Universidad de los Andes Colombia

39th International Conference of the Polymer Processing Society

ICIPC INSTITUTO DE CAPACITACIÓN E INVESTIGACIÓN DEL PLÁSTICO Y DEL CAUCHO

Introduction to OpenFOAM® Computational Library and Viscoelastic Fluid Flow Simulation

P3 - Case studies: Viscoelastic fluid flow solvers

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JKU IPPD Institute of Polymer Processing and Digital Transformation

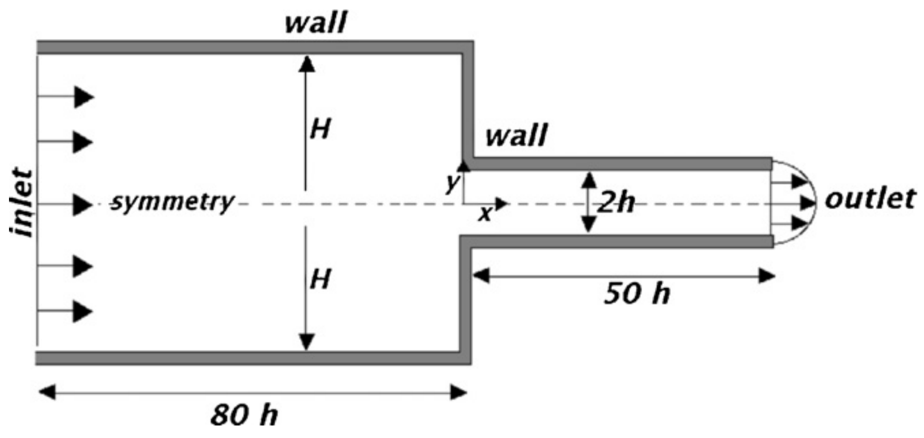
JKU JOHANNES KEPLER UNIVERSITY LINZ

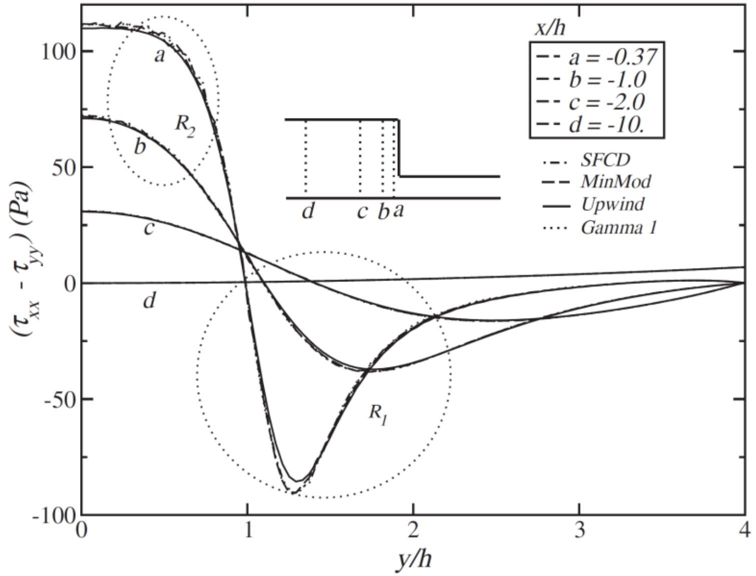
Outline

9:00 – 10:30	**Introduction to OpenFOAM (P1)**
10:30 – 12:00	**Mesh generation and post-processing (P2)**
12:00 – 13:00	Lunch break
13:00 – 14:30	**Case studies: Single- and two-phase flow solvers (P3)**
14:30 – 16:00	**Case studies: Viscoelastic fluid flow solvers (P4)**

Differential viscoelastic flow solvers in OpenFOAM

Multi mode constitutive models		
Maxwell	(L,E, Feta) PTT	FENE-(P,CR)
Oldroyd-B	Giesekus	(S,D) XPP
White Metzner	Leonov	DCPP





*JL Favero et al. (Computer Aided Chemical Engineering, **2009**, **2010** / **Journal of non-Newtonian Fluid Mechanics, 2010** / Computers & Chemical Engineering, **2010**)*

Viscoelastic Solvers in OpenFOAM

foam-extend 5.0

Differential viscoelastic flow solvers in OpenFOAM

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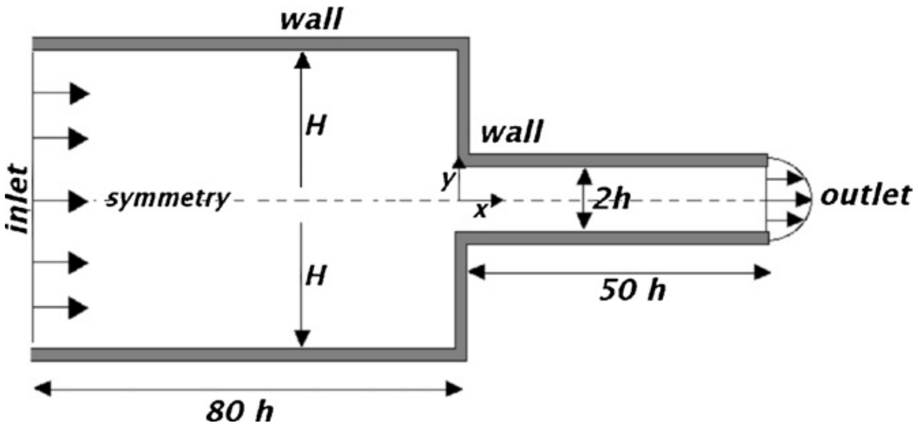

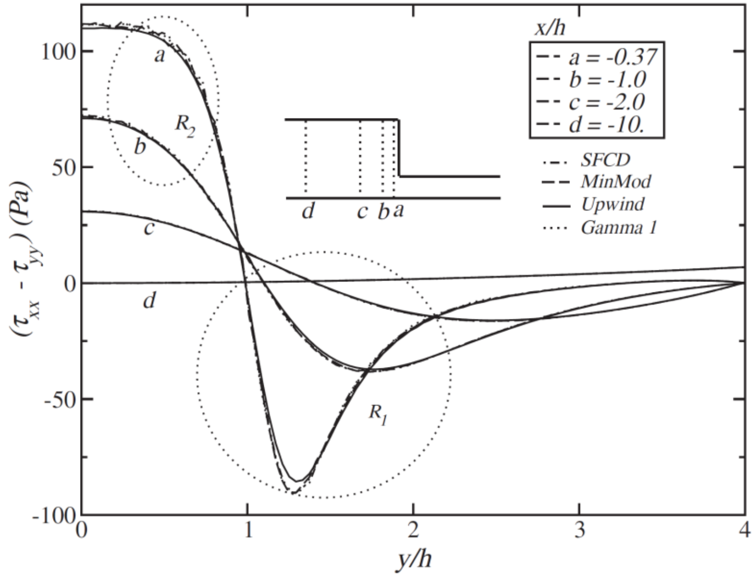

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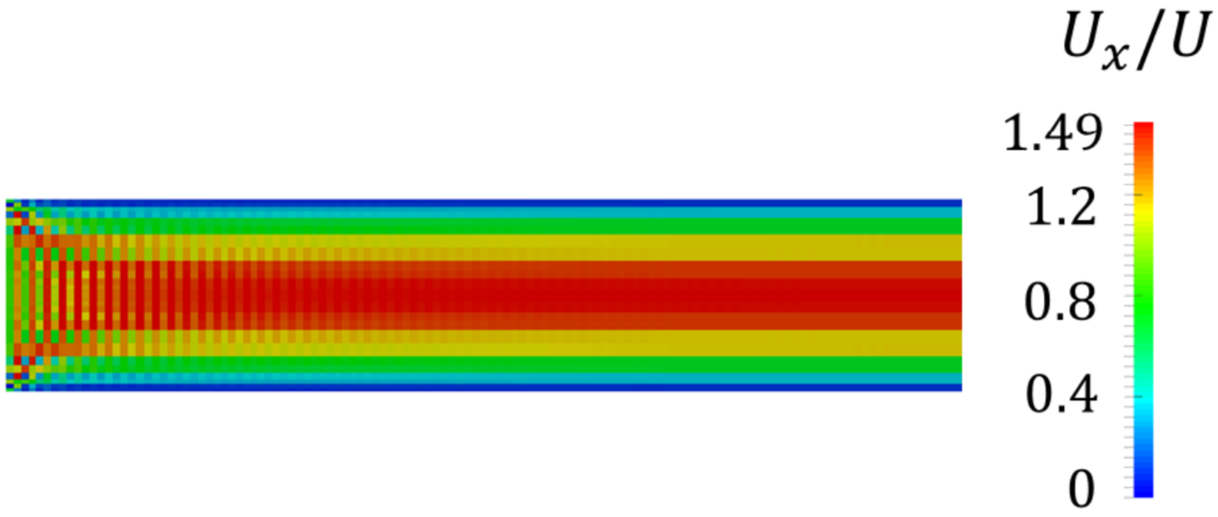
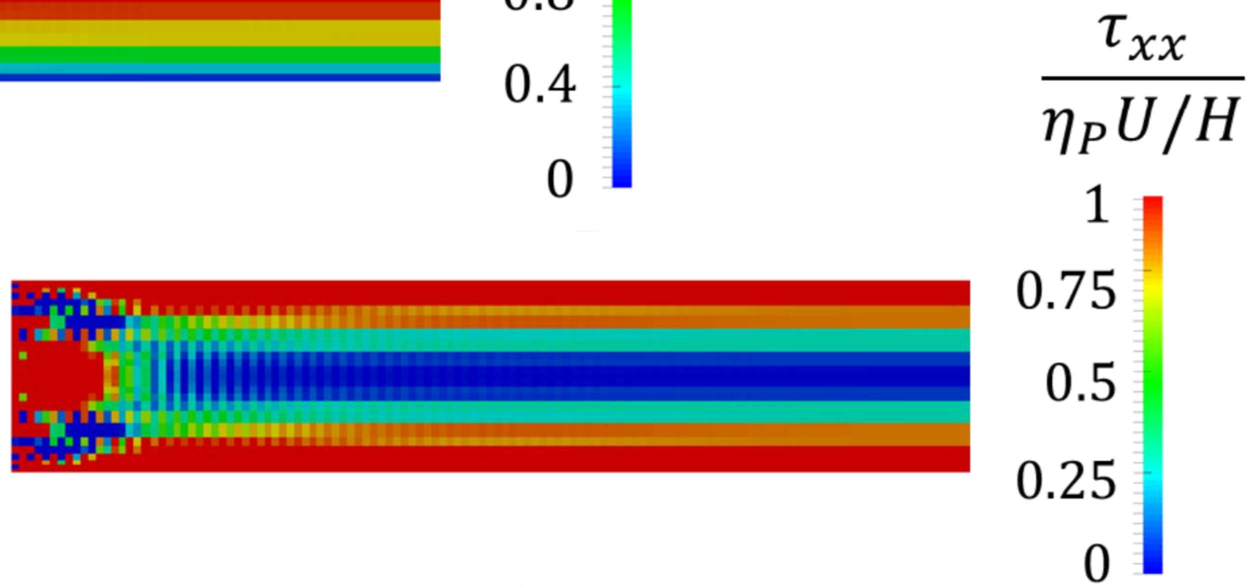
Improved Both Sides Diffusion

Poiseuille Flow + UCM (Oldroyd-B η_s=0)

Re=0.01, De=1





Improved Both Sides Diffusion

The iBSD – Improved Both-Sides-Diffusion

Conservati

Origi

```
  fvm::div(phi, U)
- fvm::laplacian(etaStr/rho, U))
==
```

BSD

```
  fvm::div(phi, U)
- fvm::laplacian(etaStr/rho, U))
==
- fvc::grad(p)
+ fvc::div(tau/rho)
- fvc::div((etaStr/rho)*fvc::grad(U))
```

iBS

$$\nabla\cdot\left(\rho\mathbf{U}\mathbf{U}\right)-\eta^{*}\nabla^{2}\mathbf{U}=-\nabla p+\nabla\cdot\boldsymbol{\tau}-\nabla\cdot\left(\eta^{*}\nabla\mathbf{U}\right)$$

Larger Stencil

C Fernandes et al., Improved Both Sides Diffusion (iBSD): a new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017

