

طراحی الگوریتم ها (CE221)

جلسه نوزدهم: شاخه و حد

سجاد شیرعلی شمرضا

بهار، 1401

شنبه، 7 خرداد 1401

اطلاع رسانی

- نظرسنجی چهارم
 - مهلت ارسال: دوشنبه هفته آینده، 16 خرداد 1401، ساعت 8 صبح
- کلاس جبرانی
 - فردا، یکشنبه، ساعت 16:30-18

روش شاخه و حد

Idea

- Sometimes it is not easy to design a DP or greedy solution
- Checking all possible answers is not feasible either
- Idea: stop exploring a path if we know it would not give us a better answer
 - Keep an estimate of how good the answer will be
 - Continue the path with the highest possible answer
 - Stop a path if it cannot provide a better solution

0/1 Knapsack with Branch and Bound

- n items
 - Each has a weight w_i and value v_i
- A knapsack (i.e., bag) with capacity W
- Goal: fill the bag such that
 - Total weight of items is less than or equal to W
 - $\sum w_i \leq W$
 - The value of selected items is maximised
 - $\sum v_i$ is maximised



Capacity: **16**

Item:
Weight:
Value:



2

40



5

30



10

50



5





10

Greedy Approach

- Sort items based on unit value
- Start with item with highest unit value
- Add item if it fits in the bag
- Final answer is not necessarily the best one
 - Will select avocado, magnet, and quala with value 80
 - Best answer: select avocado and grape with value 90



Capacity: **16**





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

Branch and Bound Algorithm

- Start with an empty answer as partial answer
- Select the partial answer with highest bound
- Expand the partial answer by deciding whether to pick the next item
 - If selecting the item, update weight and value
 - Estimate the bound by filling the remaining capacity with highest remaining items in terms of unit value
- Continue while there is a partial answer that is not eliminated



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2



0/1 Knapsack with Branch and Bound

- Start with an empty answer

```
weight = 0  
value = 0  
bound = 320
```



Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2


0/1 Knapsack with Branch and Bound

- Expand node with bound 320

```
weight = 0  
value  = 0  
bound  = 320
```

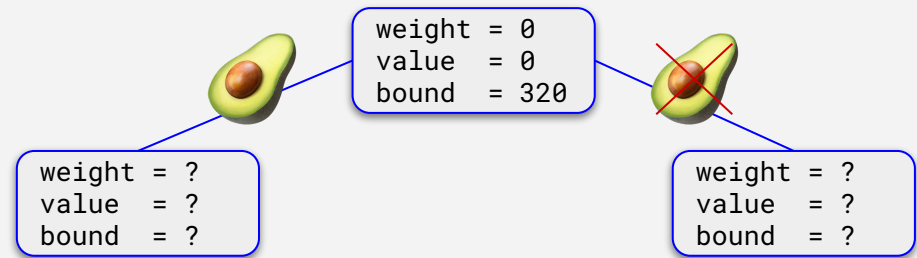


Capacity: **16**


Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 320

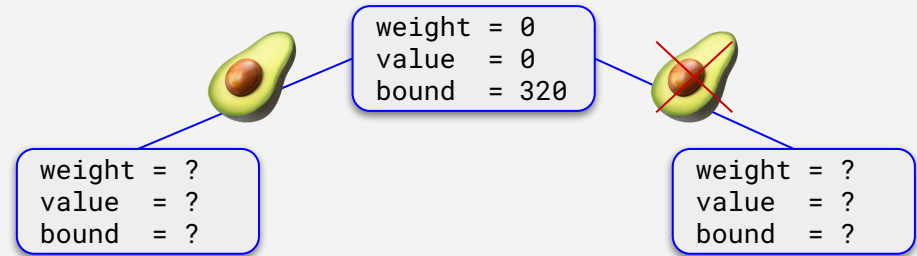


Capacity: **16**


Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Avocado

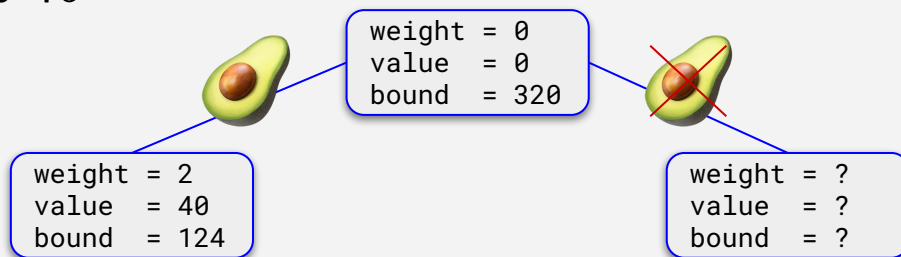


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Avocado
 - Found an answer with value 40

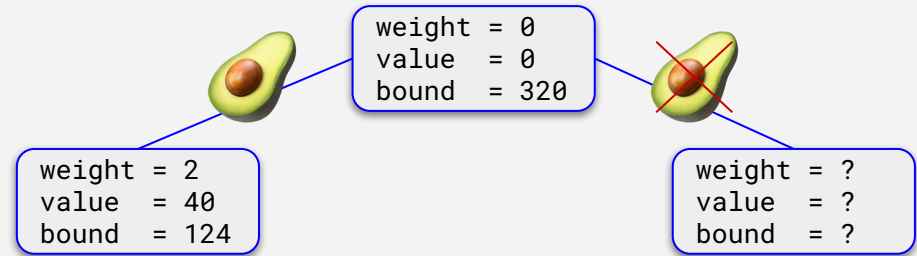


Capacity: **16**


Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Avocado

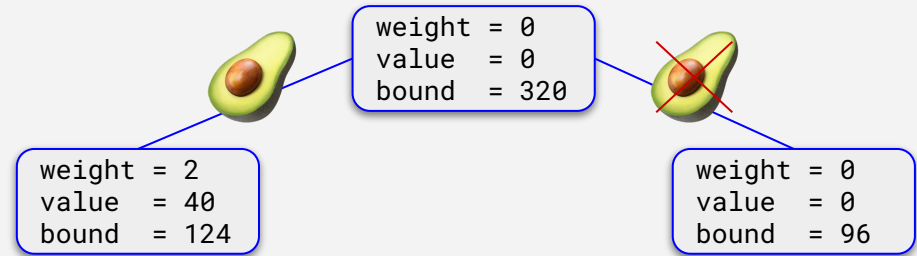


Capacity: **16**


Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Avocado

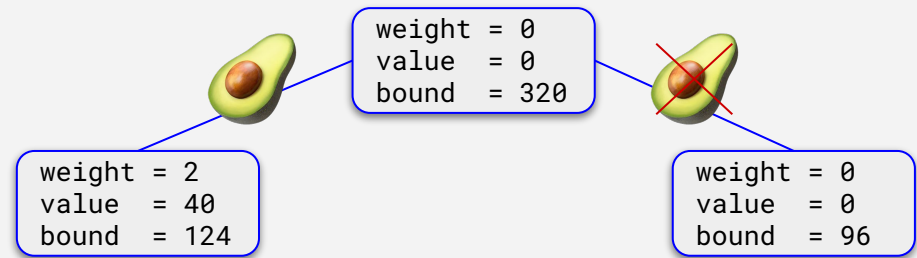


Capacity: **16**


Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 124

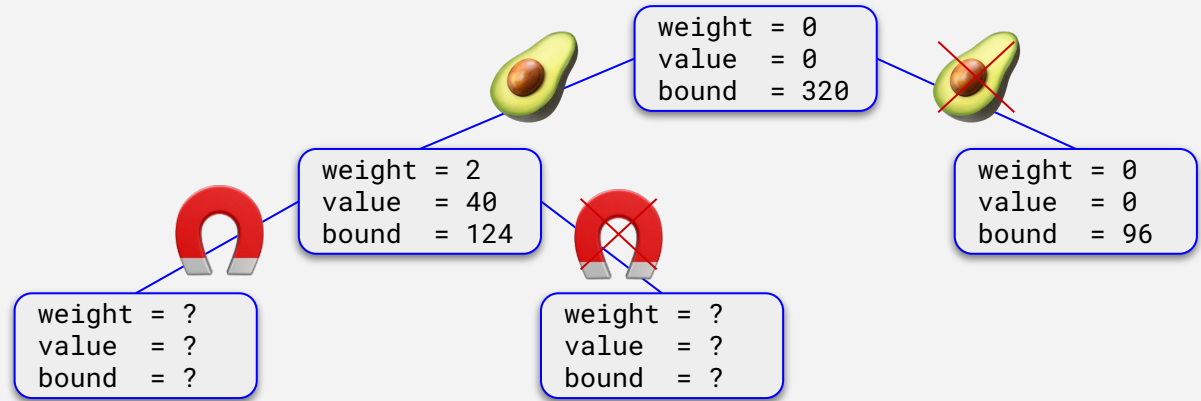


Capacity: **16**


Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 124

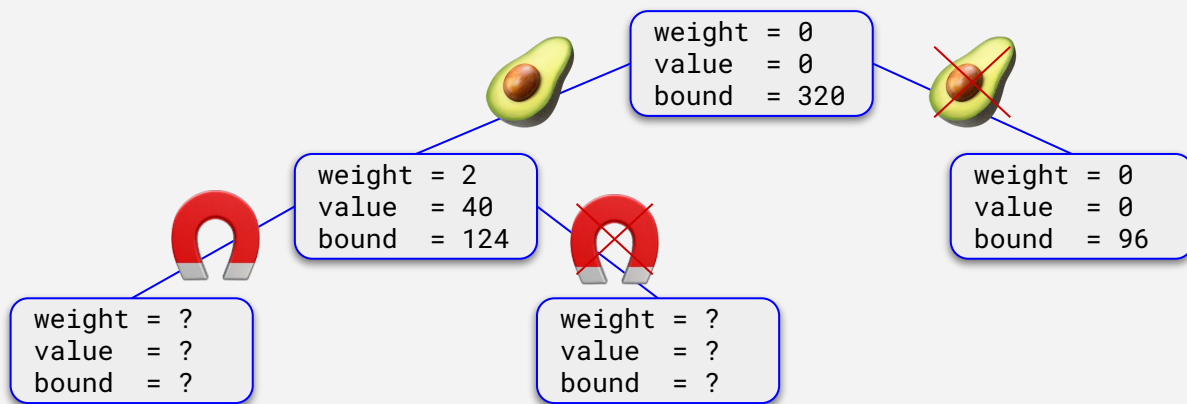


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Magnet

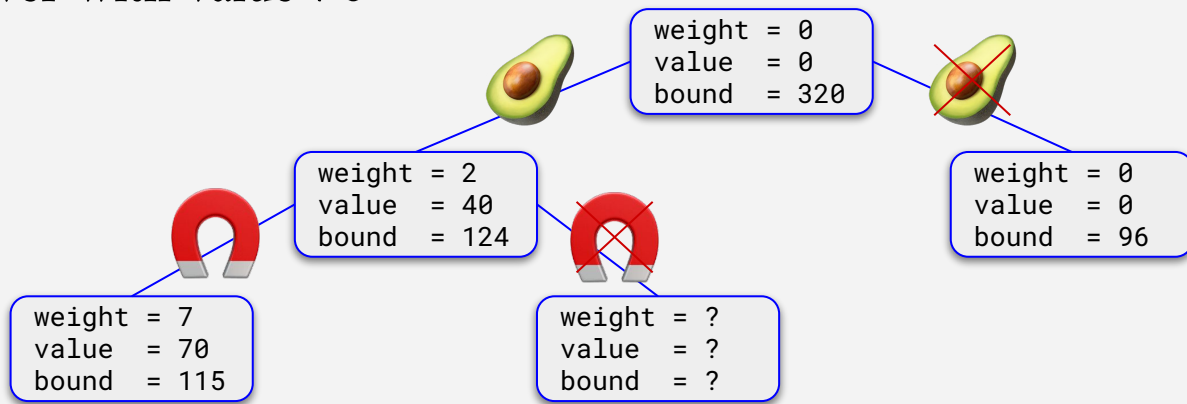


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Magnet
 - Found an answer with value 70

