2.2 Multimode devices

Waveguides, devices and sensors

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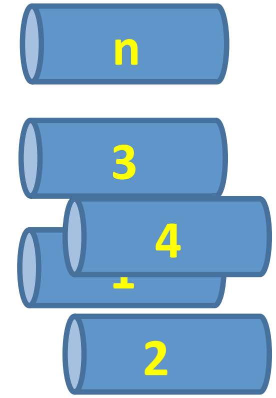
Summary of previous lectures

- In a perfect uniform optical fibre modes are orthogonal and do not exchange power.
- Tapered devices can remove orthogonality if the transition region is abrupt
- If transition region is smooth, no power exchange occurs and tapers are called adiabatic
- Couplers exchange power between different cores
- Supermodes are exact solution of Maxwell equations
- Coupled wave equations provide solution of multi-waveguide systems from solutions of single waveguides
- Perturbation theory provides coupling coefficients

Coupled wave equations



Coupling between different waveguides is described by:

$$
\begin{cases}
\dfrac{dA_1}{dz} = i\kappa_{12} A_2 e^{i(\beta_1-\beta_2)z} + i\kappa_{13} A_3 e^{i(\beta_1-\beta_3)z} + \ldots + i\kappa_{1n} A_n e^{i(\beta_1-\beta_n)z} \\
\dfrac{dA_2}{dz} = i\kappa_{21} A_2 e^{i(\beta_2-\beta_1)z} + i\kappa_{23} A_3 e^{i(\beta_2-\beta_3)z} + \ldots + i\kappa_{2n} A_n e^{i(\beta_2-\beta_n)z} \\
\ldots \\
\dfrac{dA_n}{dz} = i\kappa_{n1} A_2 e^{i(\beta_n-\beta_1)z} + i\kappa_{n3} A_3 e^{i(\beta_n-\beta_3)z} + \ldots + i\kappa_{nn-1} A_{n-1} e^{i(\beta_n-\beta_{n-1})z}
\end{cases}
$$

where k_{nm} are coupling coefficients between waveguides *n* and *m*.

Outline

- Coupled Wave Equations in multimode waveguides
- Fibre Bragg Gratings (FBGs)
- Long Period Gratings (LPGs)

Learning Outcome

- Assess coupling between different modes
- Describe the working principle of fiberised devices (FBGs, LPGs)
- Design optical devices

MM Coupled wave equations

If each of the waveguides *n*, *m* can support multiple modes *x*, coupling between all possible modes has to be considered

$$dA_{\nu} = \sum_{\mu} \sum_{\xi} \pm i \kappa_{\nu\mu\xi} A_{\mu\xi} e^{i\left(\beta_{\nu} - \beta_{\mu\xi}\right) z}$$

+ is used for copropagating,

- for counterpropagating modes

Propagation equation

Wave propagation is described by Maxwell's equations

$$\begin{cases} \nabla \times E = -\dfrac{\partial B}{\partial t} \\ \nabla \times H = J + \dfrac{\partial D}{\partial t} \\ \nabla \cdot (\varepsilon E) = \sigma \\ \nabla \cdot (\mu H) = 0 \end{cases}$$

$$B = \mu H$$

$$D = \varepsilon E$$

D=Electric displacement field

B= Magnetic induction

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t} \right)$$

$$\nabla \times (\nabla \times E) = -\mu \nabla \times \left(\frac{\partial H}{\partial t} \right)$$

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial (\nabla \times H)}{\partial t}$$

J=0

$$\nabla \times (\nabla \times E) = \nabla (\nabla \cdot E) - \nabla^2 E$$

$$\cancel{\nabla (\nabla \cdot E)} - \nabla^2 E = \mu \frac{\partial \left(\frac{\partial D}{\partial t} \right)}{\partial t} = -\mu \frac{\partial^2 D}{\partial t^2}$$

s=0

$$\nabla^2 E = \mu \frac{\partial^2 D}{\partial t^2}$$

Perturbation theory (simplified)

Perturbation is included in the definition of D:

$$D = \varepsilon E + D_{pert}$$

$$\nabla^2 E = \mu \frac{\partial^2 D}{\partial t^2} \qquad \nabla^2 E = \mu \left(\varepsilon \frac{\partial^2 E}{\partial t^2} + \frac{\partial^2 D_{pert}}{\partial t^2} \right)$$

$$\nabla^2 E - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = \mu \frac{\partial^2 D_{pert}}{\partial t^2}$$

Rough approximation!

