

Introduction to OpenFOAM® Computational Library and Viscoelastic Fluid Flow Simulation

P4 - Case studies: Viscoelastic fluid flow solvers

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Outline

9:00 - 10:30	Introduction to OpenFOAM (P1)
10:30 - 12:00	Mesh generation and post-processing (P2)
12:00 - 13:00	Lunch break
13:00 - 14:30	Case studies: Single- and two-phase flow solvers (P3)
14:30 – 16:00	Case studies: Viscoelastic fluid flow solvers (P4)

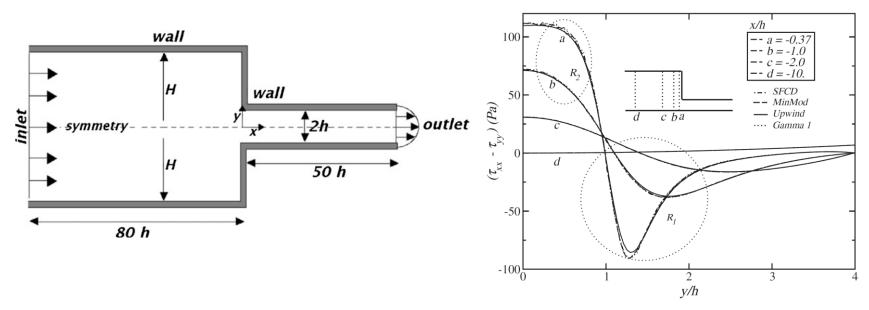


Viscoelastic Solvers in OpenFOAM

Differential viscoelastic flow solvers in OpenFOAM

Multi mode constitutive models

Maxwell (L,E, Feta) PTT FENE-(P,CR)
Oldroyd-B Giesekus (S,D) XPP
White Metzner Leonov DCPP



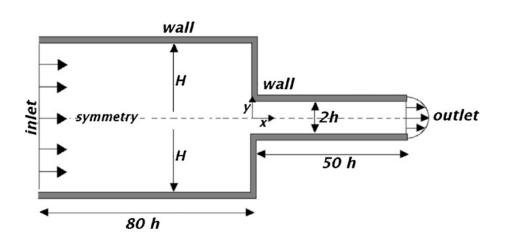


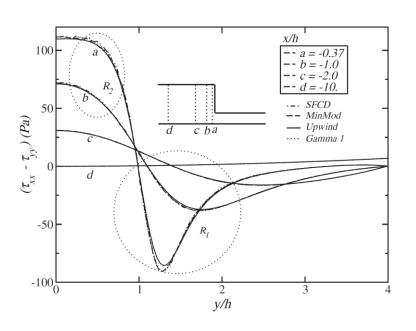


<u>Differential viscoelastic flow solvers in OpenFOAM</u>

Multi mode constitutive models

Maxwell(L,E, Feta) PTTFENE-(P,CR)Oldroyd-BGiesekus(S,D) XPPWhite MetznerLeonovDCPP



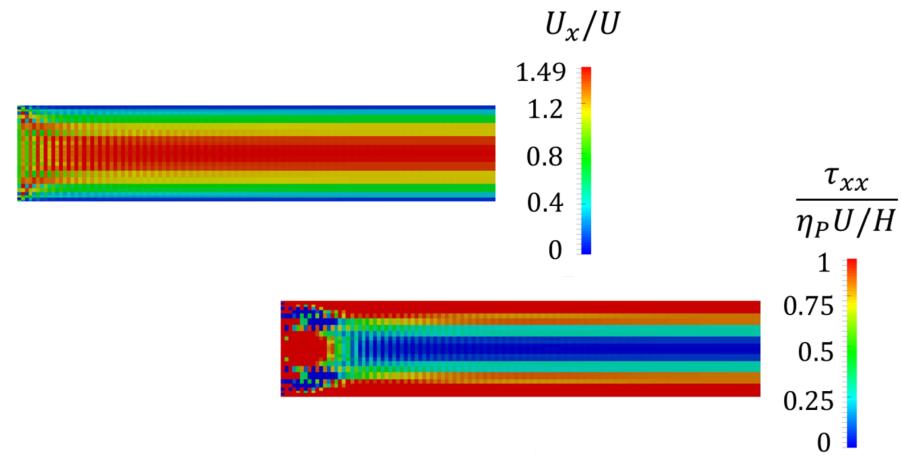




JL Favero et al. (Computer Aided Chemical Engineering, 2009, 2010 / **Journal of non-Newtonian Fluid Mechanics**, 2010 / Computers & Chemical Engineering, 2010)

Poiseuille Flow + **UCM** (Oldroyd-B η_s =0)

Re=0.01, De=1



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Improved Both Sides Diffusion The iBSD Improved Both Sides Diffusion

The iBSD – Improved Both-Sides-Diffusion

```
Conservati
                fvm::div(phi, U)
    Origi - fvm::laplacian(etaStr/rho, U))
            fvm::div(phi,U)
           - fvm::laplacian(etaStr/rho, U))
    BSD ==
           - fvc::grad(p)
          + fvc::div(tau/rho)
             fvc::div((etaStr/rho)*fvc::grad(U))
     iBS
              \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \frac{\eta * \nabla^2 \mathbf{U}}{\eta} = -\nabla p + \nabla \cdot \tau - \nabla \cdot (\eta * \nabla \mathbf{U})
```

Larger Stencil



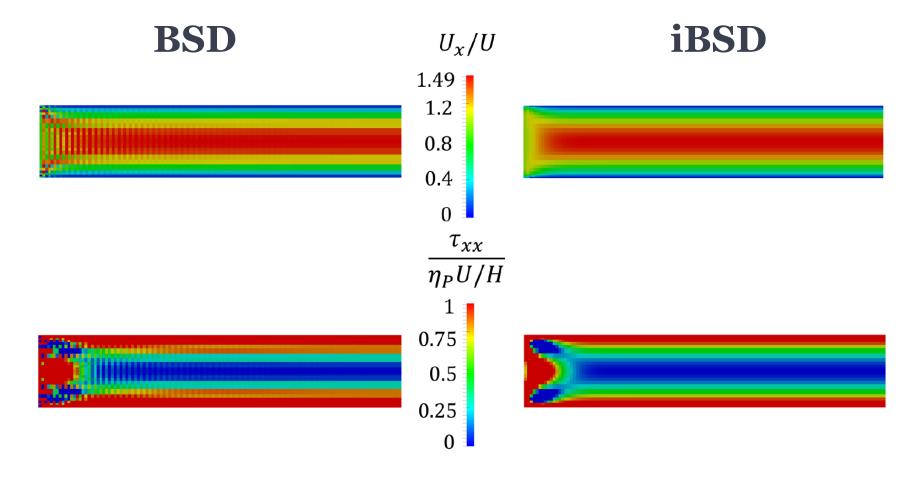
Poiseuille Flow + UCM (Oldroyd-B h_s=0)

