Message Passing

① VAPE coupling

$$\pi_i^{(k)} - \hat{\pi}_i^{(k)}$$

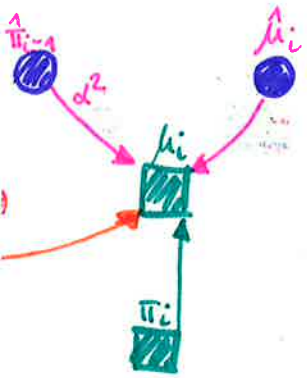
UPDATE

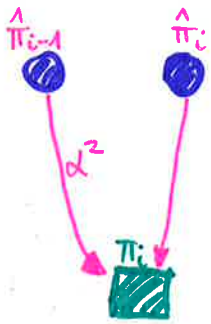
mean

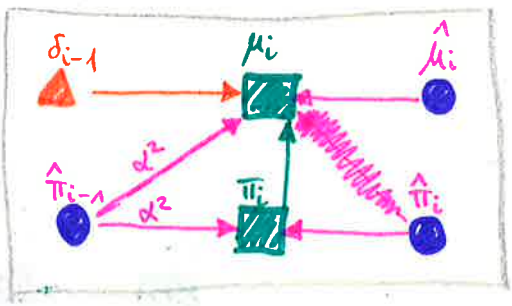
$$\mu_i^{(k)} = \hat{\mu}_i^{(k)} + \frac{\alpha_{i-1,i}^2 \, \hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \delta_{i-1}^{(k)}$$

precision

$$\pi_i^{(k)} = \hat{\pi}_i^{(k)} + \alpha_{i-1,i}^2 \, \hat{\pi}_{i-1}^{(k)}$$



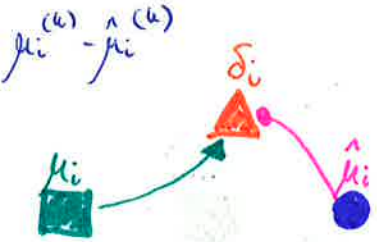




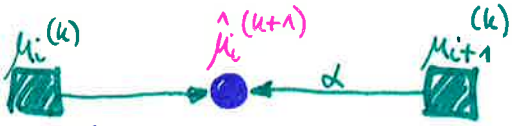
both updates

PE

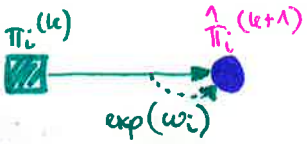
$$\mu_i^{(k)} - \hat{\mu}_i^{(k)}$$



③ PRED



$$\hat{\mu}_i^{(k+1)} = \mu_i^{(k)} + \alpha_{i,i+1} \, \mu_{i+1}^{(k)}$$



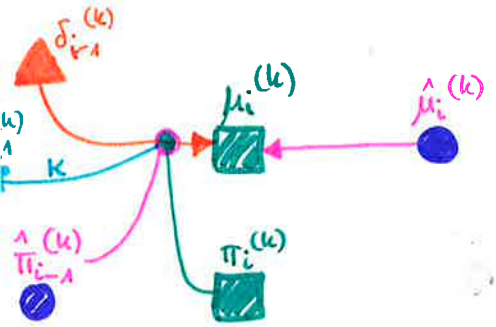
$$\hat{\pi}_i^{(k+1)} = \frac{1}{1/\pi_i^{(k)} + v_i^{(k+1)}} = \frac{1}{1/\pi_i^{(k)} + \exp(\omega_i)}$$

