Computational Physics

Topic 03 — Computational Problems involving Marko Chains

Lecture 02 — The Collector Problem

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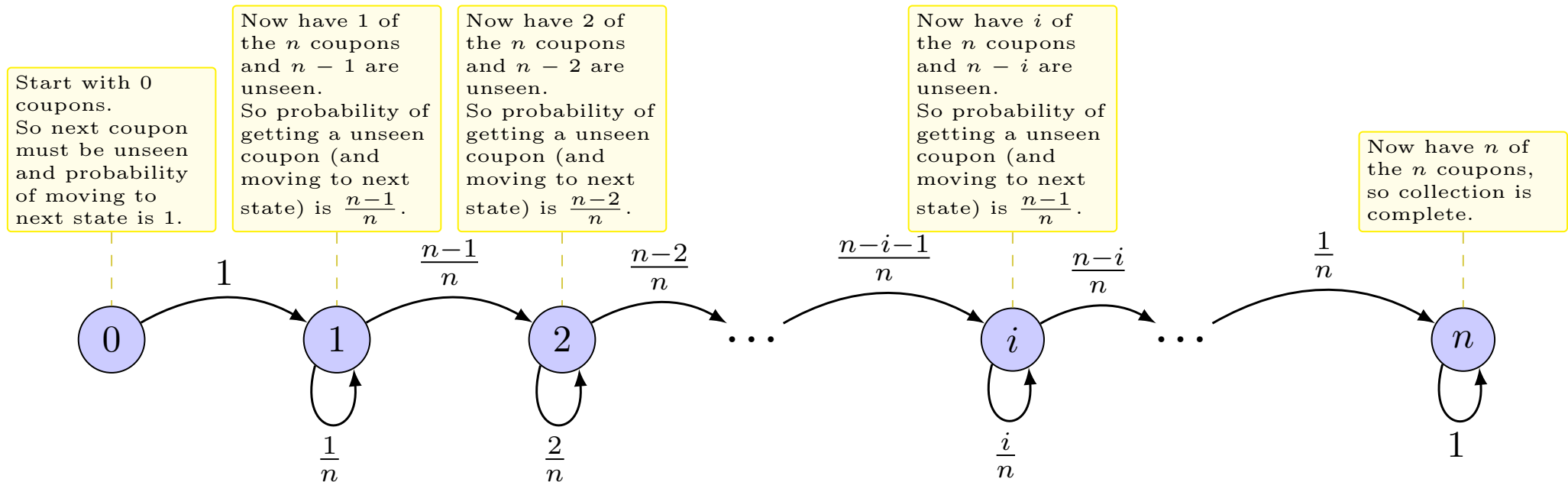
Autumn Semester, 2021
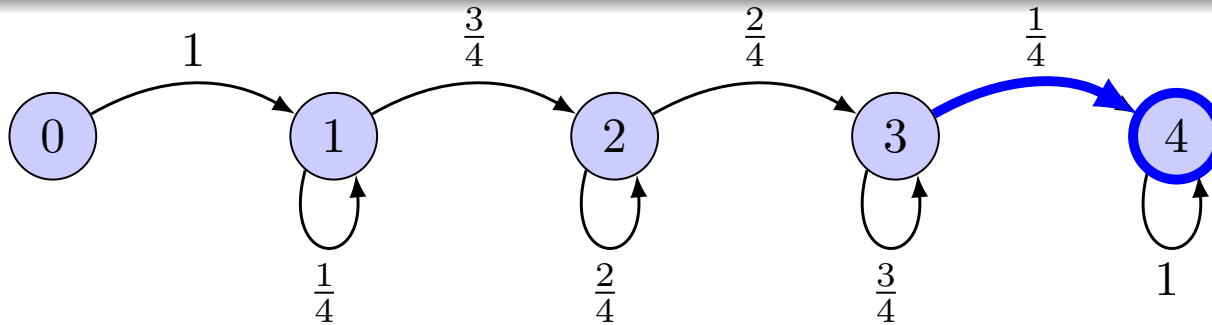
RESOURCE OUTLINE LABEL

