
Learning Algorithm Implementations

Inferring Rudimentary Rules and Decision Trees

Summary

- **Input Knowledge representation**
- **Preparing data for learning**
 - Input: Concept, Instances, Attributes
- **Output Knowledge representation**
- **Algorithms: Basic Methods**

Knowledge Representation

***Input:
Concepts, Instances, Attributes***

Preparing for learning

- **Components of the input:**
 - **Concepts:** kinds of things that can be learned
 - Intelligible and operational concept description
 - **Instances:** the individual, independent examples of a concept
 - More complicated forms of input are possible
 - **Attributes:** measuring aspects of an instance
 - Nominal and numeric
- **Practical issues: input file format**

What is a concept?

- **Concept: thing to be learned**
- **Concept description: output of learning scheme**
- **4 major styles of learning:**
 1. Classification learning: predicting a discrete class
 2. Association learning: detecting associations between features
 3. Clustering: grouping similar instances into clusters
 4. Numeric prediction: predicting a numeric quantity

What's in an example?

- **Instance: specific type of example**
 - Thing to be classified, associated, or clustered
 - Individual, independent example of target concept
 - Characterized by a predetermined set of attributes
- **Input to learning scheme:**
 - Set of instances/dataset
 - Represented as a single relation/flat file
- **Most common form in practical data mining**

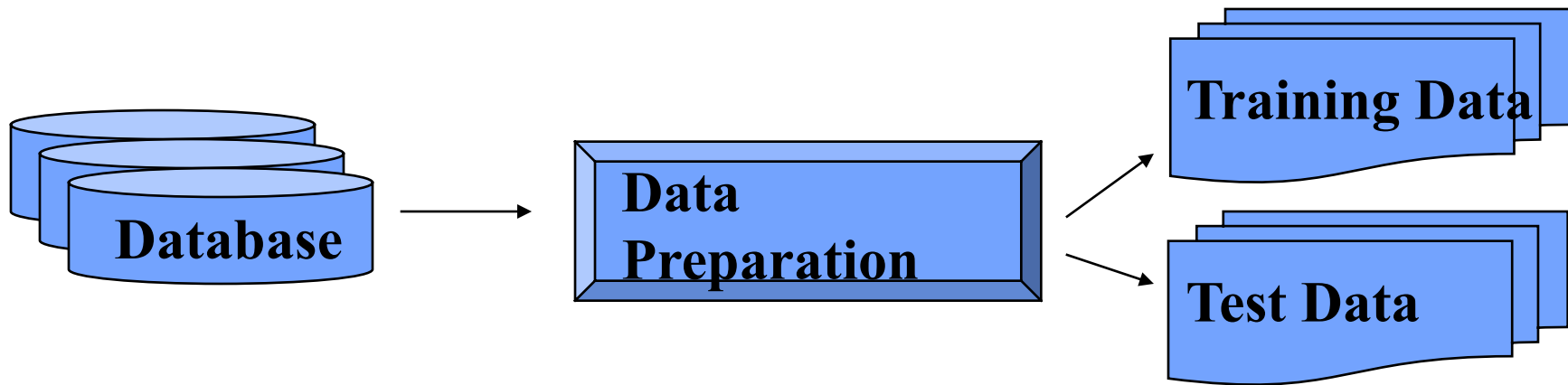
What is an attribute?

- **Each instance is described by a fixed predefined set of features or “attributes”**
- **Number of attributes may vary in practice**
- **Existence of an attribute may depend of value of another attribute**
- **Possible attribute types**
 - Nominal, ordinal, interval and ratio
 - Nominal (categorical) vs. numeric (continuous)

Jargon

- **Collection of records – training set**
 - Each record contains a set of attributes (class)
- **Model for the class attribute is a function of the values of other attributes**
- **Previously unseen data – assigned to the appropriate class**
- **Test set – determines the accuracy of the model on unseen data**

From Data Source To Algorithm Input



User Decides:

- **Selection Criteria –**
- **Joins => denormalize**
- **How much data?**

Depends on needs and domain knowledge about what's relevant

User Performs:

- **Cleaning data and Transformations**

Depends on domain knowledge, data itself and possibly on algorithms

From Data Sources To Algorithm Input

- **Instance = specific example**
 - thing to be classified, associated, or clustered
 - instances may be labeled as a class, or as an outcome
 - If no labels available you can either do unsupervised learning or try to get labels
- **Set of instances comprise the input dataset**
 - Often represented as a single flat file or *data matrix*

Terms from database to math

TABLE

Attributes (i.e. columns)

Rows	Cells ...		

Instances

...
...

DATA MATRIX

Variables

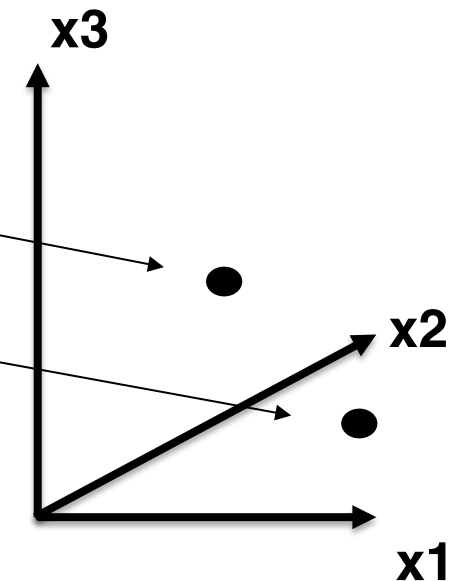
attributes in the database relate to variables in the data matrix

Terms data matrix to math

For real numbers, each variable is also a dimension in some coordinate space

Each Row is a vector, each vector is a coordinate point

x1	x2	x3	
1	1.5	1.3		
2.2	1	.25		
1				



sometimes called the
'input space'

Models enabled by Data Matrix

Instance	Customer	Item	Price	Date	Label/outcome
1	John	Acme Mower	100	Jan 2000	Used coupon
2	John	Acme Wrench	10	Sept 2000	Used coupon
3	Jane	Ace Mower	120	Mar 2003	No coupon
4	Jane	Ace Rake	20	Mar 2003	No coupon
5 ...	Fred	Ace Hammer	15	July 2002	Used coupon

If your model is:

Coupon-use is function of Price, Customer, Item, Date, etc..

You can write it as:

$coupon_1 = F (Price_1 , Customer_1 , Item_1 , Date_1 \text{ etc..})$

$coupon_2 = F (Price_2 , Customer_2 , Item_2 , Date_2 \text{ etc..})$

...

