اگر U و V یک ماتریس متعامد(orthogonal matrix) n×n باشند آنگاه:

الف) نشان دهید سطرهای ماتریس U تشکیل پایه متعامد برای R^n می‌دهد.

ب) توضیح دهید که چرا ماتریس UV نیز یک ماتریس متعامد می‌شود.

پاسخ الف)

If U is an $n \times n$ orthogonal matrix, then $I = UU^{-1} = UU^T$. Since U is the transpose of U^T, Theorem 6 applied to U^T says that U^T has orthogonal columns. In particular, the columns of U^T are linearly independent and hence form a basis for $\mathbb{R}^n$ by the Invertible Matrix Theorem. That is, the rows of U form a basis (an orthonormal basis) for $\mathbb{R}^n$.

پاسخ ب)

Since U and V are orthogonal, each is invertible. By Theorem 6 in Section 2.2, UV is invertible and $(UV)^{-1} = V^{-1}U^{-1} = V^TU^T = (UV)^T$, where the final equality holds by Theorem 3 in Section 2.1. Thus UV is an orthogonal matrix.