A Minimal Ledger Framework for Representable Physical Theories

Recursive Persistence, Dimensionality, Amplitudes, and Renormalization as Constraint Theorems

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Status: Foundational constraint framework (non-dynamical, falsifiable)
Scope: Admissibility conditions for persistent physical descriptions
Non-claims: No ultraviolet completion, no replacement of quantum field theory or general relativity, no equations of motion

Abstract

Any physical theory serves as a finite structure to record, transform, and compose distinctions in an entropic environment. This work derives minimal constraints that such descriptions must satisfy to persist and compose under finite resources, assuming no primitives, dynamics, or initial conditions.

Four structural theorems follow. (1) Persistent descriptions require a cancellative minimal ledger to support compression and composition without history-dependent ambiguity. (2) No such ledger can persist nontrivially in one or two effective dimensions; three effective dimensions are minimally sufficient. (3) When multiple interior histories are indistinguishable at the boundary, aggregation must be amplitude-like to preserve recomposition consistency. (4) Scale-dependent coarse-graining necessarily induces renormalization-group–like flows.

Two corollaries follow. Persistence is finite and register-relative, with maximal attractors that saturate and decay. The framework is falsifiable and compatible with both discrete and continuum effective theories. Together, these results frame core structural features of modern physics as consequences of representability under finite entropic budgets rather than independent postulates.

1 Introduction: What Is Being Constrained

At its most abstract level, what is being constrained here is computation: the ability to record, manipulate, and preserve distinctions.

Any physical theory, regardless of ontology, functions as a finite descriptive system operating in an entropic environment. It must encode distinctions, transform them, and compose them in ways that remain operationally meaningful under finite resources. This work therefore asks a question that is prior to dynamics or microscopic modeling:

What kinds of descriptions can persist and compose at all under such conditions?

Physical theories typically begin by postulating primitives—fields, particles, manifolds—and deriving consequences from assumed dynamics. Yet certain structural features recur across otherwise distinct frameworks. Dimensionality bounds, renormalization flows, and amplitude-based composition rules appear repeatedly, even when underlying models differ. Their persistence suggests that they may reflect constraints on representability itself, rather than contingent modeling choices.

The present approach is explicitly non-dynamical. It assumes no equations of motion, no microscopic ontology, and no initial conditions. Instead, it constrains which records can persist and compose in an entropic environment under finite resources. Any dynamics that cannot be represented under such conditions, if they exist, are empirically inaccessible.

In such environments, unattended distinctions tend toward indistinguishability. Persistent ones must incur nonzero maintenance cost, and composition must be possible without unbounded bookkeeping overhead. These requirements alone impose strong structural limits on admissible descriptions.

The approach is analogous in spirit to Wilson's renormalization framework, Landauer's thermodynamic bounds on information, and Kolmogorov's limits on descriptional complexity—but applied here as meta-constraints on physical description itself. The framework neither replaces nor competes with quantum field theory or general relativity; it instead constrains the kinds of structures any such effective theory must employ in order to be representable at all.

2 Preliminaries: Definitions, Axioms, and Overview

2.1 Core Definitions

Distinction: A realized difference maintainable and composable with finite resources.

Register: A regime bounded by resources and resolution within which distinctions can be maintained and distinguished.

Minimal Ledger: The least-structure bookkeeping framework capable of representing and composing distinctions under finite resources in an entropic environment, consisting of records, an equivalence relation identifying boundary-indistinguishable records, and a local composition rule preserving boundary distinctions under compression.

Equivalence Class / Indistinguishability: Records equivalent if no finite procedure available in the register distinguishes them at the boundary.

Boundary Record: The externally relevant equivalence class preserved for interaction at the chosen resolution.

Interior History: Any admissible internal realization yielding the same boundary record after compression.

Compression: An operation reducing representational or maintenance cost while preserving boundary distinctions.

Composition: A local rule combining records, well-defined on equivalence classes.

Compositional Closure: Composition repeatable without global recomputation or unbounded overhead.

Cancellative: A ledger is cancellative if

$$a \circ b \sim a \circ c \;\Rightarrow\; b \sim c.$$

Effective Dimension: The minimal dimension in which equivalence and composition can be realized locally, without nonlocal bookkeeping.

Maximal Attractor (MA): An equivalence class within a register that maximizes persistence per representational cost.

2.2 Admissibility Axioms

- **A0 — Finite Resources**: No infinite precision, memory, computation, or zero-cost erasure.
- **A1 — Entropic Persistence**: Distinctions degrade unless maintained at nonzero cost.
- **A2 — Operational Distinguishability**: At least two states must be distinguishable by finite procedures.
- **A3 — Boundary-Preserving Compression**: Internal redundancy may be discarded; boundary distinctions must be preserved.
- **A4 — Compositional Closure**: Local composition without global recomputation or nonlocal overhead.
- **A5 — Minimal Commitment**: No structure is admitted unless required by A0–A4.

2.3 Results Overview

From A0–A5 follow four necessity results: Ledger Necessity; Minimal Dimensional Sufficiency; Amplitude Necessity; and Renormalization Inevitability. Two corollaries follow: persistence is finite and register-relative with maximal attractors, and the framework is falsifiable while remaining compatible with discrete and continuum theories.

3 Result I: Ledger Necessity

Proposition 1 (Ledger Necessity): Systems satisfying A3 and A4 require a cancellative minimal ledger tracking equivalence classes.

Compression maps multiple internal realizations to a single boundary record. Composition must operate consistently on what survives compression. Without explicit tracking of equivalence classes, composition becomes history-dependent, violating A4. Cancellativity is therefore forced.

4 Result II: Minimal Dimensional Sufficiency

Proposition 2 (Dimensional Sufficiency): No cancellative, locally compositional ledger persists nontrivially in fewer than three effective dimensions. Three effective dimensions are minimally sufficient.

4.1 Why 1D Fails

Linear sequences support only adjacency. Nontrivial cancellation either trivializes distinctions or requires global parsing, violating admissibility axioms.

4.2 Why 2D Fails

Planar diagrams admit ambiguous crossings requiring nonlocal conventions, violating compositional closure.

4.3 Why 3D Is Minimally Sufficient

Three-dimensional embeddings resolve crossings intrinsically, enabling local equivalence and cancellation.

Effective Dimension	Native Class	Failure Mode
1D	Linear sequences	Global parsing required
2D	Planar diagrams	Nonlocal crossing resolution
3D	Embedded relations	None (minimal sufficiency)

5 Result III: Amplitudes as Forced Aggregation

Proposition 3 (Amplitude Necessity): When multiple interior histories map to the same boundary record, aggregation must preserve recomposition consistency.

Aggregation must be invariant under regrouping and refinement. The minimal aggregation rule satisfying this constraint is additive at the level of histories and quadratic in observable intensities, yielding amplitude-like structure.

6 Result IV: Renormalization as a Structural Theorem

Proposition 4 (RG Inevitability): Under A0 and A1, boundary-preserving compression induces scale-dependent flows toward attractors.

Finite resources force coarse-graining. Effective parameters must therefore vary with resolution.

7 Registers, Maximal Attractors, and Finite Collapse

Persistence is register-relative. Maximal attractors saturate under diminishing returns and ultimately decay. Registers exhibit bounded lifecycles: emergence, maximization, saturation, and collapse.

8 Conclusion

Dimensionality, amplitudes, and renormalization emerge as unavoidable consequences of representability under finite entropic resources. These results reframe core structures of modern physics as constraints on survivable descriptions rather than independent axioms.

References

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