$$f = Ae^{-\alpha t} + Be^{-\beta t}$$
$$= Ae^{-\alpha t}\left(1 + \tfrac{B}{A}\, e^{\underbrace{(-\beta+\alpha)}_{=\alpha-\beta=\gamma}t}\right)$$

$$0 = 1 + \tfrac{B}{A}e^{\gamma t} \quad \leftarrow \text{when is this} \leq 0\text{ ?}$$

$$-A/B = e^{\gamma t}$$

$$\frac{\ln(-A/B)}{\gamma} = t = \frac{\ln(-A/B)}{\alpha - \beta}$$

☆ <u>So</u> A, B must have opposite signs to force soln to $\emptyset$ (or else $\ln(-A/B)$ is imaginary)

$\ln(-A/B) < 0$ for $|A| < |B|$ so $\beta > \alpha$

$= 0$ " $|A| = |B|$ so $\emptyset$ ($\emptyset$ at $t=0$)

> 0 " $|A| > |B|$ so $\alpha > \beta$

all give solutions that $\rightarrow \emptyset$ at $t \geq 0$

<u>but</u> what is slope @ zero point ?

$$\frac{df}{dt} = -A\alpha e^{-\alpha t} - B\beta e^{-\beta t}$$
$$= -A\alpha e^{-\alpha t}\left(1 + \tfrac{B}{A}\tfrac{\beta}{\alpha}\, e^{\gamma t}\right) \qquad (\gamma = \alpha - \beta)$$

$$\frac{df}{dt}\left(t = \frac{\ln(-A/B)}{\gamma}\right) = -A\alpha\, e^{-\alpha/\gamma \ln(-A/B)}\left(1 + \frac{B}{A}\frac{\beta}{\alpha} e^{\cancel{\gamma}\frac{\ln(-A/B)}{\cancel{\gamma}}}\right)$$

$$= -A\alpha\left(-A/B\right)^{-\alpha/\gamma}\left(1 + \cancel{\tfrac{B}{A}}\,\tfrac{\beta}{\alpha}\cancel{\left(-A/B\right)}\right)$$

$$= A\left(-A/B\right)^{-\alpha/\gamma}(\beta - \alpha) \quad \text{or.}$$

$$= -A\underbrace{\left(-A/B\right)^{-\alpha/\gamma}}_{>0 \text{ for } A, B \text{ opposite signs}}\underbrace{(\alpha - \beta)}\quad [\text{more standard}]$$

So, ~~[illegible]~~

A	$\alpha - \beta$	df/dt
+	+	−
+	−	+
−	+	+
−	−	−

So, $\frac{df}{dt} < 0$ when A, $\alpha - \beta$ have <u>same sign</u>.

or $\frac{A}{\alpha - \beta} > 0$

<u>So</u> ~~[illegible]~~, combining this w/ zeros of f,
if we want $\frac{df}{dt} < 0$ for values of t $f(t) = 0$
at <u>$t > 0$</u>,

Solns that give prop. we want (generally)

$$\boxed{\begin{array}{l} A > 0,\ |A| > |B|,\ \alpha > \beta \\ A < 0,\ |A| < |B|,\ \alpha < \beta \end{array}}$$