And writing E as a superposition of modes E_j: $E(z) = \sum_j A_j(z) E_j(x,y) e^{-i(\beta_j z - \omega t)}$

$$\sum_j \left(\frac{\partial^2 A_j(z)}{\partial z^2} - 2i\beta_i \frac{\partial A_j(z)}{\partial z} \right) E_j(x,y) e^{-i(\beta_j z - \omega t)} + A_j(z) \left[\cancel{\nabla^2 \left(E_j(x,y) e^{-i(\beta_j z - \omega t)} \right) - \frac{\partial^2 \left(E_j(x,y) e^{-i(\beta_j z - \omega t)} \right)}{\partial t^2}} \right] = \mu_0 \frac{\partial^2 D_{pert}}{\partial t^2}$$

Modes are solution of Propagation equations

$$\sum_j \left(\cancel{\frac{\partial^2 A_j(z)}{\partial z^2}} - 2i\beta_i \frac{\partial A_j(z)}{\partial z} \right) E_j(x,y) e^{-i(\beta_j z - \omega t)} = \mu_0 \frac{\partial^2 D_{pert}}{\partial t^2}$$

Slowly varying envelop approximation

$$\sum_j -2i\beta_i \frac{\partial A_j(z)}{\partial z} E_j(x,y) e^{-i(\beta_j z - \omega t)} = \mu_0 \frac{\partial^2 D_{pert}}{\partial t^2}$$

D. Marcuse, Bell sys. Tech. J., 54(6) (1975) 995

B.E. Little and W.P. Huang, PIER 10 (1995) 217-70

A. Yariv, J. Quantum Electron. 9(9) (1973) 919-933

Coupled wave equations

$$\sum_j -2i\beta_i \frac{\partial A_j(z)}{\partial z} E_j(x,y) e^{-i(\beta_j z-\omega t)} = \mu_0 \frac{\partial^2 D_{pert}}{\partial t^2}$$

Each modal component can be found multiplying by $E_k{}^*(x,y)$ and integrating

$$\iint \sum_j -2i\beta_i \frac{\partial A_j(z)}{\partial z} E_k^*(x,y) e^{i(\beta_k z-\omega t)} E_j(x,y) e^{-i(\beta_j z-\omega t)} dxdy = \iint \mu_0 E_k^*(x,y) e^{i(\beta_k z-\omega t)} \frac{\partial^2 D_{pert}}{\partial t^2} dxdy$$

using the orthogonality condition and $c_{kj} = \iint E_k^*(x,y) E_j(x,y) dxdy$

$$-\frac{\partial A_k(z)}{\partial z} = \frac{-i}{2\beta_k \cancel{c_k}} \frac{\partial^2}{\partial t^2} \iint \mu_0 E_k^*(x,y) e^{i(\beta_k z-\omega t)} D_{pert} dxdy$$

c=1 for single MM fibre

Writing D_{pert} as a linear superposition of modes E_j:

$$D_{pert}(z) = \sum_j D_j(z) E_j(z) e^{-i(\beta_j z-\omega t)}$$

$$-\frac{\partial A_k(z)}{\partial z} = \frac{i\omega^2}{2\beta_k} \sum_j \iint \mu_0 E_k^*(x,y) e^{i(\beta_k z-\omega t)} E_j(x,y) e^{-i(\beta_j z-\omega t)} D_j(z) dxdy$$