$coupon_i = F (Price_i , Customer_i , Item_i , Date_i \text{ etc..})$

Q: What about instance number?

Learning About Outcomes

- **Functions**

Outcome = Function of Attributes

- **Sometimes functions are hard to interpret and describe**
- **Sometimes we can describe and interpret functions as concepts or rules**
- **Sometimes we can more directly learn simple descriptions**

Basic Methods

- **Nearest Neighbor (instance based or lazy learner)**
- **1R (rule learner)**
- **Naïve Bayes (a statistical rule)**
- **Decision Tree (divide and conquer)**
- **Hands-on Decision Trees**

Simplicity first

- **Simple algorithms often work surprisingly well**
- **Different kinds of strategies exist:**
 - One attribute
 - All attributes - equal importance
 - A linear combination (ie linear function)
- **Different kinds of conceptual descriptions**
 - An instance-based representation
 - Simple logical structures

Rudimentary rules

- **1R: learns a 1-level decision tree**
 - Set of rules that all test on one particular attribute
- **Basic version**
 - One branch for each of the attribute's values
 - Each branch assigns most frequent class
 - Error rate: proportion of instances that don't belong to the majority class of their corresponding branch
 - Choose attribute with lowest error rate

Pseudo-code for 1R

For each attribute

For each value of the attribute make a rule:

count how often each class appears

find the most frequent class

assign that class to this attribute-value

Calculate the error rate of the rules

Choose the rules with the smallest error rate

Weather Data Set

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Outlook	Temp.	Humidity	Windy	Play
Sunny	Hot	High	False	No
Sunny	Hot	High	True	No
Overcast	Hot	High	False	Yes
Rainy	Mild	High	False	Yes
Rainy	Cool	Normal	False	Yes
Rainy	Cool	Normal	True	No
Overcast	Cool	Normal	True	Yes
Sunny	Mild	High	False	No
Sunny	Cool	Normal	False	Yes
Rainy	Mild	Normal	False	Yes
Sunny	Mild	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overcast	Hot	Normal	False	Yes
Rainy	Mild	High	True	No

Attribute	Rules	Errors	Total errors
Outlook	Sunny → No	2/5	4/14
	Overcast → Yes	0/4	
	Rainy → Yes	2/5	
Temperature	Hot → No*	2/4	5/14
	Mild → Yes	2/6	
	Cool → Yes	1/4	
Humidity	High → No	3/7	4/14
	Normal → Yes	1/7	
Windy	False → Yes	2/8	5/14
	True → No*	3/6	

Summary

- **1R was described in a paper by Holte (1993)**
- **16 datasets**
- **Minimum number of instances was set to 6**
- **1R's simple rules performed not much worse than much more complex decision trees**
- **“Simplicity first” pays off!**

Statistical modeling

- **“Opposite” of 1R: use all the attributes**
- **Two assumptions: Attributes are**
 - equally important
 - statistically independent
- **Knowledge about the value of a particular attribute doesn't tell us anything about the value of another attribute (if the class is known)**
- **Assumptions that are almost never correct**
 - scheme works well in practice!

Weather Data Set

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
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D5	Rain	Cool	Normal	Weak	Yes
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D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
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D11	Sunny	Mild	Normal	Strong	Yes
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D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Weather Data Counts and Probabilities

Outlook			Temperature			Humidity			Windy		Play		
	Yes	No		Yes	No		Yes	No		Yes	No	Yes	No
Sunny	2	3	Hot	2	2	High	3	4	False	6	2	9	5
Overcast	4	0	Mild	4	2	Normal	6	1	True	3	3		
Rainy	3	2	Cool	3	1								
Sunny	2/9	3/5	Hot	2/9	2/5	High	3/9	4/5	False	6/9	2/5	9/14	5/14
Overcast	4/9	0/5	Mild	4/9	2/5	Normal	6/9	1/5	True	3/9	3/5		
Rainy	3/9	2/5	Cool	3/9	1/5								

A new day to be classified

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	?

Likelihood of the New Day Outcome

Likelihood of the two classes

$$\text{For "yes"} = 2/9 \times 3/9 \times 3/9 \times 3/9 \times 9/14 = 0.0053$$

$$\text{For "no"} = 3/5 \times 1/5 \times 4/5 \times 3/5 \times 5/14 = 0.0206$$

Conversion into a probability by normalization:

$$P(\text{"yes"}) = 0.0053 / (0.0053 + 0.0206) = 0.205$$

$$P(\text{"no"}) = 0.0206 / (0.0053 + 0.0206) = 0.795$$

Bayes's rule

- **Probability of event H given evidence E:**

$$\Pr[H | E] = \frac{\Pr[E | H] \Pr[H]}{\Pr[E]}$$

- **A priori probability of H:**
 - Probability of event before evidence has been seen

$$\Pr[H]$$

- **A posteriori probability of H:**
 - Probability of event after evidence has been seen

$$\Pr[H | E]$$

Naïve Bayes for classification

- Classification learning: what's the probability of the class given an instance?
- Evidence E = instance
- Event H = class value for instance
- Naïve Bayes assumption: evidence can be split into independent parts

$$\Pr[H | E] = \frac{\Pr[E_1 | H] \Pr[E_2 | H] \dots \Pr[E_n | H] \Pr[H]}{\Pr[E]}$$

Evidence:

Outlook	Temp.	Humidity	Windy	Play
Sunny	Cool	High	True	?

$$\frac{\begin{aligned} &\Pr[\text{yes}|E]= \\ &\Pr[\textit{Outlook}=\textit{Sunny}|\text{yes}] \times \\ &\Pr[\textit{Temperature}=\textit{Cool}|\text{yes}] \times \\ &\Pr[\textit{Humidity}=\textit{High}|\text{yes}] \times \Pr[\textit{Windy}=\textit{True}|\text{yes}] \times \Pr[\text{yes}] \end{aligned}}{\Pr[E]}$$

**Probabilities
for class YES**

$$= \frac{2/9 \times 3/9 \times 3/9 \times 3/9 \times 9/14}{\Pr[E]}$$

Summary

- **Naïve Bayes works amazingly well**
 - Violated independence assumption
- **Because classification doesn't require accurate probability estimates as long as maximum probability is assigned to correct class**
- **Problem: Adding too many redundant attributes**
 - Example: identical attributes
- **Conditional Probability visualization:**
<http://setosa.io/conditional/>

