control using a chopper

A filter inductor $\mathbf{L_d}$ is provided in series between the rectifier and the external resistor to smoothen the current $\mathbf{I_d}$**.** A higher ripple in I_d produces higher harmonics in the rotor current and hence the rotor copper losses will increase. ***The diode bridge is the main contributor for the ripple and not the Chopper Switch since it operates at a relatively higher frequency.***

The Diode Bridge output $\mathbf{E_{DC}}$ changes from its maximum value at standstill to about 5 % at near motor rated speed. Here a Thyristor is not suitable as a Switch since reliable commutation at a higher switching frequency can be obtained only by external commutating circuits which would be bulky and expensive.

The DC voltage $\mathbf{E_{DC}}$ is small because Induction motors are usually designed with stator to Rotor turns ratio of greater than 1. Hence a Transistor switch is good enough for low power drives and **GTO** can be used for ratings beyond the capability of Transistors. Self commutation capability of these devices ensures reliable commutation at all operating points and makes the Semiconductor switch compact.

Slip Power Recovery:

We have seen that In the Rotor resistance control method, the slip power which increases with decreasing speed gets dissipated in the resistance and hence the efficiency of the system gets reduced at lower speeds. The mechanical power that can be obtained from the Air gap power is with a per unit conversion efficiency of (1—s) and the overall motor efficiency would still be lesser than this. The Air gap power is almost totally dissipated as heat in the Rotor circuits at lower speeds and hence the efficiency would be very poor. Therefore the Rotor resistance method of speed control is very inefficient except for a very small speed range close to the synchronous speed.

However instead of dissipating the slip power in the resistance, if it can be conveniently returned to the mains or effectively utilized to increase the drive power then the Drive system becomes more efficient. This is achieved by means of two widely used ***slip power recovery*** methods known as **Scherbius** and **Kramer** drives. They are also called as ***cascade drives***.

Scherbius drive :

In the traditional **Scherbius** drive shown in the figure below a rotary converter rectifies the slip power and the rectified output drives a DC motor which is coupled to a squirrel cage Induction Generator. The Induction generator is driven at super synchronous speeds and returns the slip power to the same mains supply which gives supply to the Induction motor drive.

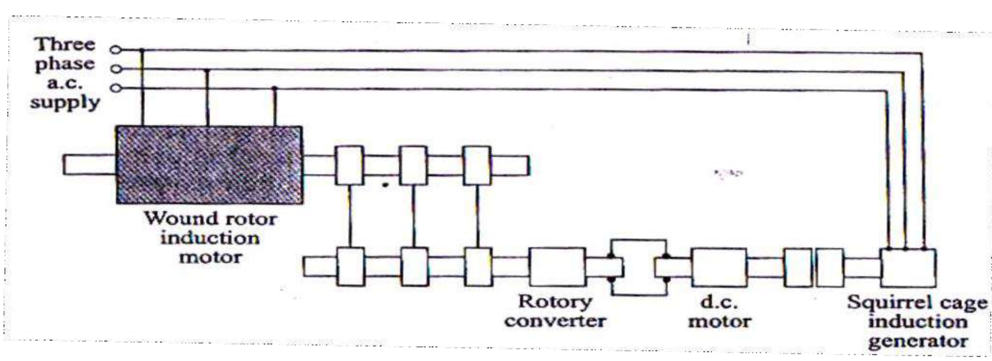


Fig: Traditional Scherbius drive.

Static Scherbius drive:

The static ***Scherbius*** drive system for the speed control of a wound rotor Induction motor is shown in the figure below. This is also known as sub synchronous converter cascade since it is capable of providing speed control only in the sub synchronous speed range.

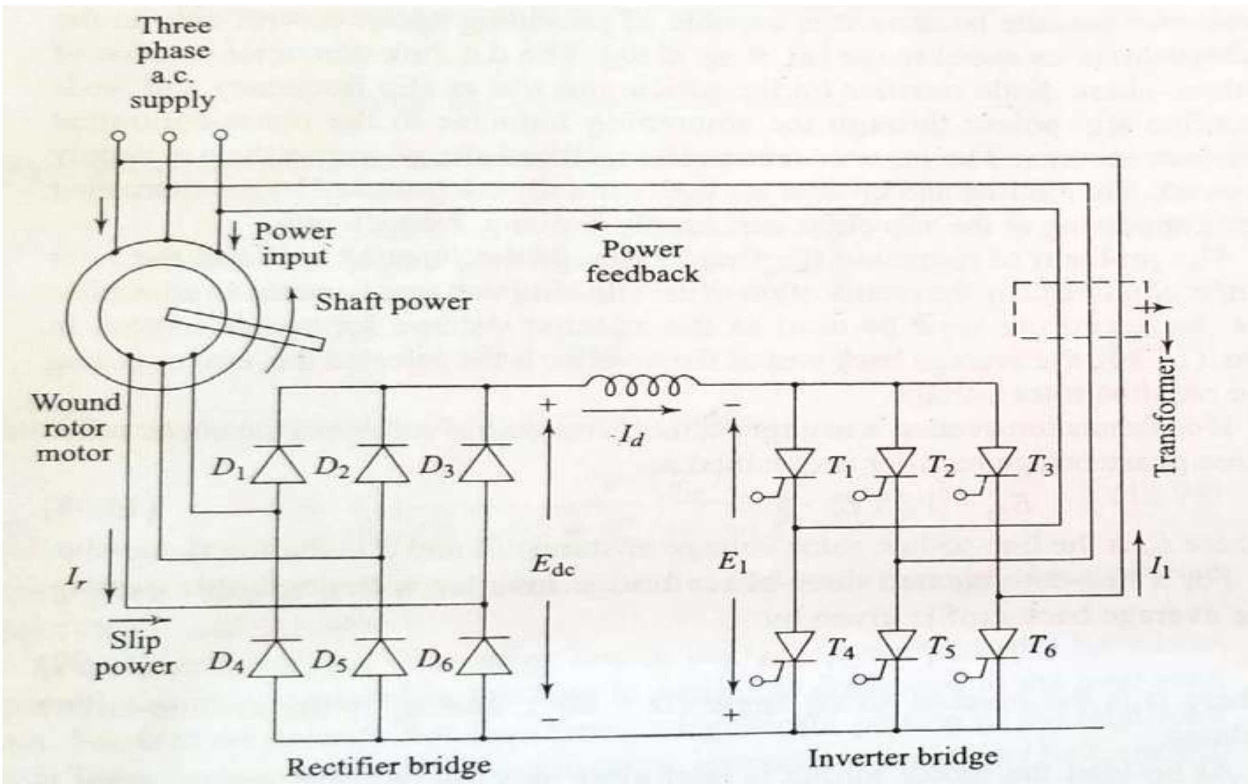


Fig: Static Scherbius drive

The DC link converter consists of a three phase diode bridge rectifier which operates at slip frequency and feeds the rectified slip power to a phase controlled three phase Inverter through a smoothing Inductor. The inverter returns the rectified slip power to the AC supply. The rectifier and the inverter are both naturally commutated by the alternating e.m.f s appearing at the slip rings of the rotor circuit and supply bus bars respectively. The problem of matching the frequencies of the injected e.m.f and the rotor e.m.f is eliminated by rectifying the rotor voltage and using the variable back e.m.f available from the controlled three phase inverter as the externally injected speed control voltage.

If commutation overlap is negligible the DC output voltage of the uncontrolled three phase rectifier is given by :

$\mathbf{E_{DC} = 3E_{rm}/\pi = 3\sqrt{2}\, E_r/\pi \quad 3 = 1.35\, E_{rs}}$

where E_{rm} is the maximum value of Rotor side line voltage at stand still

where E_r is the RMS value of Rotor side line voltage at stand still

where E_{rs} is the RMS value of Rotor side line voltage in running condition with slip 's'

For a line commutated three phase bridge inverter with negligible commutation overlap the average back e.m.f is given by:

$\mathbf{E_I = 1.35.E_L.\cos\alpha}$

Where **α** is the inverter firing angle **(α > 90°)** and E_L is the AC line to voltage.

Neglecting the drop across the inductor,

$\mathbf{E_{DC} + E_I = 0 \text{ or } 1.35\, E_{r.s} + 1.35.E_L.\cos\alpha = 0}$

And hence s = --(E_I/ E_r). Cosα = a|cosα|

Where **a = (E_I/ E_r)** is the effective stator to rotor turns ratio of the motor. Therefore speed control is obtained by simple variation of the Inverter firing angle. *If **'a'** is unity the no-load speed of the motor can be controlled from near standstill to full speed as **|cosα|** is varied from **almost unity** (since the maximum value of **α** is limited about 165° for safe commutation of Inverter thyristiors) to zero.* This is explained in simple words as below.

- As **α** is varied from 90^0 to 167^0 **|cosα|**varies from 0 to almost unity (0.96)
- Assuming **'a'** as unity we can say that slip varies from 0 to almost unity (0.96) as **|cosα|** varies from 0 to almost unity (0.96)

- So we can say that that slip varies from **0** to almost **unity** (0.96) as **α** is varied from **90^0 to 167^0**
- In other words "Speed varies from **Full speed** to almost **Stand still** as **α** is varied from **90^0 to 167^0 "**

In practice the motor turns ratio **a** is larger than unity resulting in a lower Rotor voltage. This results in the requirement of lower value of **cosα** for a given lower speed and hence a ***lower power factor*** which is not desirable. To overcome this limitation a step-down transformer is introduced in between the supply lines and the Inverter as shown by the dotted lines with a turns ratio of **m**. The governing relation between the firing angle (**α from 90° to 165°)** and the slip then becomes:

$$s = (a/m)|\cos\alpha|$$

We know that the power factor of the converter is low at low firing angles. Hence the turns ratio **'m'** of the transformer is chosen such that the drive operates always at **α = 165° (|cosα|= 0.966)** for the required lowest speed (highest slip s_{max}) so that the power factor is highest.