Coupled wave equations (Cont')

$$-\frac{\partial A_k(z)}{\partial z} = \frac{i\omega^2}{2\beta_k}\sum_j \iint \mu_0 E_k^*(x,y) e^{i(\beta_k z - \omega t)} D_j(z) E_j(x,y) e^{-i(\beta_j z - \omega t)} dxdy$$

$$-\frac{\partial A_k(z)}{\partial z} = \frac{i\omega^2}{2\beta_k}\sum_j \iint \mu_0 E_k^*(x,y) D_j(z) E_j(x,y) e^{i(\beta_k - \beta_j)z} dxdy$$

if then D is expressed as a function of the refractive index perturbations:

$$D_j(z) = \varepsilon_0 \Delta n^2(x,y)$$

$$-\frac{\partial A_k(z)}{\partial z} = i\sum_j \iint \frac{\mu_0 \varepsilon_0 \omega^2}{2\beta_k} E_k^*(x,y) \Delta n^2 E_j(x,y) dxdy e^{i(\beta_k - \beta_j)z}$$

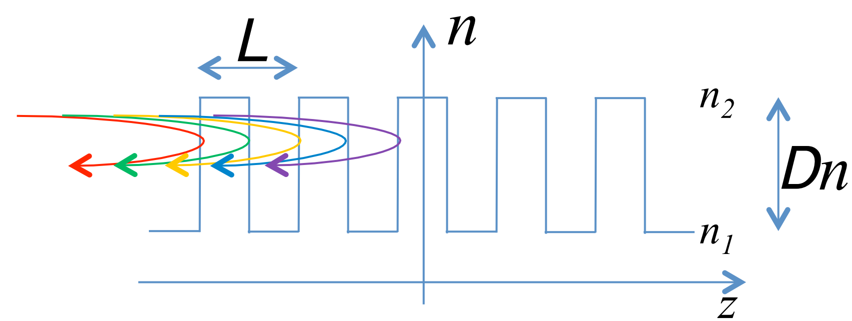
$$-\frac{\partial A_k(z)}{\partial z} = i\sum_j C_{jk} e^{i(\beta_k - \beta_j)z}$$

Bragg gratings

Gratings are periodic structures with alternated layers of different n.

Reflection at each interface is given by

$$R = \frac{n_2 - n_1}{n_2 + n_1}$$



Each interface between different refractive indices reflects a fraction of the incident light.

When the different contributions are in phase, reflected fraction of the mode is maximised. As $Dn << n_1$, the **Bragg condition** is written as:

$$\lambda/n = 2\Lambda \qquad \lambda = 2n\Lambda$$

In Bragg gratings the structure periodicity is **comparable** to the wavelength of light propagating in it.