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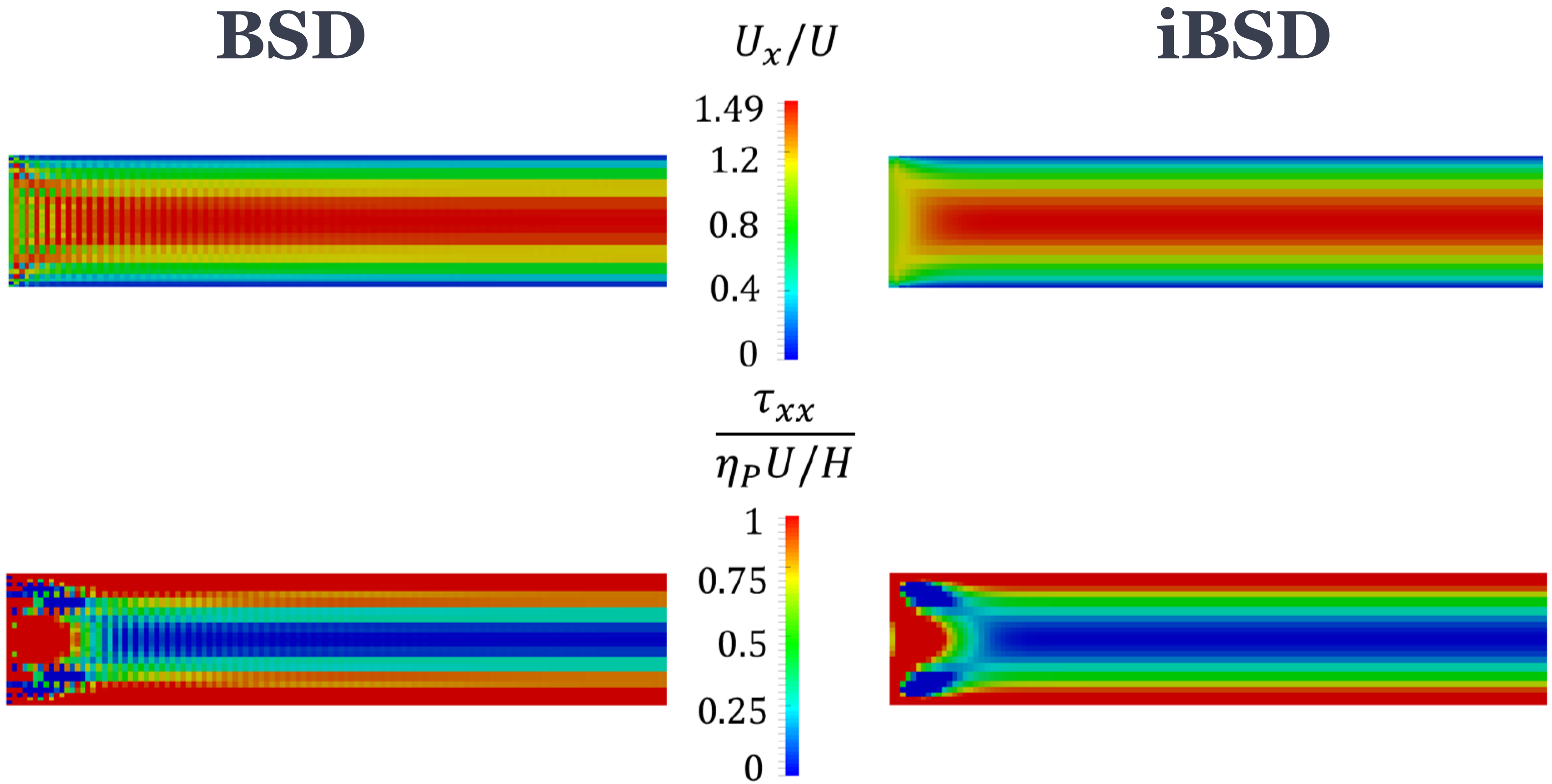
Improved Both Sides Diffusion

foam-extend 5.0

Poiseuille Flow + UCM (Oldroyd-B h_s=0)

Re=0.01, De=1



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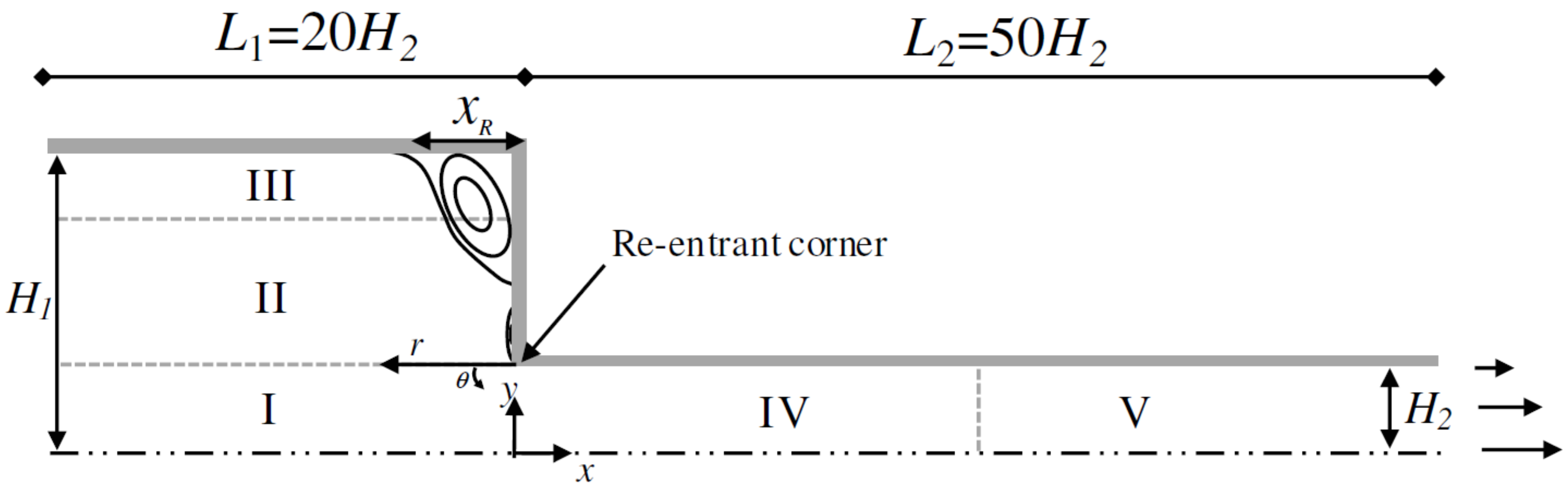
Improved Both Sides Diffusion

Case Study 1 – 4:1 Contraction (UCM)



Meshes

M1 – 228 Cells

…

M5 – 228128 Cells

Re = 0.01

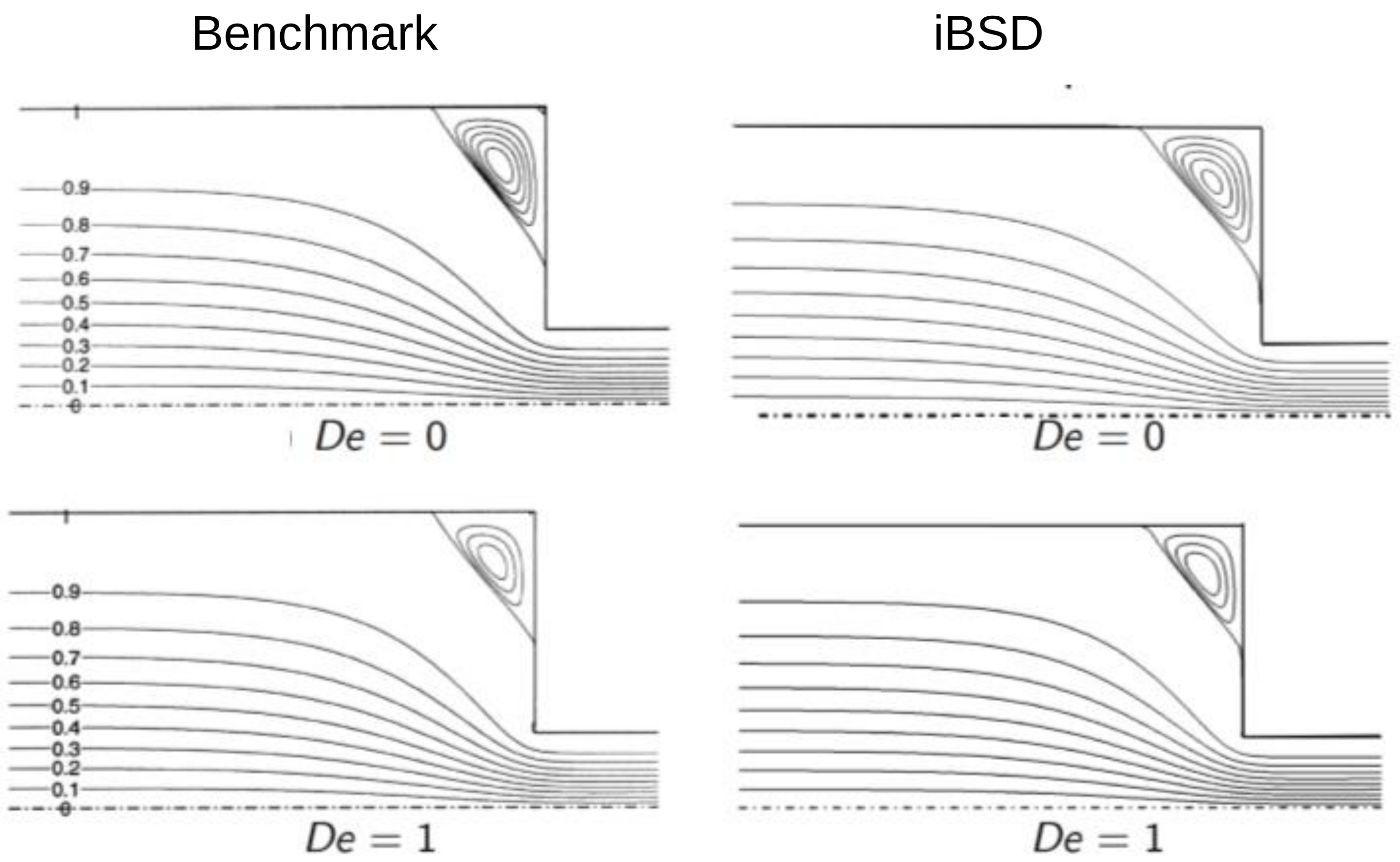
De = {0,1,2,3,4,5}

C Fernandes et al., Improved Both Sides Diffusion (iBSD): a new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017

Improved Both Sides Diffusion

Case Study 1 – 4:1 Contraction (UCM)

Streamlines



MM Alves, FT Pinho,PJ Oliveira, J. Non-Newtonian Fluid Mech., 93, 2000

C Fernandes et al., Improved Both Sides Diffusion (iBSD): a new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017