DECISION TREE INDUCTION

- **Method for approximating discrete-valued functions**
 - robust to noisy/missing data
 - can learn non-linear relationships
 - inductive bias towards shorter trees

Decision trees

- **“Divide-and-conquer” approach**
- **Nodes involve testing a particular attribute**
- **Attribute value is compared to**
 - Constant
 - Comparing values of two attributes
 - Using a function of one or more attributes
- **Leaves assign classification, set of classifications, or probability distribution to instances**
- **Unknown instance is routed down the tree**

Decision Tree Learning

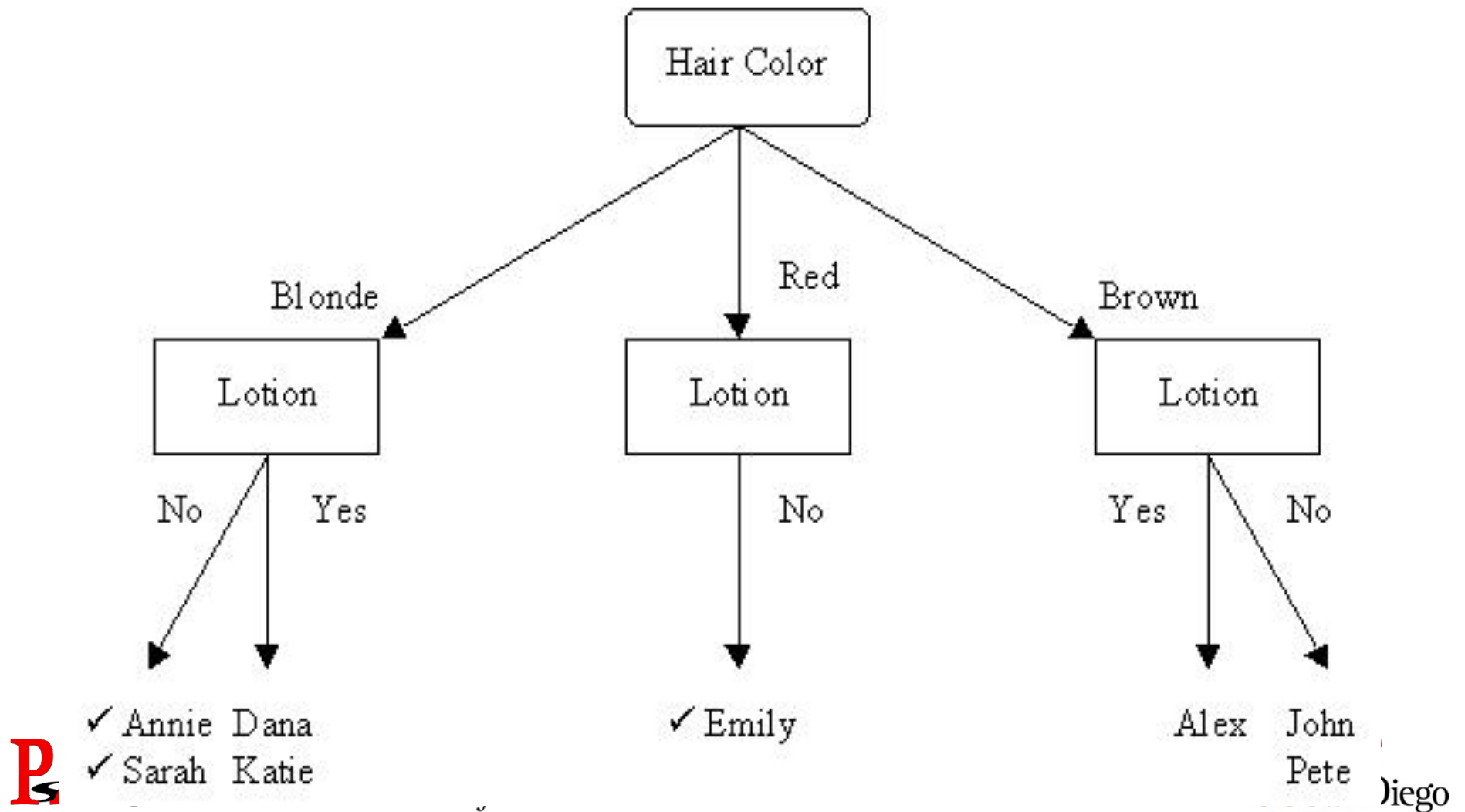
- **Applications:**

- medical diagnosis – ex. heart disease
- analysis of complex chemical compounds
- classifying equipment malfunction
- risk of loan applicants
- Boston housing project – price prediction