(2) VOPE coupling

UPDATE

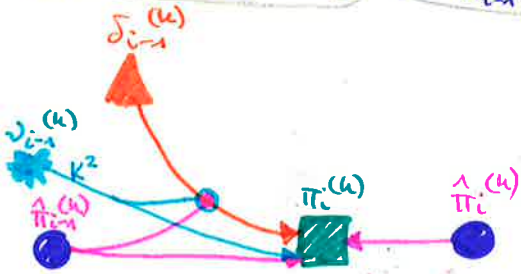
mean

$$\mu_i^{(k)} = \hat{\mu}_i^{(k)} + \frac{1}{2} K_{i-1} v_{i-1}^{(k)} \frac{\hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \delta_{i-1}^{(k)}$$



precision

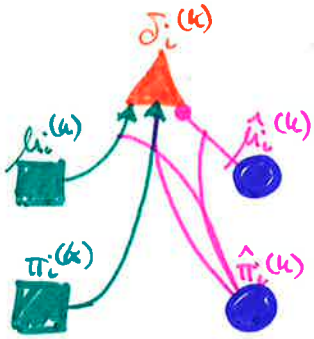
$$\pi_i^{(k)} = \hat{\pi}_i^{(k)} + \frac{1}{2}\left(K_{i-1} v_{i-1}^{(k)} \hat{\pi}_{i-1}^{(k)}\right)^2 \cdot \left[1 + \left(2 - \frac{1}{\hat{\pi}_{i-1}^{(k)} v_{i-1}^{(k)}}\right)\delta_{i-1}^{(k)}\right]$$



PE

$$\mu_i^{(k)} = \hat{\pi}_i^{(k)}\left(\frac{1}{\pi_i^{(k)}} + \left(\mu_i^{(k)} - \hat{\mu}_i^{(k)}\right)^2\right) - 1$$

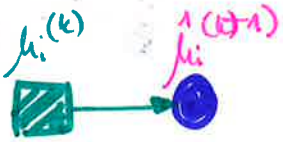
$$= \hat{\pi}_i^{(k)} + \frac{1}{2}\left(K_{i-1} v_{i-1}^{(k)} \hat{\pi}_{i-1}^{(k)}\right)^2 + \left(K_{i-1} v_{i-1}^{(k)} \hat{\pi}_{i-1}^{(k)}\right)^2 \delta_{i-1}^{(k)} - \frac{1}{2} K_{i-1}^2 v_{i-1}^{(k)} \hat{\pi}_{i-1}^{(k)} \delta_{i-1}^{(k)}$$



(III) PRED

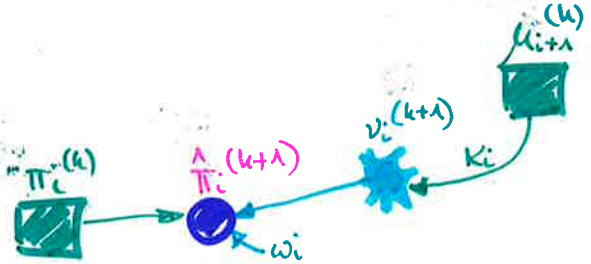
mean

$$\hat{\mu}_i^{(k+1)} = \mu_i^{(k)}$$



precision

$$\hat{\pi}_i^{(k+1)} = \frac{1}{\frac{1}{\pi_i^{(k)}} + \exp\left(K_i \mu_{i+1}^{(k)} + \omega_i\right)}$$



→ Rewrite VOPE coupling, s.th. $\delta_i^{(k)} := \mu_i^{(k)} - \hat{\mu}_i^{(k)}$

→ $\delta_i^{\text{VOPE}(k)} = \hat{\pi}_i^{(k)} \left(\frac{1}{\pi_i^{(k)}} + \left(\delta_i^{\text{VAPE}(k)} \right)^2 \right) - 1$

① UPDATE

mean

$$\mu_i^{(k)} = \hat{\mu}_i^{(k)} + \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \frac{\hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \delta_{i-1}^{\text{VOPE}(k)}$$

$$= \hat{\mu}_i^{(k)} + \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \frac{\hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \left[\hat{\pi}_{i-1}^{(k)} \left(\frac{1}{\pi_{i-1}^{(k)}} + \left(\delta_{i-1}^{\text{VAPE}(k)} \right)^2 \right) - 1 \right]$$

$$= \hat{\mu}_i^{(k)} + \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \frac{\left(\hat{\pi}_{i-1}^{(k)} \right)^2}{\pi_i^{(k)}} \left[\frac{1}{\pi_{i-1}^{(k)}} + \left(\delta_{i-1}^{\text{VAPE}(k)} \right)^2 \right]$$

$$- \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \frac{\hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}}$$