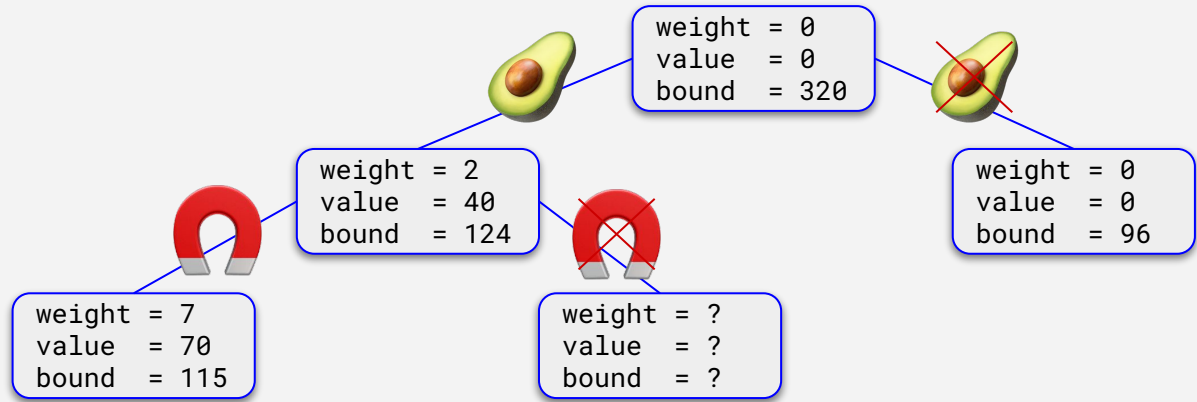


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Magnet

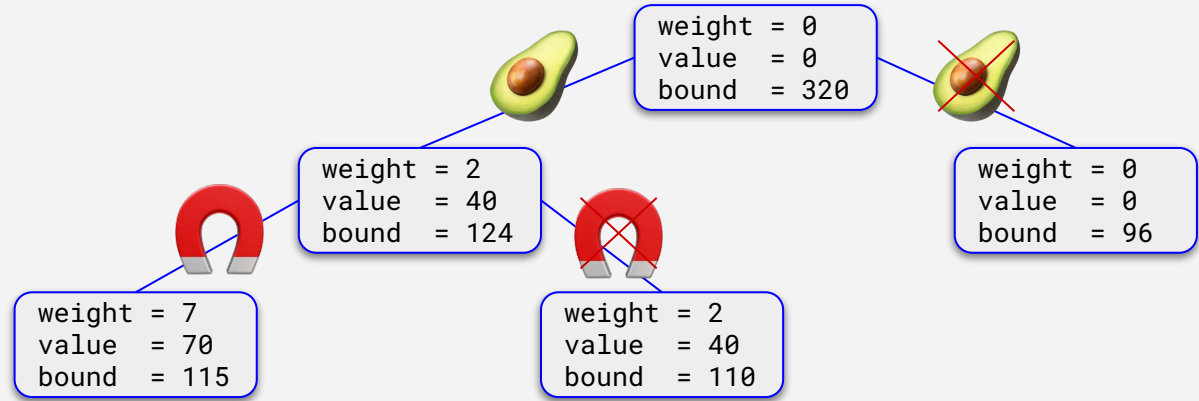


Capacity: 16



Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Magnet

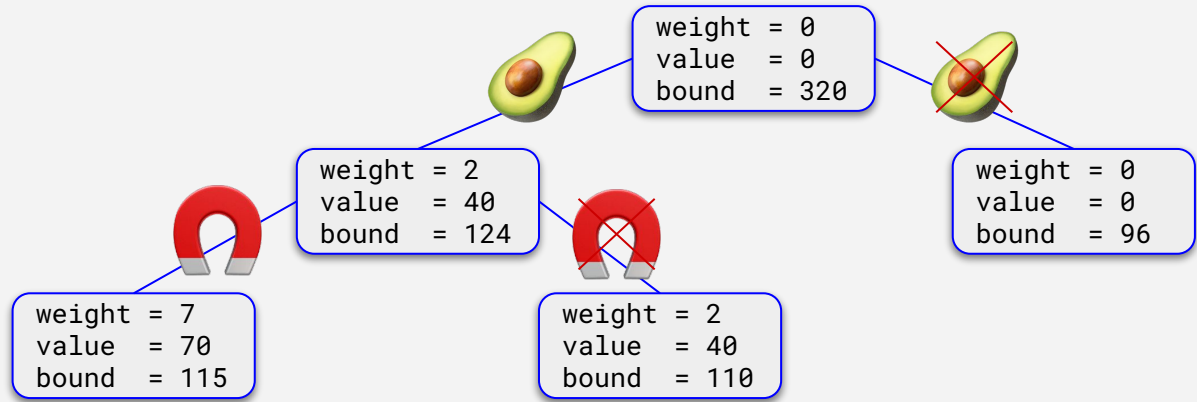


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 115

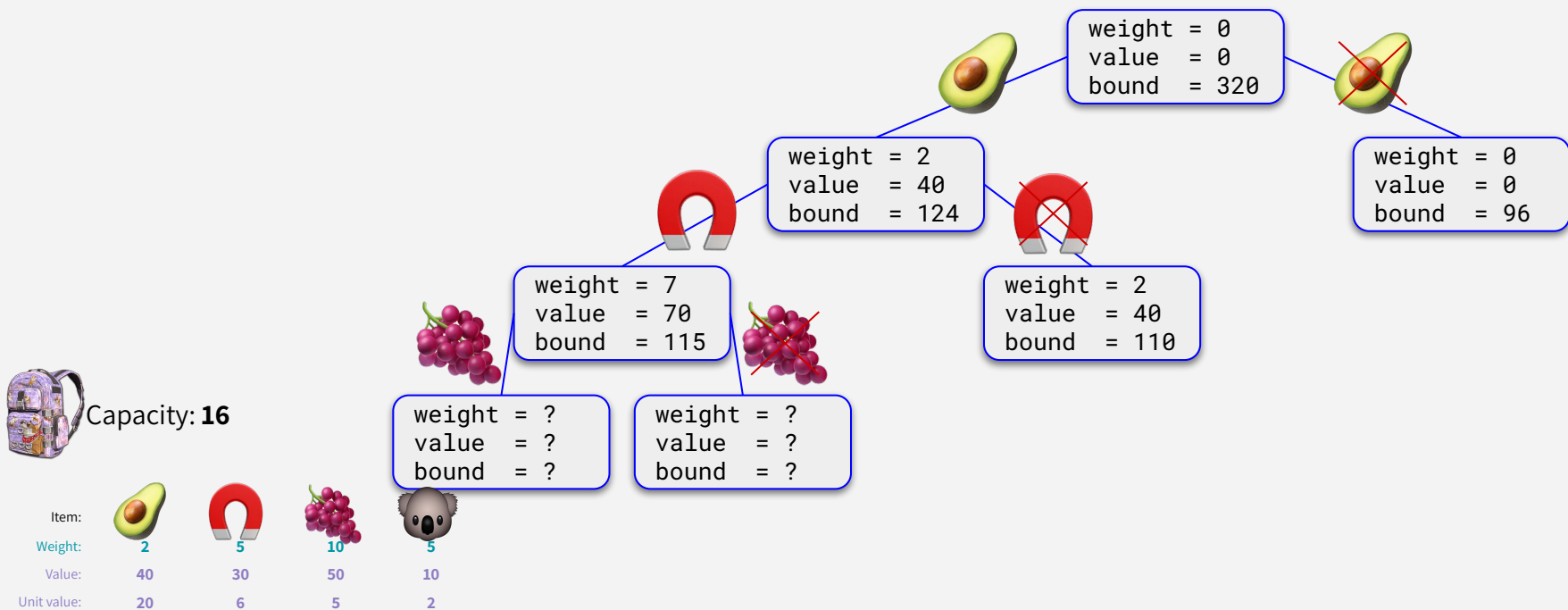


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

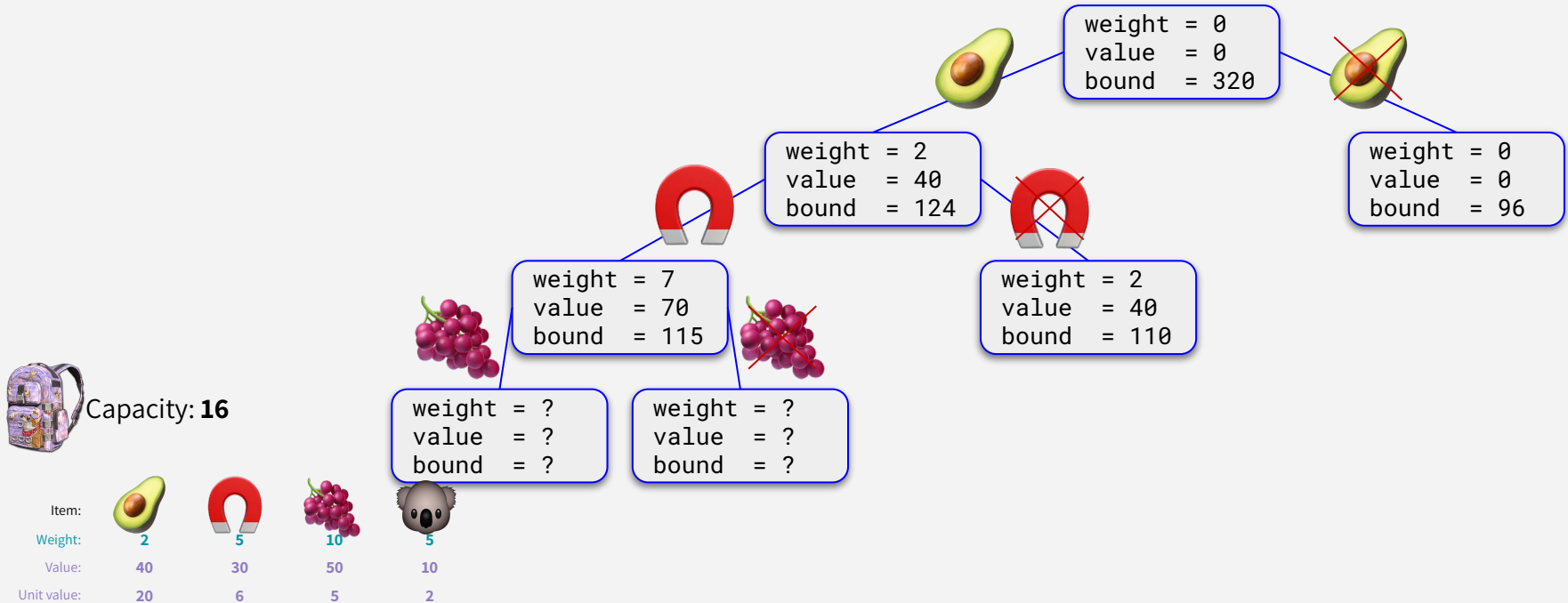
0/1 Knapsack with Branch and Bound

- Expand node with bound 115