Torque-Speed relationship:

Assuming the rotor resistance to be small:

The Rotor slip power is equal to the DC link power.i.e. $s.P_{ag} = E_1.I_d$

$$P_{ag} = E_1.I_d/ s$$

But $$P_{ag} = T.\omega_s$$

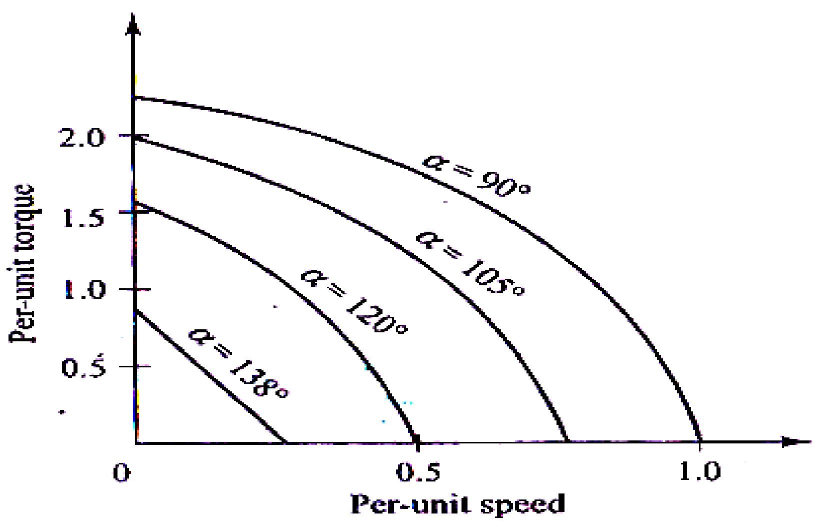
And hence $$T = E_1.I_d/s.\omega_s$$

Substituting the values of $s = a|\cos\alpha|$ and $E_l = 1.35.E_L.\cos\alpha$ from the earlier relations in to the above expression for torque viz. $T = E_1.I_d/s.\omega_s$ we finally get:

$$T = 1.35.E_L.I_d/a.\omega_s$$

Thus the steady state Torque is proportional to the rectified Rotor current **I_d** which in turn is equal to the difference between the rectified Rotor voltage and the average back e.m.f of the inverter divided by the resistance of the DC link Inductor. The inverter e.m.f is constant for a fixed firing angle and hence the Rotor slip increases linearly with load torque giving Torque- Speed characteristics similar to that of a separately excited DC motor with armature voltage control.

The complete open loop Torque-Speed characteristics of the Induction motor with a **Scherbius** drive are shown in the figure below.



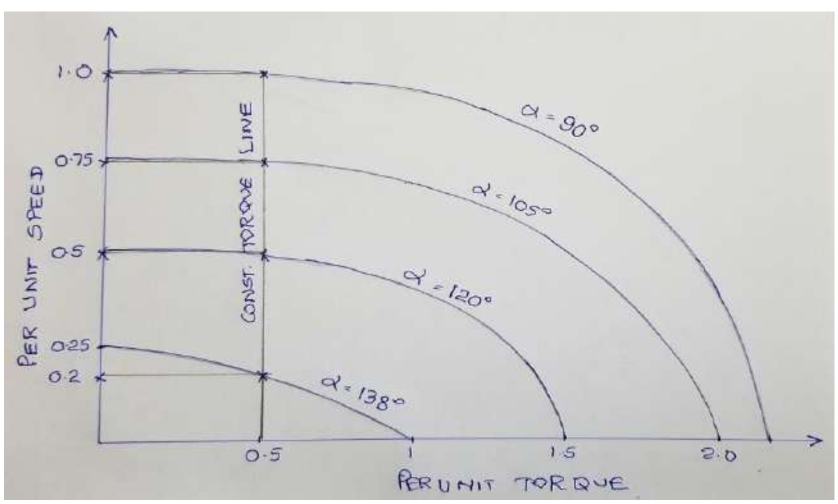


Fig: open loop Torque-Speed characteristics of an Induction motor with a Scherbius drive

Important features of Scherbius drive:

- Since power is fed back to the source, unlike in rotor resistance control where it is wasted in external resistors, drive has a high efficiency. The efficiency is even higher than the static voltage control for the same reason**.**
- Drive Input power is the difference between motor input power and the power fed back. Reactive power is the sum of the motor and inverter reactive powers. Therefore this drive has a poor power factor throughout its range of operation.

Kramer drive:

Kramer System: The Kramer system is applicable for only sub synchronous speed operation. Figure below shows a conventional Kramer system. The system consists of a 3 phase rotary converter and a DC motor. The slip power is converted into dc power by a rotary converter and fed to the armature of the DC motor. The slip ring induction motor is coupled to the shaft of the dc motor. The slip rings are connected to the rotary converter. The dc output of rotary convener is used to drive a dc motor. The rotary converter and dc motor are excited from the dc bus bars or from an exciter. The speed of slip ring induction motor is adjusted by adjusting the speed of dc motor with the help of a field regulator. This system is also called an '*electromechanical cascade'*, because the slip frequency power is returned as mechanical power to the slip ring induction motor shaft by the DC motor.

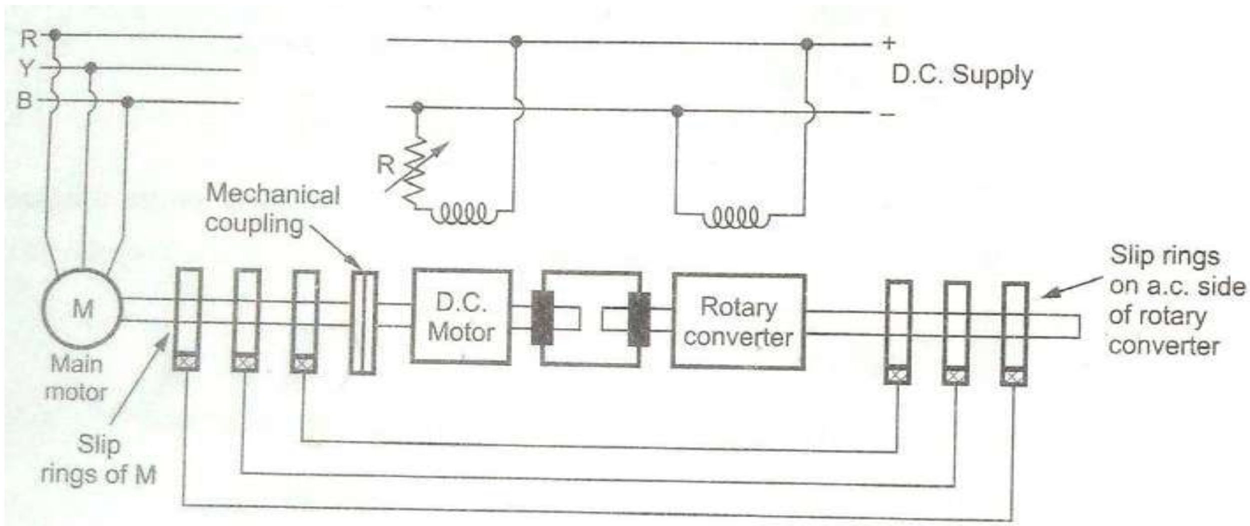


Figure: Conventional Kramer System

Static Kramer Drive:

In the Static Kramer drive the slip power is converted to DC by a Diode bridge and fed to a DC motor which is mechanically coupled to the Induction motor. Torque supplied to the motor is the sum of the torque produced by the Induction and DC motors. Speed control of the Induction motor is obtained by controlling the field current of the DC motor. A schematic diagram of this type of Static Kramer drive is shown in the figure below.