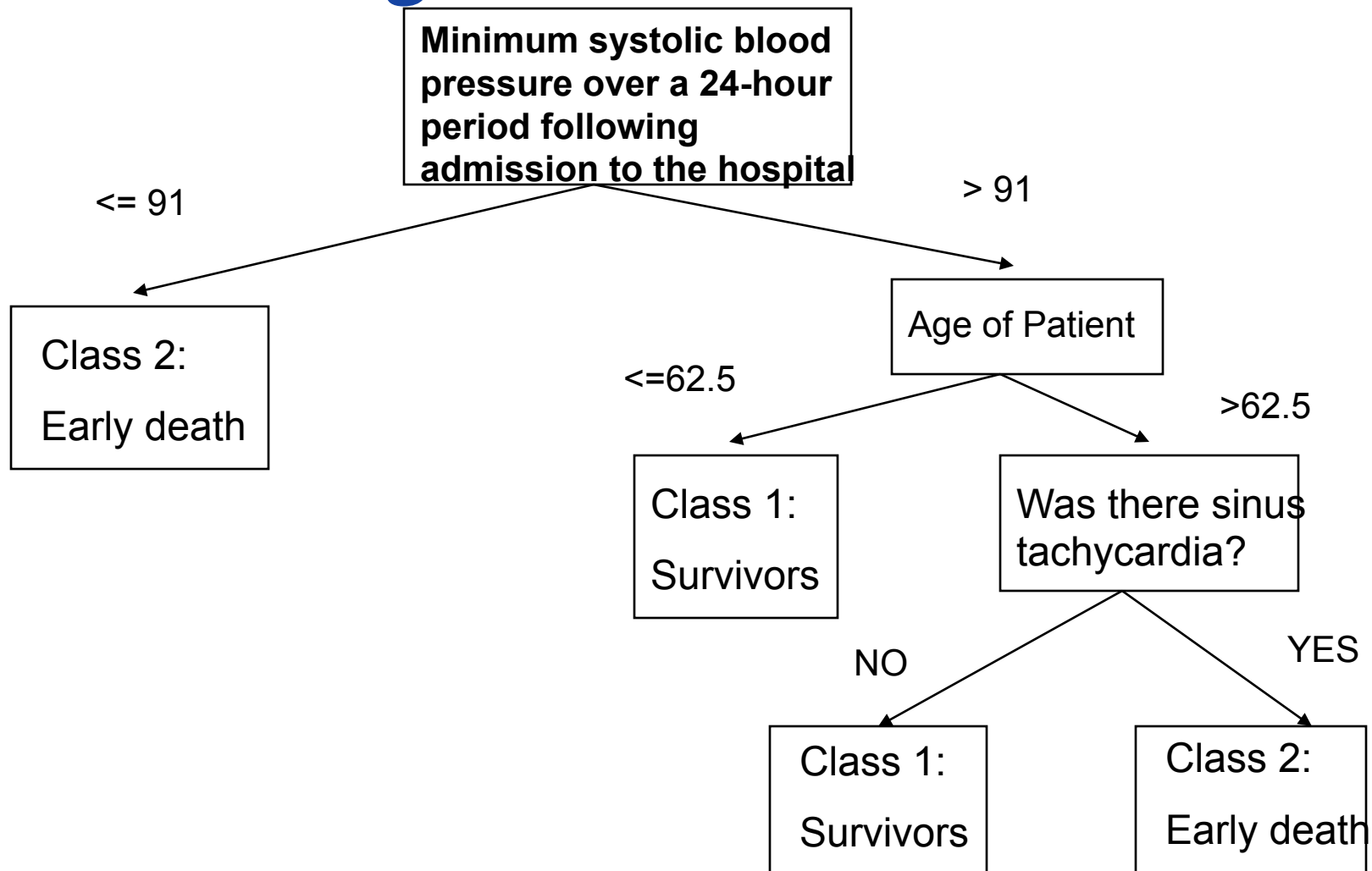
DECISION TREE FOR THE CONCEPT “Sunburn”

Name	Hair	Height	Weight	Lotion	Result
Sarah	blonde	average	light	no	sunburned (positive)
Dana	blonde	tall	average	yes	none (negative)
Alex	brown	short	average	yes	none
Annie	blonde	short	average	no	sunburned
Emily	red	average	heavy	no	sunburned
Pete	brown	tall	heavy	no	none
John	brown	average	heavy	no	none
Katie	blonde	short	light	yes	none

DECISION TREE FOR THE CONCEPT “Sunburn”



DT for Medical Diagnosis and Prognosis Heart Disease



Occam's Razor

- **“The world is inherently simple. Therefore the smallest decision tree that is consistent with the samples is the one that is most likely to identify unknown objects correctly”**

Decisions Trees Representation

- **Each internal node tests an attribute**
- **Each branch corresponds to attribute value**
- **Each leaf node assigns a classification**

When to Consider Decision Trees

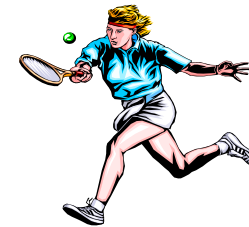
- **Instances describable by attribute--value pairs**
 - each attribute takes a small number of disjoint possible values
- **Target function has discrete output value**
- **Possibly noisy training data**
 - may contain errors
 - may contain missing attribute values

Weather Data Set-Make the Tree!

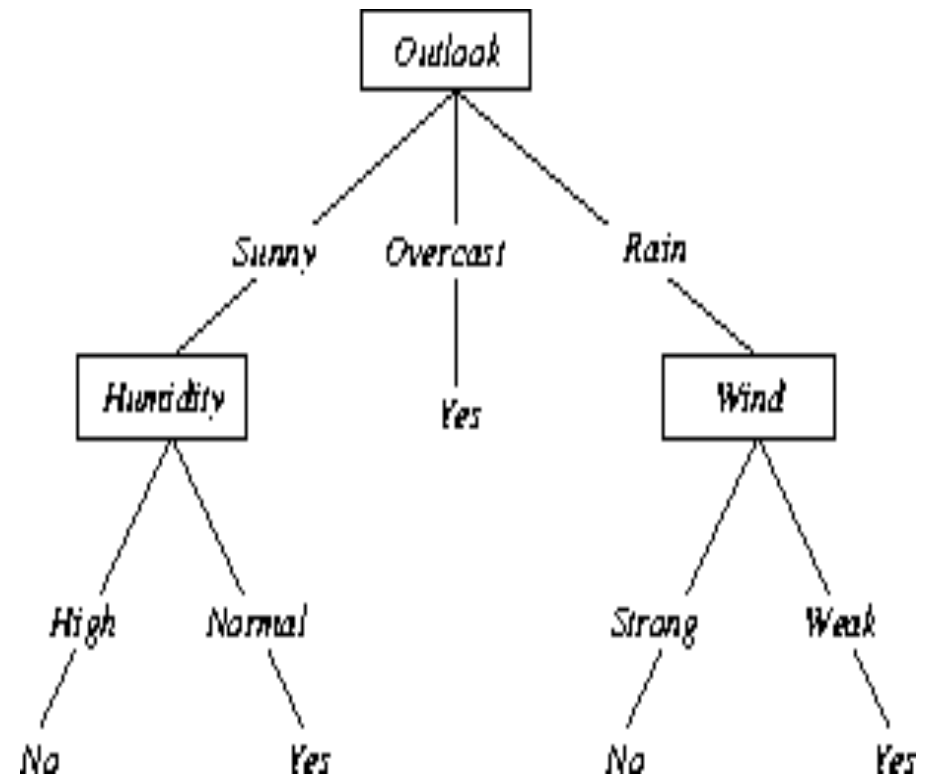
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DECISION TREE FOR THE CONCEPT

“Play Tennis”



Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
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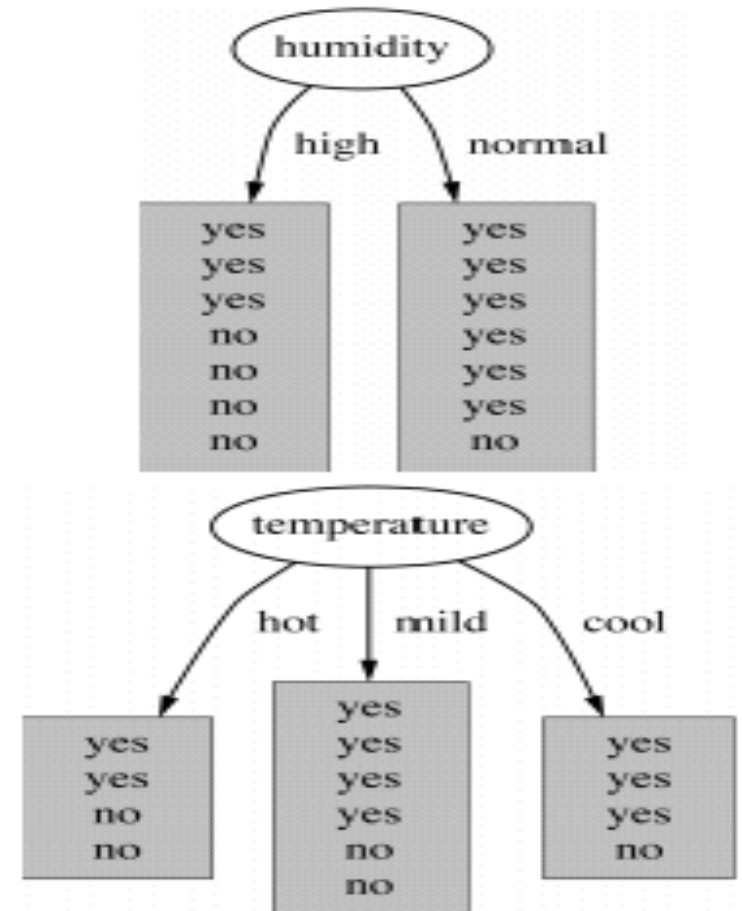
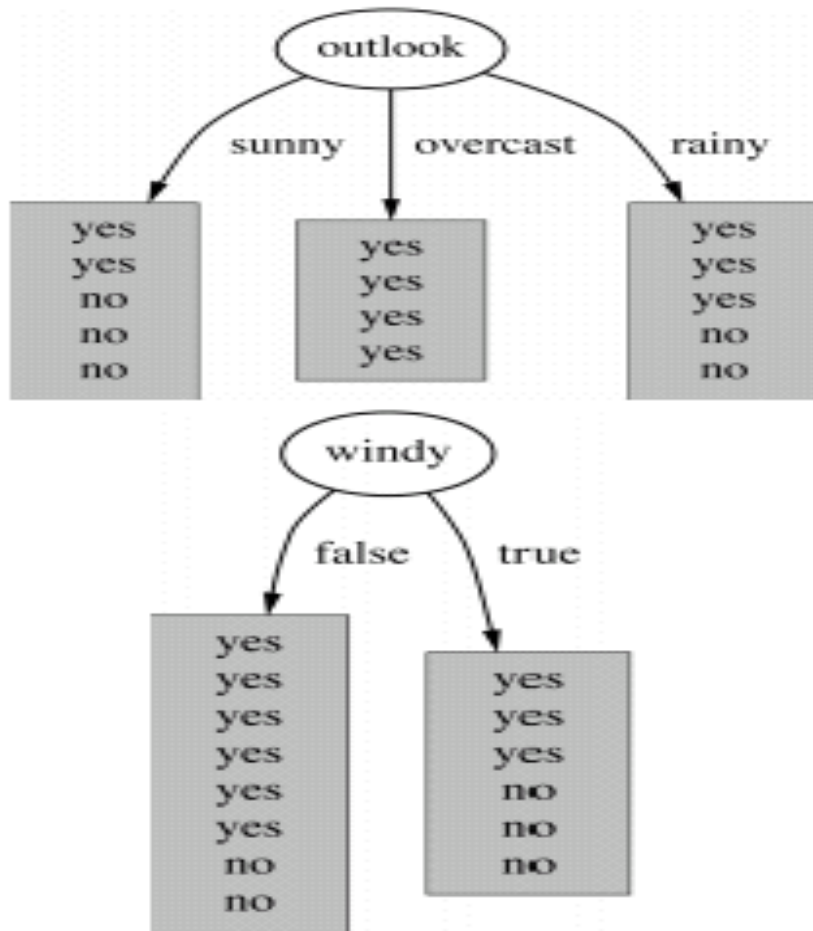
Constructing decision trees

- **Normal procedure: top down in recursive divide-and-conquer fashion**
- **First: attribute is selected for root node and branch is created for each possible attribute value**
- **Then: the instances are split into subsets (one for each branch extending from the node)**
- **Finally: procedure is repeated recursively for each branch, using only instances that reach the branch**
- **Process stops if all instances have the same class**

Induction of Decision Trees

- **Main Recursive loop:**
 - Pick the “best” attribute to split the data at current decision node, according to some measure
 - For each value of attribute, create new leaf node descendants of current node
 - Sort data to new leaf nodes
 - For each leaf node:
 - If training examples perfectly classified,
 - Then STOP
 - Else Iterate at current leaf node as next decision node

Which is the best attribute?



Attribute selection

- **How to choose the best attribute?**
 - Smallest tree
 - Heuristic: Attribute that produces the “purest” nodes
- **Impurity criterion:**
 - Information gain
 - Increases with the average purity of the subsets produced by the attribute split
- **Choose attribute that results in greatest information gain**

Computing information

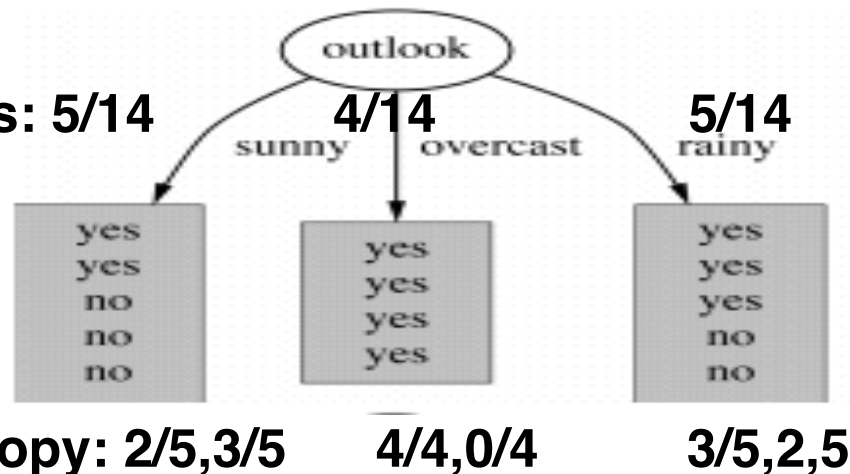
- **Information is measured in bits**
- **Given a probability distribution, the info required to predict an event is the distribution's entropy**
 - e.g. highly predictable events => low entropy
 - e.g. random probabilities => higher entropy
- **Formula for computing the entropy for n classes:**

$$\text{entropy}(p_1, p_2, \dots, p_n) = -p_1 \log p_1 - p_2 \log p_2 \dots - p_n \log p_n$$