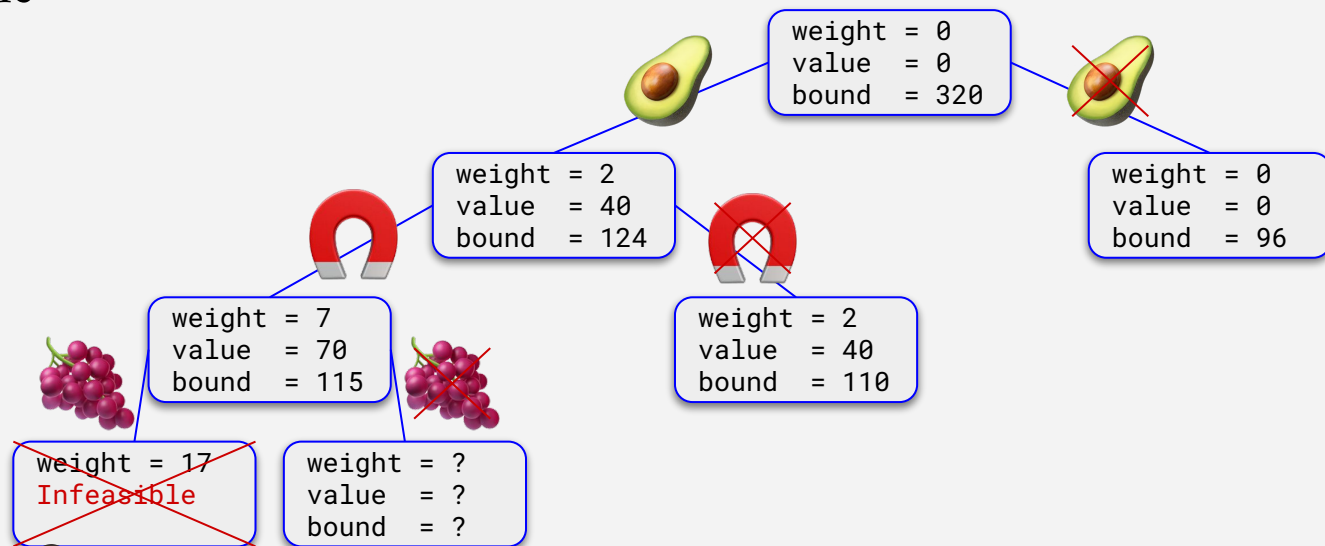
0/1 Knapsack with Branch and Bound

- Pick Grape



0/1 Knapsack with Branch and Bound

- Pick Grape
 - Infeasible

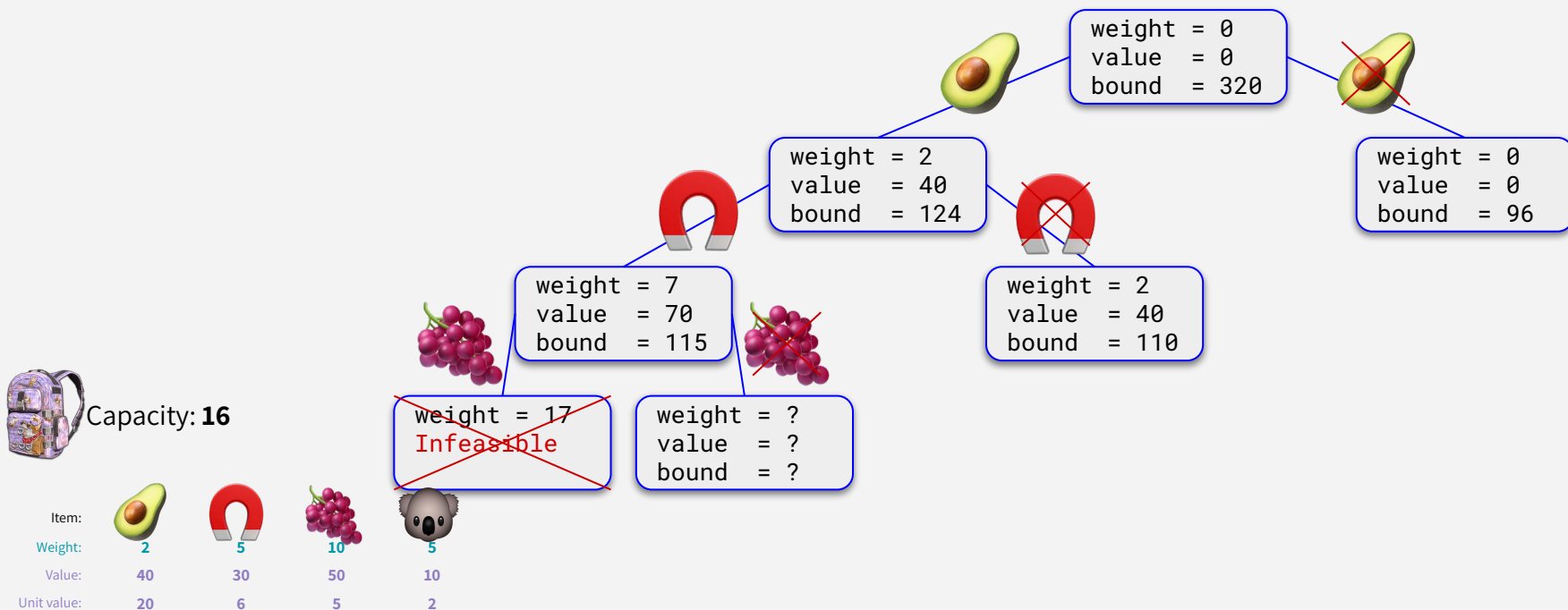


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

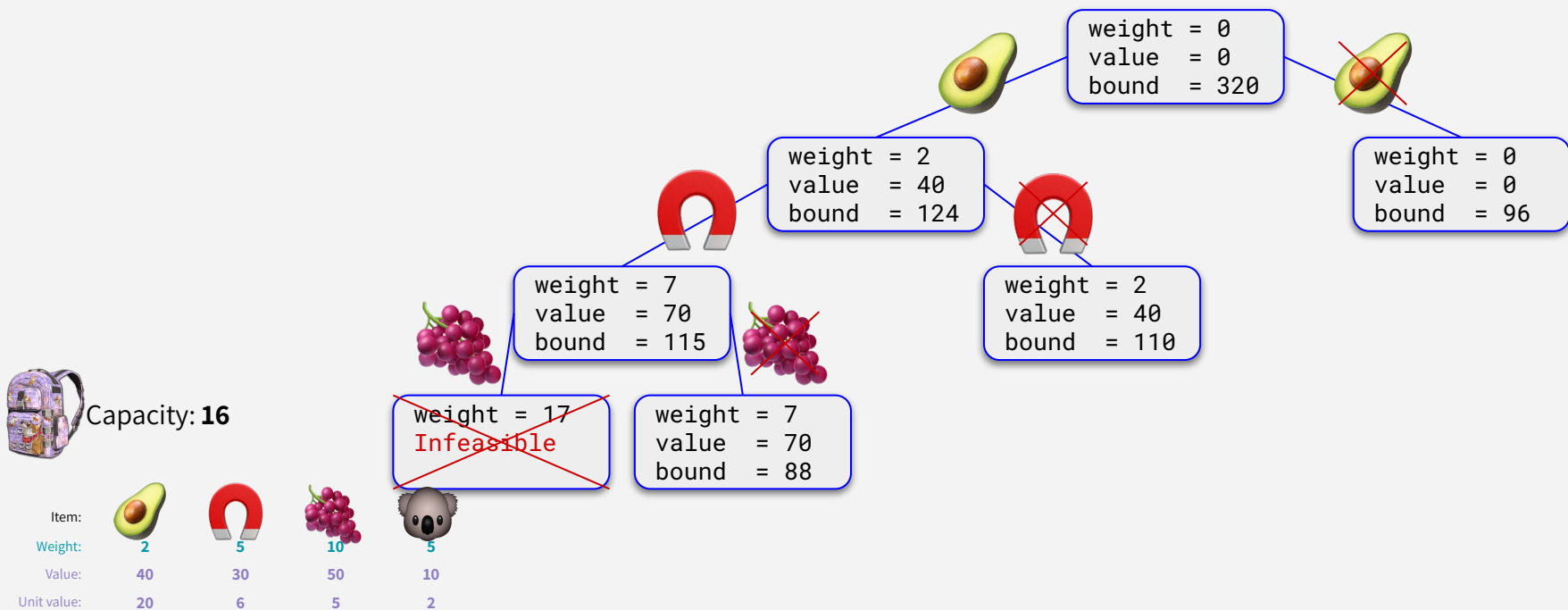
0/1 Knapsack with Branch and Bound

- Do not pick Grape



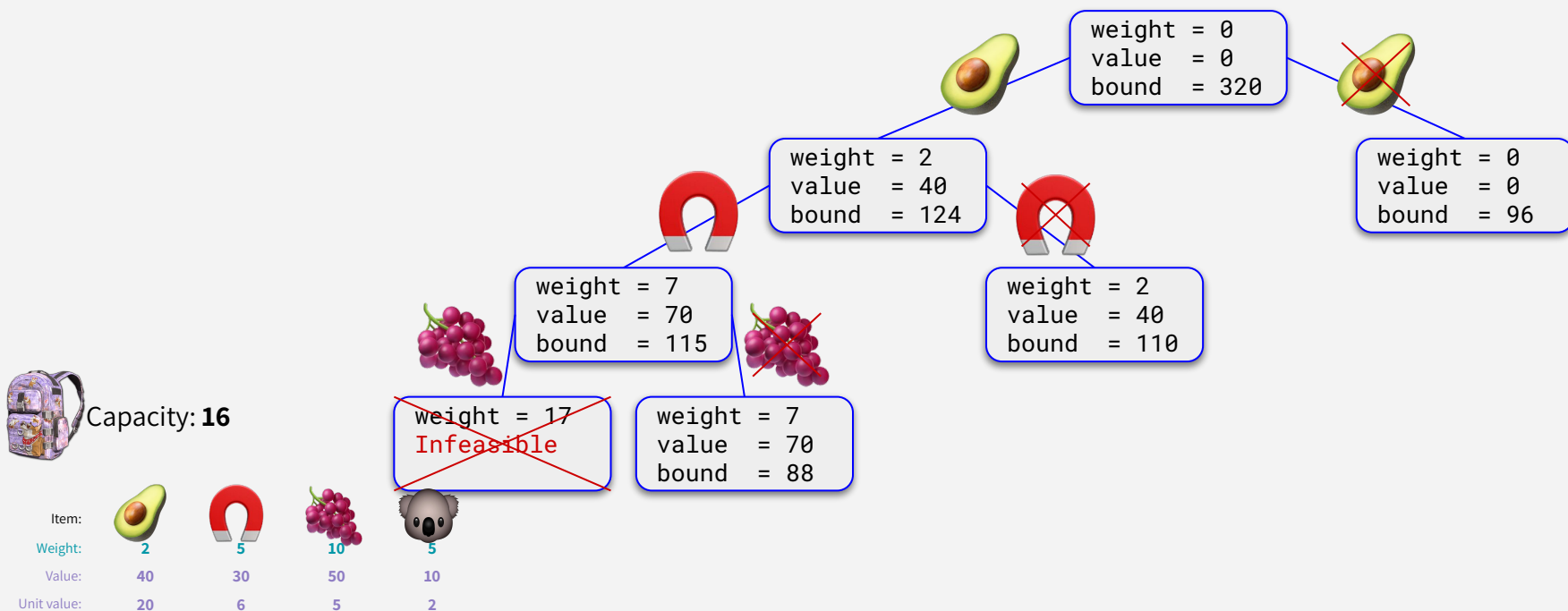
0/1 Knapsack with Branch and Bound

- Do not pick Grape



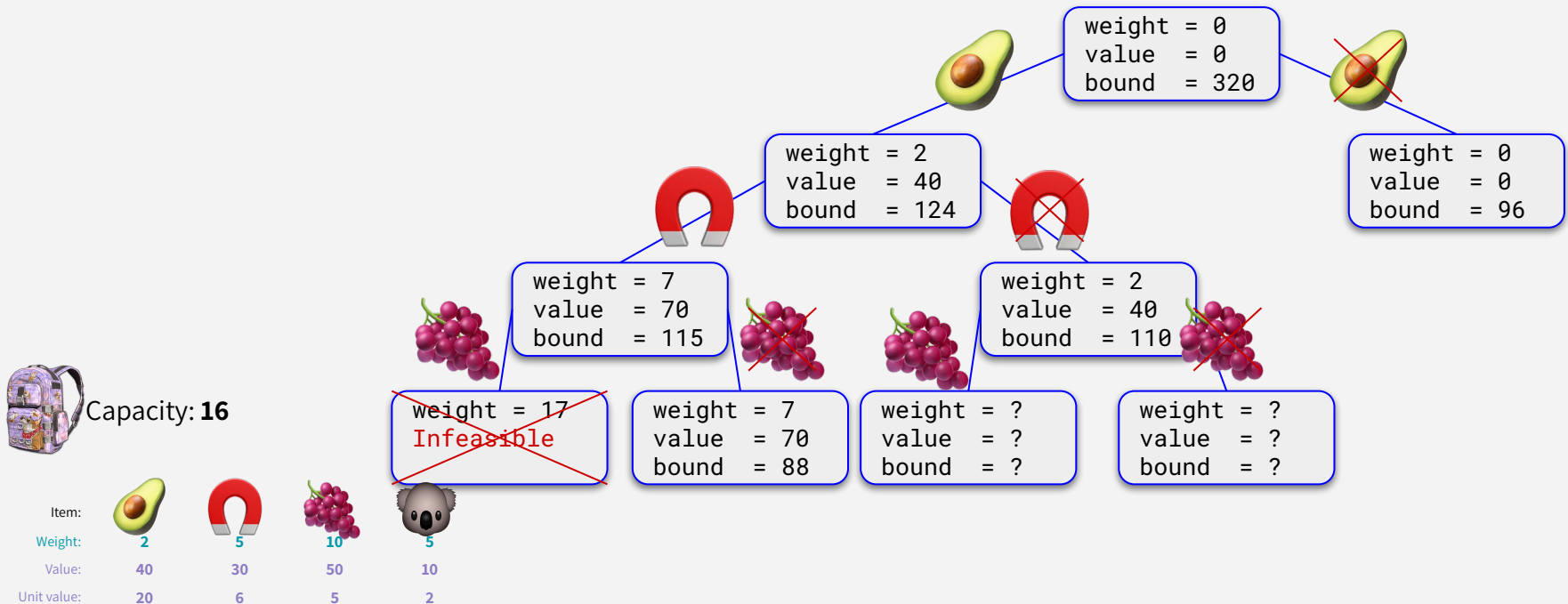
0/1 Knapsack with Branch and Bound