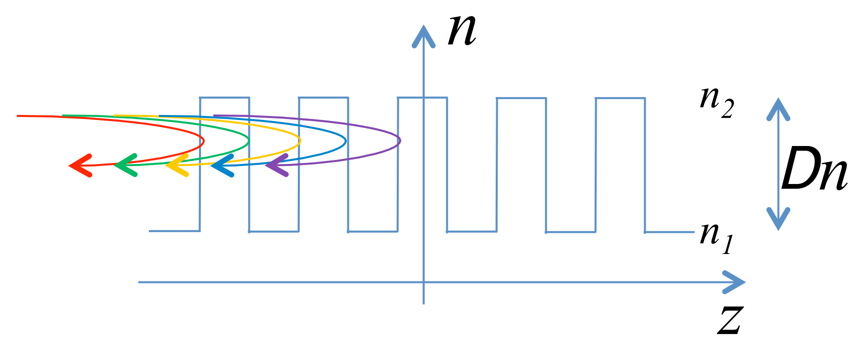
Fibre Bragg gratings

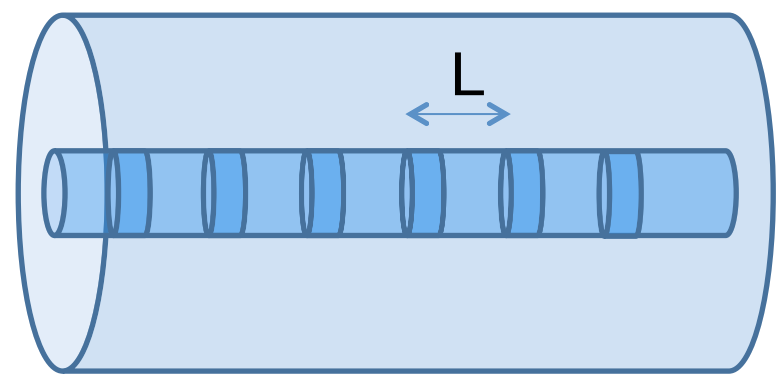
In optical fibres, only a fraction of the mode *h* propagates in the core.

Refractive index modulation is induced mainly in the core and it usually amounts to $Dn \sim 10^{-3} << n_1$

The **Bragg condition** then becomes:

$$\lambda = 2n_{eff}\Lambda$$
$$= 2\Lambda\left(\eta n_{core} + (1-\eta) n_{clad}\right)$$





Coupled wave equations

Coupling between different waveguides is described by:

$$\begin{cases} dA_1 = i\kappa_{11}A_1\cancel{e^{i(\beta_1-\beta_1)z}} + i\kappa_{12}A_2 e^{i(\beta_1-\beta_2)z} \\ dA_2 = i\kappa_{21}A_1 e^{i(\beta_2-\beta_1)z} + i\kappa_{22}A_2\cancel{e^{i(\beta_2-\beta_2)z}} \end{cases}$$

κ κ

There are self-coupling terms k$_{11}$ and k$_{22}$ because the waveguide refractive index is not constant.

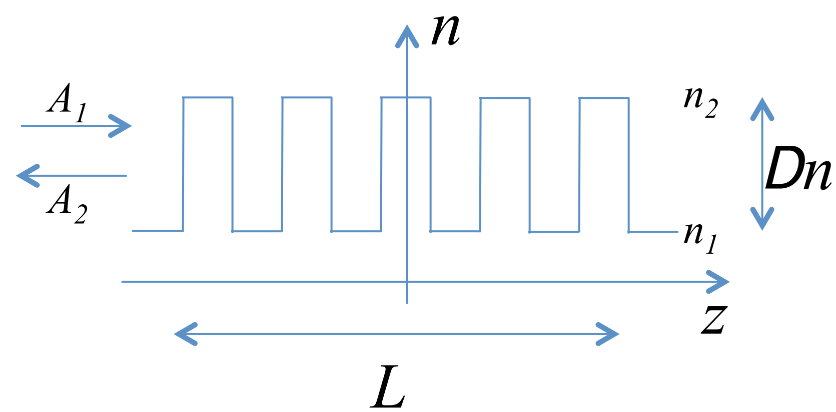
$$\kappa_{\nu\mu} = \omega c_{\nu\mu}\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty}\left(E_\nu^* \cdot \Delta n_\mu^2 \cdot E_\mu\right)dx\,dy$$

$$c_{\nu\mu} = 1$$

Bragg grating spectra

The wavelength response of Bragg gratings can be calculated using the coupled wave equations:

$$\begin{cases} dA_1 = i\kappa_{11}A_1 - i\kappa_{12}A_2 e^{i(\beta_2-\beta_1)z} \\ -dA_2 = i\kappa_{21}A_1 e^{i(\beta_1-\beta_2)z} - i\kappa_{22}A_2 \end{cases}$$