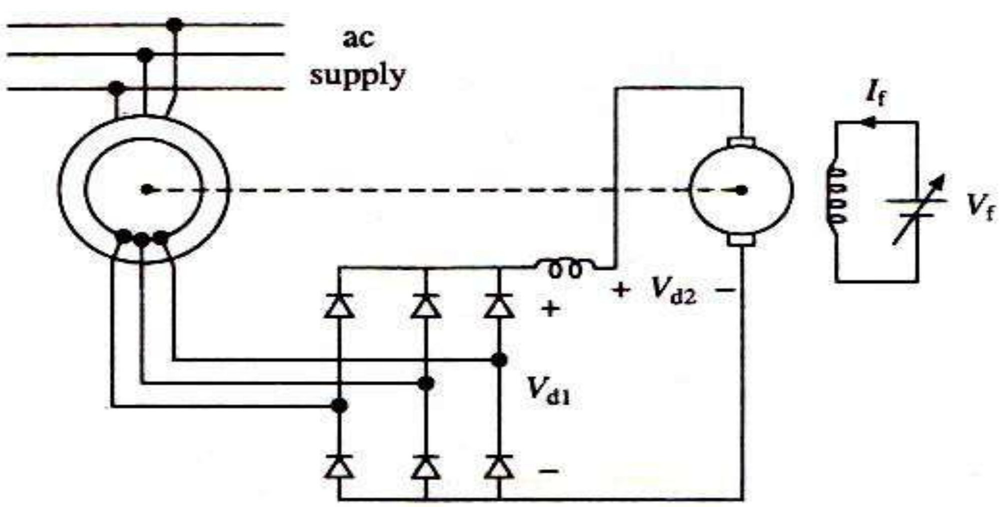
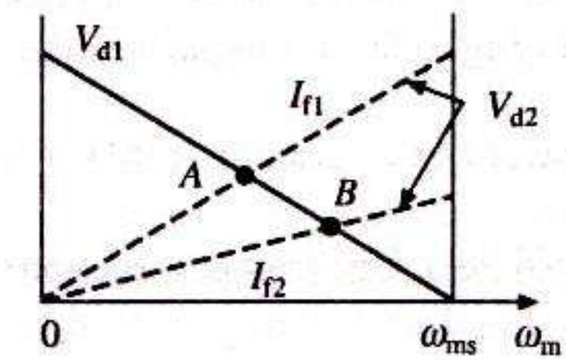


Fig: Static Kramer drive circuit

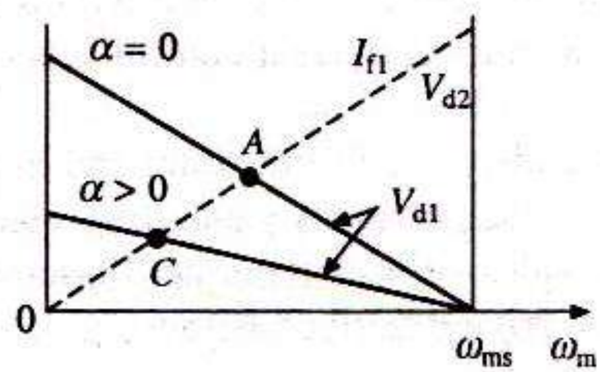
Figure (a) below shows the variations of V_{d1} and V_{d2} with speed for two values of field current. Steady state operation is obtained when $V_{d1} = V_{d2}$ i.e. at points **A** and **B** for field currents I_{f1} and I_{f2}. With this method speed control is possible from synchronous speed to around half of synchronous speed. Below this the speed cannot be brought down. This limitation is mainly because: To increase the Speed on the lower side ***either***

- The slope of the line V_{d1} **vs. Speed** is to be decreased. For this, the maximum DC voltage V_{d1} **is to be reduced but** it is not possible from the **Diode Bridge**.
- ***Or*** the slope of the line V_{d2} **vs. Speed** is **to be increased**. i.e. the maximum value of V_{d2} is to be increased. This is also not possible because for a given DC motor with the maximum ratings the maximum value of speed and hence the maximum back e.m.f V_{d2} are fixed.

This can be clearly seen in figure (a) below.



(a)



(b)

Fig: (a) Field control with Diode Bridge (b) Firing angle control of Thyristor Bridge with constant Motor field.

When larger speed range is required, the above limitation is overcome (lower limit can be brought down) by replacing the **Diode Bridge with a Thyristor bridge**. With this the maximum value of V_{d1} can be brought down and the slope of the line V_{d1} **vs. Speed** can be reduced. This increases the lower speed limit as shown in figure (b) above. As can be seen, with this change, the speed can now be controlled almost up to standstill.

Summary:

Important concepts and conclusions:

- **In Rotor resistance control:**
    - For a given Load torque, the motor speed is reduced (since slip **s** Increases) as the Rotor resistance is increased. However the no load speed remains the same with the variation of the Rotor Resistance.
    - The increase in rotor resistance does not affect the value of the maximum Torque but increases the value of Slip at which it occurs.
    - With increase in Rotor resistance the starting torque increases and the starting current reduces. Hence the Torque to current ratio improves.
- **In a Scherbius drive**: The slip **S** is a function of the firing angle α of the Inverter as given by: $S = a|\cos\alpha|$

Where **a** is the effective stator to rotor turns ratio of the induction motor and is given by $a = n/m$

Where **n** is the actual stator to Rotor turns ratio and **m** is the turns ratio of the Transformer from supply side to inverter side.

- **In a Kramer drive:**
    - The speed on the lower side is limited to about half of the synchronous speed. This is due to the fact that the maximum value of the DC output from the Diode Bridge V_{d1} cannot be brought down and maximum value of the back e.m.f of the DC motor V_{d2} cannot be increased. .
    - This problem is eliminated by the use of a fully controlled rectifier in place of the diode bridge whose maximum value of DC output V_{d1} can be reduced by increasing the firing angle.

Illustrative Examples:

Example-1: A 4 pole, 50 Hz, 3 phase induction motor has rotor has resistance of 0.2 Ω per phase and rotor standstill reactance of 1 Ω per phase. On full load it is running with a slip of 4 %. Calculate the extra resistance required in the rotor circuit per phase to reduce the speed to 1260 r.p.m., on the same load condition.

Solution: From the given data: $P = 4$, $f = 50Hz$, $R_2 = 0.2\ \Omega$, $X_2 = 1\ \Omega$, $s_f = 4\ \% = 0.04$

Synchronous speed is given by : $N_s = 120f/P = 120 \times 50/4 = 1500$ RPM

Let N_1 = full load speed. Then $N_1 = N_s\ (1-s_f)$

$= 1500\ (1 – 0.04) = 1440$ RPM

Let N_2 = Reduced speed at the same load = 1260 r.p.m.

Let the new rotor resistance with extra resistance **(R_{EX})** added be **R'_2** for achieving the reduced speed **N_2**.

i.e. $R'_2 = R_2 + R_{ex}$ Where R_{ex} = Extra resistance

Let $T = T_1$ for $N = N_1$ and $T = T_2$ for $N = N_2$ and S_2 be the slip at reduced speed 1260 RPM. Then $S_2 = N_s\text{-}N_2/N_s = 1500\text{-}1260/1500 = 0.16$

We know that : $\mathbf{T\ \alpha\ sE^2{}_2R_2/[R^2{}_2+(sX_2)^2]}$

Let $T = T_1$ for $N = N_1$ and $T = T_2$ for $N = N_2$

and $R_2 = R_2$ for $N = N_1$ and $R_2 = R'_2$ for $N = N_2$

Then we get : $\mathbf{T_1/T_2 = s_fE^2{}_2R_2/[R^2{}_2+(s_fX_2)^2]}$

$\mathbf{\times{*}(R'_2)^2+(s_2X_2)^2]/s_2E^2{}_2R'_2}$

But $T_1 = T_2$ as load is as same

Hence: $\mathbf{s_fE^2{}_2R_2{*}(R'_2)^2+(s_2X_2)^2] = s_2E^2{}_2R'_2[R^2{}_2+(s_fX_2)^2]}$

Cancelling **$E^2{}_2$** on both sides**:** $\mathbf{s_fR_2{*}(R'_2)^2+(s_2X_2)^2] = s_2R'_2[R^2{}_2+(s_fX_2)^2]}$

Substituting the values from the given data we get :

i.e. $0.04 \times 0.2[(R'_2)^2+(0.16 \times 1)^2] = 0.16 \times R'_2\ [(0.2)^2 + (0.04 \times 1)^2]$

Simplifying we get: $(R'_2)^2 – 0.832\ R'_2 + 0.0256 = 0$

$R'_2 = [\ 0.832 + \sqrt{(0.832)^2 - 4 \times 0.0256}]/2 = 0.032\ \Omega,\ 0.8\ \Omega$

After adding R_{ex} , R'_2 cannot be less than R_2. So neglecting smaller value of 0.032

we get $R'_2 = 0.8 = R_2 + R_{ex}$

Hence finally: $R_{ex} = 0.8 – R_2 = 0.8 – 0.2 =$ **0.6 Ω per phase.**

Example-2: A Three phase, 440 V, 6 pole, 50 Hz, delta connected SRIM has rotor resistance of 0.3 Ω and leakage reactance of 1Ω per phase referred to stator. When driving a fan load it runs at full load at 3% slip. What resistance must be inserted in the rotor circuit to obtain a speed of 850 rpm if stator to rotor turns ratio is 2?

Solution: Given data: $V_{ph} = 440$ V (since delta connected input line voltage = Phase voltage) P=6, f= 50 Hz, R_2= 0.3 Ω, X_2= 1 Ω, full load slip $s_f = 0.03$

Synchronous speed N_s = 120 x50/6 =1000 RPM

and $\omega_s = 2\pi \times 1000/60 = 104.72$ Rad/sec

In this problem we can use the simple equivalent circuit with Rotor side circuit parameters alone since they are only given and stator side parameters have been neglected. Then we can use the expression we know for full load torque as

$$\mathbf{T = (3/2\pi n_s)\ [s\, E_2{}^2 R_2 / \{R_2{}^2 + (sX_2)^2\}]\quad N\text{-}m}$$

In this problem the values of R_2 and X_2 are given referred to stator side and hence we can work on the stator side itself except that we have to take $\mathbf{V_{ph}}$ in place of $\mathbf{E_2}$ in the above equation and finally take the answer back to the rotor side since the Stator to Rotor side turns ratio is 2 (and not unity). Then taking the full load values we get the above equation as

$$\mathbf{T_{FL} = (3/\omega_s)\ [s\, V_{ph}{}^2 R_2 / \{R_2{}^2 + (sX_2)^2\}]\quad N\text{-}m\ \text{----------}(1)}$$

Substituting the values from the given data we get:

$$T_{FL} = (3/\ 104.72).[440^2.\ (0.03 \times 0.3)\ /\ \{(0.3)^2 + (0.03 \times 1)^2\}]$$

$$T_{FL} = 0.0286\ [193600 \times 0.009/\ 0.0909] = 548.21\ \text{N-m}$$

Since the load is a fan load we know that the $T_L \alpha N^2$ or $T_L = k.N^2$

So, at the rated conditions:

$T_{FL} = 548.21 = k.N^2 = k.[N_s\ (1\text{-}s)\ ]^2 = k.[1000\ (1\text{-}0.03)]^2$ from which we get :

$k = 548.21/(1000 \times 0.97)^2 \quad = 5.826 \times 10^{-4}$ N-m/RPM^2

Now let us find out the load torque at 800 RPM using the fact that $T_L = k.N^2$ and using the value of k as obtained above.