as $f(\hat{\pi}_{i-1}^{(k)})$ $\pi_{i-1}^{(k)} = \frac{1}{\frac{1}{\hat{\pi}_{i-1}^{(k+1)}} - \nu_{i-1}^{(k+1)}}$

$$= \hat{\mu}_i^{(k)} + \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \frac{\left(\hat{\pi}_{i-1}^{(k)} \right)^2}{\pi_i^{(k)} \pi_{i-1}^{(k)}} + \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \frac{\left(\hat{\pi}_{i-1}^{(k)} \right)^2}{\pi_i^{(k)}} \left(\delta_{i-1}^{\text{VAPE}(k)} \right)^2$$

$$- \frac{1}{2} K_i \nu_{i-1}^{(k)} \frac{\hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}}$$

$$= \hat{\mu}_i^{(k)} + \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \left[\frac{\left(\hat{\pi}_{i-1}^{(k)} \right)^2}{\pi_i^{(k)} \pi_{i-1}^{(k)}} + \frac{\left(\pi_{i-1}^{(k)} \right)^2}{\pi_i^{(k)}} \left(\delta_{i-1}^{\text{VAPE}(k)} \right)^2 - \frac{\hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \right]$$

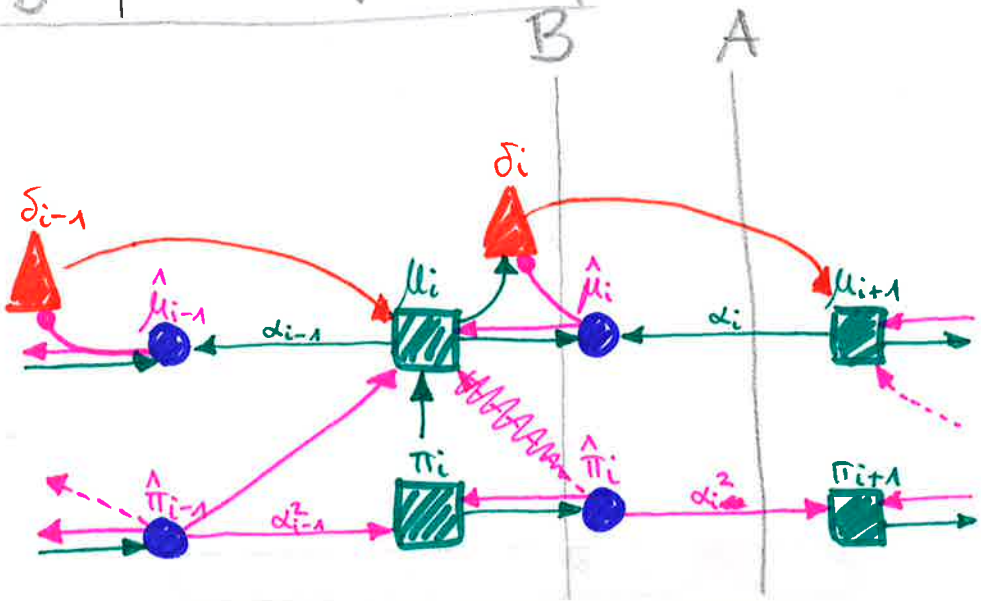
$$= \hat{\mu}_i^{(k)} + \frac{1}{2} K_{i-1} \nu_{i-1}^{(k)} \frac{\hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \left[\frac{\hat{\pi}_{i-1}^{(k)}}{\pi_{i-1}^{(k)}} + \hat{\pi}_{i-1}^{(k)} \left(\delta_{i-1}^{\text{VAPE}(k)} \right)^2 - 1 \right]$$

→ no advantage.