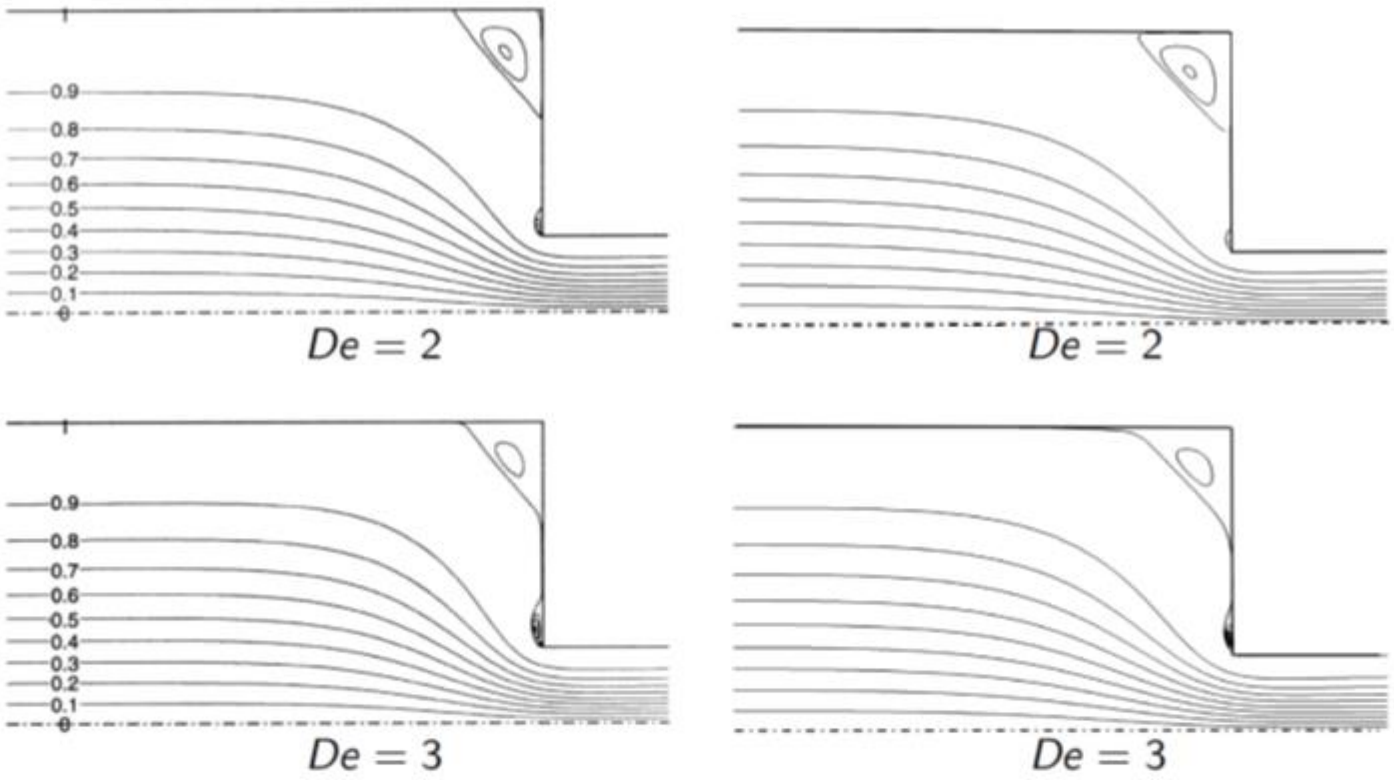
Improved Both Sides Diffusion

Case Study 1 – 4:1 Contraction (UCM)

Streamlines

Benchmark

iBSD



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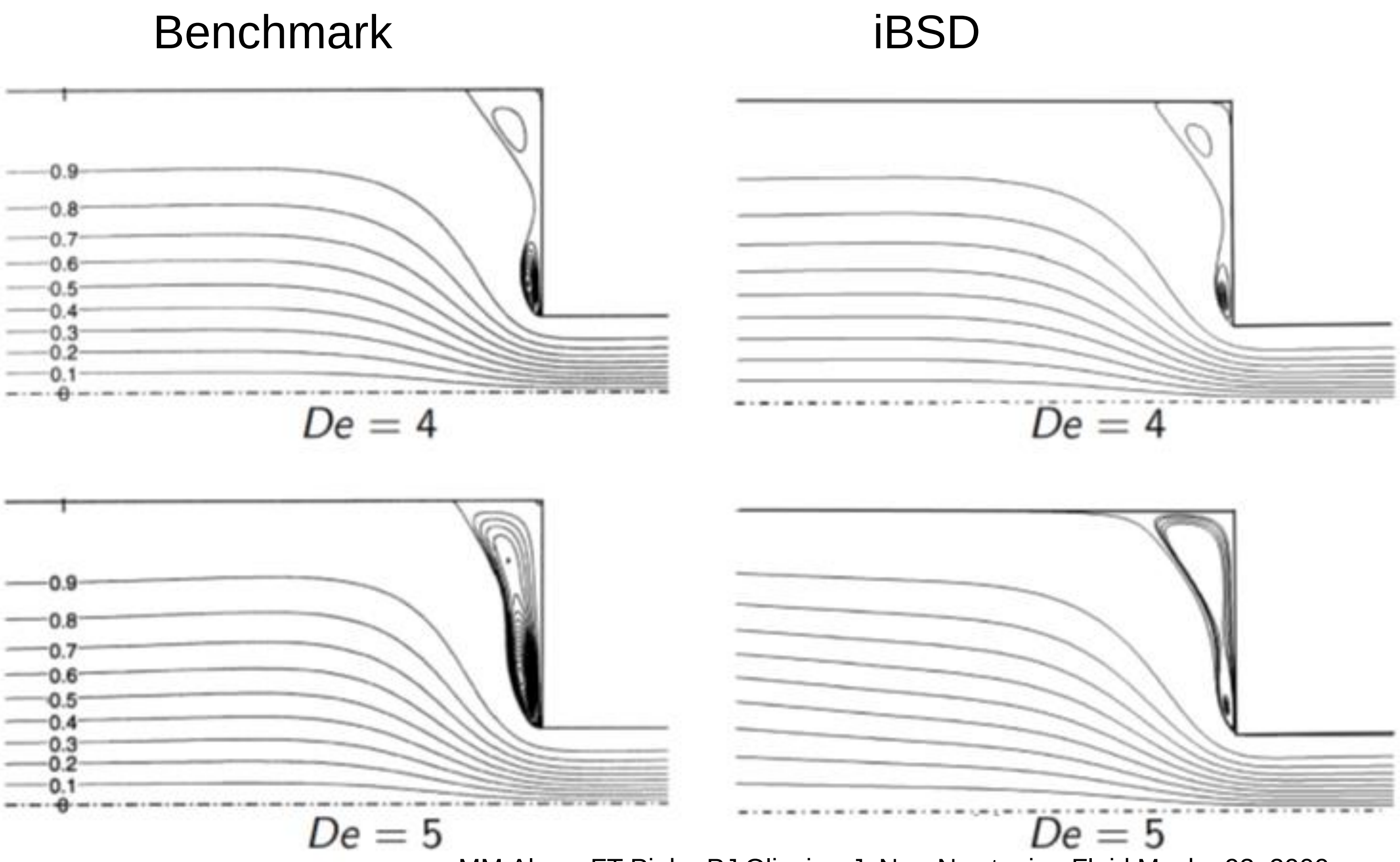
Case Study 1 – 4:1 Contraction (UCM)

Streamlines



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Improved Both Sides Diffusion

Case Study 1 – 4:1 Contraction (UCM)

Primary vortex length (Xr=xr/H2)

Developed code in *OpenFOAM*®							
De	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Mesh 5	Extrapolated	Difference (%)
0	1.438	1.479	1.492	1.495	1.496	1.4965	0.03
1	1.293	1.336	1.326	1.32711	1.32696	1.32694	0.002
2	1.207	1.200	1.130	1.101	1.091	1.0857	0.5
3	1.333	1.118	0.958	0.900	0.885	0.880	0.6
4	1.391	1.088	0.845	0.75	0.732	0.728	0.6
5	1.469	1.101	0.758	0.636	0.617	0.613	0.6

Alves et al. (2000)						
De	Mesh 1	Mesh 2	Mesh 3	Mesh 4	Extrapolated	Difference (%)
0	1.472	1.488	1.494	1.495	1.496	0.1
1	1.349	1.371	1.349	1.339	1.335	0.3
2	1.631	1.259	1.154	1.118	1.105	1.2
3	1.517	1.266	1.014	0.946	0.923	2.5
4	1.644	1.337	0.987	–	0.87	13.4
5	1.687	1.517	1.127	–	0.997	13

MM Alves, FT Pinho,PJ Oliveira, J. Non-Newtonian Fluid Mech., 93, 2000

C Fernandes et al., Improved Both Sides Diffusion (iBSD): a new and straightforward stabilization approach for viscoelastic fluid flows, JNNFM, 249, 63-78, 2017

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Integral Viscoelastic Solver

$$\boldsymbol{\tau}_p = \int_{-\infty}^{t} M\left(t-t'\right) f\left(\mathbf{B}_{t'}\right) dt'$$

$$M\left(t-t'\right) = \sum_k \frac{a_k}{\lambda_k} e^{-\frac{\left(t-t'\right)}{\lambda_k}}$$

$$f\left(\mathbf{B}_{t'}\right) = \begin{cases} \mathbf{B}_{\mathbf{t}'} & \text{- UCM} \\ \dfrac{\alpha}{\alpha + \beta I_1 + \left(1-\beta\right) I_2} \mathbf{B}_{\mathbf{t}'} & \text{- K-BKZ (PSM)} \end{cases}$$

Deformation Fields approach

$$\frac{\partial \mathbf{B}_i}{\partial t} + \nabla \cdot \left(\mathbf{U}\mathbf{B}_i\right) - \left(\left(\nabla \mathbf{U}\right)^T \cdot \mathbf{B}_i + \mathbf{B}_i \cdot \nabla \mathbf{U}\right) = 0$$

Araujo, MSB., et al,. "A stable numerical implementation of integral viscoelastic models in the OpenFOAM® computational library. Computers & Fluids, 2018

Integral Viscoelastic Solver

Stress Tensor Field Calculation

- Define **cutoff time** (s_{max}) and **number of deformation fields** (nf)

- At each time step t:

1. Update velocity and pressure fields (PISO)
2. Transport all the previously created deformation fields ($\mathbf{B}_{\mathbf{i}=1,nf}$)

$$\frac{\partial \mathbf{B}_i}{\partial t} + \nabla \cdot \left(\mathbf{U}\mathbf{B}_i \right) - \left(\left(\nabla \mathbf{U} \right)^T \cdot \mathbf{B}_i + \mathbf{B}_i \cdot \nabla \mathbf{U} \right) = 0$$

3. Create a new deformation field ($\mathbf{B}_{\boldsymbol{t}}$=$\mathbf{I}$)
4. If required redistribute/interpolate the deformation fields
5. Compute the stress field

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Stre

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OpenFOAM Code:

```
volTensorField L = fvc::grad(U);

for (int fieldI = 0; fieldI <= nActFields ; fieldI++)
       {
         volTensorField SB = B[fieldI] & L;
         fvSymmTensorMatrix BEqn
           (
              fvm::ddt(B[fieldI])
              + fvm::div(phi, B[fieldI], "div(phi,B)")
               ==
              twoSymm(SB)
           );
           BEqn.solve(mesh.solutionDict().solver("B"));
        }
```

$$\frac{\partial \mathbf{B}_i}{\partial t} + \nabla \cdot \left(\mathbf{U}\mathbf{B}_i\right) - \left(\left(\nabla \mathbf{U}\right)^T \cdot \mathbf{B}_i + \mathbf{B}_i \cdot \nabla \mathbf{U}\right) = 0$$

3. Create a new deformation field ($\mathbf{B}_t = \mathbf{I}$)
4. If required redistribute/interpolate the deformation fields
5. Compute the stress field