- Expand node with bound 110



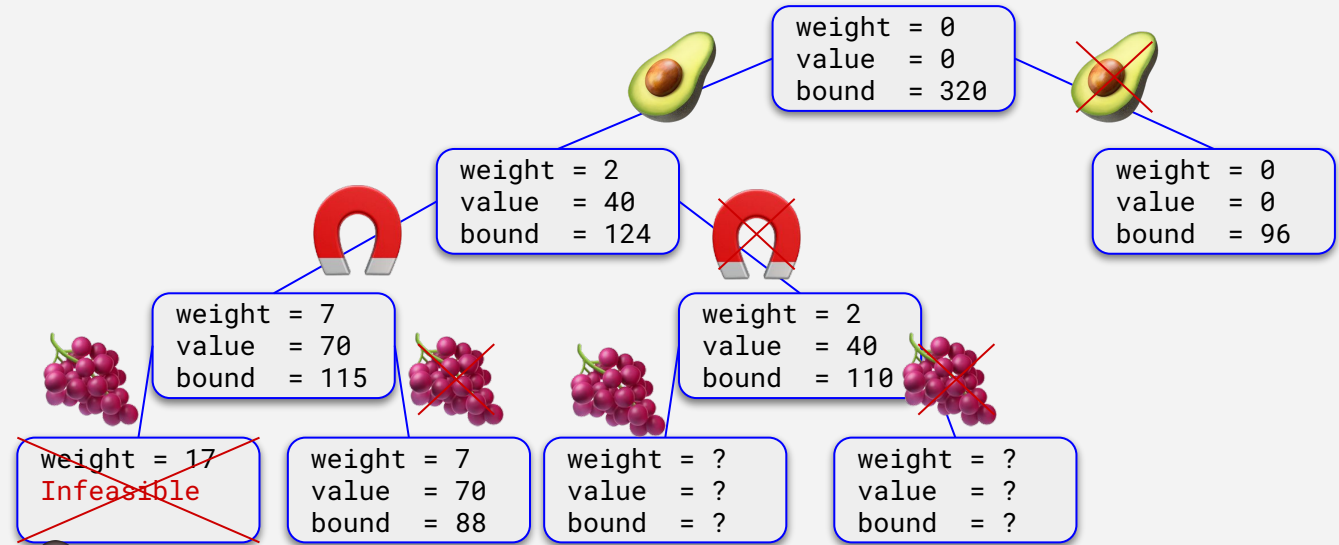
0/1 Knapsack with Branch and Bound

- Expand node with bound 110



0/1 Knapsack with Branch and Bound

- Pick Grape

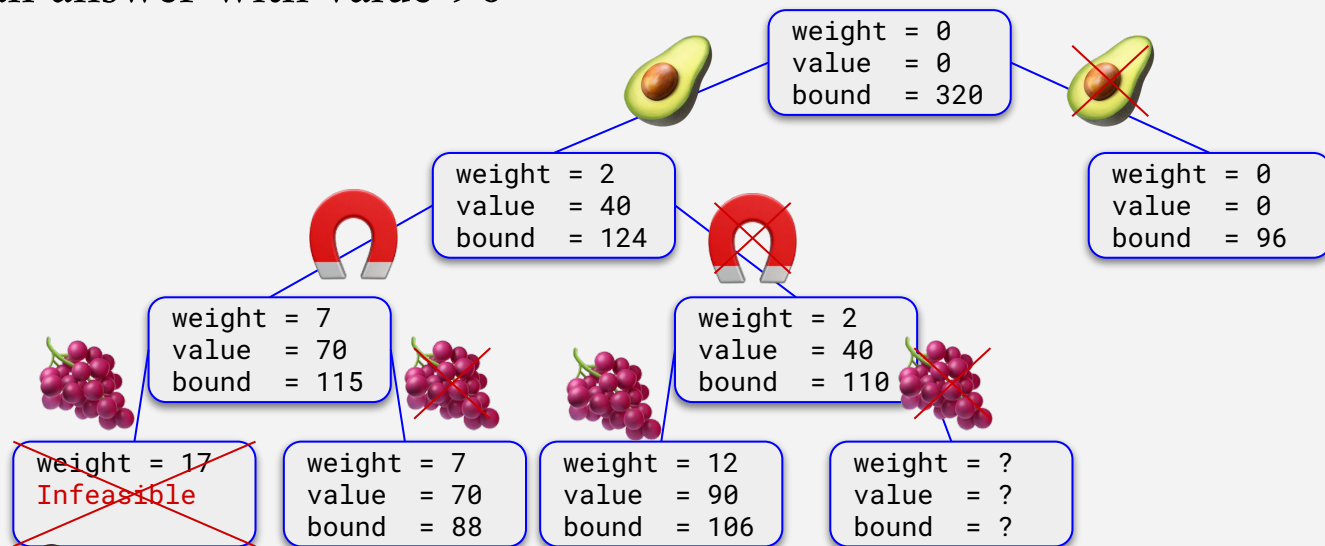


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape
 - Found an answer with value 90

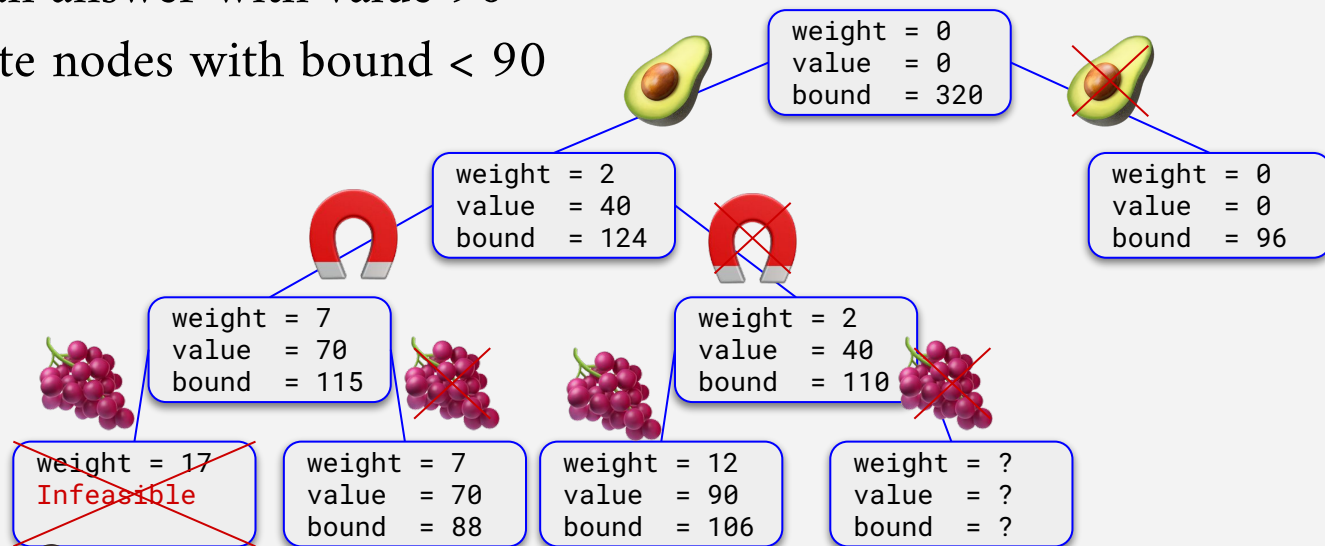


Capacity: **16**

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape
 - Found an answer with value 90
 - Eliminate nodes with bound < 90

