MATH96023/MATH97032/MATH97140 - Computational Linear Algebra

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CONTENTS

CHAPTER

ONE

PRELIMINARIES

In this preliminary section we revise a few key linear algebra concepts that will be used in the rest of the course, emphasising the column space of matrices.

1.1 Matrices, vectors and matrix-vector multiplication

We will consider the multiplication of a vector

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots x_n \end{pmatrix}, \quad x_i \in \mathbb{C}, \ i = 1, 2, \ldots, n, \text{ i.e. } x \in \mathbb{C}^n,$$

by a matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix},$$

i.e. $A \in \mathbb{C}^{m \times n}$. A has m rows and n columns so that the product

$$b = Ax$$

produces $b \in \mathbb{C}^m$. In this course it is important to consider the general case where $m \neq n$, which has many applications in data analysis, curve fitting etc. We will usually state generalities in this course for vectors over the field $\mathbb{C}$, noting where things specialise to $\mathbb{R}$.

We can quickly check that the map $x \to Ax$ given by matrix multiplication is a linear map from $\mathbb{C}^n \to \mathbb{C}^m$, since

$$A(\alpha x + y) = \alpha Ax + Ay,$$

for all $x, y \in \mathbb{C}^n$ and $\alpha \in \mathbb{C}$.