$T_{L@800RPM} = 5.826 \times 10^{-4} \times 800^2 = 372.86$ N-m
Slip @800 RPM = (1000-800)/1000 = 0.2
The equation (1) given above for the T_L @800 RPM then becomes:
$T_{L@800RPM} = (3/\ \omega_s).[V^2_{ph}.\ s.k\ /\ \{(\ k)^2 + (sX_2)^2\}]$ where s is now the slip @800 RPM = and k is the new rotor resistance with external resistance R_E added to theexisting R of 0.3Ω i.e. k = $(R_E + 0.3)$
Now substituting the corresponding torque, new rotor resistance and the slip in the above equation we get:

$$372.86 = T_{FL}= (3/\ 104.72).[440^2.\ (0.2\ k)\ /\ \{(k)^2 + (0.2 \times 1)^2\}]$$

Then the above equation becomes:

$372.86 = 5547 \times 0.2k/\ (k^2+ 0.2^2)\}]$ or

$372.86\ k^2 – 1109k+ 372.86 \times 0.04 = 0$ or

$k^2 – 2.97\ k + 0.04 = 0$ from which we get
$k = [2.97 +/– (2.97\ ^2 – 4 \times 0.04)^{1/2}]\ /\ 2 = (2.97 +/– 2.94)/2 = 2.955$ or 0.015
i.e. $k = (R_E + 0.3) = 2.955$ or 0.015
From which $R_E = 2.955 – 0.3 = 2.655$ Ω or
$R_E = 0.015 – 0.3 = – 0.015$ Ω
But resistance cannot be negative and hence $R_E = 2.655$ Ω
But this is the resistance to be added to the Rotor resistance referred to stator.
Hence value of external resistance to be added in the Rotor circuit = R_E/(turns ratio)^2 = 2.655/2^2 = 0.664 Ω

PART-2: CONTROL OF SYNCHRONOUS MOTORS

Introduction:

A synchronous motor is one in which the alternating current flows in the armature winding and DC excitation is supplied to the field winding. The armature winding is on the stator and is usually a three phase winding. The armature is identical to that of the stator in an Induction motor but there is no Induction into the Rotor. The field winding is on the rotor which is a solid forging and the slots are milled on the surface in which the DC field windings are placed.

The balanced three phase armature currents establish a rotating magnetic field at the synchronous speed corresponding to the supply frequency (N_s = 120f/P) just like in an Induction motor. If the Rotor which is supplied with a DC excitation is also made to rotate at the same synchronous speed, then the magnetic fields of stator and rotor are stationary relative to each other and a steady Torque is developed due to the tendency of the two magnetic fields to align with each other and this torque sustains the synchronous speed of the rotor. The process of initially bringing the rotor to the synchronous speed is called ***Starting***.

Unlike an Induction motor Synchronous motor runs only at synchronous speed until the load Torque exceeds the ***Pull out torque*** which is the Torque beyond which the motor slips out of synchronism and comes to a halt.

There are several types of synchronous motors like cylindrical Rotor motors, salient pole motors, Reluctance motors, Permanent magnet motors etc. But to understand the basic control methodology we will briefly study the equivalent circuit of a cylindrical rotor motor.

Equivalent circuit of a Synchronous Motor with cylindrical rotor:

A simplified per phase Equivalent circuit of a Synchronous Motor with cylindrical rotor is shown in the figure below.

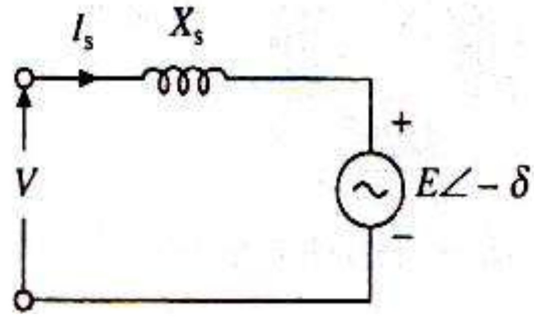


Fig: Equivalent circuit of a synchronous motor with cylindrical rotor

Xs is the synchronous reactance and E is the excitation e.m.f.The power in put to the motor is given by :

$P_{in} =$ **3** $VI_s \cos\phi$ where ϕ is the phase angle of I_s with respect to **V**

Neglecting the stator loss which is small the power developed by the synchronous motor is given by :

$P_m =$ **3** $VI_s \cos\phi$

$$I_s = \frac{V\angle 0 - E\angle -\delta}{jX_s} = \frac{V}{X_s}\angle -\pi/2 - \frac{E}{X_s}\angle -(\pi/2+\delta)$$

$$I_s \cos\phi = \frac{V}{X_s}\cos(\pi/2) - \frac{E}{X_s}\cos(\pi/2+\delta)$$

$$I_s \cos\phi = \frac{E}{X_s}\sin\delta$$

Substituting this in the equation for P_m we get

$$P_m = \frac{3VE\sin\delta}{X_s}$$

The rotating field produced by the stator moves at a synchronous speed given by :

$$\omega_{ms} = 4\pi f/P \text{ rad/sec}$$

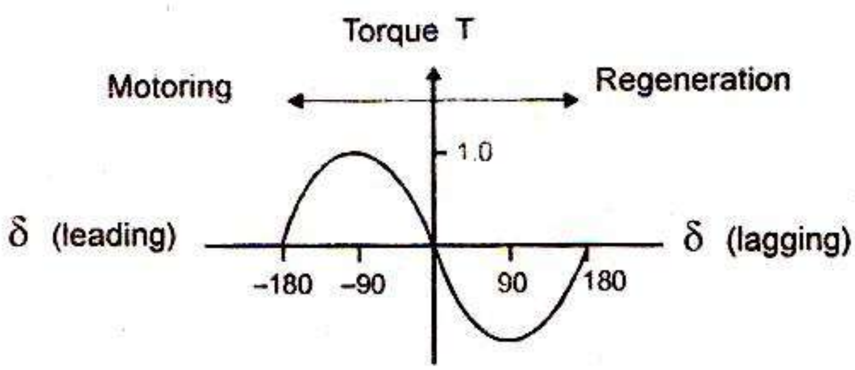
Where f is the supply frequency and **P** is the number of poles.

For a steady torque to be produced, rotor field must move at the same speed as the stator field. Since rotor field has the same speed as that of the Rotor the Rotor also runs at the same synchronous speed. Therefore torque is given by :

$$T = \frac{P_m}{\omega_m} = \frac{3VE}{X_s \omega_{ms}} \sin\delta$$

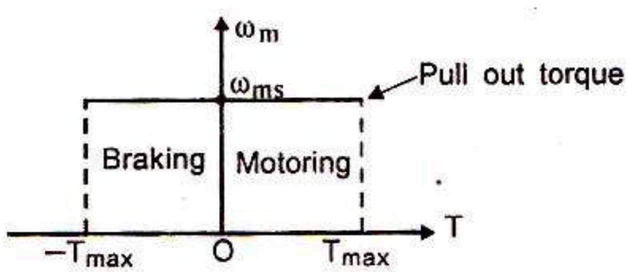
For a given field excitation **E** is constant. Therefore $\mathbf{P_m}$ and **T** are proportional to **sinδ.** The angle **δ** is called ***Torque (or Power) angle***.

The Pull out torque $\mathbf{T_{pull\ out}}$ (same as maximum Torque $\mathbf{T_{max}}$) is reached at **δ = +/-- 90°.** If the load Torque exceeds $\mathbf{T_{pull\ out}}$ the motor pulls out of synchronism. The plot of developed torque vs. the torque angle **δ** is shown in the figure (a) below.



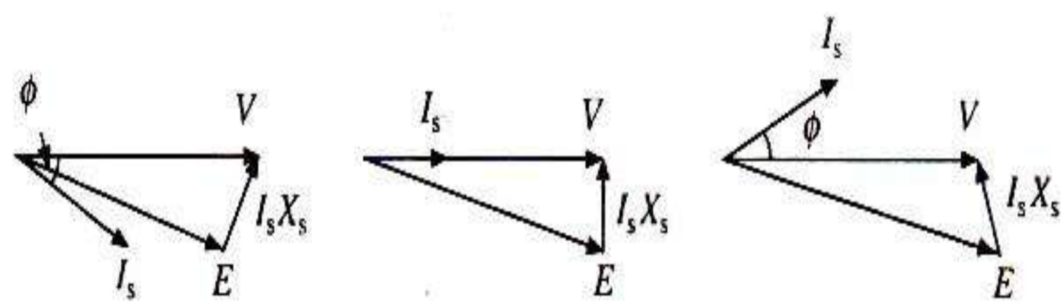
(a) Torque versus torque angle with cylindrical rotor

The Speed-Torque curve is shown in figure (b) below. Motoring operation is obtained when **δ** is positive i.e **E** lags behind **V.** Regenerative braking is obtained when **δ** is negative or **E** leads **V.**



(b) Speed-torque characteristics with a fixed frequency supply

The important feature of wound field synchronous motor is that its power factor can be controlled by varying the field current which in turn varies the excitation voltage **E.** The phasor diagrams of a synchronous motor for a given developed power are shown in the figure below. As can be seen when the field excitation is small the motor operates with a lagging power factor. The power factor can be made unity or leading by increasing the field excitation.