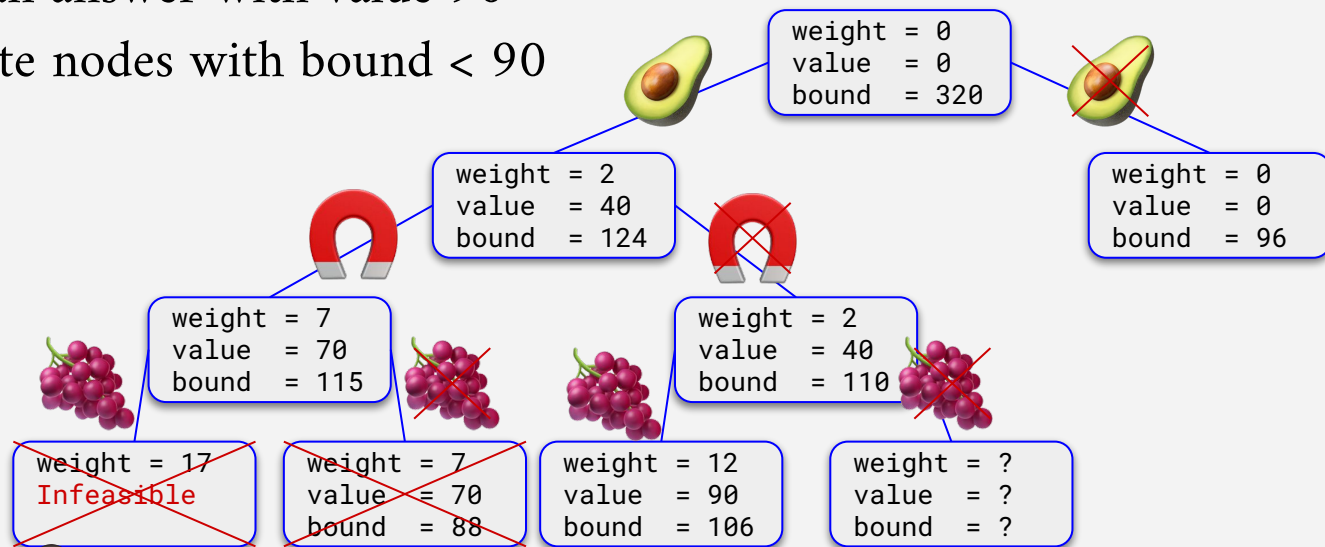


Capacity: 16





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Pick Grape
 - Found an answer with value 90
 - Eliminate nodes with bound < 90

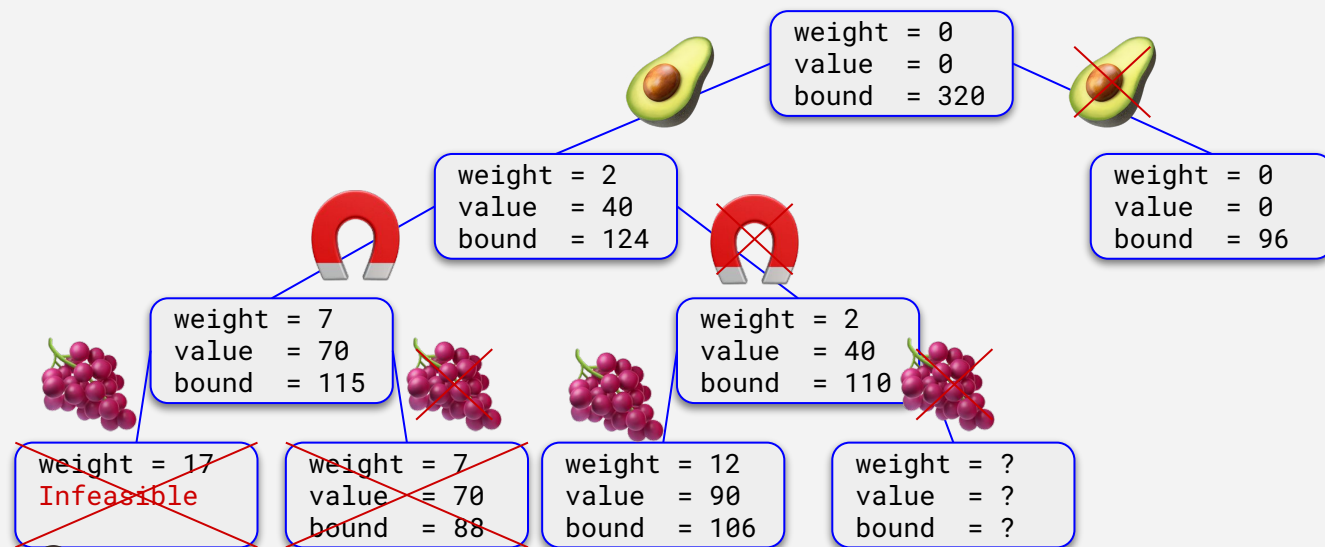


Capacity: 16



Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Grape

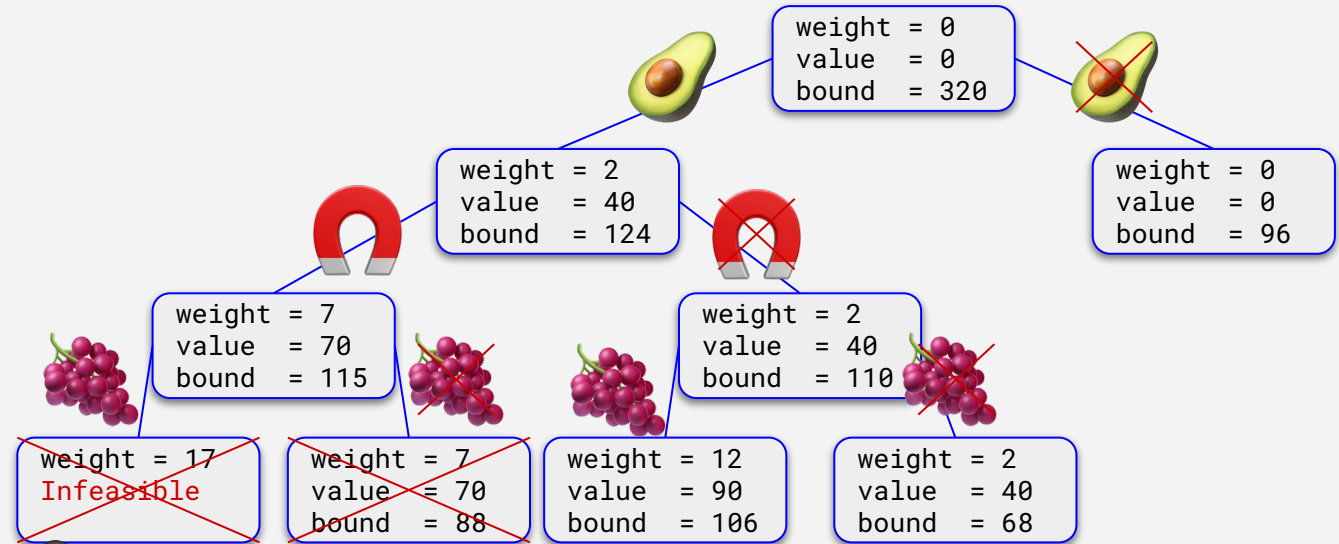


Capacity: 16





Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Grape

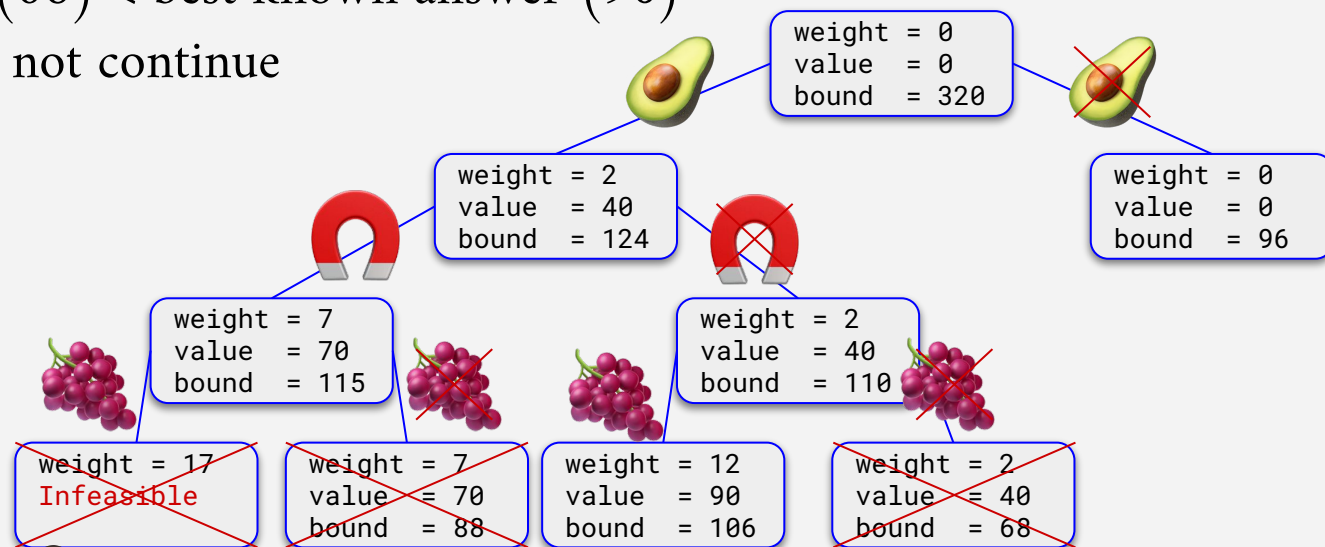


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Do not pick Grape
 - Bound (68) < best known answer (90)
 - Do not continue