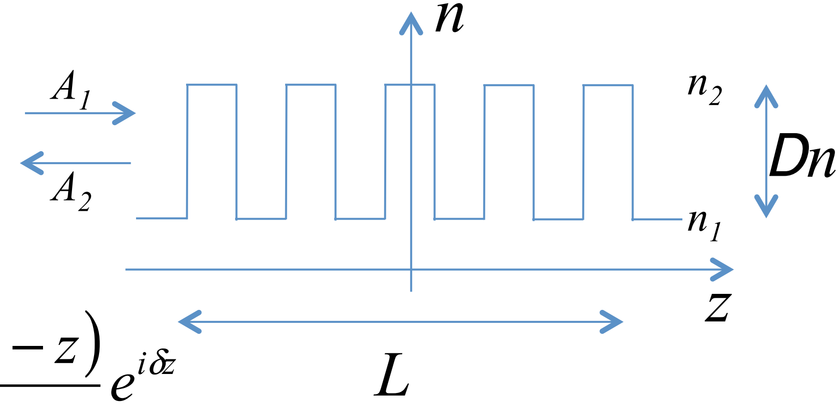
where

$$\kappa_{\nu\mu} = \omega \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left(E_\nu^* \cdot \Delta n^2 \cdot E_\mu\right) dxdy$$

c_{nm}=1 and Dn_m= Dn

Bragg grating spectra

If $A_1(0)=A_0$ is the boundary condition, the analytical solution gives $A_1(z)$ and $A_2(z)$:



$$\begin{cases} A_1(z) = A_0 \dfrac{\alpha_c \cosh \alpha_c (L-z) + i\delta \sinh \alpha_c (L-z)}{\alpha_c \cosh \alpha_c L + i\delta \sinh \alpha_c L} e^{i\delta z} \\ A_2(z) = A_0 \dfrac{i\kappa_{21} \sinh \alpha_c (L-z)}{\alpha_c \cosh \alpha_c L + i\delta \sinh \alpha_c L} e^{-i\delta z} \end{cases}$$

where $\alpha_c = \sqrt{\kappa_{12}\kappa_{21} - \delta^2}$

And $\delta = (\beta_2 + \kappa_{22}) - (\beta_1 + \kappa_{11})$ is the **detuning**

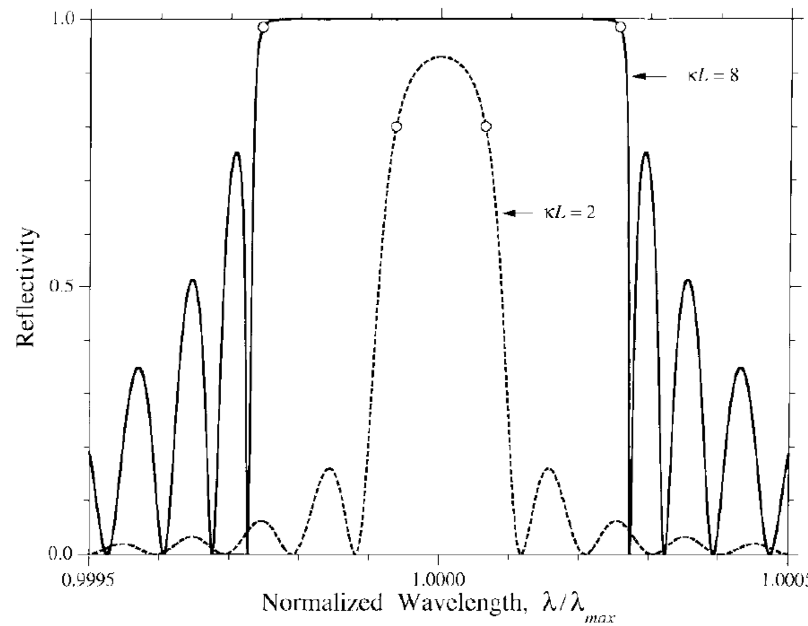
T. Erdogan, J. Lightwave Technol. 15, (1997) 1277-1294

Bragg grating spectra

In practice, instruments measure powers which are proportional to $\left|A_v\right|^2$

Reflected and transmitted powers are given by $\left|\frac{A_1(L)}{A_1(0)}\right|^2$ and $\left|\frac{A_2(0)}{A_1(0)}\right|^2$

$$\begin{cases} T = \left|\dfrac{\alpha_c}{\alpha_c \cosh \alpha_c L + i\delta \sinh \alpha_c L}\right|^2 \\ R = \left|\dfrac{\kappa_{21} \sinh \alpha_c L}{\alpha_c \cosh \alpha_c L + i\delta \sinh \alpha_c L}\right|^2 \end{cases}$$