- Problem statement
- Sample run
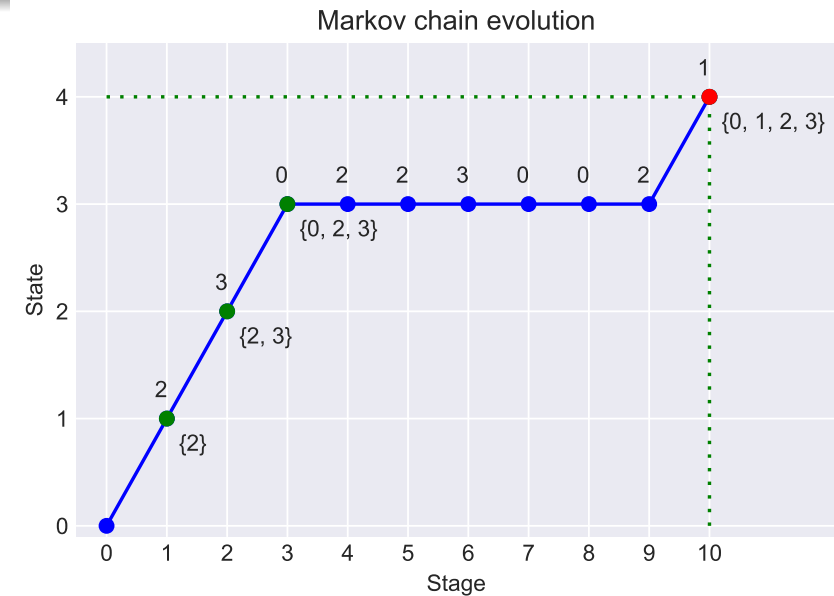
Markov chain approach

⟩Model⟩

- **State**: i, $i = 0, \ldots, n$, where i is number of collected coupons.
  Have i of the n available coupons, so $(n - i)$ coupons are unseen.
- Initial state is 0. State n is terminal.
- **Stage**: Number of coupons purchased.
  How long does it take to travel from 0 to n?

Start with 0 coupons. So next coupon must be unseen and probability of moving to next state is 1.

Now have 1 of the n coupons and $n - 1$ are unseen. So probability of getting a unseen coupon (and moving to next state) is $\frac{n-1}{n}$.

Now have 2 of the n coupons and $n - 2$ are unseen. So probability of getting a unseen coupon (and moving to next state) is $\frac{n-2}{n}$.

Now have i of the n coupons and $n - i$ are unseen. So probability of getting a unseen coupon (and moving to next state) is $\frac{n-1}{n}$.

Now have n of the n coupons, so collection is complete.

$$0 \xrightarrow{1} 1 \xrightarrow{\frac{n-1}{n}} 2 \xrightarrow{\frac{n-2}{n}} \cdots \xrightarrow{\frac{n-i-1}{n}} i \xrightarrow{\frac{n-i}{n}} \cdots \xrightarrow{\frac{1}{n}} n$$

Self-loops: state 1: $\frac{1}{n}$, state 2: $\frac{2}{n}$, state i: $\frac{i}{n}$, state n: 1.

Viewing our first simulation run as a Markov chain . . .


```
count:    0      found:       collected: set()
count:    1      found: {2}   collected: {2}
count:    2      found: {3}   collected: {2, 3}
count:    3      found: {0}   collected: {0, 2, 3}
count:    4      found: {2}   collected: {0, 2, 3}
count:    5      found: {2}   collected: {0, 2, 3}
count:    6      found: {3}   collected: {0, 2, 3}
count:    7      found: {0}   collected: {0, 2, 3}
count:    8      found: {0}   collected: {0, 2, 3}
count:    9      found: {2}   collected: {0, 2, 3}
count:   10      found: {1}   collected: {0, 1, 2, 3}
```

Number of stages needed = 10