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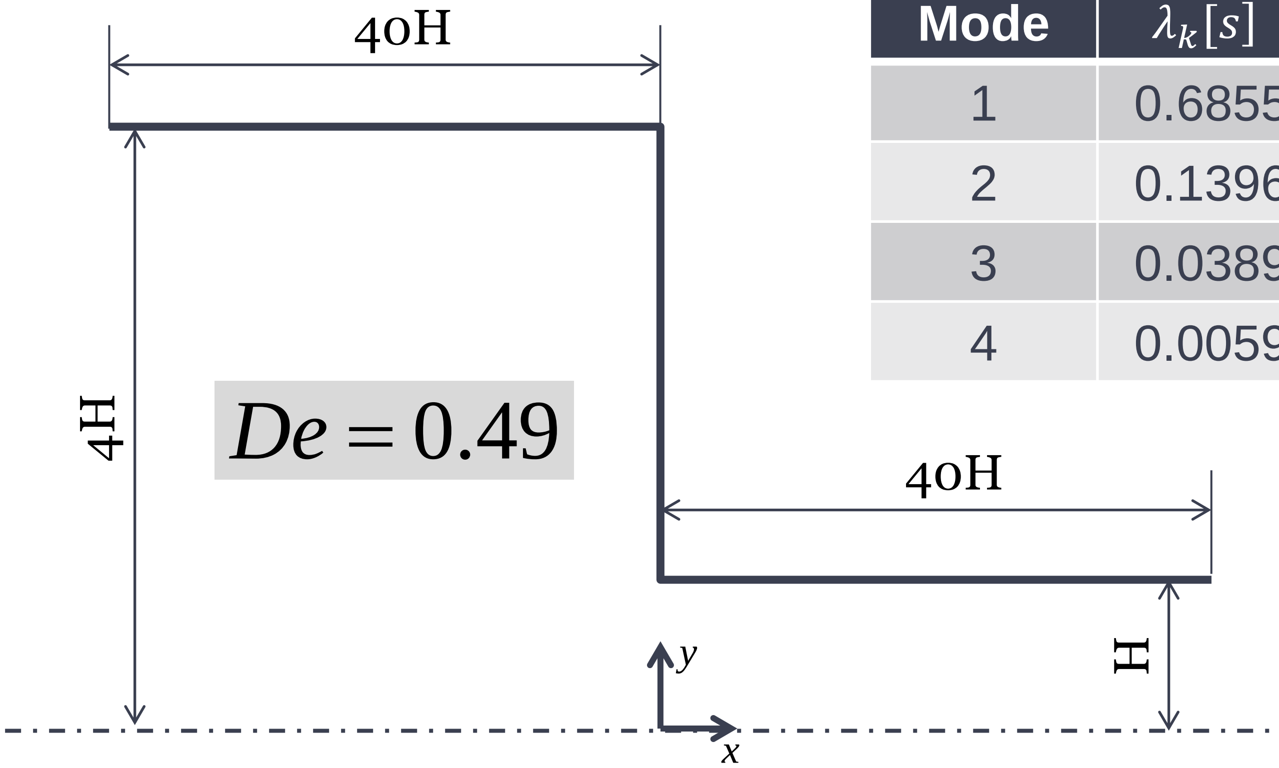
Integral Viscoelastic Solver

Case Study - 4:1 Abrupt Planar Contraction - K- BKZc

Solution of 5% wt. polyisobutylene (PIB) in Tetradecane(C14)

$s_{max} = 1.5$ s
$n_f = 150$



Mode	$\lambda_k[s]$	$a_k[Pa]$
1	0.6855	0.0584
2	0.1396	1.6648
3	0.0389	14.5604
4	0.0059	99.1525

$\alpha = 10$

$\beta = 0.7$

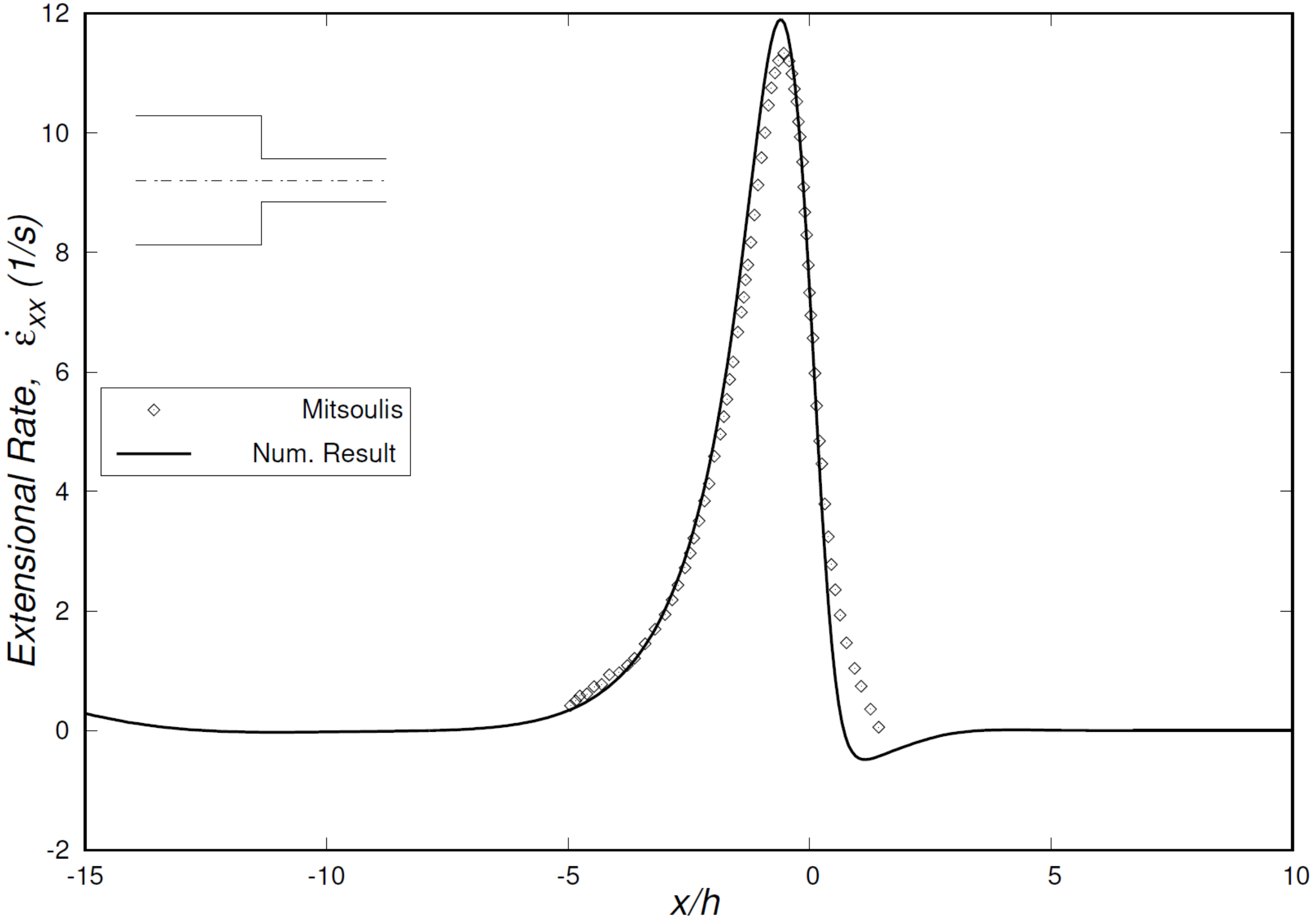
$\theta = 0$

- LM Quinzani, RC Armstrong, RA Brown, Journal non-Newtonian Fluid Mechanics, 52, 1994
- E Mitsoulis, Journal of Rheology, 37, 1993

Araujo, MSB., et al,. "A stable numerical implementation of integral viscoelastic models in the OpenFOAM® computational library. Computers & Fluids, 2018

Integral Viscoelastic Solver

Case Study - 4:1 Abrupt Planar Contraction - K- BKZ



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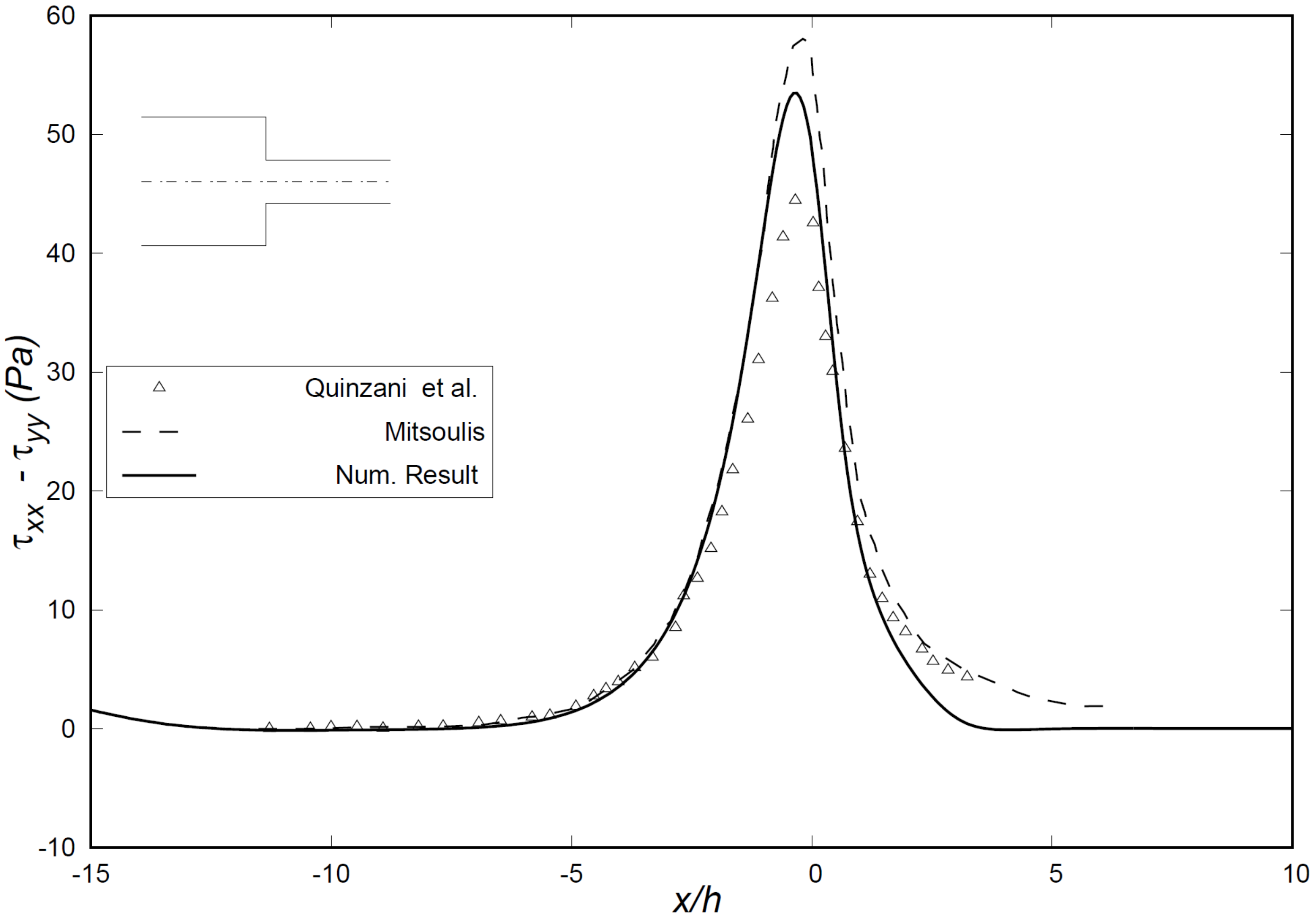
Integral Viscoelastic Solver