Re=0.01, De=1

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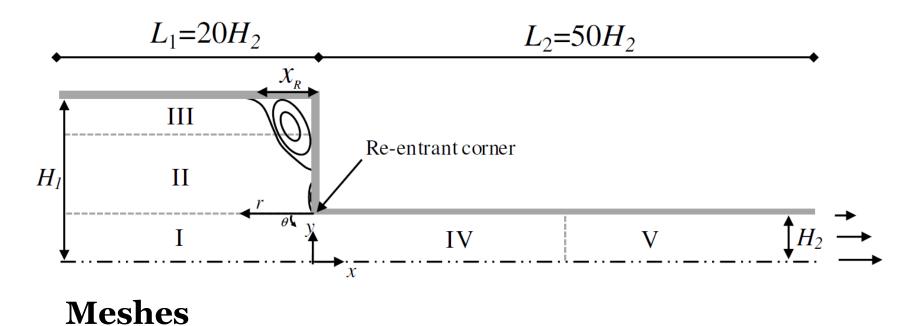
iPC

Digital Transformation





Case Study 1 – 4:1 Contraction (UCM)









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$$M_5$$
 – 228128 Cells

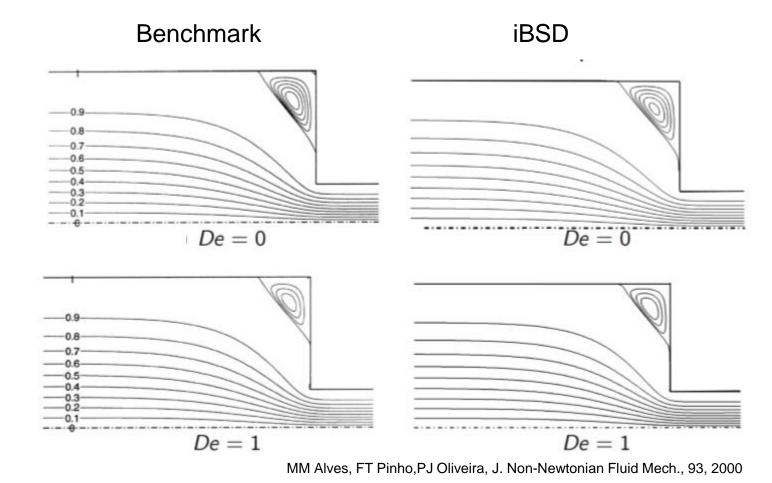
M1 – 228 Cells

$$Re = 0.01$$

$$\mathbf{De} = \{0,1,2,3,4,5\}$$

Case Study 1 – 4:1 Contraction (UCM)

Streamlines



C Fernandes et al., Improved Both Sides Diffusion (iBSD): a new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017 IPPD Institute of

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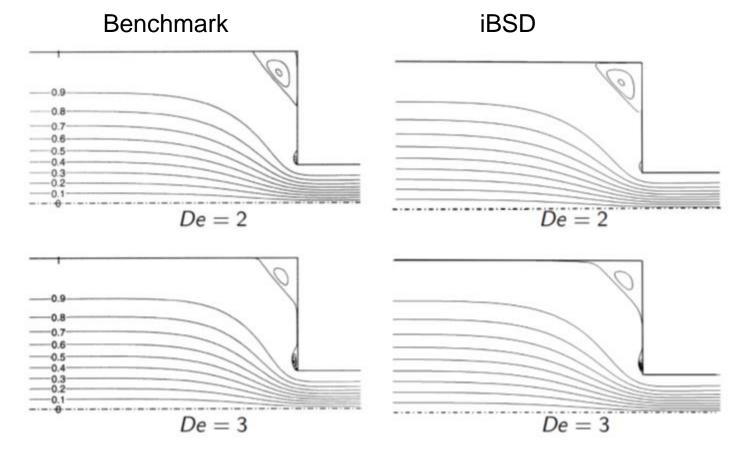
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MM Alves, FT Pinho, PJ Oliveira, J. Non-Newtonian Fluid Mech., 93, 2000

C Fernandes et al., Improved Both Sides Diffusion (iBSD): a new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017 IPPD Institute of Polymer Processing and

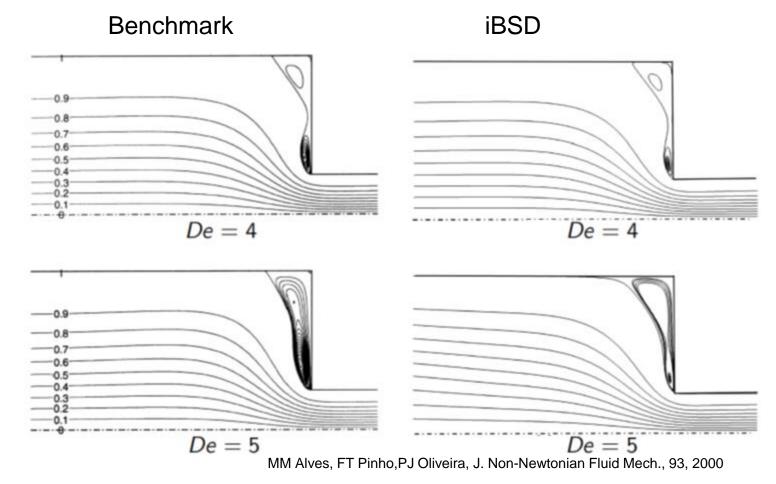
Case Study 1 – 4:1 Contraction (UCM)

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C Fernandes et al., Improved Both Sides Diffusion (iBSD): a new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017 **IPPD Institute of Polymer Processing and**

Case Study 1 – 4:1 Contraction (UCM)

Primary vortex length (Xr=xr/H2)

