Assignment 01

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1 Ray-plane intersection

A ray equation can be formulated as follows:

$$\mathbf{s}(t) = \mathbf{o} + t\mathbf{d}$$

For a plane $\mathbf{n}$, the ray-plane intersection can be formulated as:

$$\mathbf{n}^\top(\mathbf{o} + t\mathbf{d}) - b = 0$$

solving for t, we get

$$\begin{aligned} \mathbf{n}^\top(\mathbf{o} + t\mathbf{d}) - b &= 0 \\ \mathbf{n}^\top\mathbf{o} + t\mathbf{n}^\top\mathbf{d} - b &= 0 \\ \implies t\mathbf{n}^\top\mathbf{d} &= b - \mathbf{n}^\top\mathbf{o} \\ \implies t &= \frac{b - \mathbf{n}^\top\mathbf{o}}{\mathbf{n}^\top\mathbf{d}} \end{aligned}$$

2 Ray-cylinder intersection

The implicit equation for a cylinder centered on z with radius r is

$$x^2 + y^2 = r^2$$

where

$$\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned}$$

Now we can write

$$\|\mathbf{q} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{q} - \mathbf{p}_a))\mathbf{v}_a\|^2 - r^2 = 0$$

- r: radius of a cylinder
- $\mathbf{p}_a + t\mathbf{v}_a$: line orientation of the cylinder
- $\mathbf{q}$: point on the cylinder

to find intersection points with a ray $\mathbf{p} + t\mathbf{v}$, substitute $\mathbf{q} = \mathbf{p} + t\mathbf{v}$

$$\begin{aligned}
\|\mathbf{p} + t\mathbf{v} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} + t\mathbf{v} - \mathbf{p}_a))\mathbf{v}_a\|^2 - r^2 &= 0 \\
\|\mathbf{p} + t\mathbf{v} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a) + \mathbf{v}_a \cdot t\mathbf{v}))\mathbf{v}_a\|^2 - r^2 &= 0 \\
\|\mathbf{p} + t\mathbf{v} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a - (\mathbf{v}_a \cdot t\mathbf{v}))\mathbf{v}_a\|^2 - r^2 &= 0 \\
\|t\underbrace{(\mathbf{v} - (\mathbf{v}_a \cdot \mathbf{v}))\mathbf{v}_a)}_{\mathbf{F}} + \underbrace{\mathbf{p} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a}_{\mathbf{G}}\|^2 - r^2 &= 0 \\
\|t\mathbf{F} + \mathbf{G}\|^2 - r^2 &= 0 \\
t^2\underbrace{\mathbf{F}^\top\mathbf{F}}_{a} + t\underbrace{2\mathbf{F}^\top\mathbf{G}}_{b} + \underbrace{\mathbf{H}^\top\mathbf{H} - r^2}_{c} &= 0
\end{aligned}$$

This is a quadratic equation with the following coefficients:

- $a = \|\mathbf{v} - (\mathbf{v}_a \cdot \mathbf{v}))\mathbf{v}_a\|^2$
- $b = 2\Big(\mathbf{v} - (\mathbf{v}_a \cdot \mathbf{v}))\mathbf{v}_a\Big)^\top\Big(\mathbf{p} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a\Big)$
- $c = \|\mathbf{p} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a\|^2 - r^2$

References

[1] Denis Zorin, https://mrl.nyu.edu/~dzorin/rend05/lecture2.pdf