Case Study - 4:1 Abrupt Planar Contraction - K- BKZ



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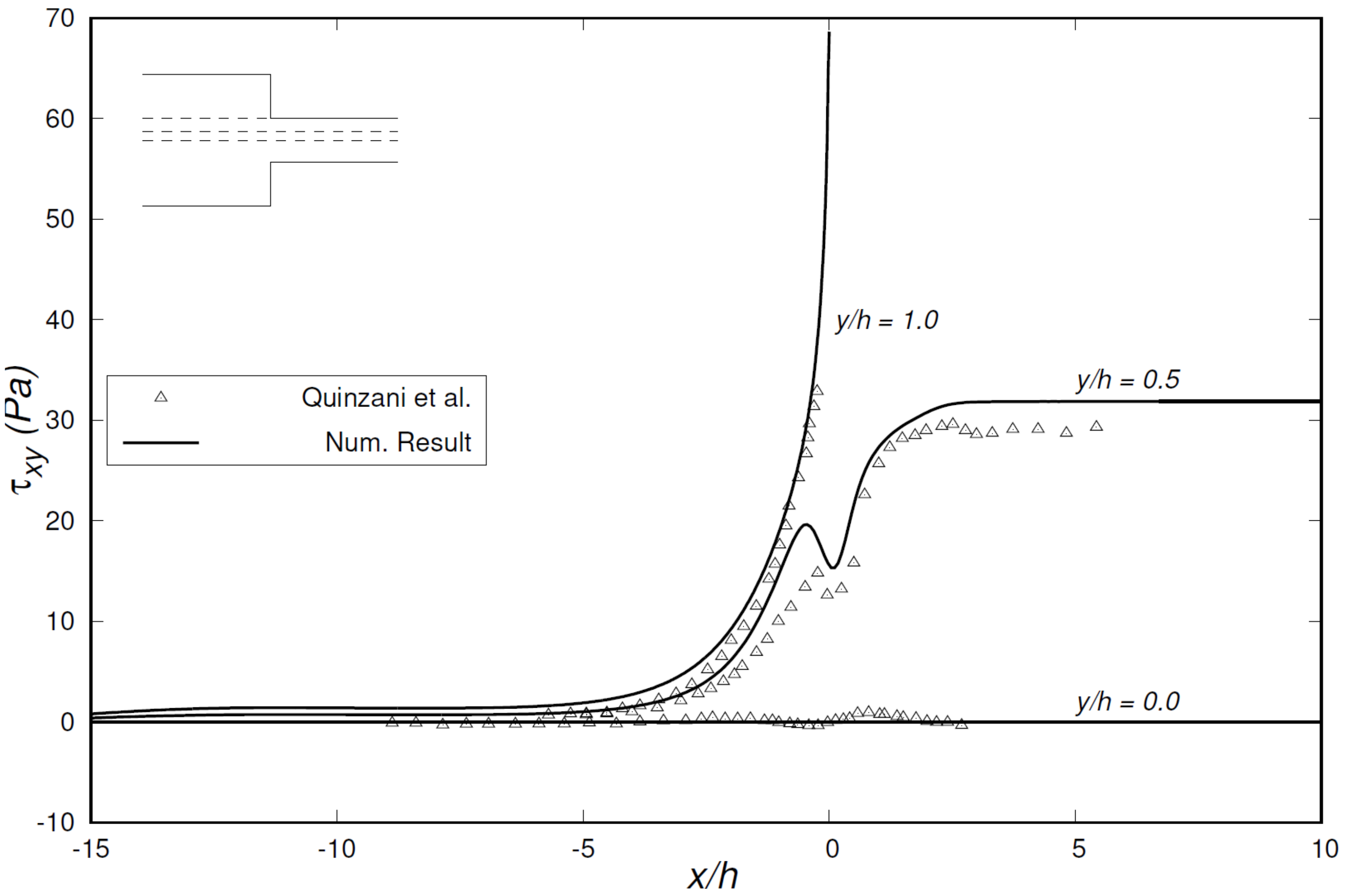
Integral Viscoelastic Solver

Case Study - 4:1 Abrupt Planar Contraction - K- BKZ



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Coupled Approaches - Viscoelastic Semi-coupled solver

exaFOAM

$$\begin{pmatrix} \mathbf{A}_u & \nabla \\ \nabla\cdot & -\nabla\cdot(\boldsymbol{\Gamma}_u^{-1}\nabla) \end{pmatrix}\begin{bmatrix} \mathbf{u} \\ p \end{bmatrix} = \begin{bmatrix} b_{\mathbf{u}} \\ b_p \end{bmatrix}$$

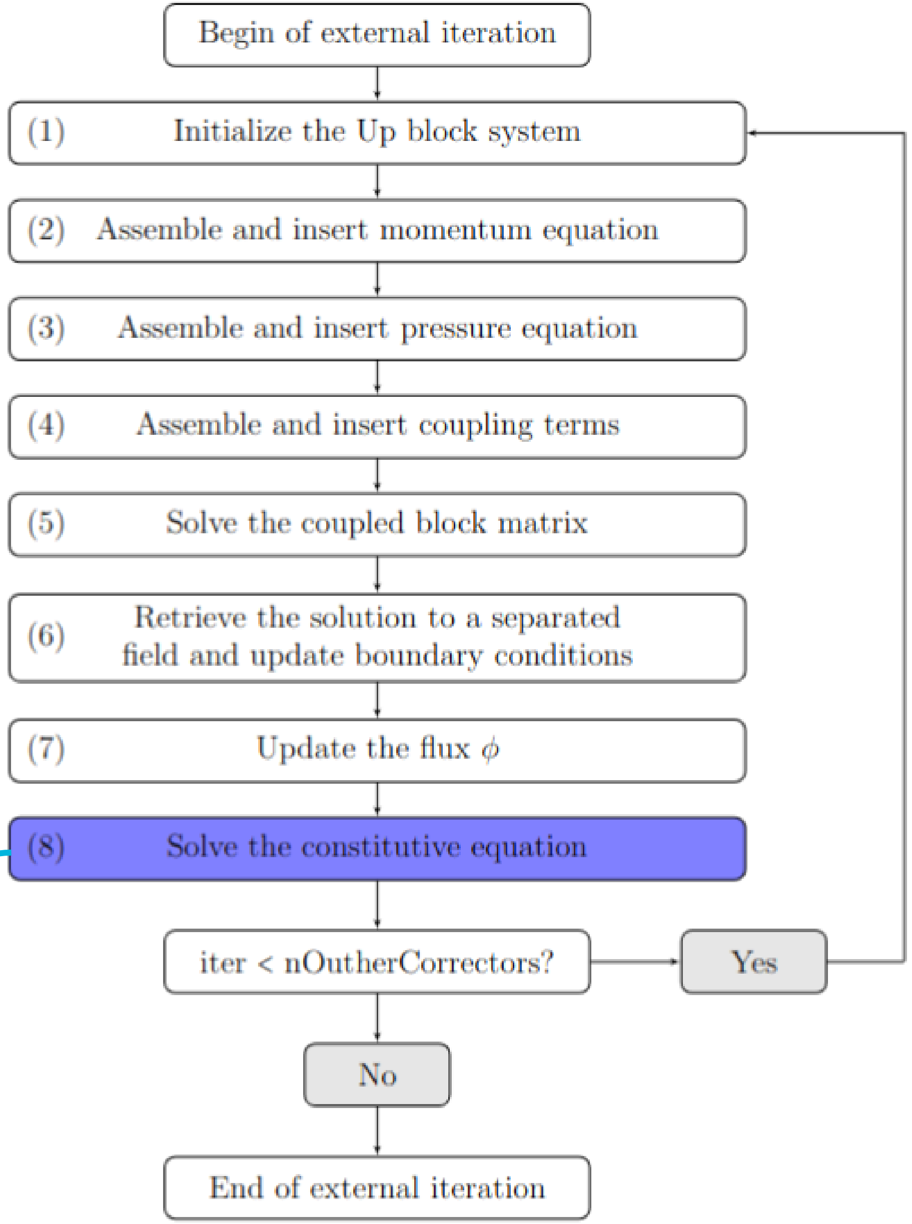
$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u}) - \nabla^2\left[\left(\eta_S + \sum_{m=1}^{N_m}\eta_{Stab_m}\right)\mathbf{u}\right] = -\nabla p$$

$$+\sum_{m=1}^{N_m}(\nabla\cdot\boldsymbol{\tau}_{P_m}) - \nabla\cdot\left[\nabla\left(\sum_{m=1}^{N_m}\eta_{Stab_m}\mathbf{u}\right)\right]$$

$$\nabla\cdot\mathbf{u} \; -\nabla\cdot[(a_{ii}^u)^{-1}\nabla p] = -\nabla\cdot[\overline{(a_{ii}^u)^{-1}\nabla p_f}],$$

- For each mode:

$$\frac{\partial\boldsymbol{\tau}_P}{\partial t} + \nabla\cdot(\mathbf{u}\boldsymbol{\tau}_P) - \boldsymbol{\tau}_P(\nabla\mathbf{u})^T - \nabla\mathbf{u}\boldsymbol{\tau}_P = \frac{\eta_P}{\lambda}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right) - g_R(\boldsymbol{\tau}_P)$$



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Coupled Approaches - Viscoelastic fully-coupled solver

exaFOAM

$$\begin{pmatrix} \mathbf{A}_u & \nabla & \nabla\cdot \\ \nabla\cdot & -\nabla\cdot\left[\boldsymbol{\Gamma}_u^{-1}\nabla\right] & 0 \\ \nabla\cdot\left(\boldsymbol{\tau}^{\star 0}\right) & 0 & \mathbf{A}_\tau \end{pmatrix}\begin{bmatrix}\mathbf{u}\\ p\\ \boldsymbol{\tau}^\star\end{bmatrix}=\begin{bmatrix}b_\mathbf{u}\\ b_p\\ b_{\boldsymbol{\tau}^\star}\end{bmatrix}$$

$$\frac{\partial(\rho\mathbf{u})}{\partial t}+\nabla\cdot(\rho\mathbf{u}\mathbf{u})-\nabla^2\left[\left(\eta_S+\sum_{m=1}^{N_m}\eta_{Stab_m}\right)\mathbf{u}\right]=-\nabla p$$

$$+\left(\nabla\cdot\boldsymbol{\tau}^\star\right)+\sum_{\substack{m=1\\ m\neq m_C}}^{N_m}\left(\nabla\cdot\boldsymbol{\tau}_{P_m}\right)-\nabla\cdot\left[\nabla\left(\sum_{m=1}^{N_m}\eta_{Stab_m}\mathbf{u}\right)\right]$$

$$\nabla\cdot\mathbf{u}\ -\nabla\cdot\left[\left(a_{ii}^{u}\right)^{-1}\nabla p\right]=-\nabla\cdot\left[\overline{\left(a_{ii}^{u}\right)^{-1}\nabla p_f}\right],$$

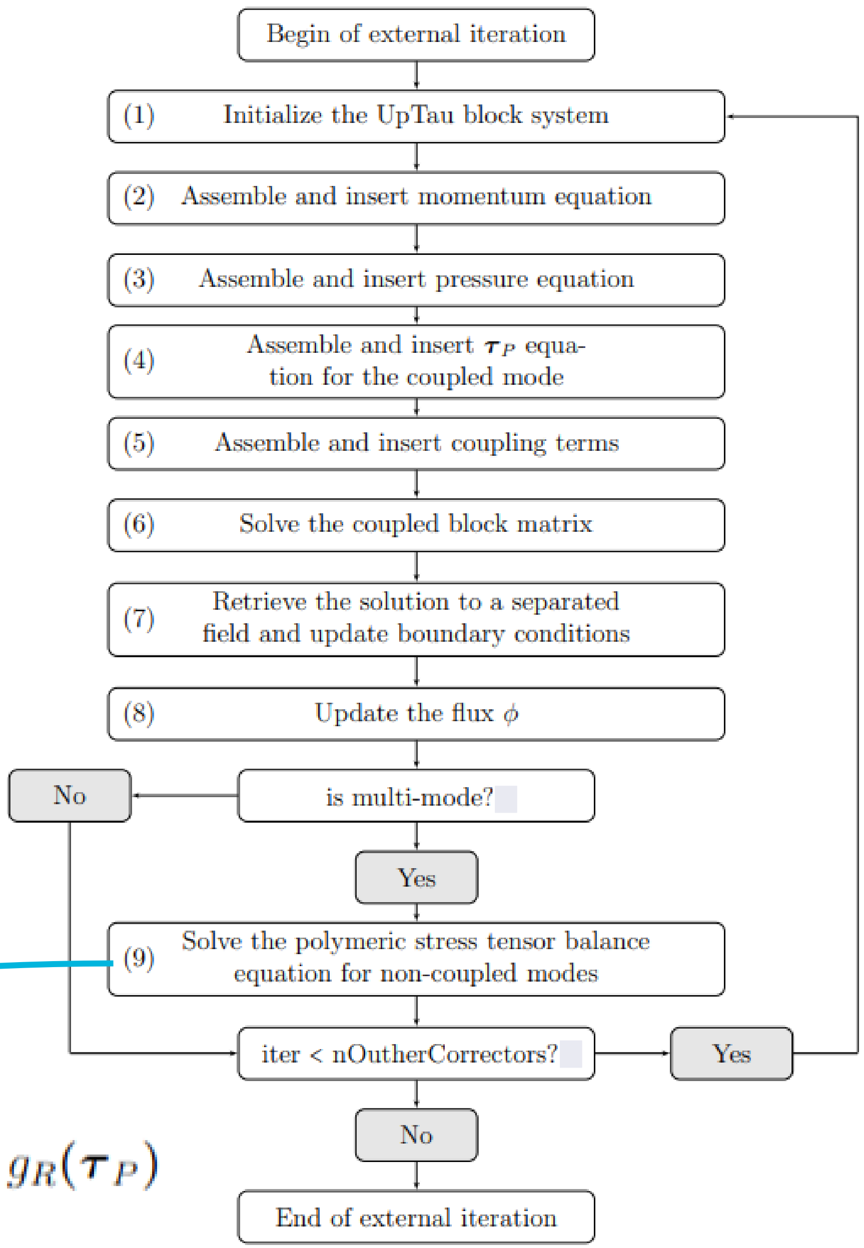
- For the **coupled** mode:

$$\frac{\partial\boldsymbol{\tau}_P}{\partial t}+\nabla\cdot\left(\mathbf{u}^0\boldsymbol{\tau}_P^n\right)-\nabla\cdot\left(\mathbf{u}^0\boldsymbol{\tau}_P^0\right)+\nabla\cdot\left(\mathbf{u}^n\boldsymbol{\tau}_P^0\right)$$

$$-\boldsymbol{\tau}_P(\nabla\mathbf{u})^T-\nabla\mathbf{u}\boldsymbol{\tau}_P=\frac{\eta_P}{\lambda}\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)-g_R(\boldsymbol{\tau}_P)$$