Developed code in OpenFOAM®							
De	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5	Extrapolated	Difference (%)
0	1.438	1.479	1.492	1.495	1.496	1.4965	0.03
1	1.293	1.336	1.326	1.32711	1.32696	1.32694	0.002
2	1.207	1.200	1.130	1.101	1.091	1.0857	0.5
3	1.333	1.118	0.958	0.900	0.885	0.880	0.6
4	1.391	1.088	0.845	0.75	0.732	0.728	0.6
5	1.469	1.101	0.758	0.636	0.617	0.613	0.6
Alves et al. (2000)							
				Alves et a	al. (2000)		
De	Mesh 1	Mesh 2	Mesh 3	Alves et a	Extrapolated	Difference (%)	
De 0	Mesh 1 1.472	Mesh 2 1.488	Mesh 3 1.494			Difference (%)	
				Mesh 4	Extrapolated	\	
0	1.472	1.488	1.494	Mesh 4 1.495	Extrapolated 1.496	0.1	
0 1	1.472 1.349	1.488 1.371	1.494 1.349	Mesh 4 1.495 1.339	Extrapolated 1.496 1.335	0.1 0.3	
0 1 2	1.472 1.349 1.631	1.488 1.371 1.259	1.494 1.349 1.154	Mesh 4 1.495 1.339 1.118	Extrapolated 1.496 1.335 1.105	0.1 0.3 1.2	

MM Alves, FT Pinho, PJ Oliveira, J. Non-Newtonian Fluid Mech., 93, 2000



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$$\mathbf{\tau}_{p} = \int_{-\infty}^{t} M(t - t') f(\mathbf{B}_{t'}) dt'$$

$$M(t-t') = \sum_{k} \frac{a_k}{\lambda_k} e^{-\frac{(t-t')}{\lambda_k}}$$

$$f\left(\mathbf{B}_{\mathbf{t}'}\right) = \begin{cases} \mathbf{B}_{\mathbf{t}'} & -\text{UCM} \\ \frac{\alpha}{\alpha + \beta I_1 + (1 - \beta)I_2} \mathbf{B}_{\mathbf{t}'} & -\text{K-BKZ (PSM)} \end{cases}$$



Deformation Fields approach

$$\frac{\partial \mathbf{B}_{i}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{B}_{i}) - ((\nabla \mathbf{U})^{T} \cdot \mathbf{B}_{i} + \mathbf{B}_{i} \cdot \nabla \mathbf{U}) = 0$$



Araujo, MSB., et al,. "A stable numerical implementation of integral viscoelastic models in the OpenFOAM® computational library. Computers & Fluids, 2018

Stress Tensor Field Calculation

- Define **cutoff time** (s_{max}) and **number of deformation fields** (nf)
- At each time step t:
 - 1. Update velocity and pressure fields (PISO)
 - 2. Transport all the previously created deformation fields $(\mathbf{B}_{i=1,n_f})$

$$\frac{\partial \mathbf{B}_{i}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{B}_{i}) - ((\nabla \mathbf{U})^{T} \cdot \mathbf{B}_{i} + \mathbf{B}_{i} \cdot \nabla \mathbf{U}) = 0$$

- 3. Create a new deformation field ($\mathbf{B}_t = \mathbf{I}$)
- 4. If required redistribute/interpolate the deformation fields
- 5. Compute the stress field



Integra OpenFOAM Code:

Stre

```
volTensorField L = fvc::grad(U);
for (int fieldI = 0; fieldI <= nActFields ; fieldI++)</pre>
         volTensorField SB = B[fieldI] & L;
         fvSymmTensorMatrix BEqn
              fvm::ddt(B[fieldI])
              + fvm::div(phi, B[fieldI], "div(phi,B)")
              twoSymm(SB)
           BEqn.solve(mesh.solutionDict().solver("B"));
```

$$\frac{\partial \mathbf{B}_{i}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{B}_{i}) - ((\nabla \mathbf{U})^{T} \cdot \mathbf{B}_{i} + \mathbf{B}_{i} \cdot \nabla \mathbf{U}) = 0$$

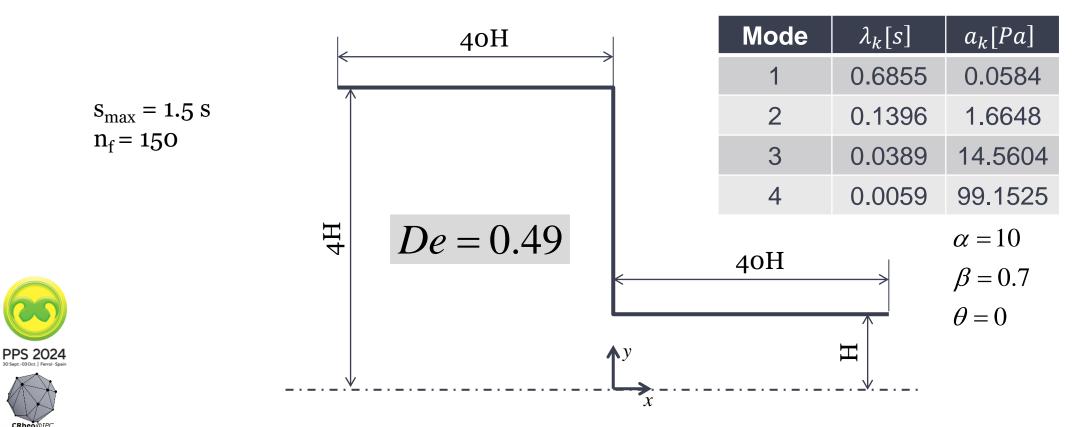
- Create a new deformation field ($\mathbf{B}_t = \mathbf{I}$)
- If required redistribute/interpolate the deformation fields
- Compute the stress field

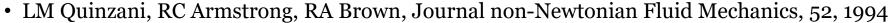




Case Study - 4:1 Abrupt Planar Contraction - K- BKZc

Solution of 5% wt. polyisobutylene (PIB) in Tetradecane(C14)



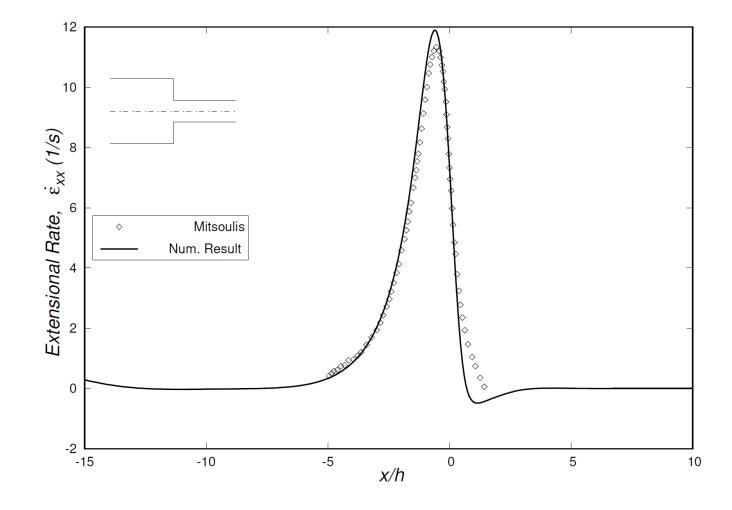


• E Mitsoulis, Journal of Rheology, 37, 1993

Araujo, MSB., et al,. "A stable numerical implementation of integral viscoelastic models in the OpenFOAM® computational library. Computers & Fluids, 2018

