فرض کنید که u یک بردار ناتهی در R^n باشد. اگر L = Span{u} باشد، نشان دهید که $x \mapsto proj_L x$ یک تبدیل خطی است.

پاسخ)

Let $L = \text{Span}\{\mathbf{u}\}$, where $\mathbf{u}$ is nonzero, and let $T(\mathbf{x}) = \dfrac{\mathbf{x} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u}$. For any vectors $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^n$ and any scalars c and d, the properties of the inner product (Theorem 1) show that

$$\begin{aligned}
T(c\mathbf{x} + d\mathbf{y}) &= \frac{(c\mathbf{x} + d\mathbf{y}) \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} \\
&= \frac{c\mathbf{x} \cdot \mathbf{u} + d\mathbf{y} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} \\
&= \frac{c\mathbf{x} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} + \frac{d\mathbf{y} \cdot \mathbf{u}}{\mathbf{u} \cdot \mathbf{u}}\mathbf{u} \\
&= cT(\mathbf{x}) + dT(\mathbf{y})
\end{aligned}$$

Thus T is a linear transformation. Another approach is to view T as the composition of the following three linear mappings: $\mathbf{x} \mapsto a = \mathbf{x} \cdot \mathbf{v}$, $a \mapsto b = a / \mathbf{v} \cdot \mathbf{v}$, and $b \mapsto b\mathbf{v}$.