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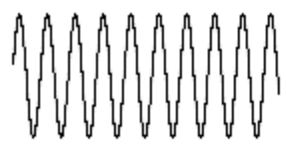
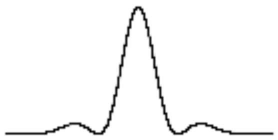
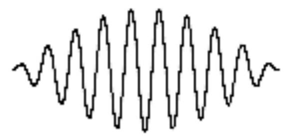
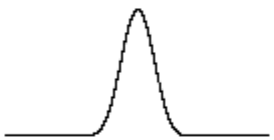
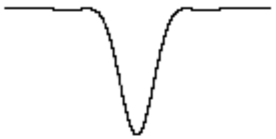
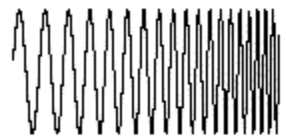
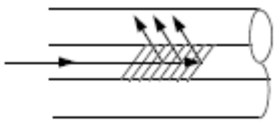
At l_B, d=0 and k_{21}=k_{12}=a_c=k

$$\boxed{R = \tanh^2 \kappa L}$$

$$\kappa = \frac{\pi \Delta n}{\lambda}$$

Grating spectra

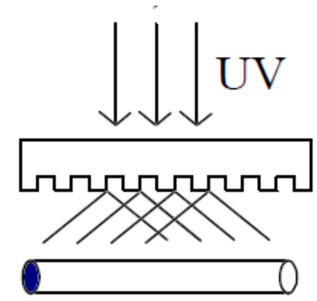
FBG can have complex spectra if refractive index modulation is not uniform along the longitudinal direction:

	Grating	Δn_{mod}	Reflection	Transmission
a)	uniform			
b)	apodized			
c)	chirped			
d)	blazed			

Bragg grating writing

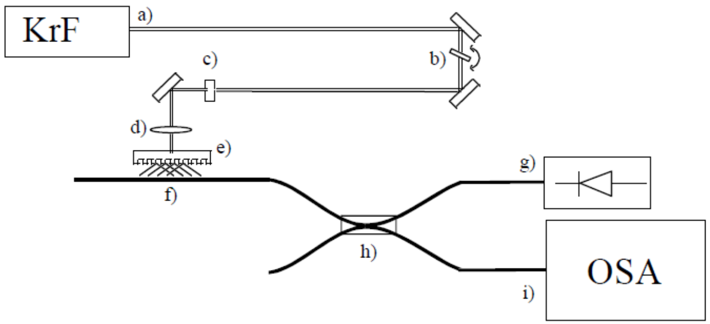
Most commonly used techniques:

- Phase mask
- Interferometric
- Point by point



Most commonly used sources:

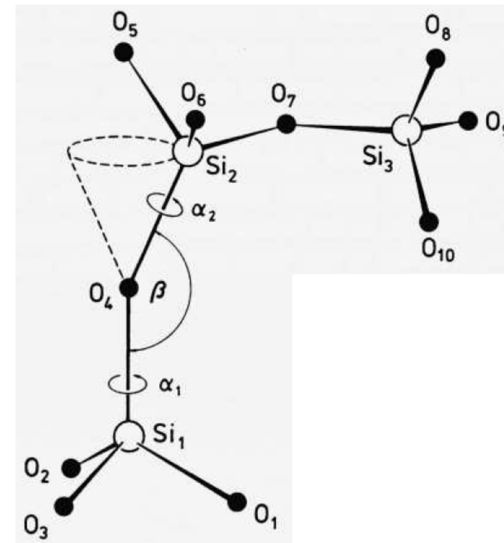
- Frequency doubled Ar+ laser (244nm)
- Excimer lasers: KrF @ 248nm or ArF @ 193nm



FBGs are usually written exploiting material photosensitivity at selected wavelengths

Photosensitivity

Photosensitivity: an induced refractive index change Dn_{mod} as a consequence to exposure to light.