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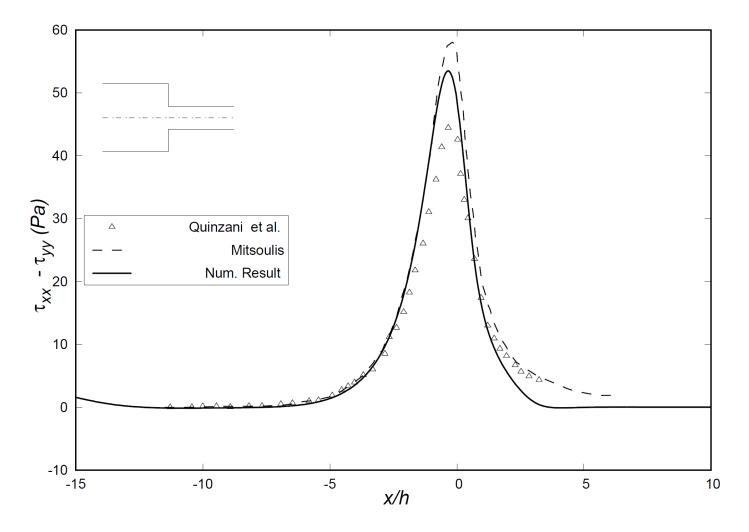
Case Study - 4:1 Abrupt Planar Contraction - K- BKZ







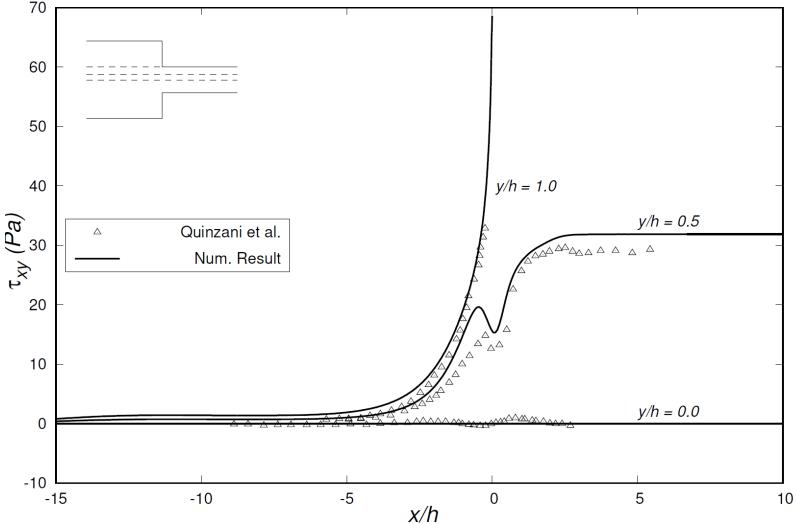
Case Study - 4:1 Abrupt Planar Contraction - K- BKZ







Case Study - 4:1 Abrupt Planar Contraction - K- BKZ





Araujo, MSB., et al,. "A stable numerical implementation of integral viscoelastic models in the OpenFOAM® computational library. Computers & Fluids, 2018

Coupled Approaches - Viscoelastic Semi-coupled solver

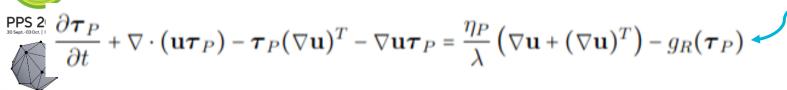
$$\begin{pmatrix} \mathbf{A}_u & \nabla \\ \nabla \cdot & -\nabla \cdot (\mathbf{\Gamma}_u^{-1} \nabla) \end{pmatrix} \begin{bmatrix} \mathbf{u} \\ p \end{bmatrix} = \begin{bmatrix} b_{\mathbf{u}} \\ b_p \end{bmatrix}$$

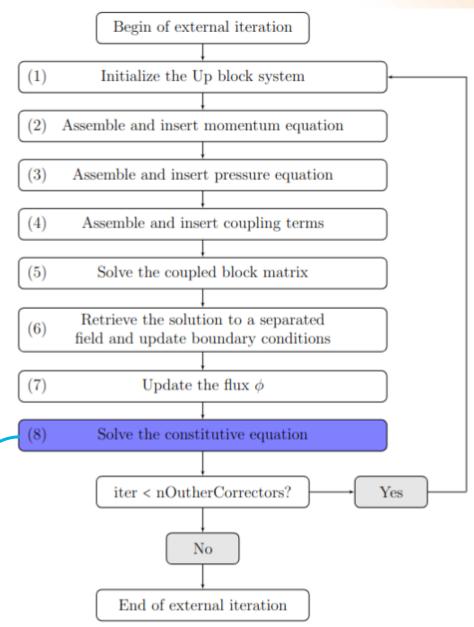
$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla \cdot (\rho\mathbf{u}\mathbf{u}) - \nabla^{2} \left[\left(\eta_{S} + \sum_{m=1}^{N_{m}} \eta_{Stab_{m}} \right) \mathbf{u} \right] = -\nabla p$$

$$+ \sum_{m=1}^{N_{m}} \left(\nabla \cdot \boldsymbol{\tau}_{P_{m}} \right) - \nabla \cdot \left[\nabla \left(\sum_{m=1}^{N_{m}} \eta_{Stab_{m}} \mathbf{u} \right) \right]$$

$$\nabla \cdot \mathbf{u} \left[-\nabla \cdot \left[\left(a_{ii}^{u} \right)^{-1} \nabla p \right] \right] = -\nabla \cdot \left[\overline{\left(a_{ii}^{u} \right)^{-1} \nabla p_{f}} \right],$$