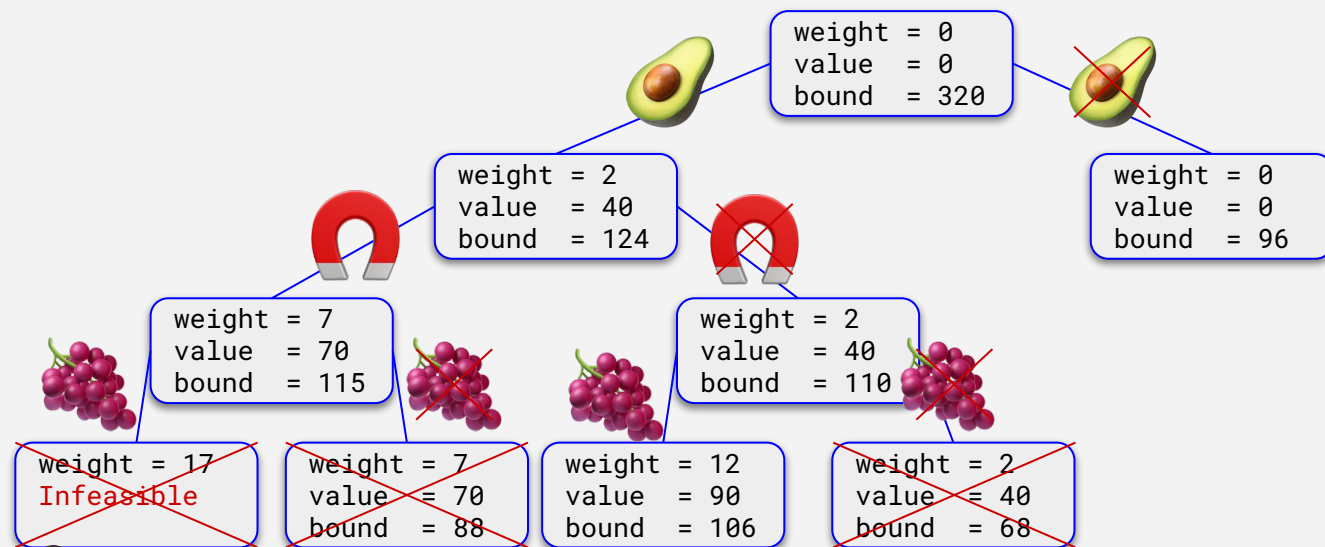


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand the node with bound 106

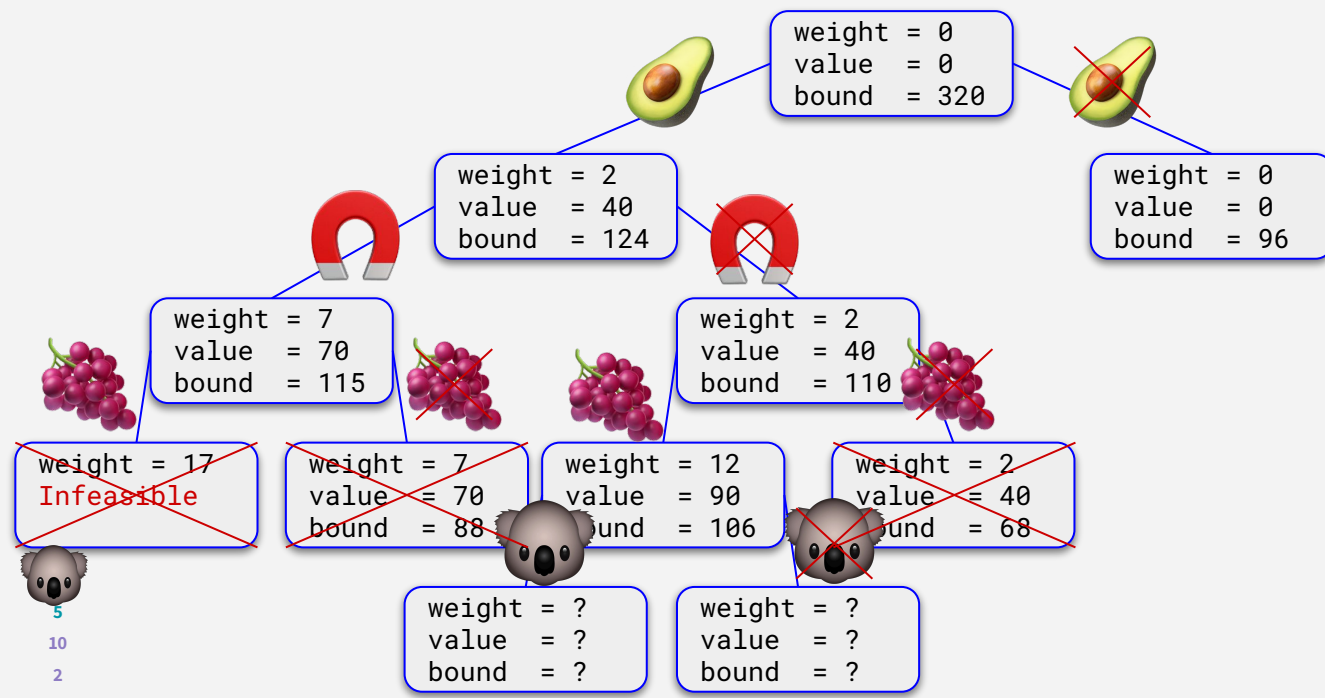


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand the node with bound 106

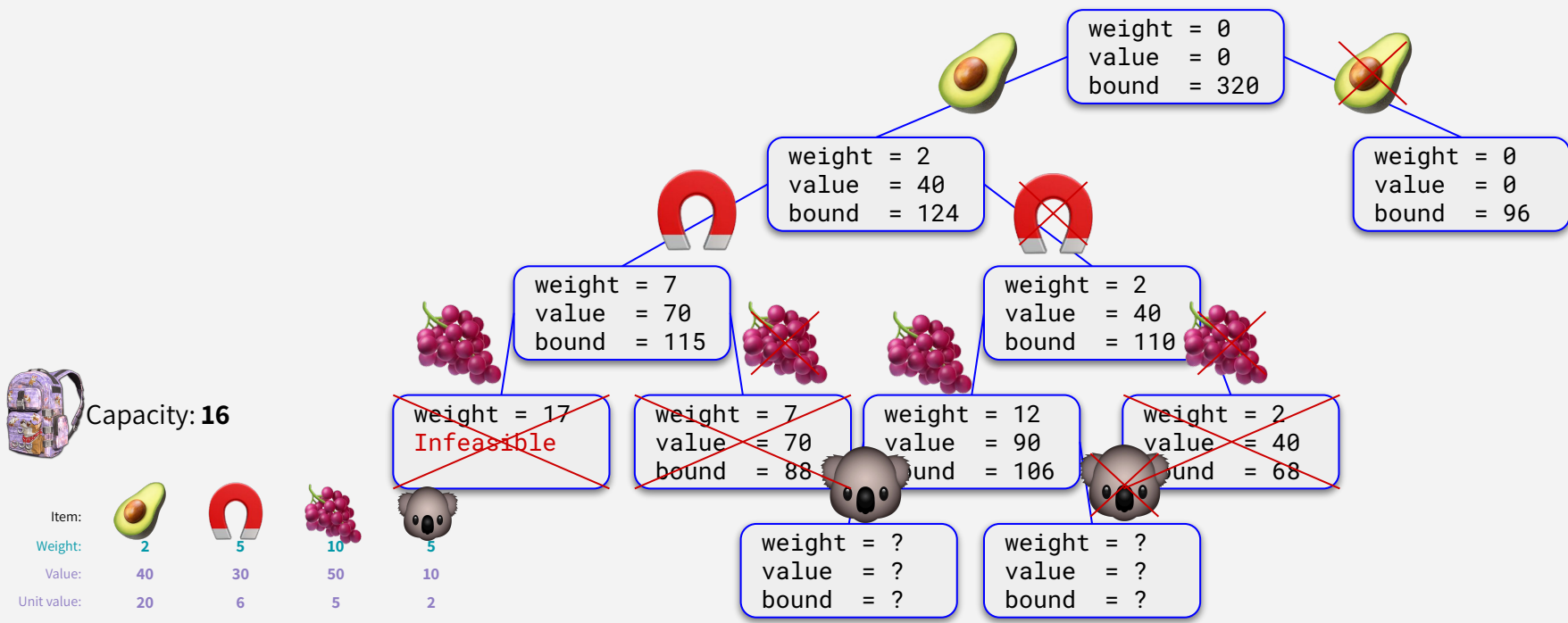


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

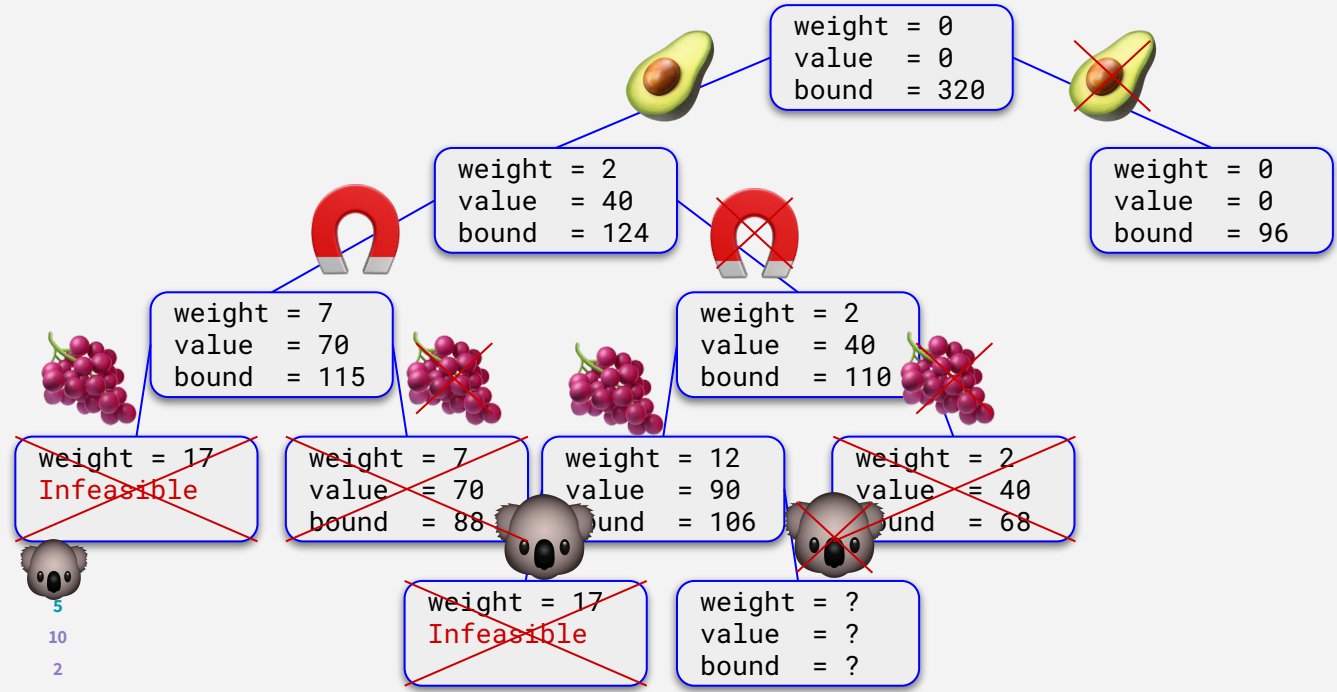
0/1 Knapsack with Branch and Bound

- Pick Quala



0/1 Knapsack with Branch and Bound

- Pick Quala
 - Infeasible

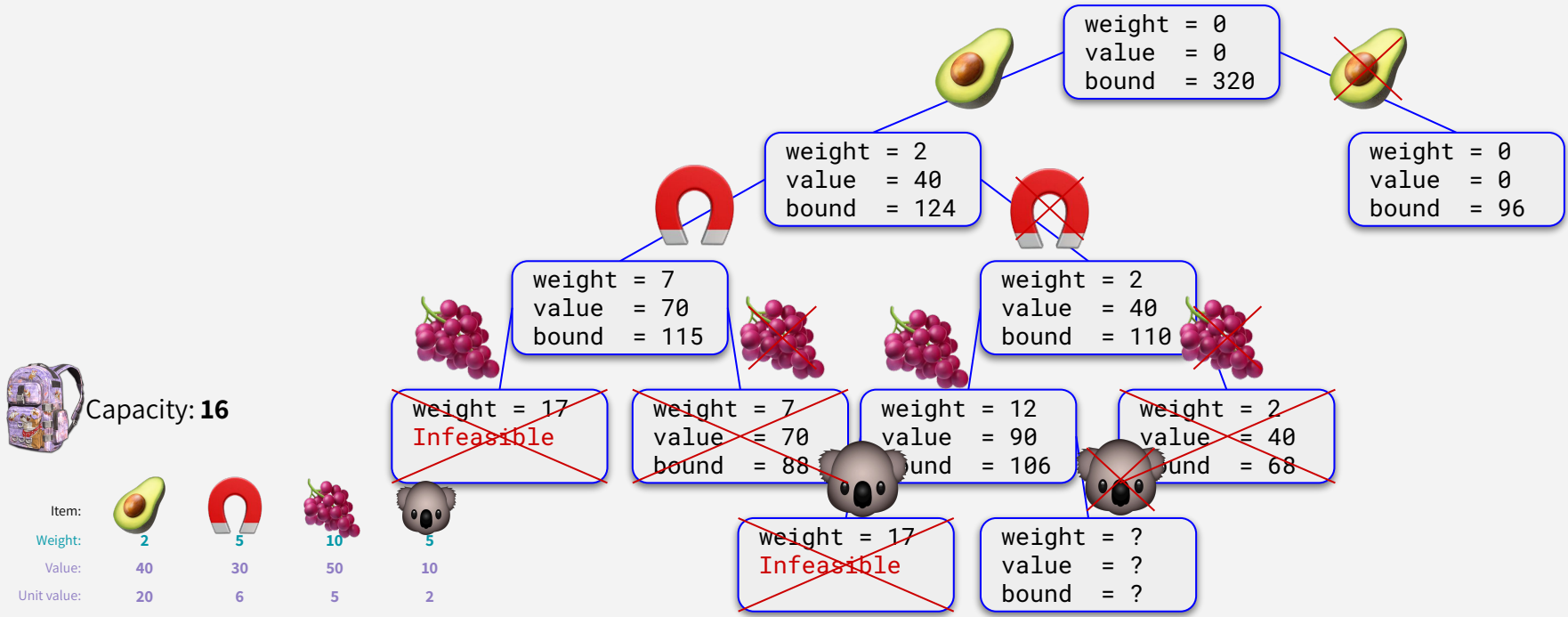


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

