Assignment 01

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Contents

1   Ray-plane intersection

A ray equation can be formulated as follows:

$$\mathbf{s}(t) = \mathbf{o} + t\mathbf{d}$$

For a plane $\mathbf{n}$, the ray-plane intersection can be formulated as:

$$\mathbf{n}^{\top}(\mathbf{o} + t\mathbf{d}) - b = 0$$

solving for t, we get

$$\mathbf{n}^{\top}(\mathbf{o} + t\mathbf{d}) - b = 0$$
$$\mathbf{n}^{\top}\mathbf{o} + t\mathbf{n}^{\top}\mathbf{d} - b = 0$$
$$\implies t\mathbf{n}^{\top}\mathbf{d} = b - \mathbf{n}^{\top}\mathbf{o}$$
$$\implies t = \frac{b - \mathbf{n}^{\top}\mathbf{o}}{\mathbf{n}^{\top}\mathbf{d}}$$

2   Ray-cylinder intersection + normal derivations

The implicit equation for a cylinder centered on z with radius r is

$$x^2 + y^2 = r^2$$

where

$$x = r\cos\phi$$
$$y = r\sin\phi$$

We can now substitute the ray equation in the implicit cylinder equation:

$$(o_x + td_x)^2 + (o_y + td_y)^2 = r^2$$
$$(o_x + td_x)^2 + (o_y + td_y)^2 - r^2 = 0$$
$$o_x^2 + 2td_xo_x + t^2d_x^2 + o_y^2 + 2td_yo_y + t^2d_y^2 - r^2 = 0$$
$$t^2(d_x^2 + d_y^2) + t(2d_xo_x + 2d_yo_y) + o_x^2 + o_y^2 + -r^2 = 0$$

which is a form of the quadratic equation:

$$at^2 + bt + c$$

References

[1] Pharr et al. Physically Based Rendering: From Theory To Implementation, http://www.pbr-book.org/3ed-2018/Shapes/Cylinders.html