Expected information for attribute “Outlook”

- “*Outlook*” = “Sunny”
- “*Outlook*” = “Overcast”
- “*Outlook*” = “Rainy”
- Total expected information: is weighted avg. of entropy at each node branch,

Branch probs
are the weights: 5/14



Class probs

used for each Entropy: 2/5, 3/5

4/4, 0/4

3/5, 2/5

Computing the information gain

- **Information gain: information before splitting – information after splitting**

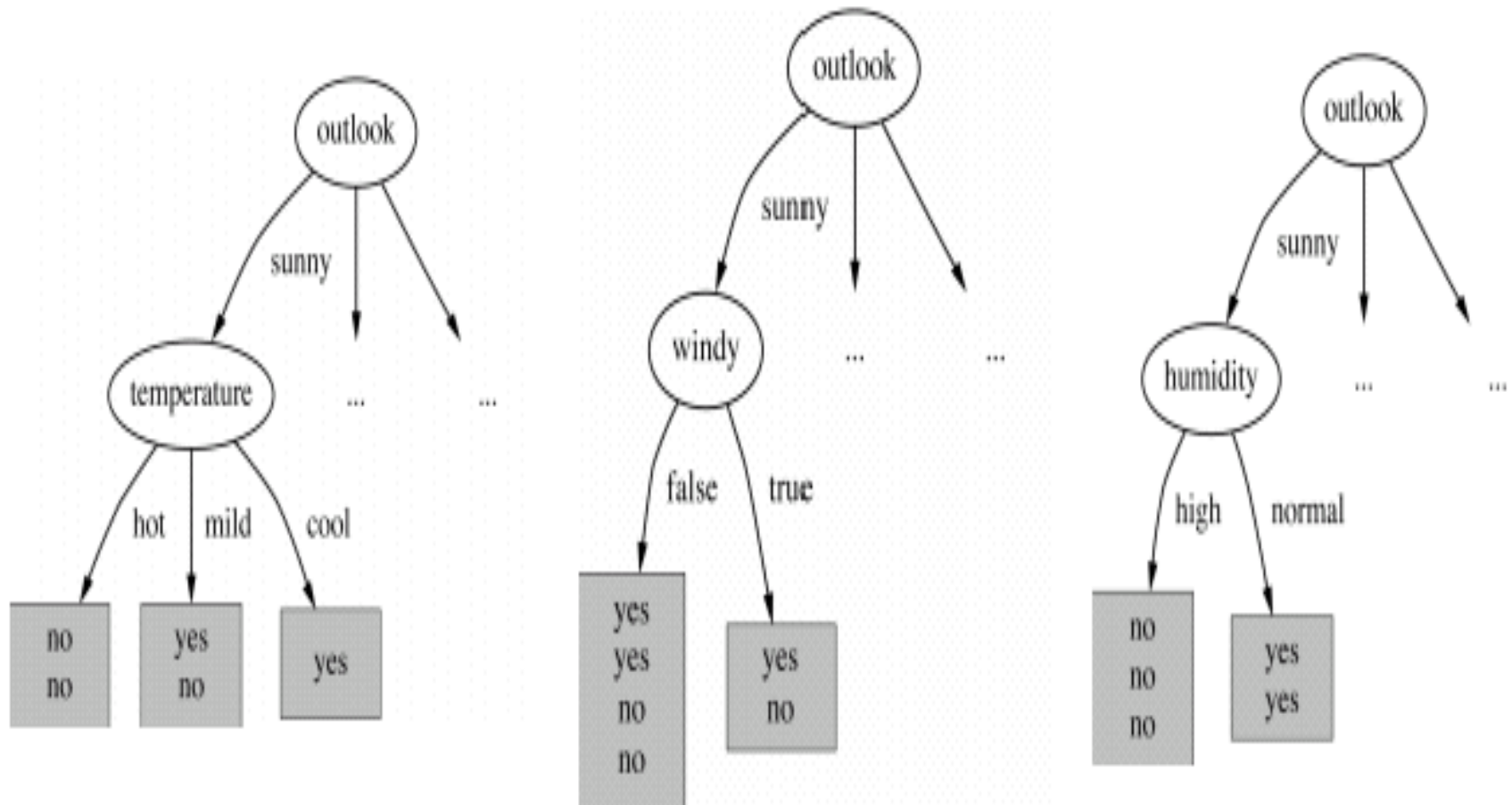
- **Test the Info Gain for outlook at root node:
Gain(“Outlook”)=entropy of classes –
avg. entropy of classes at new leaf nodes**

$$\begin{aligned} &= \text{Ent}(9/14, 5/14) - \\ &5/14 * \text{Ent}(2/5, 3/5) + 4/14 * \text{Ent}(4/4, 0/4) + \\ &5/14 * \text{Ent}(3/5, 2/5) \end{aligned}$$