Two contributions: compaction and colour centre

$$\frac{\Delta n_{mod}}{n} = \frac{(2+n^2)(1-n^2)}{6n^2}\left[\frac{\Delta V}{V} - \frac{\Delta R_m}{R_m}\right]$$

Compaction increases material density

$$\frac{\Delta \rho}{\rho} = \left(-\frac{\Delta V}{V} =\right) \mathrm{a} \left(\frac{I_p^2 N}{\tau}\right)^b$$

Colour centre are new chemical species absorbing in the UV, thus changing refractive index at longer lD.

Dn can be evaluated from Kramers-Krönig:

$$\Delta n(\lambda') = \frac{1}{2\pi^2}\int_0^\infty \frac{\Delta\alpha(\lambda)}{1-\left(\frac{\lambda}{\lambda'}\right)^2} d\lambda.$$

Formula	Defect	Acronym	UV band	EPR
Ge—Si	Neutral Oxygen Mono-Vacancy	NOMV	242 nm	-
Ge	Germanium E'	GeE'	270 + 244 [60] + 214 nm	YES
Si O Ge	Self Trapped Hole	STH	-	YES
Ge	Germanium Electron Centre	GEC	-	YES
O Ge O	Neutral Oxygen Di-Vacancy	NODV	241.2 nm	-
Si O Ge	Bridging Oxygen	BO	-	-

Long Period Grating (LPG)

If L>>l the grating is called Long Period Grating.

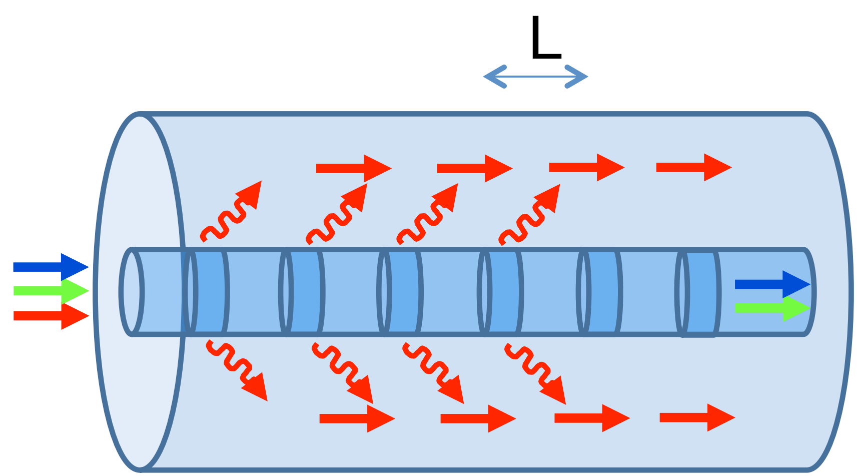
Bragg gratings couple counterpropagating modes, LPGs propagating.

If modes propagating in the core and in the cladding have propagation constants b_n and b_m, coupling is maximised when:

$$\beta_\nu - \beta_\mu = n\frac{2\pi}{\Lambda}$$

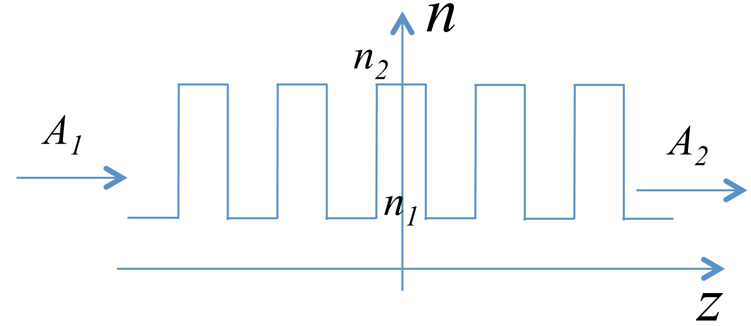
m is integer

LPGs usually have only few pitches, but with high Dn (10^{-3}-10^{-2})