(a) Lagging power factor (b) Unity power factor (c) Leading power factor

Fig: Variation of power factor with field excitation

Introduction to speed control of synchronous motors:

In synchronous motors also,in steady state, the speed is directly proportional to the supply frequency and the control methodology is same like in Induction motors. Constant flux operation below base speed is achieved by constant V/f control. Above base speed once the rated voltage is reached, the terminal voltage is kept constant and frequency is increased. The pull out Torque **(** T_{max} **)** is constant during the constant flux operation where as it decreases with increase in frequency for higher speeds.

Unlike an Induction motor the synchronous motor either runs at the synchronous speed or it does not run at all. Hence the variable frequency control adopts any of the following two methods.

1. True Synchronous Mode or Separate Control Mode
2. Self control Mode

Separate Control Mode:

This is an open loop control mode in which the stator supply frequency is controlled from an Independent oscillator. Hence the frequency is gradually increased from its initial value to the final desired value so that the difference between the synchronous and rotor speed is always very small. This enables the rotor to track the changes in synchronous speed and catch up without pulling out. When the desired synchronous speed is reached, the rotor pulls into step, after hunting oscillations. This method can be used for smooth starting and regenerative braking. This method is best suited for multiple synchronous, reluctance or Permanent magnet (PM) motor drives where close speed tracking is essential among a number of machines in applications such as fiber spinning mills, paper and textile mills where accurate speed tracking is required.

The block diagram of such an open loop control system using this separate control method for multiple synchronous motors is shown in the figure below.

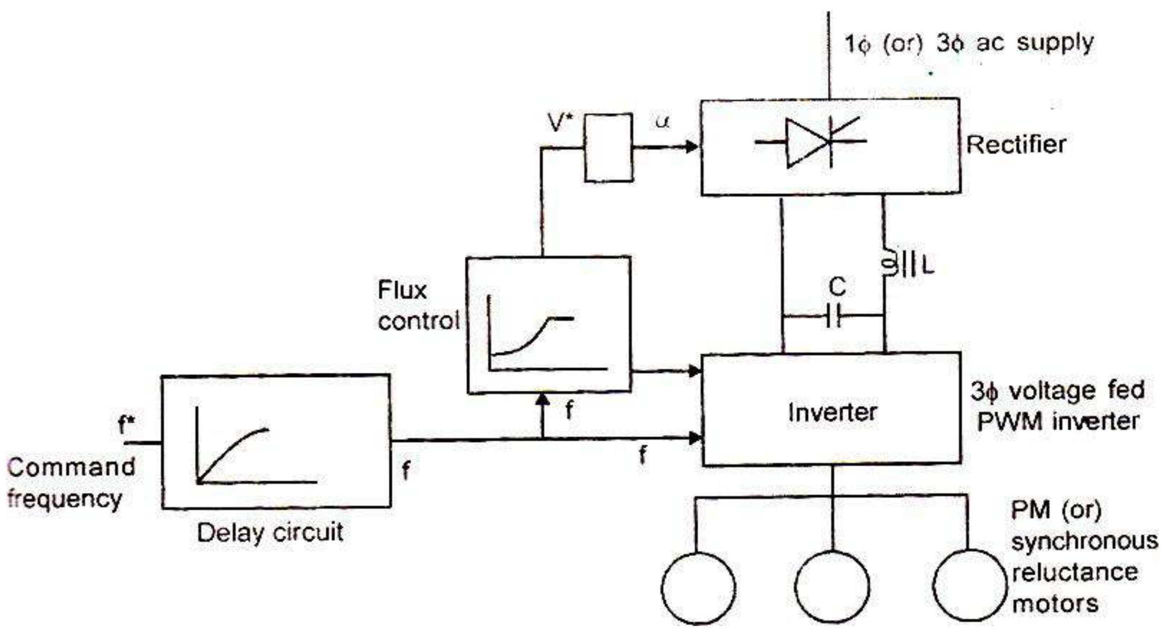


Fig: Open loop speed control of multiple PM synchronous motors.

Here all the machines are connected to the same Inverter and they move in response to the command frequency **f***at the input to the Ramp/delay circuit. The Input speed command is given through a ramp generator with a finite delay to ensure that the rotor gradually picks up speed and pulls into synchronism with the stator magnetic field and settles at the final synchronous speed.The frequency command **f*** after passing through the ramp/delay circuit generates the required **V** and **f** control signals just like in a VSI with a PWM Inverter as shown in the figure. The **V** control is applied to the DC converter through a flux control block so as to generate the required Voltage to generate a constant flux with varying frequency. The Rectifier output then gets applied to the PWM inverter through **L**& **C** filter as required for a VSI type drive. The frequency command is directly applied to the PWM inverter. The synchronous motor can be built with damper winding to prevent oscillations.

Self controlled mode:

In *Self controlled mode*, the stator supply frequency is changed in proportion to the rotor speed, so that the rotating magnetic field produced by the stator always moves at the same speed as the rotor (Or rotor field). This ensures that the rotor runs at synchronous speed at all operating points. (In all Load conditions)

Consequently, a self controlled synchronous motor does not pull-out out of step and does not suffer from hunting oscillations & instability associated with a step change in torque or frequency when controlled from an independent oscillator (Separate control Mode). Hence Synchronous motors working in *Self controlled mode* of operation do not require a damper winding.

Absolute Position Sensors are mounted on the Rotor shaft to track the rotor position and speed. These sensors are called rotor position sensors. The frequency and Phase of the Inverter output power are controlled by taking feedback from the Absolute position sensor. Hence, the stator supply frequency can be made to track the frequency of these signals.

Alternatively, since the voltage induced in the stator phase has a frequency proportional to rotor speed, self control can also be realized by making the stator supply frequency track the frequency of induced voltages.

The basic block diagram of a ***self controlled synchronous motor*** fed from a three phase inverter and working with Rotor Position sensors is shown in the figure below.

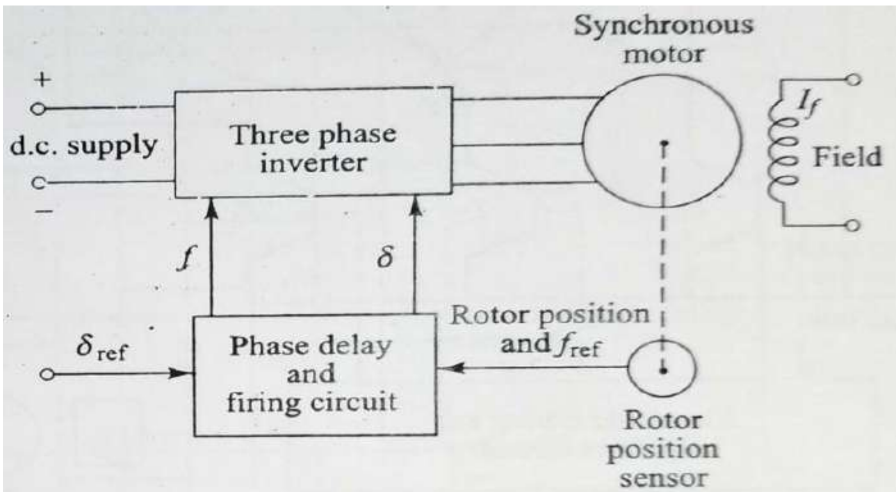


Figure: Self Controlled Synchronous Motor (Brush Less DC Motor)

When an inverter is used the input is a DC source. The stator winding of the machine is fed from the inverter which generates a variable voltage variable frequency sinusoidal supply.

Here the frequency and phase angle **δ** of the control signal required to generate the required input to the synchronous motor is produced by comparing the Position output and frequency (**f_{ref}**) of the absolute position sensor , thus giving it the self control characteristic. Here the phase angle of the pulse train from the position sensor can be delayed by an external **δ_{ref}** command as shown in the figure.

Operation of the drive is similar to that of a DC motor. The rotor position sensor and inverter now perform the same function as brushes and commutator in a DC motor.

Due to similarity in operation of a DC motor, an inverter fed Synchronous motor drive as shown in this figure is also known as a *Commutator Less DC Motor* (CLDM). If the synchronous motor is a permanent magnet motor or a reluctance motor or wound field motor with a brushless excitation, then it is a known as a Brush Less and Commutator Less DC Motor or simply a *Brush Less DC Motor* (BLDC) . This type of Self controlled systems driving synchronous motors offer the linear Torque speed characteristics of DC motors and are finding increasing applications in servo drives.

In this kind of control the machine behavior is decided by the torque angle and voltage/current. Such a motor can be considered as a DC motor with its commutator replaced by a fully controlled converter connected to the stator. Such a self controlled motor has the properties of a DC motor both under steady state and dynamic conditions. Hence it is called a Commutator Less Motor (CLM).These motors have better stability performance.