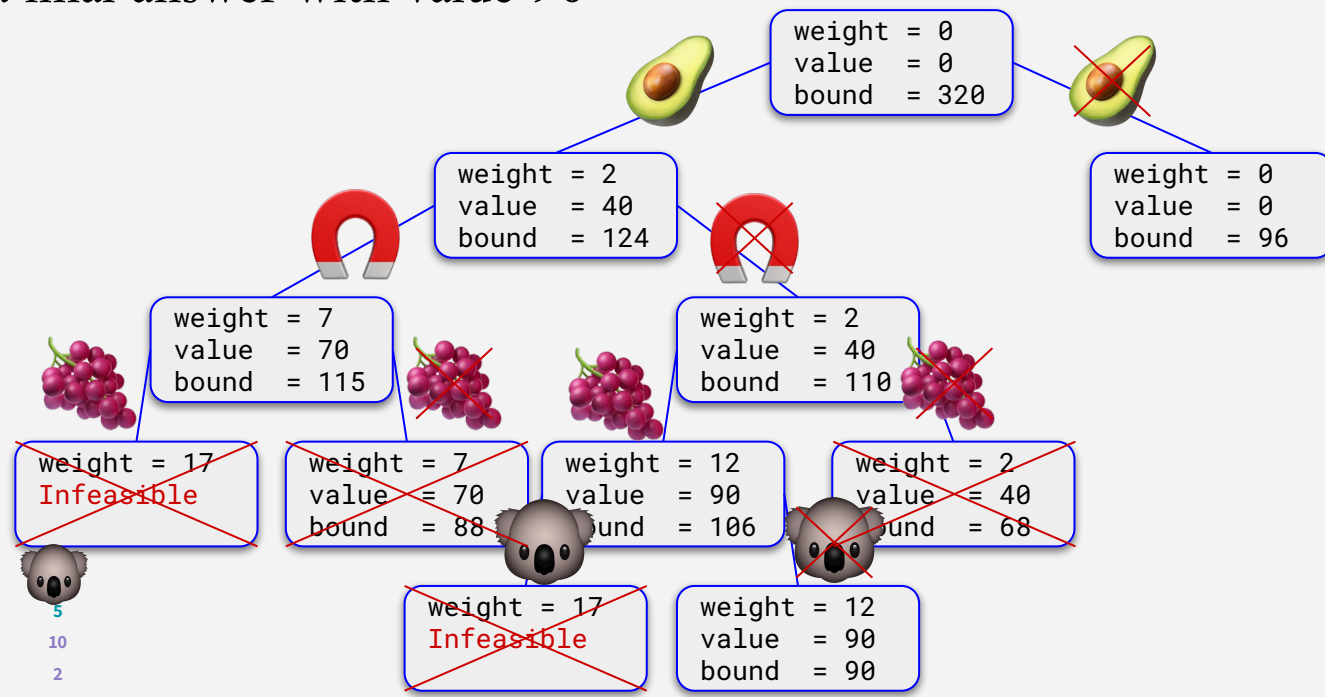
0/1 Knapsack with Branch and Bound

- Do not pick Quala



0/1 Knapsack with Branch and Bound

- Do not pick Quala
 - Found a final answer with value 90

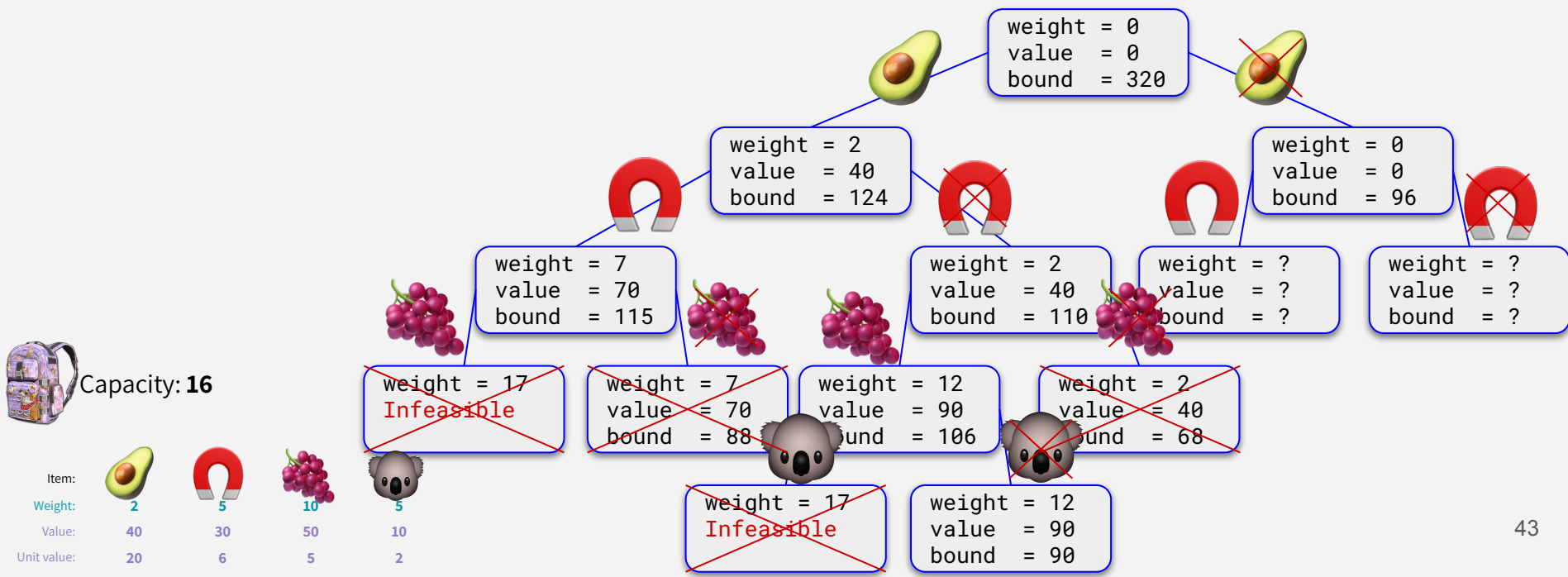


Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2

0/1 Knapsack with Branch and Bound

- Expand node with bound 96



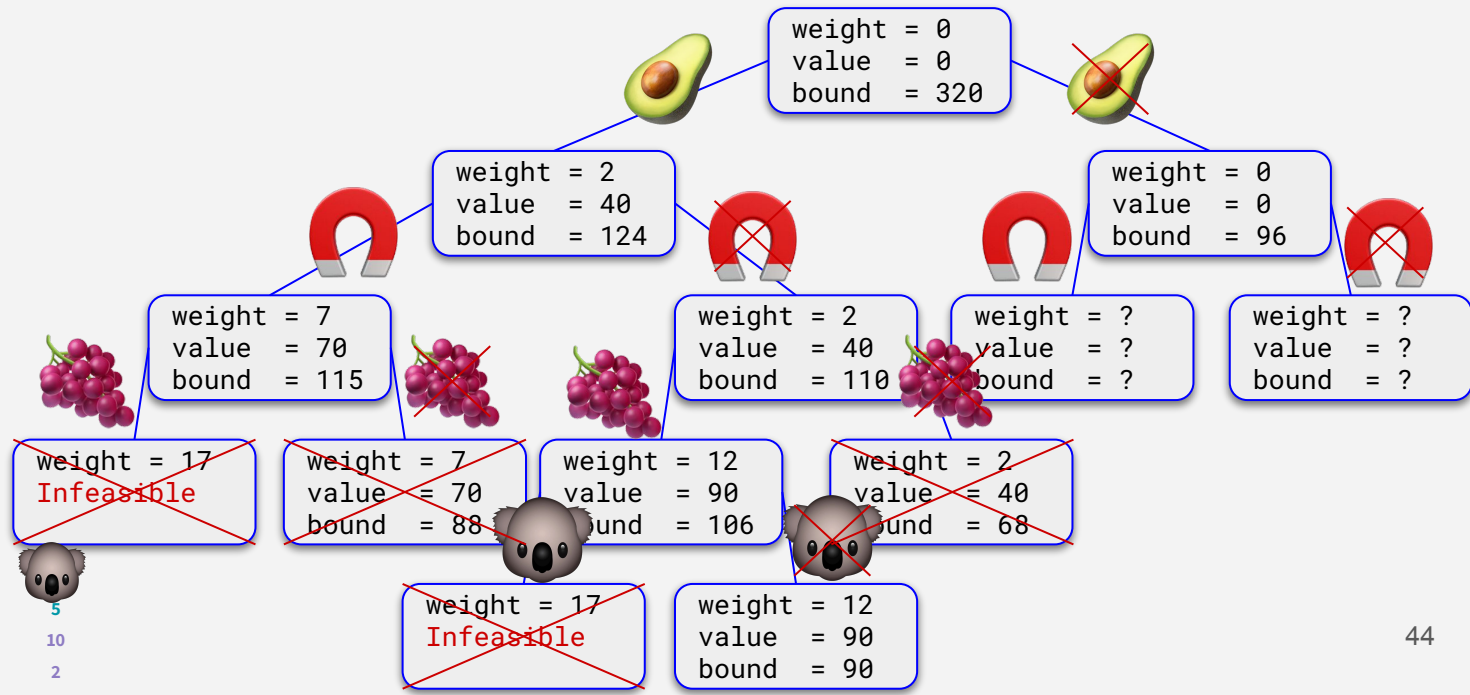
0/1 Knapsack with Branch and Bound

- Pick Magnet



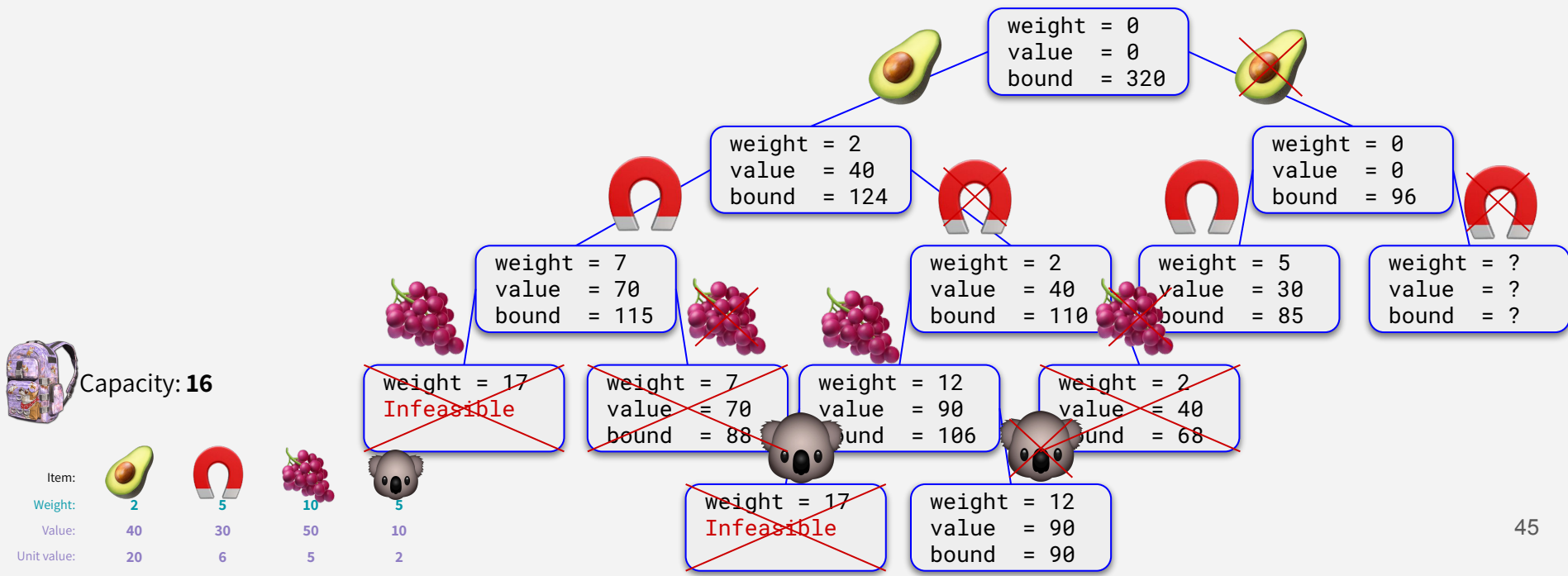
Capacity: 16

Item:				
Weight:	2	5	10	5
Value:	40	30	50	10