Coupled Approaches - Viscoelastic fully-coupled solver

exaFOAM

$$\begin{pmatrix} \mathbf{A}_{u} & \nabla & \nabla \cdot \\ \nabla \cdot & -\nabla \cdot \begin{bmatrix} \mathbf{\Gamma}_{u}^{-1} \nabla \end{bmatrix} & 0 \\ \nabla \cdot (\boldsymbol{\tau}^{\star 0}) & 0 & \mathbf{A}_{\tau} \end{pmatrix} \begin{bmatrix} \mathbf{u} \\ p \\ \boldsymbol{\tau}^{\star} \end{bmatrix} = \begin{bmatrix} b_{\mathbf{u}} \\ b_{p} \\ b_{\boldsymbol{\tau}^{\star}} \end{bmatrix}$$

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla \cdot (\rho\mathbf{u}\mathbf{u}) - \nabla^{2} \left[\left(\eta_{S} + \sum_{m=1}^{N_{m}} \eta_{Stab_{m}} \right) \mathbf{u} \right] = -\nabla p$$

$$+ \left(\nabla \cdot \boldsymbol{\tau}^{\star} \right) + \sum_{\substack{m=1\\m \neq m_{C}}}^{N_{m}} \left(\nabla \cdot \boldsymbol{\tau}_{P_{m}} \right) - \nabla \cdot \left[\nabla \left(\sum_{m=1}^{N_{m}} \eta_{Stab_{m}} \mathbf{u} \right) \right]$$

$$\nabla \cdot \mathbf{u} \left[-\nabla \cdot \left[(a_{ii}^u)^{-1} \nabla p \right] \right] = -\nabla \cdot \left[(a_{ii}^u)^{-1} \nabla p_f \right],$$

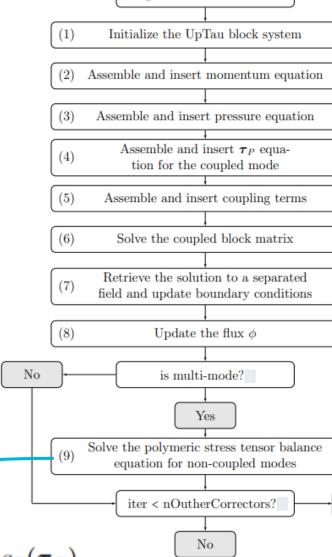
• For the **coupled** mode:



$$\frac{\partial \boldsymbol{\tau}_P}{\partial t} + \nabla \cdot (\mathbf{u}^0 \boldsymbol{\tau}_P^n) - \nabla \cdot (\mathbf{u}^0 \boldsymbol{\tau}_P^0) + \nabla \cdot (\mathbf{u}^n \boldsymbol{\tau}_P^0)$$

$$-\boldsymbol{\tau}_P(\nabla \mathbf{u})^T - \nabla \mathbf{u} \boldsymbol{\tau}_P = \frac{\eta_P}{\lambda} \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right) - g_R(\boldsymbol{\tau}_P)$$

For the other modes (decoupled):



End of external iteration

Begin of external iteration





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 $\frac{\partial \boldsymbol{\tau}_{P}}{\partial t} + \nabla \cdot (\mathbf{u}\boldsymbol{\tau}_{P}) - \boldsymbol{\tau}_{P}(\nabla \mathbf{u})^{T} - \nabla \mathbf{u}\boldsymbol{\tau}_{P} = \frac{\eta_{P}}{\lambda} \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^{T} \right) - g_{R}(\boldsymbol{\tau}_{P})$

Yes

Coupled Approaches - Application cases exaFOAM Memory requirement

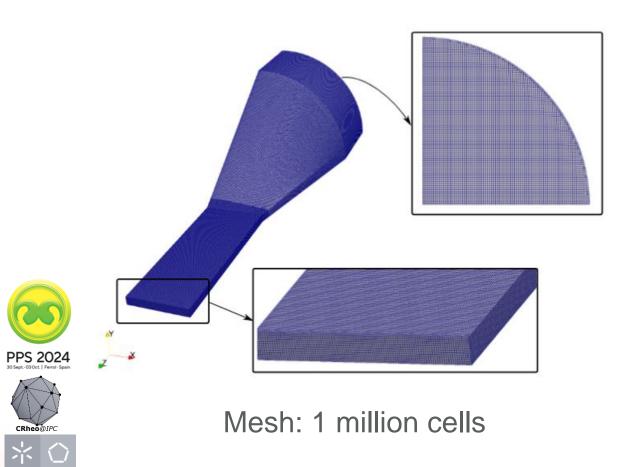
- Number of coupled variables:
 - Segregated: 0
 - Semi-coupled: 4 (p, U)
 - Fully-coupled: 10 (p, U, τ^*)

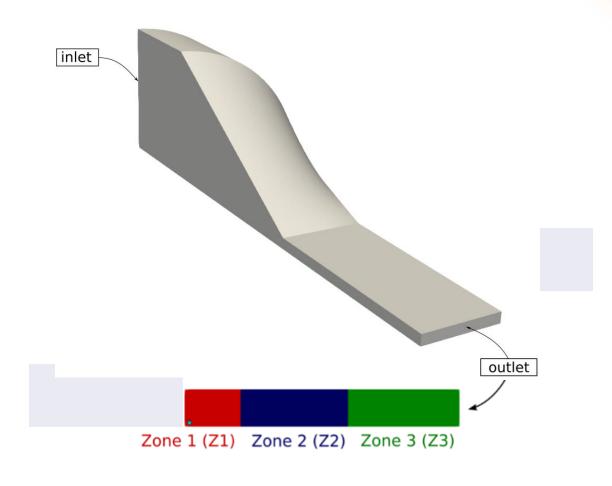
Solver	Number of cells [million]	Memory requirement [Gb]	Ratio to Segregated
Segregated	1	9.2	-
Semi-coupled	1	14.8	1.61
Fully-coupled	1	43.6	4.74
Segregated	20	173.9	-
Semi-coupled	20	561.6	3.23
Fully-coupled	20	845.6	4.86