Alternately the firing pulses for the inverter can be obtained from the phase angle of the stator voltages in which case the rotor position sensor can be dispensed with. When synchronous motors are over excited (field current is large) they will work with a leading power factor and can supply the reactive power required for commutation of thyristors. In such a case the induced voltages in the synchronous motor provide the required voltages for commutation of the thyristors in the inverter just as in a line commutated Inverter.

Here the firing angles are synchronized with the motor induced voltages and hence they serve both for control as well as commutation. Hence the frequency of the inverter will be same as that of the motor induced voltages. This type of inverters are called load commutated Inverters (LCI). Hence the commutation is simple due to the absence of diodes, capacitors and auxiliary thyristors.

But this natural commutation is not possible at low speeds upto 10% of base speed as the motor voltages are not sufficient to provide satisfactory commutation. At that time forced commutation must be employed.

Load commutated CSI fed synchronous motor:

The circuit diagram of a self controlled synchronous motor drive employing a load commutated thyristor Inverter is shown in the figure below. This drive consists of two parts: Source side converter and load side converter.

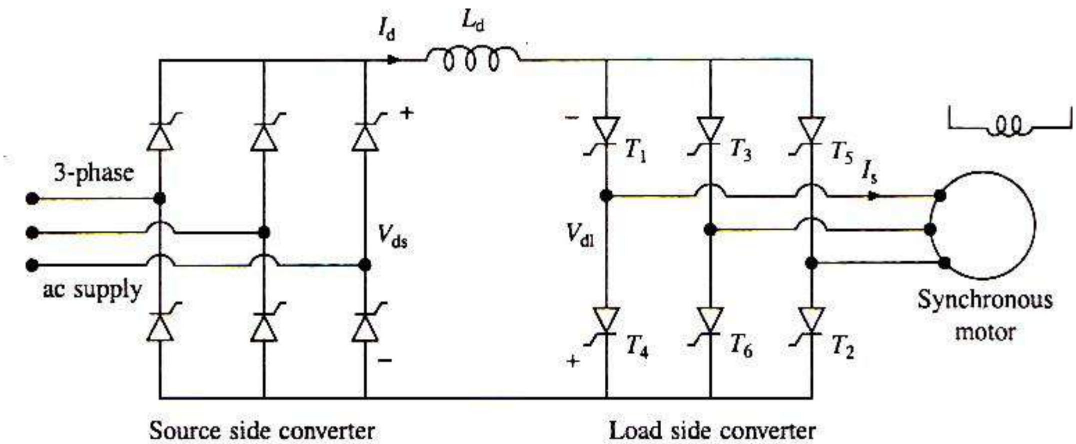


Fig: Self controlled Synchronous Motor Drive employing Load Commutated Inverter

The source side converter is a 3 phase 6 pulse line commutated fully controlled converter. When the firing angle range is **$0° < \alpha_s < 90°$** the converter acts as a line commutated fully controlled rectifier. During this mode the output voltage **V_{ds}** and output current **I_{ds}** are both positive.

When the firing angle range is **$90° < \alpha_s < 180°$** the converter acts as a line commutated fully controlled inverter. During this mode the output voltage **V_{ds}** is negative and output current **I_{ds}** is positive.

When the synchronous motor operates at a leading power factor, thyristors of the load side converter are commutated by the motor induced voltages just as the thyristors in a line commutated converter are commutated by the supply voltages. This is called Load commutation (here load is synchronous motor).Firing(triggering) angles are referred to the induced voltages just like the triggering angles in a line commutated inverter are referred to the supply voltages.

When the firing angle range is **$0° < \alpha_l < 90°$** the **load side** converter acts as a line commutated fully controlled rectifier. During this mode the output voltage **V_{dl}** and output current **I_d** are both positive.

When the firing angle range is **$90° < \alpha_l < 180°$** the **load side** converter acts as a line commutated fully controlled inverter. During this mode the output voltage **V_{dl}** is negative and current **I_d** is positive.

For **$0° < \alpha_s < 90°$ & $90° < \alpha_l < 180°$** and with **$V_{ds} > V_{dl}$** the source side converter acts like a line commutated Rectifier and load side Converter acts like a line commutated Converter causing power to flow from the source to the motor thus giving motoring operation.

When the firing angles are changed such that **$90° < \alpha_s < 180°$ and $0° < \alpha_l < 90°$** the Load Side Converter acts like a line commutated Rectifier and Source Side Converter acts like a line commutated Inverter causing power to flow from the motor to the source thus giving regenerative braking operation.

The magnitude of Torque depends on **(V_{ds} -- V_{dl})**. The motor speed can be controlled by control of line side converter firing angles.

When working as an Inverter, the firing angle has to be less than 180° to take care of commutation overlap and turn off of thyristors. It is common to define a commutation lead angle for load side converter as

$$\beta_l = 180° - \alpha_l$$

If commutation overlap is ignored, the input AC current of the converter will lag behind the input AC voltage by an angle **α_l.** Since motor input current has an opposite phase to converter input current, the motor current will lead its terminal voltage by an angle **β_l.** Therefore the motor operates at a leading power factor.Lower the value of **β_l,** higher the motor power factor and lower the Inverter rating.

In a simple control scheme ,the drive is operated at a fixed value of commutation lead angle **β_{lc}** for the load side converter working as an Inverter and at **β_l = 180°(or**

α_l = 0°) when working as a rectifier. *When good power factor is required to minimize converter rating, the load side converter when working as an inverter is operated with **constant margin angle control.***

Closed loop operation of Synchronous drives:

A closed loop speed control scheme of a Load Commutated Inverter (LCI) Synchronous Drive is shown in the figure below.

- It employs outer speed control loop and inner current control loop with a limiter just as in a DC motor speed control system.
- The phase controlled Thyristor rectifier on the supply side of the DC link has a constant current regulating loop and operates as a controlled current source.
- The regulated DC current is delivered through the DC link inductor to the Thyristors in the LCI (Load Commutated Inverter) (shown in the figure as Load side Inverter) which supplies square-wave line currents to the synchronous motor.
- The terminal voltage sensors generate reference pulses of same frequency as the motor-induced voltages. The phase delay circuit shifts the reference pulses suitably to obtain control at a constant commutation lead angle **β_{lc}.**
- Depending on the sign of speed error, **β_{lc}** is set to provide motoring or braking operation. Speed **ω_m** can be sensed either from the terminal voltage sensor or from a separate tachometer.

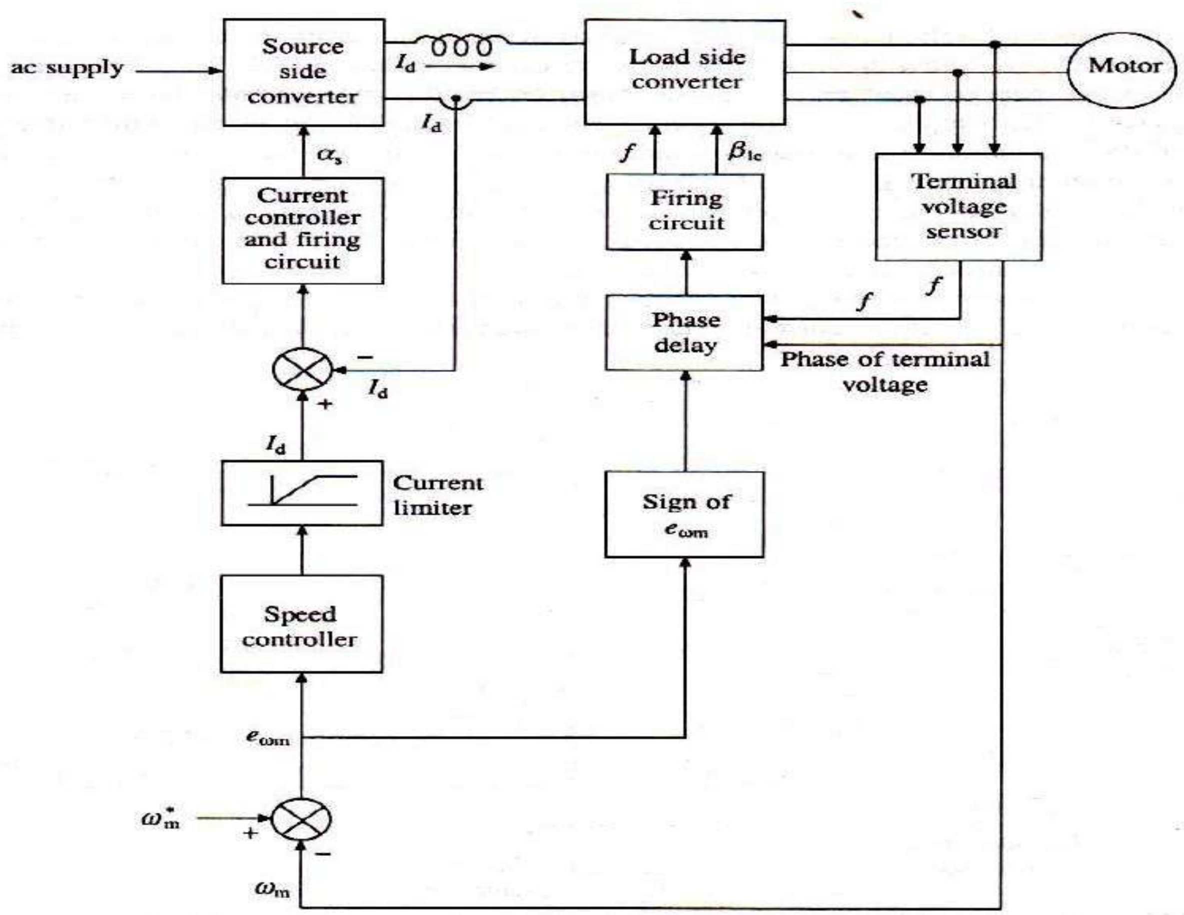


Fig: closed loop speed control scheme of a Load Commutated Inverter (LCI) Synchronous Drive

An increase in reference speed $\boldsymbol{\omega_m}$ produces a positive speed error. $\boldsymbol{\beta_{lc}}$ value is then set for motoring operation. The speed controller and the current limiter set the DC link current reference at the maximum permissible value. The motor accelerates fast. When close to the desired speed the current limiter desaturates and the drive settles at the desired speed and at a DC link current which balances motor and load torques .