Computing the information gain

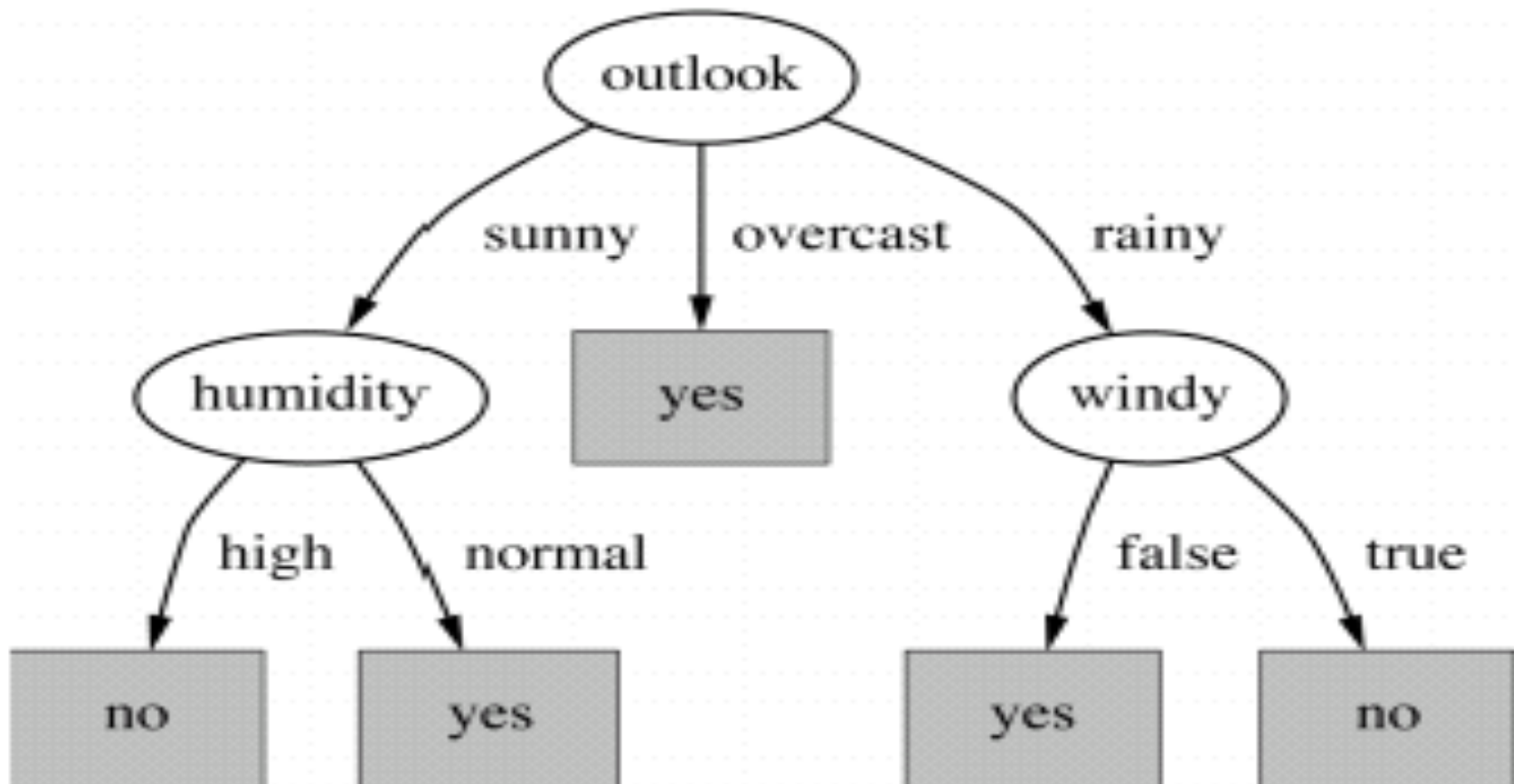
- **Information gain for attributes from weather data:**
 - Gain (“Outlook”) = 0.247 bits
 - Gain (“Temp”) = 0.029 bits
 - Gain (“Humidity”) = 0.152 bits
 - Gain (“Windy”) = 0.048 bits

Further splits



Gain (“Temp”)=0.571 bits Gain (“Humidity”)=0.971 Gain(“Windy”)=0.020 bits

Final product



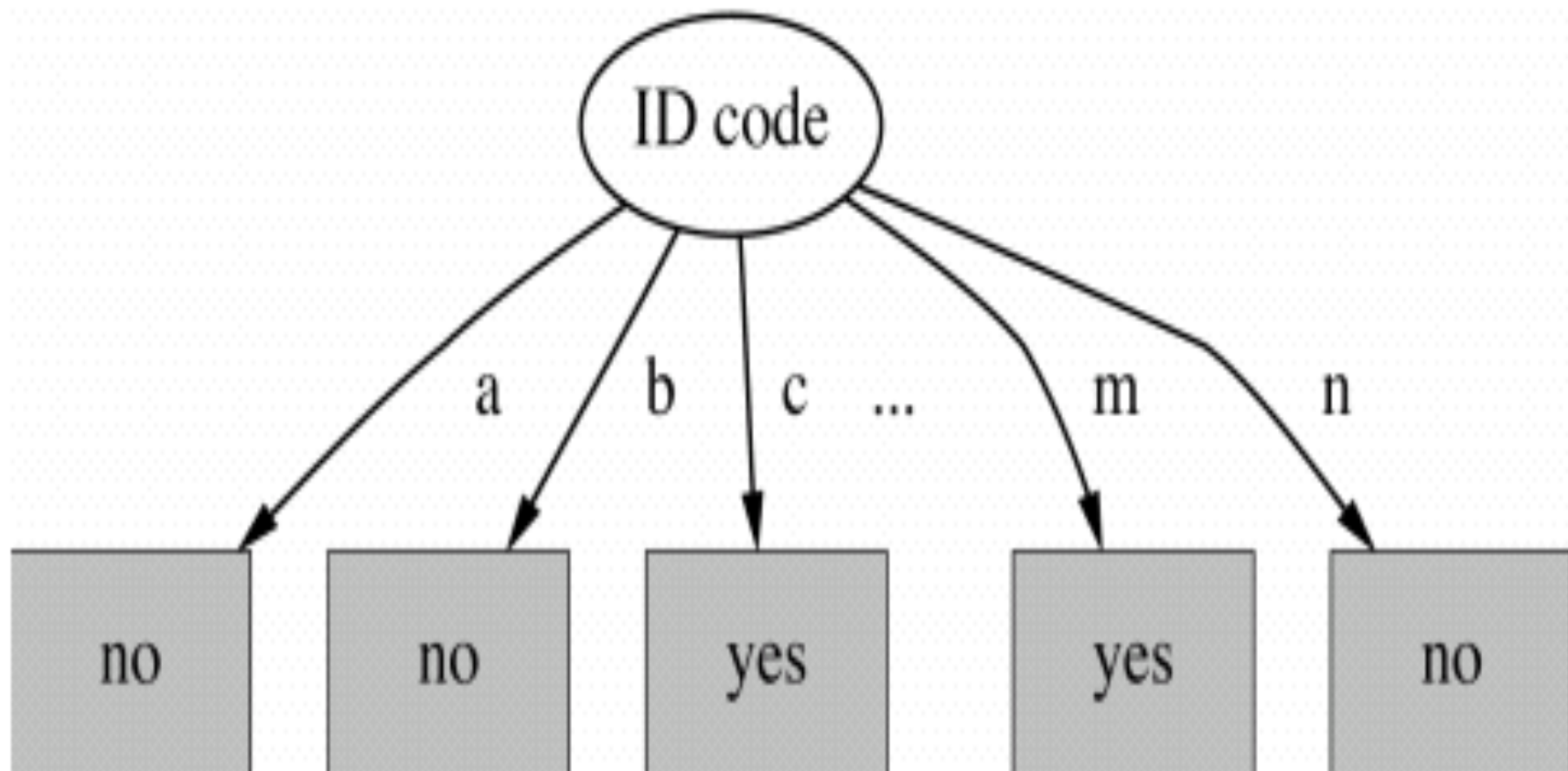
Highly-branching attributes

- **Attributes with a large number of values**
 - example: ID code
- **Subsets more likely to be pure if there is a large number of values**
 - Information gain biased towards attributes with a large number of values, (especially as leaf node training examples get smaller)
 - Overfitting