- For the other modes (decoupled):

$$\frac{\partial\boldsymbol{\tau}_P}{\partial t}+\nabla\cdot\left(\mathbf{u}\boldsymbol{\tau}_P\right)-\boldsymbol{\tau}_P(\nabla\mathbf{u})^T-\nabla\mathbf{u}\boldsymbol{\tau}_P=\frac{\eta_P}{\lambda}\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)-g_R(\boldsymbol{\tau}_P)$$



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exaFOAM

Memory requirement

- Number of coupled variables:
  - Segregated: 0
  - Semi-coupled: 4 (p, U)
  - Fully-coupled: 10 (p, U, $\boldsymbol{\tau}^*$)

Solver	Number of cells [million]	Memory requirement [Gb]	Ratio to Segregated
Segregated	1	9.2	-
Semi-coupled	1	14.8	1.61
Fully-coupled	1	43.6	4.74
Segregated	20	173.9	-
Semi-coupled	20	561.6	3.23
Fully-coupled	20	845.6	4.86

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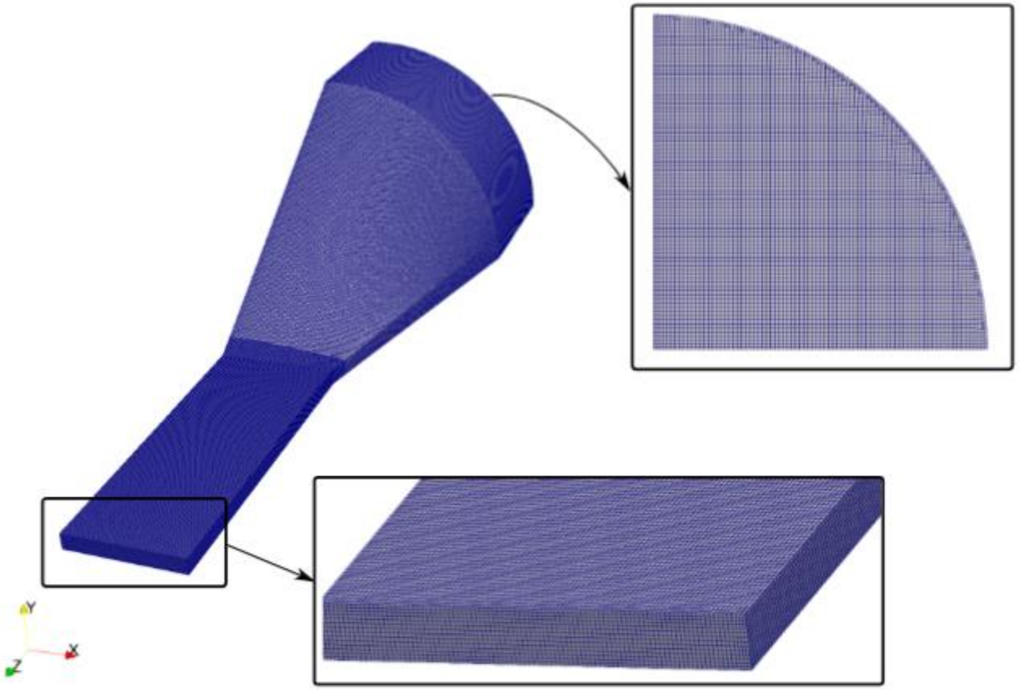
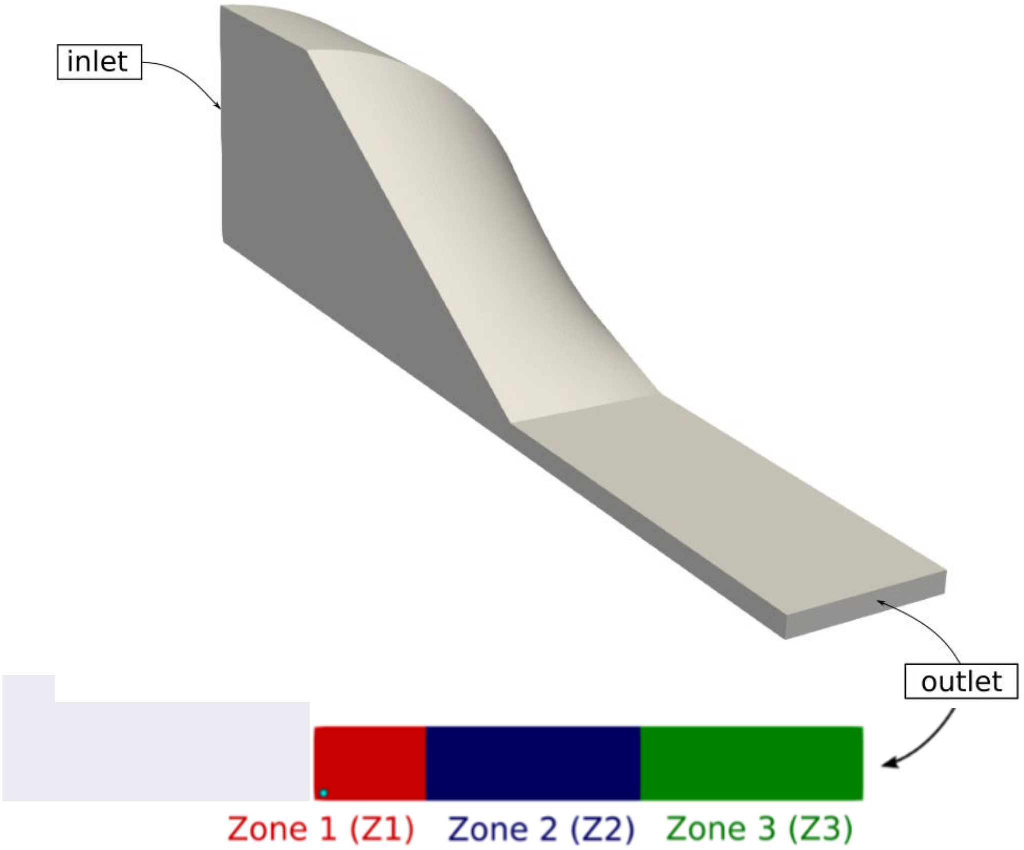
Coupled Approaches - Application case (MB19)

exaFOAM

MB19 benchmark case: profile extrusion [4]



Mesh: 1 million cells



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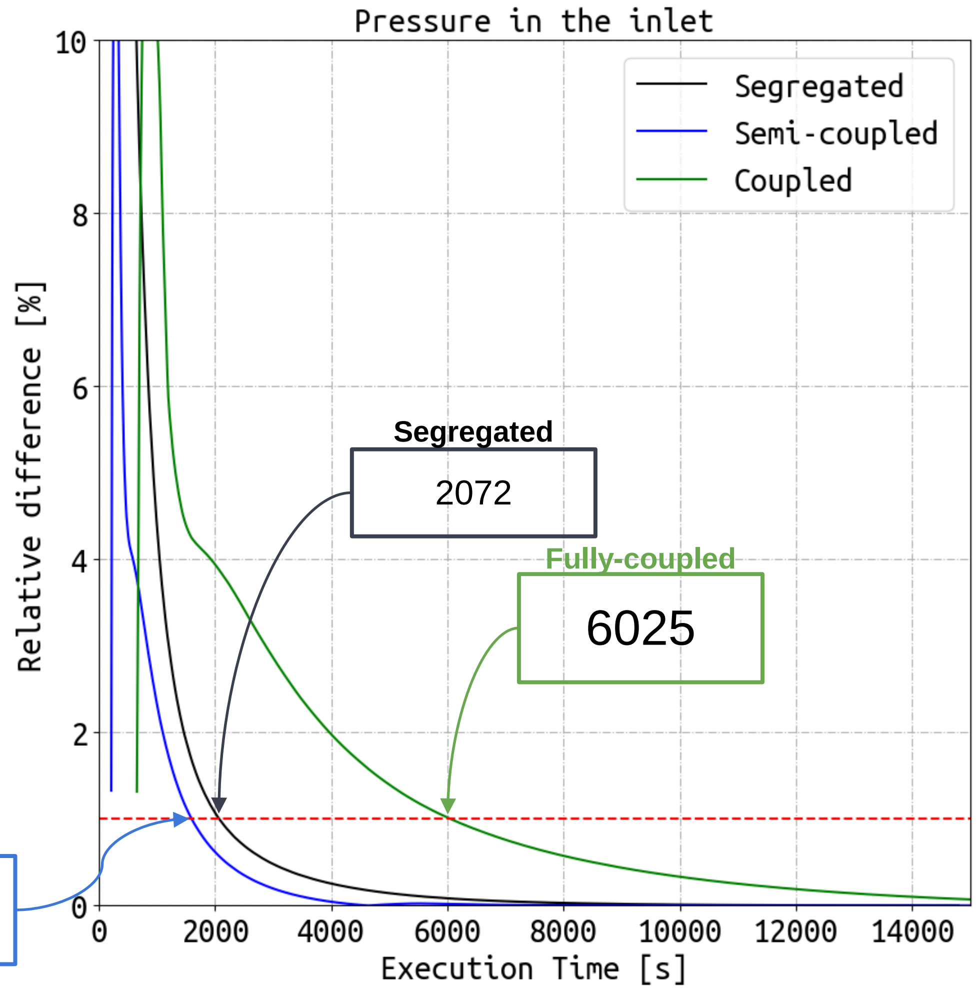
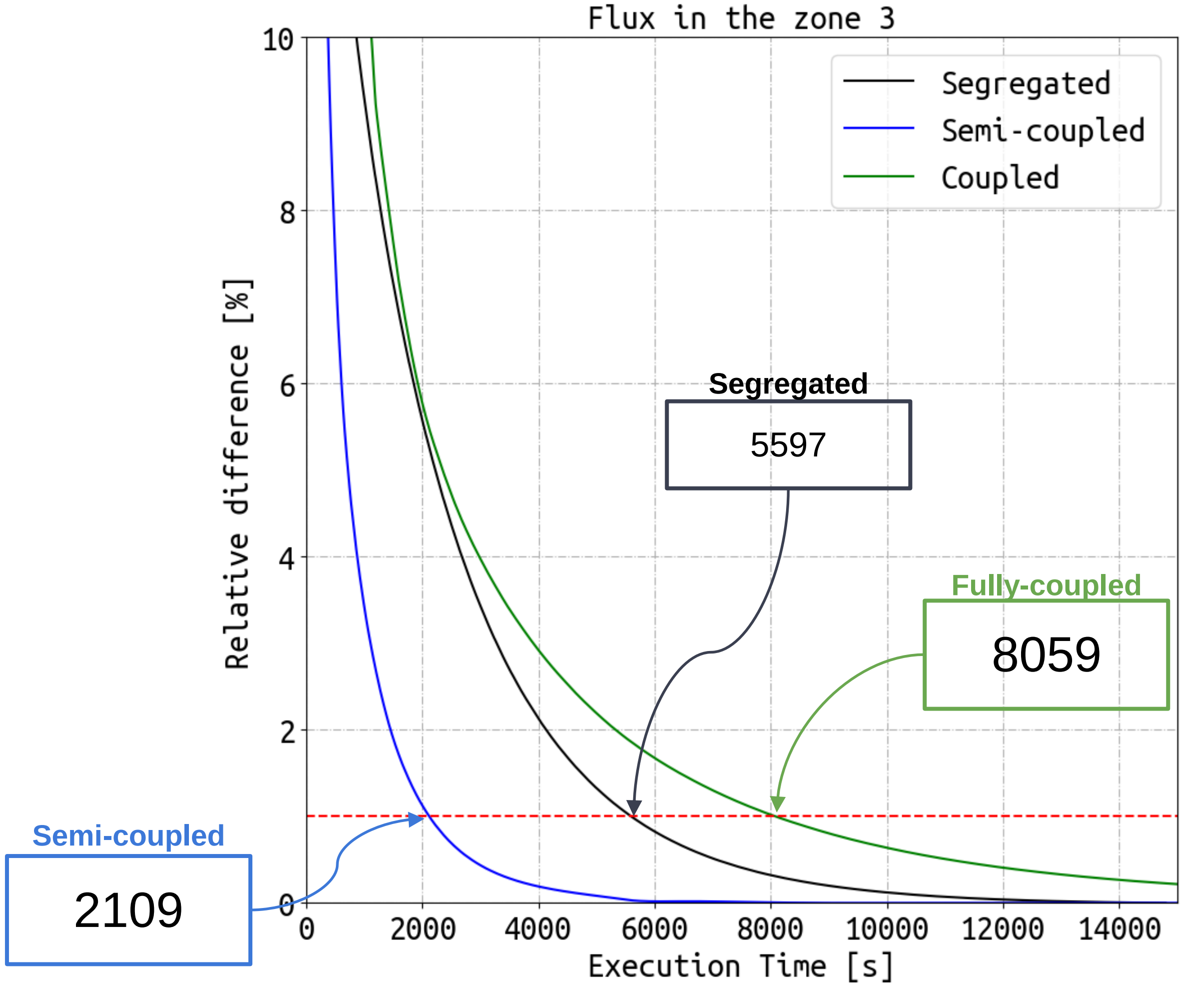
Coupled Approaches - Application case (MB19)