Similarly a reduction in reference speed produces a negative speed error. This sets $\boldsymbol{\beta_{lc}}$ for regenerative braking operation (i.e. 180 °) and the motor decelerates. When speed error changes sign $\boldsymbol{\beta_{lc}}$ value is set for motoring operation and the drive settles at the desired speed.

Advantages:

- High efficiency, four quadrant operation with regenerative braking ,high power ratings (up to 100Mw) and ability to run at high speeds (6000 RPM) are some important advantages of this drive.

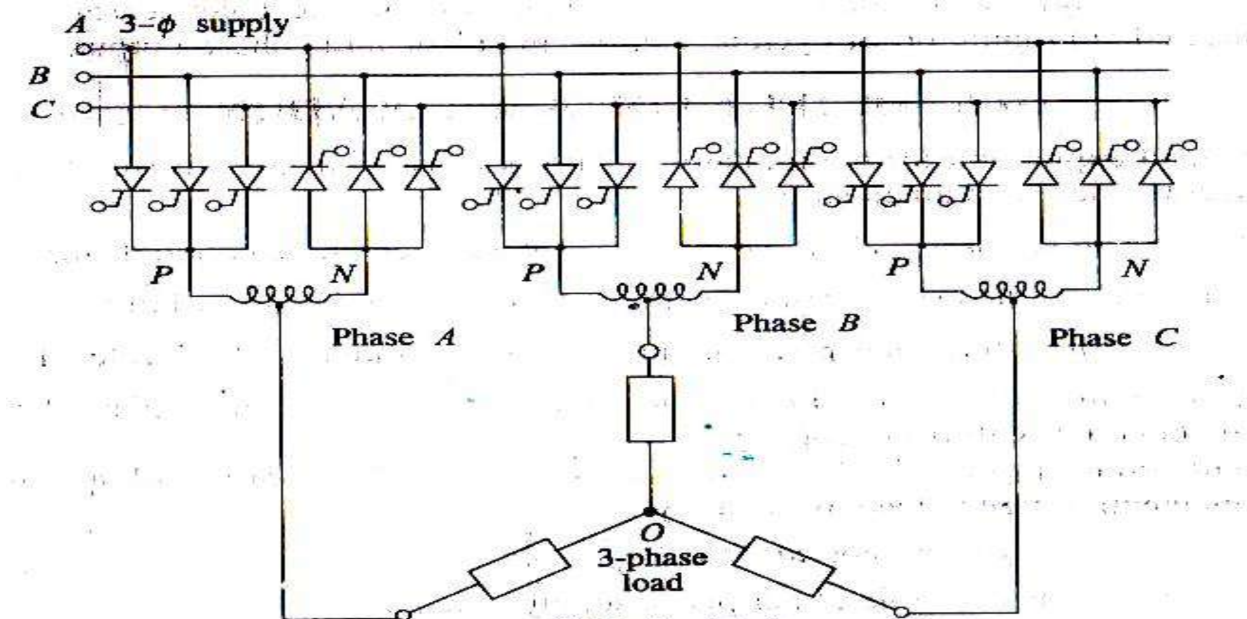
Applications:

- Wound field Synchronous motors are used in large power drives.
- Permanent motor synchronous motors are used in medium power drives.
- Some prominent applications are high speed and high power drives for compressors, blowers, fans, pumps, conveyors, steel rolling mills, main line traction, ship propulsion and aircraft test facilities.

Cycloconverter fed Synchronous Motor:

In a synchronous motor fed from a VSI or a CSI, the DC link converter has two stage conversion devices that produce variable voltage and variable frequency. But with a Cycloconverter both variable voltage and variable frequency can be obtained using a single stage conversion. A Cycloconverter gives high quality sinusoidal output voltage and hence the resulting current is also sinusoidal. Consequently the effects of harmonic current such as heating losses and torque pulsations are minimal compared to VSI or CSI fed drives. The power circuit diagram of a Cycloconverter feeding a synchronous motor and total drive system operating in a Self Control mode/Commutator less Motor (CLM) Mode are shown in the figures (a) and (b) below..

The Cycloconverter can be Line commutated or Load commutated. In Line commutated mode it provides a variable frequency, variable voltage source. It works in self controlled mode and receives its firing pulses from rotor position sensors or armature voltage sensor. Due to its limitations in the output frequency, a line commutated Cycloconverter speed control range is limited to zero to about one third of the base speed. In Load commutated mode the motor operates on trapezoidal excitation as a current source fed motor. In this mode the motor can run up to and even beyond its base speed. The other features of four quadrant mode of operation and good power factor remain same as in a Line commutated Cycloconverter.



(b) Basic circuit

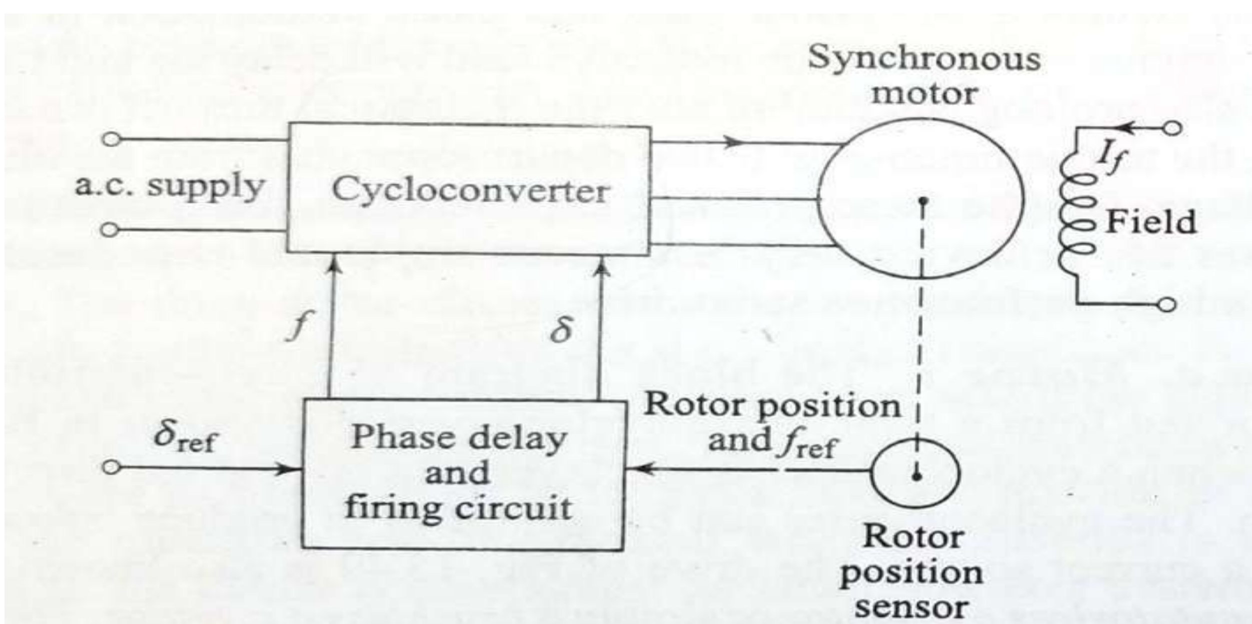


Fig (b) : Cycloconverter feeding a 3 Phase Synchronous Motor in Self Control Mode /Commutator less Motor (CLM) Mode

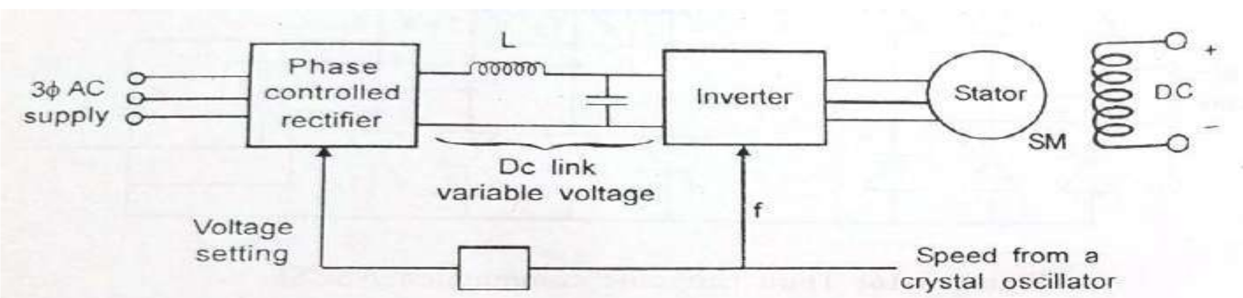
Voltage Source Converter (VSI) fed Synchronous Motor:

The basic principle of operation of VSI drives for Synchronous Motors is same as that of VSIs we have studied for Induction Motor Drives. Just like in Induction Motor drives three basic configurations are possible to provide variable voltage/variable frequency supply to synchronous motors fed from VSI.