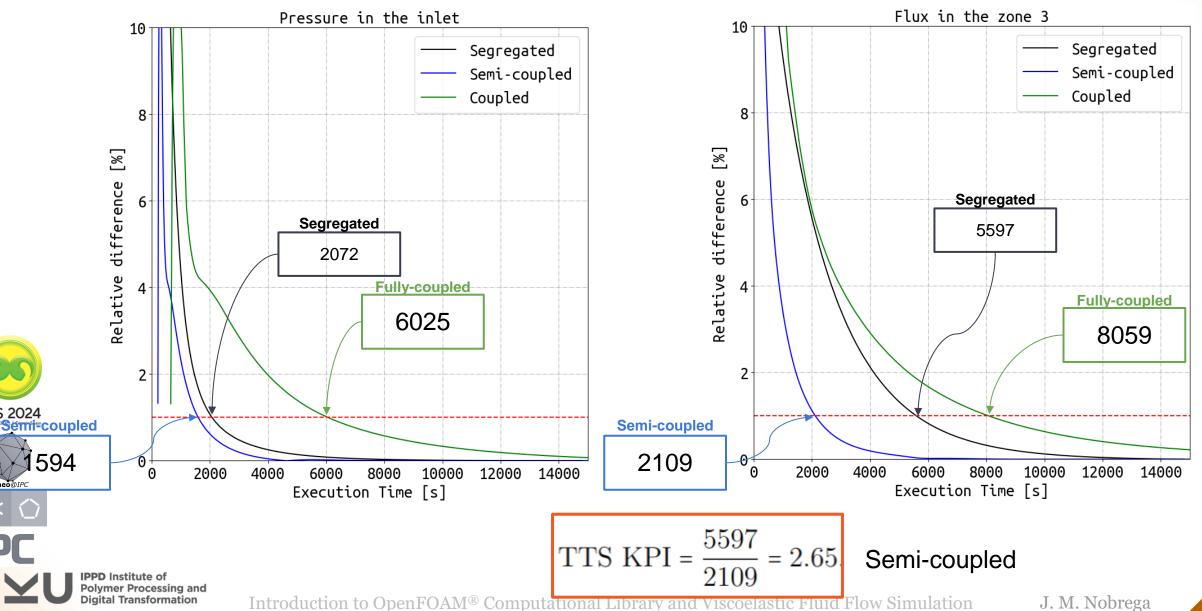
Coupled Approaches - Application case (MB19)

MB19 benchmark case: profile extrusion [4]



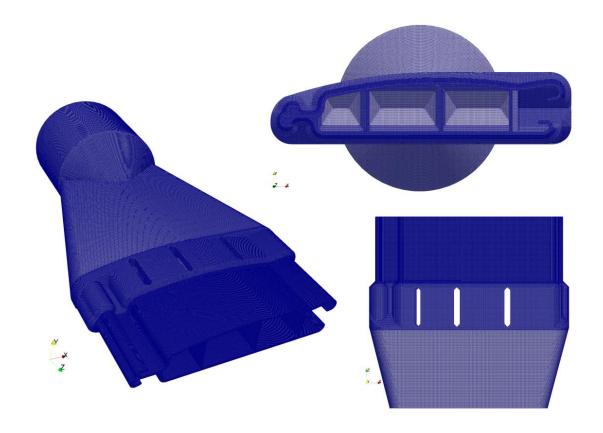


Coupled Approaches - Application case (MB19)

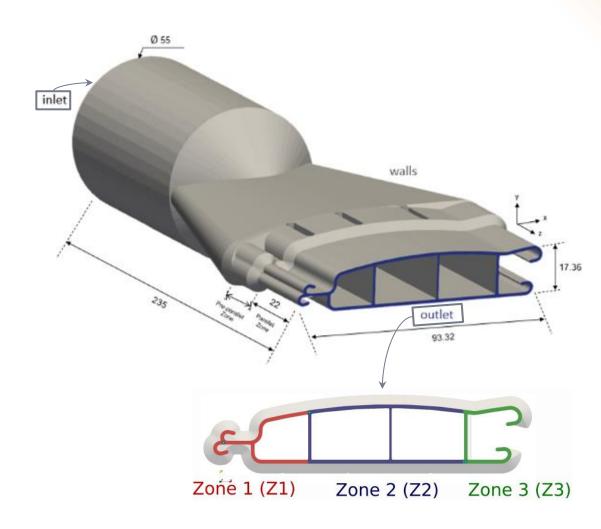


Coupled Approaches - Application case (B4)

B4 benchmark case: complex profile extrusion [4]







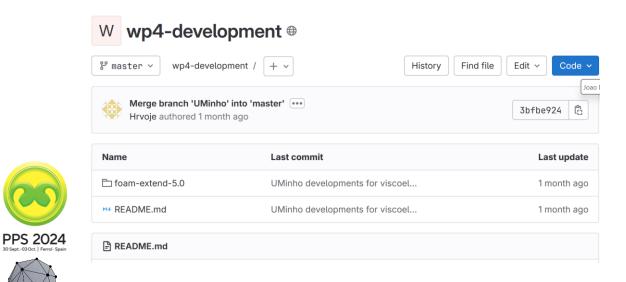


Coupled Approaches

exaFOAM

CODE

https://develop.openfoam.com/exafoam/wp4-development



CASE Studies

https://develop.openfoam.com/committees/hpc/-/tree/develop/viscoelastic/viscoelasticFluidFoam

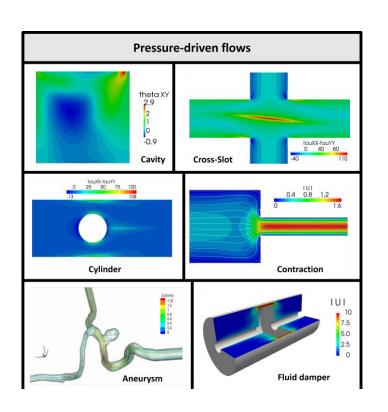
lame	Last commit
□ complexProfile	Update README.md
□ complexProfileExtrusion	Changes to visco-elastic cases
□ lidDrivenCavity	Update README.md
□ profileExtrusion	Update README.md
□ testedCases	Changes to visco-elastic cases
* README.md	Changes to visco-elastic cases
generateCase.sh	Changes to visco-elastic cases

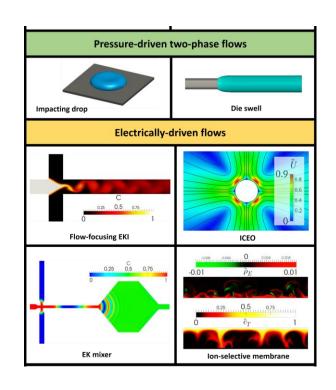
foam-extend 5.0

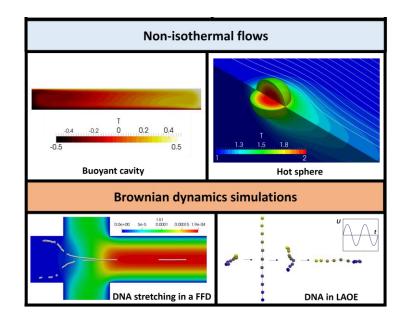
RheoTool



https://github.com/fppimenta/rheoTool









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RheoTool



https://github.com/fppimenta/rheoTool

Viscoelastic models solved in the standard extra-stress or conformation tensor variables

