Assignment 01

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1 Ray-plane intersection

A ray equation can be formulated as follows:

$$\mathbf{s}(t) = \mathbf{o} + t\mathbf{d}$$

For a plane $\mathbf{n}$, the ray-plane intersection can be formulated as:

$$\mathbf{n}^\top(\mathbf{o} + t\mathbf{d}) - b = 0$$

solving for t, we get

$$\begin{aligned}
\mathbf{n}^\top(\mathbf{o} + t\mathbf{d}) - b &= 0 \\
\mathbf{n}^\top\mathbf{o} + t\mathbf{n}^\top\mathbf{d} - b &= 0 \\
\implies t\mathbf{n}^\top\mathbf{d} &= b - \mathbf{n}^\top\mathbf{o} \\
\implies t &= \frac{b - \mathbf{n}^\top\mathbf{o}}{\mathbf{n}^\top\mathbf{d}}
\end{aligned}$$

2 Ray-cylinder intersection + normal derivations

The implicit equation for a cylinder centered on z with radius r is

$$x^2 + y^2 = r^2$$

where

$$\begin{aligned}
x &= r\cos\phi \\
y &= r\sin\phi
\end{aligned}$$

We can now substitute the ray equation in the implicit cylinder equation:

$$\begin{aligned}
(\mathbf{o}_x + t\mathbf{d}_x)^2 + (\mathbf{o}_y + t\mathbf{d}_y)^2 &= r^2 \\
(\mathbf{o}_x + t\mathbf{d}_x)^2 + (\mathbf{o}_y + t\mathbf{d}_y)^2 - r^2 &= 0 \\
\mathbf{o}_x^2 + 2t\mathbf{d}_x\mathbf{o}_x + t^2\mathbf{d}_x^2 + \mathbf{o}_y^2 + 2t\mathbf{d}_y\mathbf{o}_y + t^2\mathbf{d}_y^2 - r^2 &= 0 \\
t^2(\mathbf{d}_x^2 + \mathbf{d}_y^2) + t(2\mathbf{d}_x\mathbf{o}_x + 2\mathbf{d}_y\mathbf{o}_y) + \mathbf{o}_x^2 + \mathbf{o}_y^2 + -r^2 &= 0
\end{aligned}$$

which is a form of the quadratic equation:

$$at^2 + bt + c$$

References

[1] Pharr et al. Physically Based Rendering: From Theory To Implementation, http://www.pbr-book.org/3ed-2018/Shapes/Cylinders.html