VAfE coupling – full microcircuit

A – HGF so far

B – predictions of i on i+1



roblem with (A):

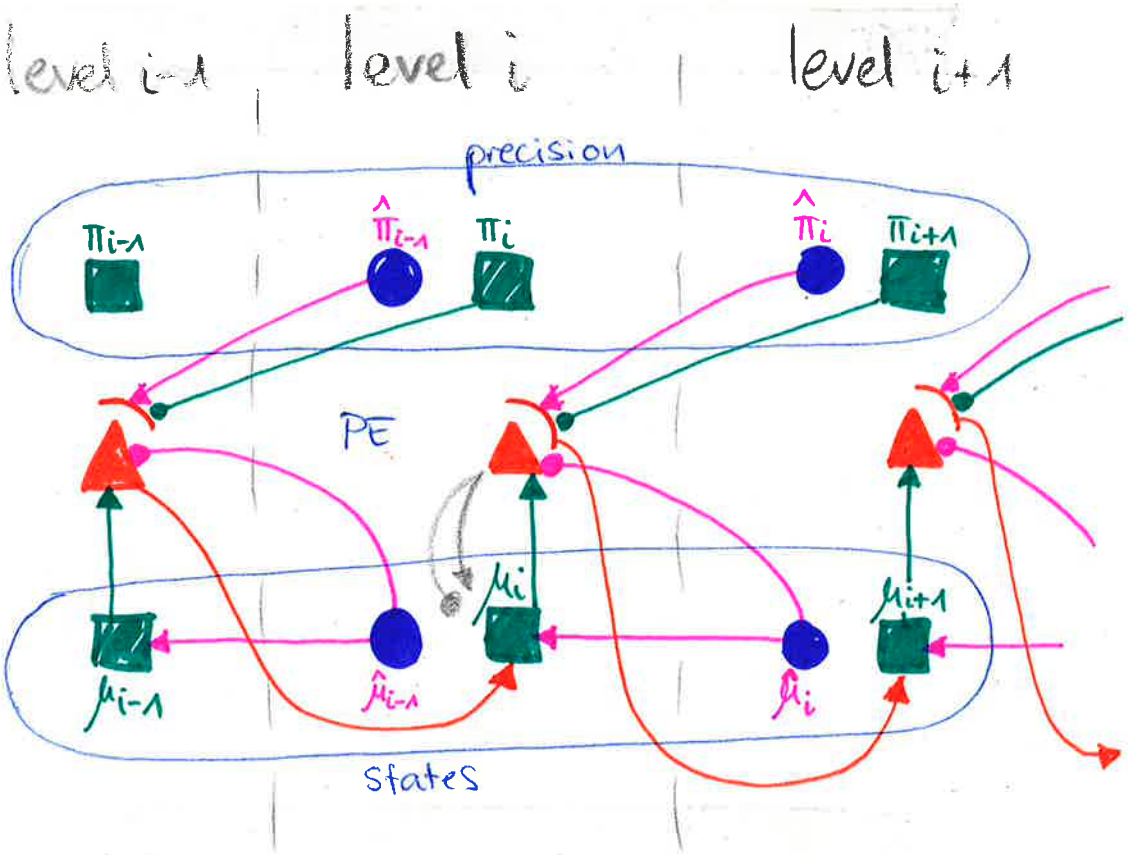
in addition to δ_i, predicted precision $\hat{\pi}_i$ needs to be signalled bottom-up (level i to i+1)

roblem with (B):

in addition to δ_i, posteriors μ_i and π_i need to be signalled bottom-up (level i to i+1)

VAPE coupling – update and prediction error

(no prediction) for the mean



- not showing calculation of predictions $\hat{\mu}_i$
- not showing update of predictions of π
- main difference to Shipp [redacted] 2016, figure (1):

> update of μ_i is driven by δ_{i-1} and $\hat{\mu}_i$
>
> in Shipp: —"— by δ_{i-1} and δ_i

$$\delta_i = \mu_i^{(k)} - \hat{\mu}_i^{(k)} \qquad \mu_i^{(k)} = \hat{\mu}_i^{(k)} + \frac{\alpha^2 \hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \delta_{i-1}^{(k)}$$

$$\rightarrow \delta_i = \frac{\alpha^2 \hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \delta_{i-1}^{(k)} \rightarrow \mu_i^{(k)} = \mu_i^{(k)} - \delta_i^{(k)} + \frac{\alpha^2 \hat{\pi}_{i-1}^{(k)}}{\pi_i^{(k)}} \delta_{i-1}^{(k)}$$