Model	TypeName	$\eta_{ m s}(\dot{\gamma})$	$\eta_{ m p}(\dot{\gamma})$	$\lambda(\dot{\gamma})$	Constitutive Equation
Oldroyd-B	$Oldroyd ext{-}B$	$\eta_{ m s}$	$\eta_{ m p}$	λ	$oldsymbol{ au} + \lambda \stackrel{ abla}{oldsymbol{ au}} = \eta_{ m P}(abla {f u} + abla {f u}^{ m T})$
WhiteMetzner (Carreau-Yasuda)	White Metzner CY	$\eta_{ m s}$	$\eta_{\rm p}[1+(K\dot{\gamma})^a]^{\frac{n-1}{a}}$	$\lambda[1+(L\dot{\gamma})^b]^{\frac{m-1}{b}}$	$oldsymbol{ au} + \lambda(\dot{\gamma}) \stackrel{ abla}{oldsymbol{ au}} = \eta_{ m p}(\dot{\gamma})(abla {f u} + abla {f u}^{ m T})$
Giesekus	Giesekus	$\eta_{ m s}$	$\eta_{ m p}$	λ	$\mathbf{\tau} + \lambda \stackrel{\nabla}{\mathbf{\tau}} + \alpha \frac{\lambda}{\eta_{\mathrm{p}}} (\mathbf{\tau} \cdot \mathbf{\tau}) = \eta_{\mathrm{p}} (\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}})$
⁴ PTT	PTT	$\eta_{ m s}$	$\eta_{ m P}$	λ	$f\boldsymbol{\tau} + \lambda \overset{\square}{\boldsymbol{\tau}} = \eta_{\mathbf{p}}(\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}})$ where $f = \left[1 + \frac{\varepsilon \lambda}{\eta_{\mathbf{p}}} \mathrm{tr}(\boldsymbol{\tau})\right]$ (linear), $f = \left[e^{\frac{\varepsilon \lambda}{\eta_{\mathbf{p}}} \mathrm{tr}(\boldsymbol{\tau})}\right]$ (exponential) or $f = \Gamma(\beta) E_{\alpha,\beta} \left(\frac{\varepsilon \lambda}{\eta_{\mathbf{p}}} \mathrm{tr}(\boldsymbol{\tau})\right)$ (generalized)
FENE-CR	FENE-CR	$\eta_{ m s}$	$\eta_{ m p}$	λ	$\begin{bmatrix} 1 + \lambda \frac{D}{Dt} \left(\frac{1}{f} \right) \end{bmatrix} \mathbf{\tau} + \frac{\lambda}{f} \overset{\nabla}{\mathbf{\tau}} = \eta_{\mathrm{p}} (\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}})$ where $f = \frac{L^{2} + \frac{\lambda}{\eta_{\mathrm{p}}} \operatorname{tr}(\mathbf{\tau})}{L^{2} - 3}$
FENE-P	FENE-P	$\eta_{ m s}$	$\eta_{ m P}$	λ	$\mathbf{\tau} + \frac{\lambda}{f} \overset{\nabla}{\mathbf{\tau}} = \frac{a\eta_{\mathrm{p}}}{f} (\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}}) - \frac{\mathrm{D}}{\mathrm{D}t} \left(\frac{1}{f}\right) [\lambda \mathbf{\tau} + a\eta_{\mathrm{p}} \mathbf{I}]$ where $f = \frac{L^2 + \frac{\lambda}{a\eta_{\mathrm{p}}} \operatorname{tr}(\mathbf{\tau})}{L^2 - 3}$ and $a = \frac{L^2}{L^2 - 3}$
⁵ Rolie-Poly	$Rolie ext{-}Poly$	$\eta_{ m s}$	$\eta_{ m p}$	$\lambda_{ m D}$	$\lambda_{\mathrm{D}} \overset{\nabla}{\mathbf{A}} = -(\mathbf{A} - \mathbf{I}) - 2k \frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{R}}} \left(1 - \sqrt{3/\mathrm{tr}(\mathbf{A})} \right) \left[\mathbf{A} + \beta \left(\frac{\mathrm{tr}(\mathbf{A})}{3} \right)^{\delta} (\mathbf{A} - \mathbf{I}) \right]$ where $k = \frac{\left(3 - \frac{\chi^{2}}{\chi_{\mathrm{max}}^{2}} \right) \left(1 - \frac{1}{\chi_{\mathrm{max}}^{2}} \right)}{\left(1 - \frac{\chi^{2}}{\chi_{\mathrm{max}}^{2}} \right) \left(3 - \frac{1}{\chi_{\mathrm{max}}^{2}} \right)}$ and $\chi = \sqrt{\frac{\mathrm{tr}(\mathbf{A})}{3}}$
eXtended Pom-Pom	XPomPom	$\eta_{ m s}$	$\eta_{ m p}$	$\lambda_{ m B}$	$f\mathbf{\tau} + \lambda_{\mathrm{B}} \overset{\nabla}{\mathbf{\tau}} + \alpha \frac{\lambda_{\mathrm{B}}}{\eta_{\mathrm{p}}} (\mathbf{\tau} \cdot \mathbf{\tau}) + \frac{\eta_{\mathrm{p}}}{\lambda_{\mathrm{B}}} (f - 1) \mathbf{I} = \eta_{\mathrm{p}} (\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}})$ where $f = 2 \frac{\lambda_{\mathrm{B}}}{\lambda_{\mathrm{S}}} e^{\frac{2}{q} (\Lambda - 1)} \left(1 - \frac{1}{A^{n+1}} \right) + \frac{1}{A^{2}} \left[1 - \frac{\alpha}{3} \frac{\operatorname{tr}(\mathbf{\tau} \cdot \mathbf{\tau})}{(\eta_{\mathrm{p}} / \lambda_{\mathrm{B}})^{2}} \right]$ and $A = \sqrt{1 + \frac{\operatorname{tr}(\mathbf{\tau})}{3\eta_{\mathrm{p}} / \lambda_{\mathrm{B}}}}$