exaFOAM





$$\text{TTS KPI} = \frac{5597}{2109} = 2.65$$

Semi-coupled

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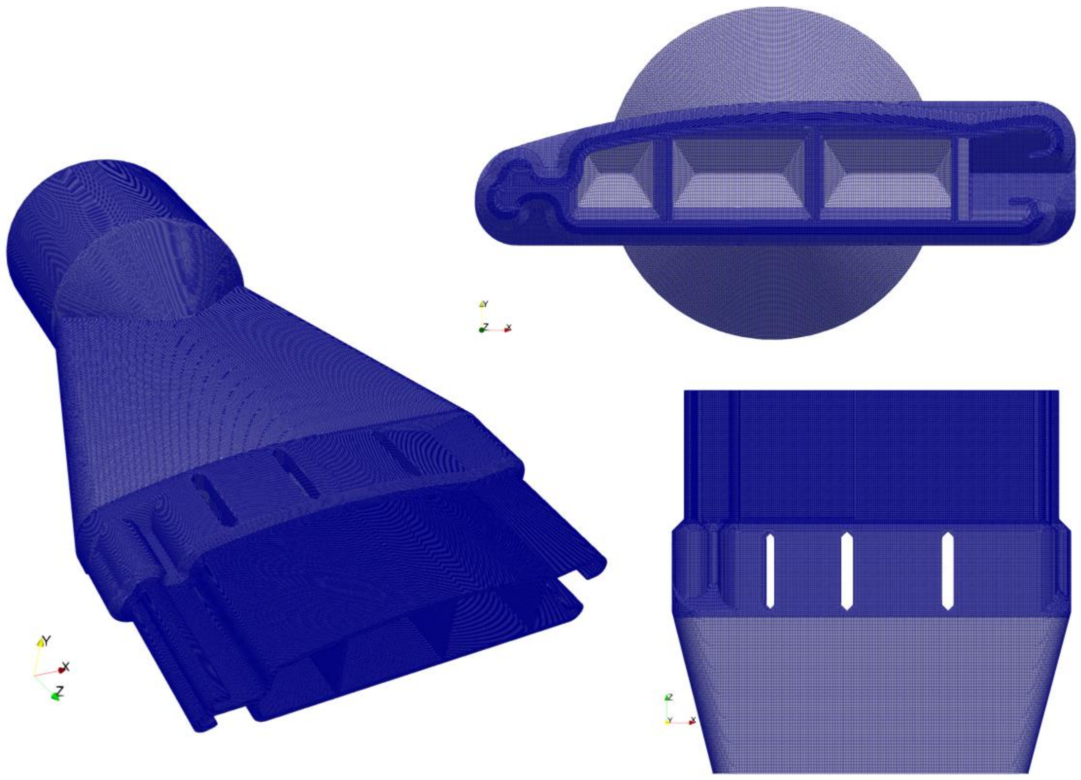
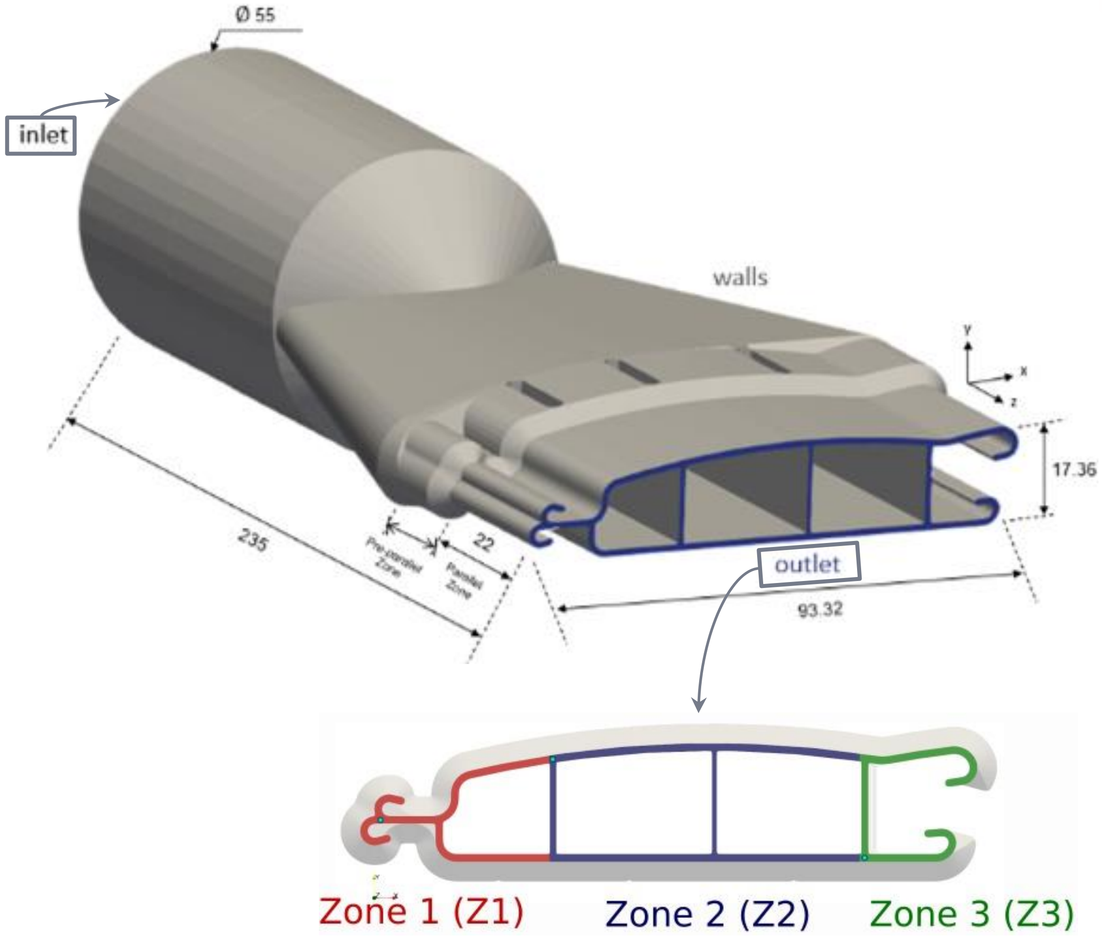
Coupled Approaches - Application case (B4)

B4 benchmark case: complex profile extrusion [4]





Mesh: 20 million cells

Coupled Approaches

exaFOAM

CODE

https://develop.openfoam.com/exafoam/wp4-development

W **wp4-development**

master · wp4-development / + · History · Find file · Edit · Code

Merge branch 'UMinho' into 'master'
Hrvoje authored 1 month ago · 3bfbe924

Name	Last commit	Last update
foam-extend-5.0	UMinho developments for viscoel...	1 month ago
README.md	UMinho developments for viscoel...	1 month ago

README.md

CASE Studies

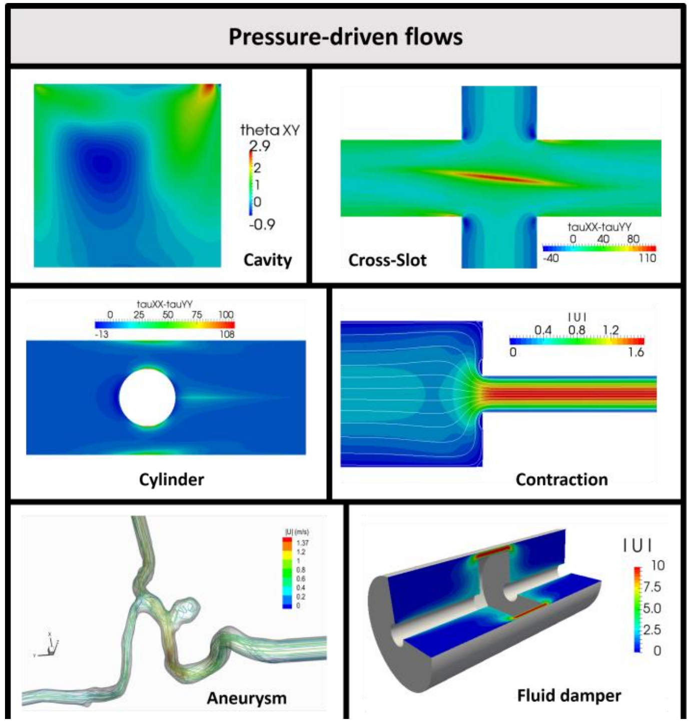
https://develop.openfoam.com/committees/hpc/-/tree/develop/viscoelastic/viscoelasticFluidFoam

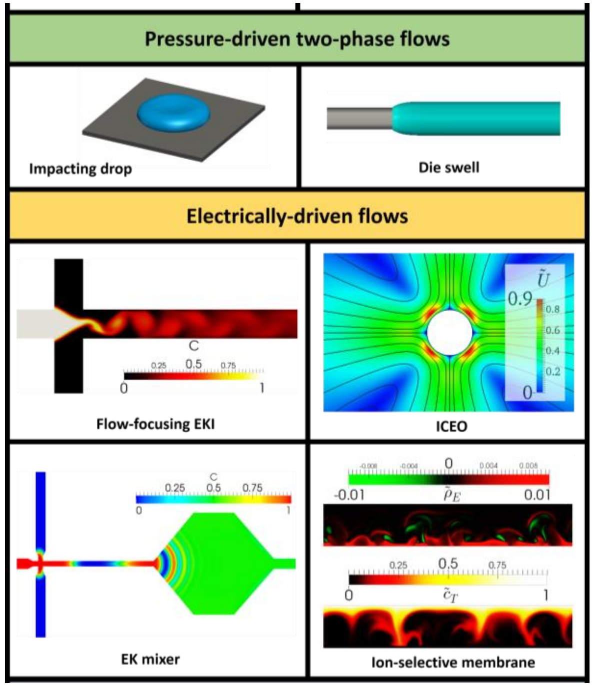
Name	Last commit
..	
complexProfile	Update README.md
complexProfileExtrusion	Changes to visco-elastic cases
lidDrivenCavity	Update README.md
profileExtrusion	Update README.md
testedCases	Changes to visco-elastic cases
README.md	Changes to visco-elastic cases
generateCase.sh	Changes to visco-elastic cases