LPG spectra

The wavelength response of LPG can be calculated using coupled wave equations for co-propagating modes:



$$\begin{cases} dA_1 = i\kappa_{11}A_1 + i\kappa_{12}A_2 e^{i(\beta_2-\beta_1)z} \\ dA_2 = i\kappa_{21}A_1 e^{i(\beta_1-\beta_2)z} + i\kappa_{22}A_2 \end{cases}$$

which has the analytical solution for $A_1(0)=A_0$ and $A_2(0)=0$:

$$\begin{cases} A_1(z) = A_0 e^{-i\delta z}\left(e^{-i\Delta\beta z}\cos(\gamma_c z) + i\dfrac{\delta}{\gamma_c} e^{-i\Delta\beta z}\sin\gamma_c z \right) \\ A_2(z) = A_0 i\dfrac{\kappa_{21}}{\gamma_c} e^{-i\kappa_{22}z}\sin(\gamma_c z) e^{-i\delta z} \end{cases}$$

where $\gamma_c = \sqrt{\kappa_{12}\kappa_{21} + \delta^2}$ $\quad \Delta\beta = \beta_1 - \beta_2$

and *d* is still the **detuning** $\delta = \dfrac{(\Delta\beta + \kappa_{22} - \kappa_{11})}{2}$

LPG spectra

In practice, instruments measure powers which are proportional to $\left|A_\nu\right|^2$

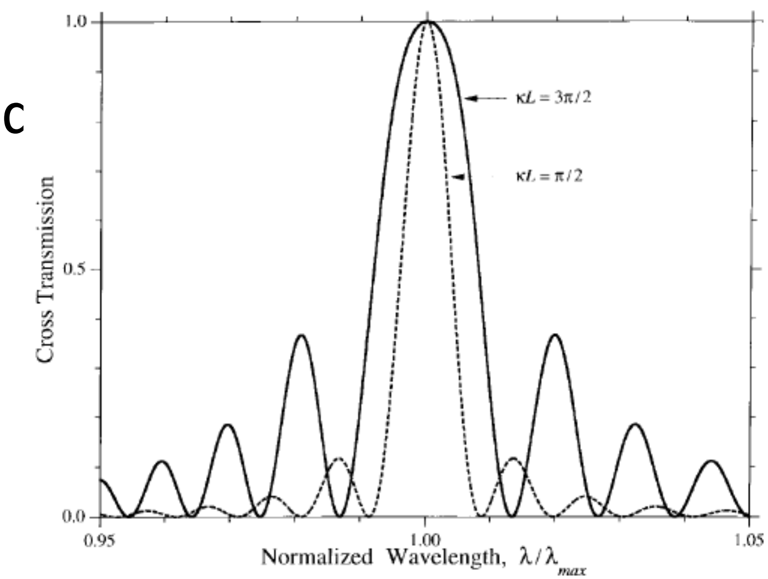
Cross transmitted power (power coupled to cladding mode) is given by $\left|\frac{A_1(L)}{A_1(0)}\right|^2$

$$T = \left|\frac{A_1(z)}{A_0}\right|^2 = \frac{\kappa_{12}\kappa_{21}}{\gamma_c}\cos^2\gamma_c z + \left(\frac{\delta}{\gamma_c}\right)^2$$

$$\kappa = \frac{\pi\Delta n}{\lambda}$$

At l_B, d=0 and $k_{21}=k_{12}=g_c=k$

$$T = \kappa\cos^2\kappa z$$



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LPG writing

Most commonly used techniques:

- Amplitude mask
- Point by point

Most commonly used sources:

- CO_2 laser
- Arc discharge
- Frequency doubled Ar+ laser (244nm)
- Excimer lasers: KrF @ 248nm or ArF @ 193nm

CO_2 laser and arc discharge exploit change in material density after solidification, lasers photosensitivity at selected wavelengths