RheoTool



https://github.com/fppimenta/rheoTool

‡Viscoelastic models solved with the log-conformation approach

Model	TypeName	$\Theta \rightarrow \tau$	^{6,7} Constitutive Equation
⁸ Oldroyd-B	$Oldroyd ext{-}BLog$	$ au = rac{\eta_{ m p}}{\lambda}(e^{oldsymbol{\Theta}} - {f I})$	$\mathbf{\Upsilon} = \frac{1}{\lambda} \left(e^{-\mathbf{\Theta}} - \mathbf{I} \right)$
⁹ WhiteMetzner (Carreau-Yasuda)	White Metzner CYLog	$oldsymbol{ au} = rac{\eta_{ m p}}{\lambda}(e^{oldsymbol{\Theta}} - \mathbf{I})$	$oldsymbol{\Upsilon} = rac{1}{\lambda(\dot{\gamma})} \left(e^{-oldsymbol{\Theta}} - \mathbf{I} ight)$
Giesekus	GiesekusLog	$\mathbf{\tau} = \frac{\eta_{\mathrm{p}}}{\lambda} (e^{\mathbf{\Theta}} - \mathbf{I})$	$\mathbf{\Upsilon} = \frac{1}{\lambda} \left[\left(e^{-\mathbf{\Theta}} - \mathbf{I} \right) - \alpha e^{\mathbf{\Theta}} \left(e^{-\mathbf{\Theta}} - \mathbf{I} \right)^2 \right]$
⁴ PTT	PTTLog	$oldsymbol{ au} = rac{\eta_{ m p}}{\lambda(1-\zeta)}(e^{oldsymbol{\Theta}} - \mathbf{I})$	$\mathbf{\Upsilon} = \frac{f}{\lambda}(e^{-\mathbf{\Theta}} - \mathbf{I}), \text{ where } f = 1 + \frac{\varepsilon}{1-\zeta} \left[\operatorname{tr}(e^{\mathbf{\Theta}}) - 3 \right] \text{ (linear)},$ $f = e^{\frac{\varepsilon}{1-\zeta}(\operatorname{tr}(e^{\mathbf{\Theta}}) - 3)} \text{ (exponential), or } f = \Gamma(\beta) E_{\alpha,\beta} \left[\frac{\varepsilon}{1-\zeta}(\operatorname{tr}(e^{\mathbf{\Theta}}) - 3) \right] \text{ (generalized)}$
FENE-CR	$FENE ext{-}CRLog$	$oldsymbol{ au} = rac{\eta_{ ext{P}} f}{\lambda} (e^{oldsymbol{\Theta}} - \mathbf{I})$	$\mathbf{\Upsilon} = \frac{f}{\lambda} \left(e^{-\mathbf{\Theta}} - \mathbf{I} \right)$, where $f = \frac{L^2}{L^2 - \operatorname{tr}(e^{\mathbf{\Theta}})}$
FENE-P	$FENE ext{-}PLog$	$\mathbf{\tau} = \frac{\eta_{\mathbf{p}}}{\lambda} (f e^{\mathbf{\Theta}} - a\mathbf{I})$	$\Upsilon = \frac{1}{\lambda} \left(ae^{-\Theta} - f\mathbf{I} \right)$, where $a = \frac{L^2}{L^2 - 3}$ and $f = \frac{L^2}{L^2 - \text{tr}(e^{\Theta})}$
¹⁰ Rolie-Poly	$Rolie ext{-}PolyLog$	$oldsymbol{ au} = rac{\eta_{ exttt{p}}}{\lambda_{ exttt{D}}} k(e^{oldsymbol{\Theta}} - \mathbf{I})$	$\mathbf{\Upsilon} = -\frac{1}{\lambda_{\mathrm{D}}} e^{-\mathbf{\Theta}} \left\{ \left(e^{\mathbf{\Theta}} - \mathbf{I} \right) + 2k \frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{R}}} \left(1 - \sqrt{3/\mathrm{tr}(e^{\mathbf{\Theta}})} \right) \left[e^{\mathbf{\Theta}} + \beta \left(\frac{\mathrm{tr}(e^{\mathbf{\Theta}})}{3} \right)^{\delta} \left(e^{\mathbf{\Theta}} - \mathbf{I} \right) \right] \right\}$
eXtended Pom-Pom	XPomPomLog	$oldsymbol{ au} = rac{\eta_{ ext{P}}}{\lambda_{ ext{B}}}(e^{oldsymbol{\Theta}} - \mathbf{I})$	$\mathbf{\Upsilon} = -\frac{1}{\lambda_{\rm B}} e^{-\mathbf{\Theta}} \left[(f - 2\alpha) e^{\mathbf{\Theta}} + \alpha e^{\mathbf{\Theta}} e^{\mathbf{\Theta}} + (\alpha - 1) \mathbf{I} \right]$ where $f = 2\frac{\lambda_{\rm B}}{\lambda_{\rm S}} e^{\frac{2}{q}(\Lambda - 1)} \left(1 - \frac{1}{\Lambda^{n+1}} \right) + \frac{1}{\Lambda^2} \left[1 - \alpha - \frac{\alpha}{3} \text{tr} (e^{\mathbf{\Theta}} (e^{\mathbf{\Theta}} - 2\mathbf{I})) \right]$ and $\Lambda = \sqrt{\frac{\text{tr}(e^{\mathbf{\Theta}})}{3}}$



foam-extend 4 / OpenFOAM 7 / OpenFOAM 9

RheoTool – Reduced Version



Solvers

RheoInterFoam Solver (Two-phase flow)

Constitutive Models

- Giesekus
- Giesekus-Log
- PTT
- PTT-Log
- Newtonian
- Carreau-Yasuda



P4 – Case 41



Extrudate Swell



- Carreau-Yasuda (CY)
- Giesekus-Log (GL)
- Giesekus-Log Multimode illustrative (MGL)

P4 – Case 41

- 1. Open Ubuntu terminal
- 2. of 2206 //Load OpenFOAM variables
- 3. >> run
- 4. >> cd case41
- 5. >> code.
- 6. Study the CY and GL cases files namely file constant/constitutiveProperties and dictionaries fvSolution and fvSchemes.
- 7. Study the Allrun files for both cases
- 8. Run both cases
- 9. Post-process the data in paraview to compare the extrudate swell
- 10. Study the MGL case files namely file constant/constitutiveProperties, dictionaries fvSolution and fvSchemes and 0 folder to understand how to define the multimode model