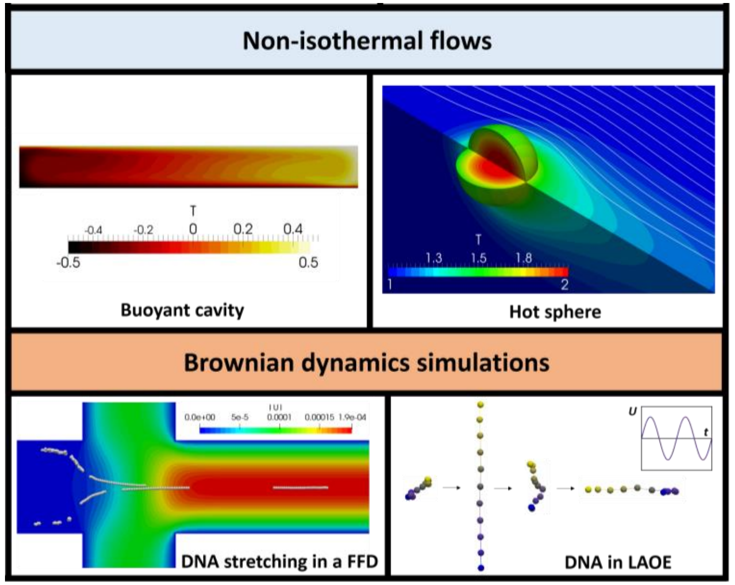
foam-extend 5.0

RheoTool

https://github.com/fppimenta/rheoTool







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RheoTool

RheoTool

https://github.com/fppimenta/rheoTool

Viscoelastic models solved in the standard extra-stress or conformation tensor variables

Model	TypeName	$\eta_s(\dot\gamma)$	$\eta_p(\dot\gamma)$	$\lambda(\dot\gamma)$	Constitutive Equation
Oldroyd-B	*Oldroyd-B*	η_s	η_p	λ	$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = \eta_p(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$
WhiteMetzner (Carreau-Yasuda)	*WhiteMetznerCY*	η_s	$\eta_p[1+(K\dot\gamma)^a]^{\frac{n-1}{a}}$	$\lambda[1+(L\dot\gamma)^b]^{\frac{m-1}{b}}$	$\boldsymbol{\tau} + \lambda(\dot\gamma) \overset{\nabla}{\boldsymbol{\tau}} = \eta_p(\dot\gamma)(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$
Giesekus	*Giesekus*	η_s	η_p	λ	$\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} + \alpha\frac{\lambda}{\eta_p}(\boldsymbol{\tau}\cdot\boldsymbol{\tau}) = \eta_p(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$
[4]PTT	*PTT*	η_s	η_p	λ	$f\boldsymbol{\tau} + \lambda\overset{\square}{\boldsymbol{\tau}} = \eta_p(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ where $f = \left[1+\frac{\varepsilon\lambda}{\eta_p}\mathrm{tr}(\boldsymbol{\tau})\right]$ (linear), $f = \left[e^{\frac{\varepsilon\lambda}{\eta_p}\mathrm{tr}(\boldsymbol{\tau})}\right]$ (exponential) or $f = \Gamma(\beta)E_{\alpha,\beta}\left(\frac{\varepsilon\lambda}{\eta_p}\mathrm{tr}(\boldsymbol{\tau})\right)$ (generalized)
FENE-CR	*FENE-CR*	η_s	η_p	λ	$\left[1+\lambda\frac{D}{Dt}\left(\frac{1}{f}\right)\right]\boldsymbol{\tau} + \frac{\lambda}{f}\overset{\nabla}{\boldsymbol{\tau}} = \eta_p(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ where $f = \frac{L^2+\frac{\lambda}{\eta_p}\mathrm{tr}(\boldsymbol{\tau})}{L^2-3}$
FENE-P	*FENE-P*	η_s	η_p	λ	$\boldsymbol{\tau} + \frac{\lambda}{f}\overset{\nabla}{\boldsymbol{\tau}} = \frac{a\eta_p}{f}(\nabla\mathbf{u} + \nabla\mathbf{u}^T) - \frac{D}{Dt}\left(\frac{1}{f}\right)[\lambda\boldsymbol{\tau} + a\eta_p\mathbf{I}]$ where $f = \frac{L^2+\frac{\lambda}{a\eta_p}\mathrm{tr}(\boldsymbol{\tau})}{L^2-3}$ and $a = \frac{L^2}{L^2-3}$
[5]Rolie-Poly	*Rolie-Poly*	η_s	η_p	λ_D	$\lambda_D \overset{\nabla}{\mathbf{A}} = -(\mathbf{A}-\mathbf{I}) - 2k\frac{\lambda_D}{\lambda_R}\left(1-\sqrt{3/\mathrm{tr}(\mathbf{A})}\right)\left[\mathbf{A} + \beta\left(\frac{\mathrm{tr}(\mathbf{A})}{3}\right)^\delta(\mathbf{A}-\mathbf{I})\right]$ where $k = \frac{\left(3-\frac{\chi^2}{\chi^2_{max}}\right)\left(1-\frac{1}{\chi^2_{max}}\right)}{\left(1-\frac{\chi^2}{\chi^2_{max}}\right)\left(3-\frac{1}{\chi^2_{max}}\right)}$ and $\chi = \sqrt{\frac{\mathrm{tr}(\mathbf{A})}{3}}$
eXtended Pom-Pom	*XPomPom*	η_s	η_p	λ_B	$f\boldsymbol{\tau} + \lambda_B\overset{\nabla}{\boldsymbol{\tau}} + \alpha\frac{\lambda_B}{\eta_p}(\boldsymbol{\tau}\cdot\boldsymbol{\tau}) + \frac{\eta_p}{\lambda_B}(f-1)\mathbf{I} = \eta_p(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ where $f = 2\frac{\lambda_B}{\lambda_S}e^{\frac{2}{q}(\Lambda-1)}\left(1-\frac{1}{\Lambda^{n+1}}\right) + \frac{1}{\Lambda^2}\left[1-\frac{\alpha}{3}\frac{\mathrm{tr}(\boldsymbol{\tau}\cdot\boldsymbol{\tau})}{(\eta_P/\lambda_B)^2}\right]$ and $\Lambda = \sqrt{1+\frac{\mathrm{tr}(\boldsymbol{\tau})}{3\eta_P/\lambda_B}}$

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‡Viscoelastic models solved with the log-conformation approach

Model	TypeName	$\boldsymbol{\Theta} \rightarrow \boldsymbol{\tau}$	[6,7]Constitutive Equation
[8]Oldroyd-B	*Oldroyd-BLog*	$\boldsymbol{\tau} = \frac{\eta_p}{\lambda}(e^{\boldsymbol{\Theta}} - \mathbf{I})$	$\boldsymbol{\Upsilon} = \frac{1}{\lambda}\left(e^{-\boldsymbol{\Theta}} - \mathbf{I}\right)$
[9]WhiteMetzner (Carreau-Yasuda)	*WhiteMetznerCYLog*	$\boldsymbol{\tau} = \frac{\eta_p}{\lambda}(e^{\boldsymbol{\Theta}} - \mathbf{I})$	$\boldsymbol{\Upsilon} = \frac{1}{\lambda(\dot{\gamma})}\left(e^{-\boldsymbol{\Theta}} - \mathbf{I}\right)$
Giesekus	*GiesekusLog*	$\boldsymbol{\tau} = \frac{\eta_p}{\lambda}(e^{\boldsymbol{\Theta}} - \mathbf{I})$	$\boldsymbol{\Upsilon} = \frac{1}{\lambda}\left[\left(e^{-\boldsymbol{\Theta}} - \mathbf{I}\right) - \alpha e^{\boldsymbol{\Theta}}\left(e^{-\boldsymbol{\Theta}} - \mathbf{I}\right)^2\right]$
[4]PTT	*PTTLog*	$\boldsymbol{\tau} = \frac{\eta_p}{\lambda(1-\zeta)}(e^{\boldsymbol{\Theta}} - \mathbf{I})$	$\boldsymbol{\Upsilon} = \frac{f}{\lambda}(e^{-\boldsymbol{\Theta}} - \mathbf{I})$, where $f = 1 + \frac{\varepsilon}{1-\zeta}\left[\mathrm{tr}(e^{\boldsymbol{\Theta}}) - 3\right]$ (linear), $f = e^{\frac{\varepsilon}{1-\zeta}(\mathrm{tr}(e^{\boldsymbol{\Theta}})-3)}$ (exponential), or $f = \Gamma(\beta)E_{\alpha,\beta}\left[\frac{\varepsilon}{1-\zeta}(\mathrm{tr}(e^{\boldsymbol{\Theta}}) - 3)\right]$ (generalized)
FENE-CR	*FENE-CRLog*	$\boldsymbol{\tau} = \frac{\eta_p f}{\lambda}(e^{\boldsymbol{\Theta}} - \mathbf{I})$	$\boldsymbol{\Upsilon} = \frac{f}{\lambda}\left(e^{-\boldsymbol{\Theta}} - \mathbf{I}\right)$, where $f = \frac{L^2}{L^2 - \mathrm{tr}(e^{\boldsymbol{\Theta}})}$
FENE-P	*FENE-PLog*	$\boldsymbol{\tau} = \frac{\eta_p}{\lambda}(f e^{\boldsymbol{\Theta}} - a\mathbf{I})$	$\boldsymbol{\Upsilon} = \frac{1}{\lambda}\left(a e^{-\boldsymbol{\Theta}} - f\mathbf{I}\right)$, where $a = \frac{L^2}{L^2-3}$ and $f = \frac{L^2}{L^2 - \mathrm{tr}(e^{\boldsymbol{\Theta}})}$
[10]Rolie-Poly	*Rolie-PolyLog*	$\boldsymbol{\tau} = \frac{\eta_p}{\lambda_D}k(e^{\boldsymbol{\Theta}} - \mathbf{I})$	$\boldsymbol{\Upsilon} = -\frac{1}{\lambda_D}e^{-\boldsymbol{\Theta}}\left\{\left(e^{\boldsymbol{\Theta}} - \mathbf{I}\right) + 2k\frac{\lambda_D}{\lambda_R}\left(1 - \sqrt{3/\mathrm{tr}(e^{\boldsymbol{\Theta}})}\right)\left[e^{\boldsymbol{\Theta}} + \beta\left(\frac{\mathrm{tr}(e^{\boldsymbol{\Theta}})}{3}\right)^{\delta}\left(e^{\boldsymbol{\Theta}} - \mathbf{I}\right)\right]\right\}$
eXtended Pom-Pom	*XPomPomLog*	$\boldsymbol{\tau} = \frac{\eta_p}{\lambda_B}(e^{\boldsymbol{\Theta}} - \mathbf{I})$	$\boldsymbol{\Upsilon} = -\frac{1}{\lambda_B}e^{-\boldsymbol{\Theta}}\left[(f - 2\alpha)e^{\boldsymbol{\Theta}} + \alpha e^{\boldsymbol{\Theta}}e^{\boldsymbol{\Theta}} + (\alpha - 1)\mathbf{I}\right]$ where $f = 2\frac{\lambda_B}{\lambda_S}e^{\frac{2}{q}(\Lambda-1)}\left(1 - \frac{1}{\Lambda^{n+1}}\right) + \frac{1}{\Lambda^2}\left[1 - \alpha - \frac{\alpha}{3}\mathrm{tr}(e^{\boldsymbol{\Theta}}(e^{\boldsymbol{\Theta}} - 2\mathbf{I}))\right]$ and $\Lambda = \sqrt{\frac{\mathrm{tr}(e^{\boldsymbol{\Theta}})}{3}}$

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RheoTool – Reduced Version

Solvers

- RheoInterFoam Solver (Two-phase flow)

Constitutive Models

- Giesekus
- Giesekus-Log
- PTT
- PTT-Log
- Newtonian
- Carreau-Yasuda

P4 – Case 41

Extrudate Swell

- Carreau-Yasuda (CY)
- Giesekus-Log (GL)
- Giesekus-Log Multimode – illustrative (MGL)

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P4 – Case 41

1. Open Ubuntu terminal
2. of2206   //Load OpenFOAM variables
3. >> run
4. >> cd case41
5. >> code .
6. Study the CY and GL cases files namely file constant/constitutiveProperties and dictionaries fvSolution and fvSchemes.
7. Study the Allrun files for both cases
8. Run both cases
9. Post-process the data in paraview to compare the extrudate swell
10. Study the MGL case files namely file constant/constitutiveProperties, dictionaries fvSolution and fvSchemes and 0 folder to understand how to define the multimode model