New version of Weather Data

ID code	Outlook	Temp.	Humidity	Windy	Play
A	Sunny	Hot	High	False	No
B	Sunny	Hot	High	True	No
C	Overcast	Hot	High	False	Yes
D	Rainy	Mild	High	False	Yes
E	Rainy	Cool	Normal	False	Yes
F	Rainy	Cool	Normal	True	No
G	Overcast	Cool	Normal	True	Yes
H	Sunny	Mild	High	False	No
I	Sunny	Cool	Normal	False	Yes
J	Rainy	Mild	Normal	False	Yes
K	Sunny	Mild	Normal	True	Yes
L	Overcast	Mild	High	True	Yes
M	Overcast	Hot	Normal	False	Yes
N	Rainy	Mild	High	True	No

ID Code Attribute Split



Info([9,5]) = 0.940 bits

Gain ratio

- **Modification that reduces its bias**
Info Gain / Intrinsic Info
- **Intrinsic information:**
Entropy of attribute based on sample values at that node
(not the class values at leaf node)
e.g. An ID variable will have high intrinsic info, thereby
shrinking the Info Gain

It helps usually (but not necessarily)

Summary

- **Algorithm for top-down induction of decision trees**
- **“ID3” was developed by Ross Quinlan**
- **C4.5 incorporate**
 - numeric attributes, missing values, and noisy data
 - J48 is the WEKA java implementation
- **CART Breiman, Friedman, Olshen, Stone**
- **Many other variations**

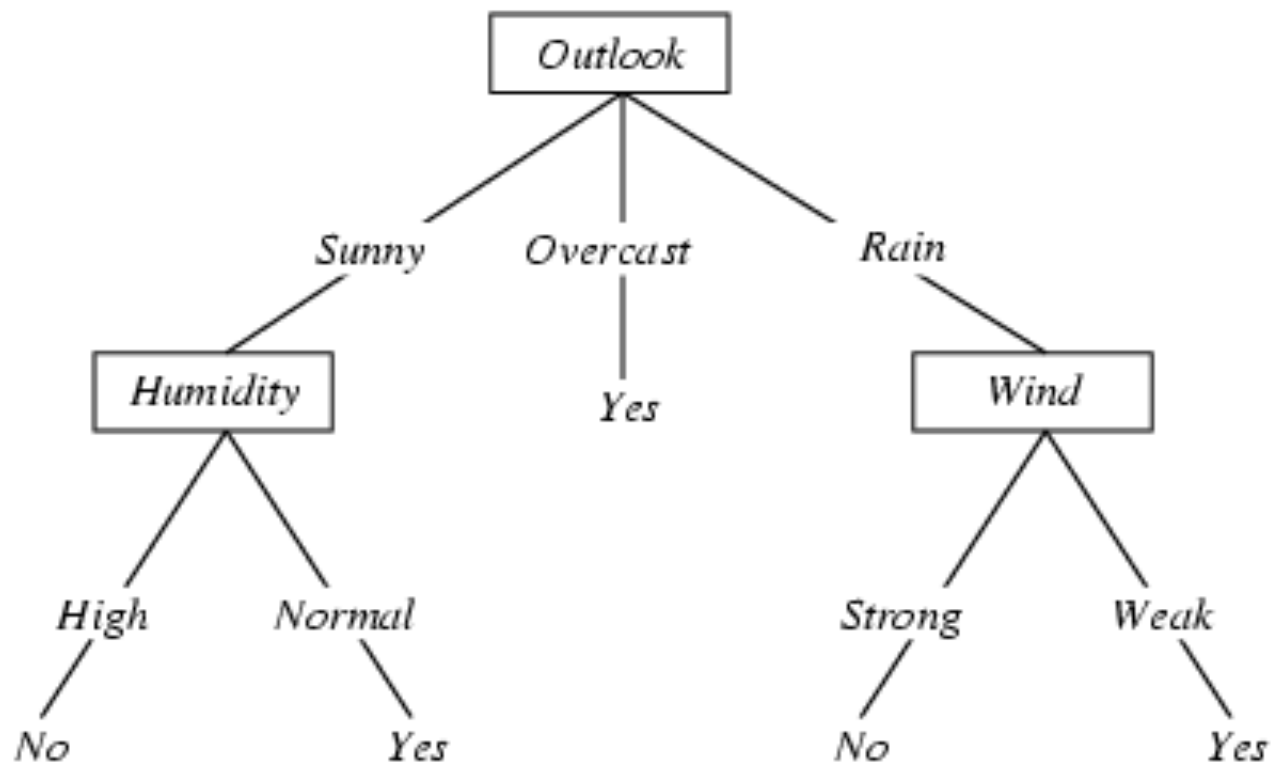
Avoid Overfitting

- **How can we avoid Overfitting:**
 - Stop growing when data split not statistically significant
 - Grow full tree then post-prune
- **How to select best tree?**
 - Measure performance over training data
 - Measure performance over separate validation data set

Pruning

- **Pruning simplifies a decision tree to prevent overfitting to noise in the data**
- **Pre-pruning:**
 - stops growing a branch when information becomes unreliable
- **Post-pruning:**
 - discard subtrees if they don't improve error estimates
 - Error estimates are derived from cross-validation or statistical assumptions
- **Post-pruning preferred in practice because of early stopping in pre-pruning**

Converting Tree to rules



Converting Tree to rules

IF $(Outlook = Sunny) \wedge (Humidity = High)$
THEN $PlayTennis = No$

IF $(Outlook = Sunny) \wedge (Humidity = Normal)$
THEN $PlayTennis = Yes$

...

Thank you!