1) Square wave inverters 2) PWM inverters 3) Chopper with square wave inverters

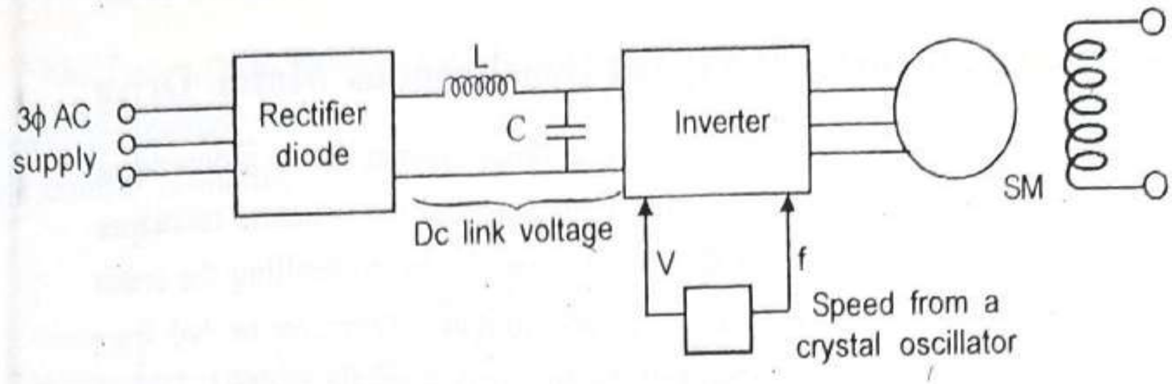
In all these cases the Synchronous motors can be operated in either self control or separate control modes. The above three schemes in these two modes are depicted in the figures (a) to (e) below and explained briefly.

(a) ***Separate control of a Synchronous Motor with a square wave inverter*****:** The Phase Controlled rectifier varies the DC voltage to the inverter and at the same time the inverter output frequency is varied based on a speed control signal from a crystal oscillator . The section between the DC source and the Inverter is known as the DC link and it includes a series Inductance and large capacitance which smoothes the DC voltage to an almost constant value. The above system cannot regenerate since current flow cannot be reversed in a phase controlled converter. If regeneration is required it can be obtained by replacing the phase controlled rectifier with a Dual Converter.



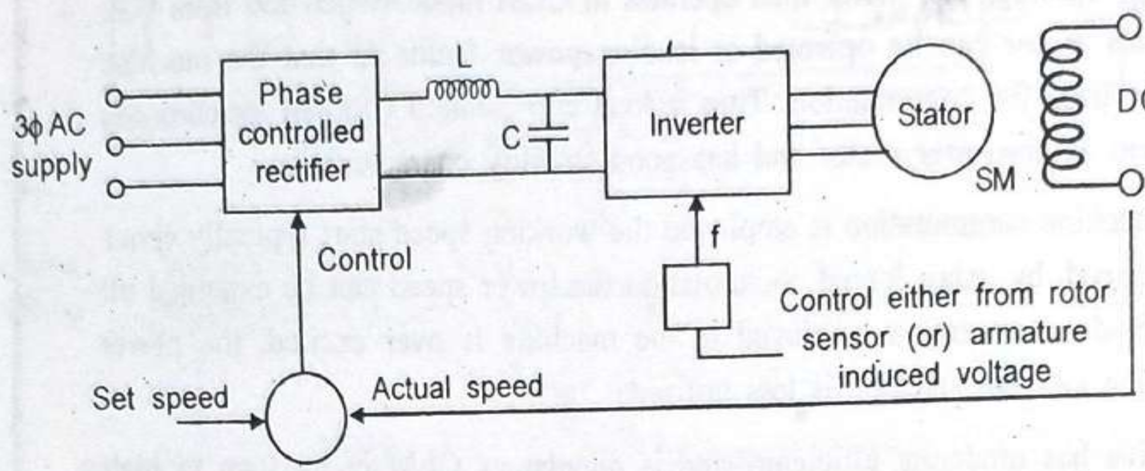
(a) Separate control of SM fed from square wave inverter

(b) ***Separate control of a Synchronous Motor with a PWM inverter*****:** A system in which the DC link voltage is constant as obtained from a simple Diode rectifier is shown in figure (b)**.** In this scheme the Inverter is a PWM based system and it varies both the voltage and the frequency as controlled from an external oscillator.



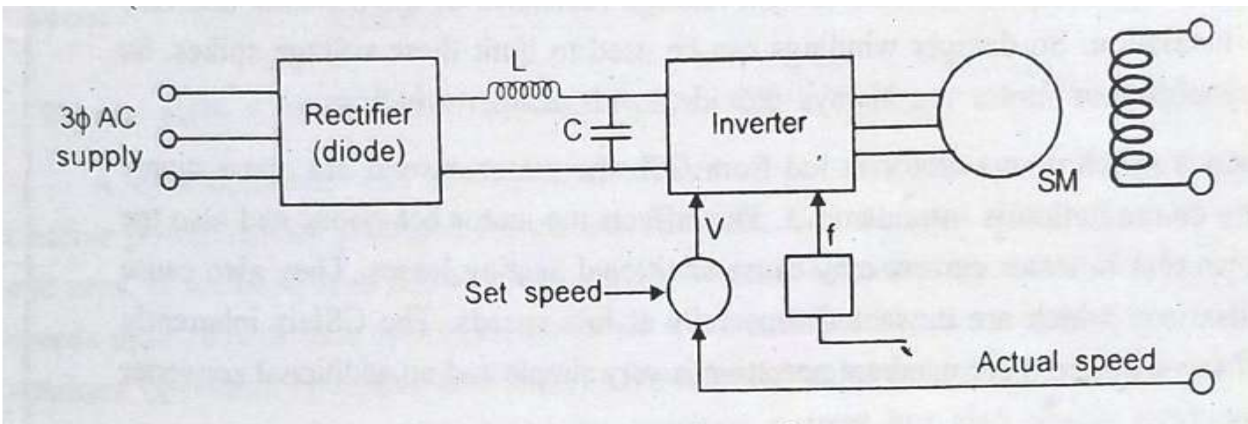
(b) Separate control of SM fed from PWM inverter

(c) ***Self control of a Synchronous Motor with a square wave inverter:*** The Phase Controlled rectifier output DC voltage to the inverter is varied with a speed control loop as shown in the figure and at the same time the inverter output frequency is varied both based on a control signal from a rotor sensor or armature induced voltage so as to maintain a constant v/f ratio. The section between the DC source and the Inverter is the DC link as already explained.



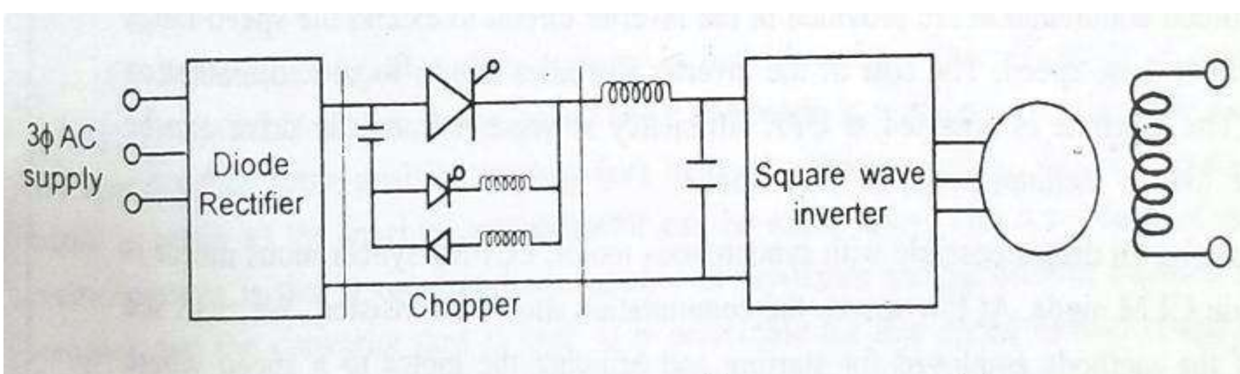
(c) Self control of SM fed from square wave inverter

(d) ***Self control of a Synchronous Motor with a PWM inverter:*** A system in which the DC link voltage is constant as obtained from a simple Diode rectifier is shown in figure (d). In this scheme the Inverter is a PWM based system and it varies the voltage with a speed control loop as shown in the figure and at the same time the inverter output frequency is varied both based on a control signal from a rotor sensor or armature induced voltage so as to maintain a constant v/f ratio.



(d) Self control of synchronous motor fed from a PWM inverter

(e) In the fifth scheme the variation of voltage is obtained by a chopper. Due to the chopper the harmonic injection into the AC supply is reduced. This scheme is a combination that is used when a high frequency output is required and hence a PWM inverter is not used and a normal square wave inverter is used.



(e) Fed from Chopper with a square wave inverter

Fig: Possible combinations of VSI fed Synchronous Motor.