Unit value:	20	6	5	2



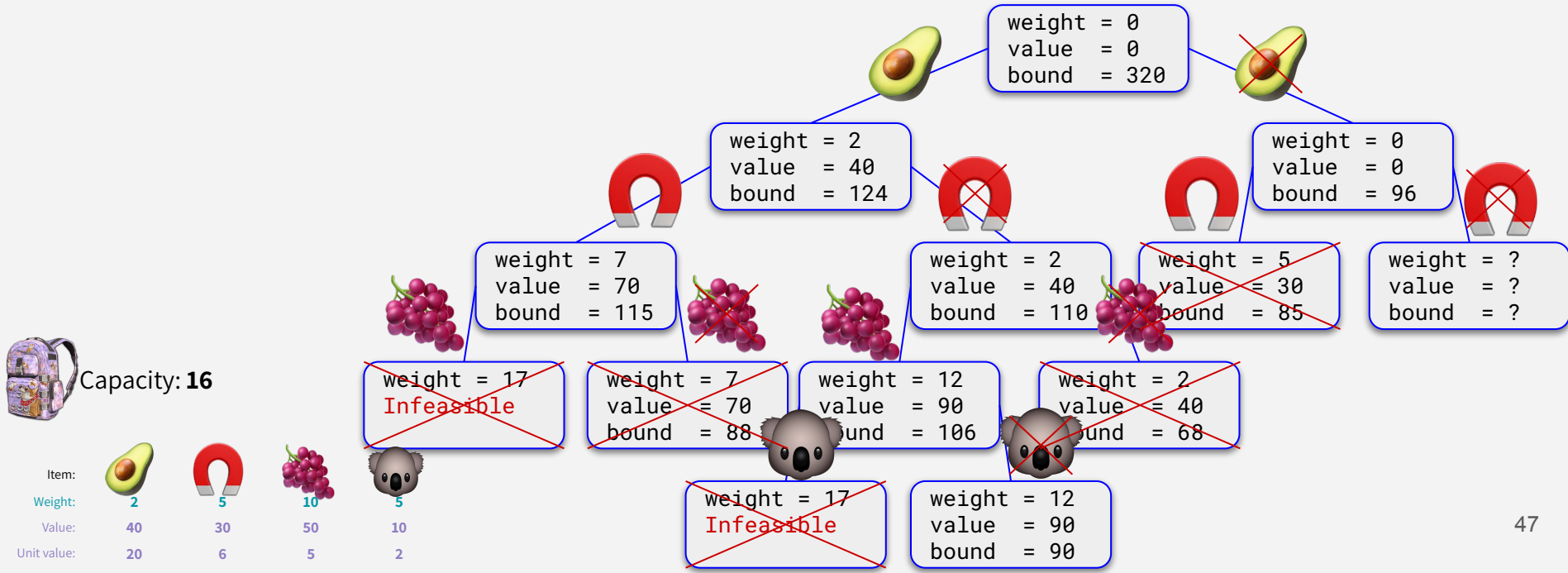
0/1 Knapsack with Branch and Bound

- Pick Magnet



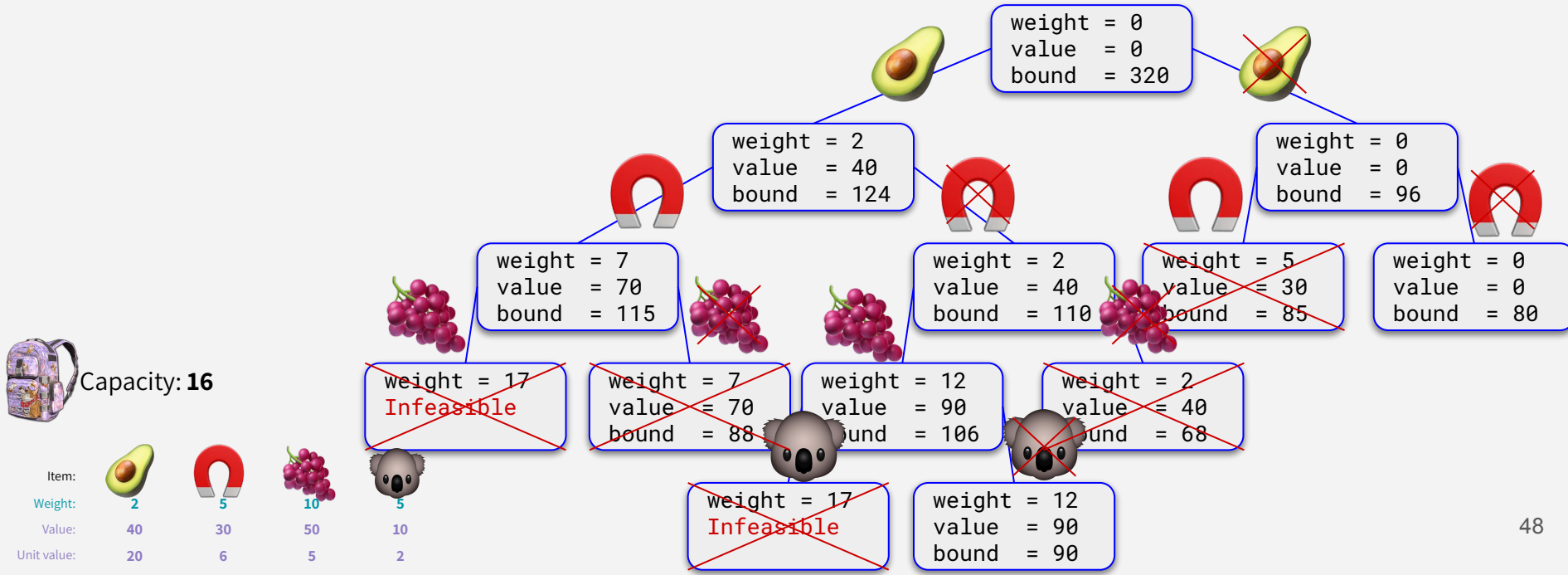
0/1 Knapsack with Branch and Bound

- Do not pick Magnet



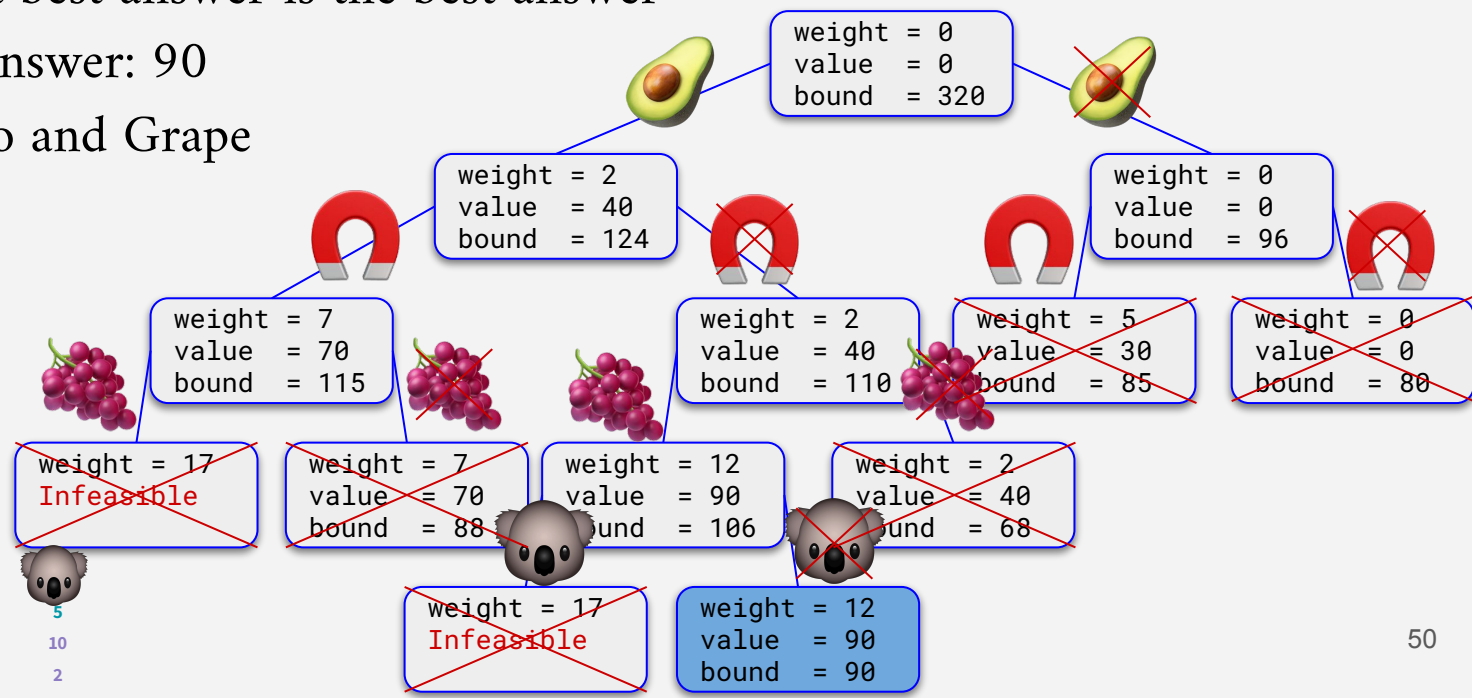
0/1 Knapsack with Branch and Bound

- Do not pick Magnet



0/1 Knapsack with Branch and Bound

- No more partial answer to expand
 - Current best answer is the best answer
- Best Final Answer: 90
 - Avocado and Grape



Capacity: 16

Item:				
Weight:	2	5	10	17
Value:	40	30	50	100
Unit value:	20	6	5	6



سوال؟