# Assignment 01

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# 1 Ray-plane intersection

A ray equation can be formulated as follows:

$$\mathbf{s}(t) = \mathbf{o} + t\mathbf{d}$$

For a plane $\mathbf{n}$, the ray-plane intersection can be formulated as:

$$\mathbf{n}^\top(\mathbf{o} + t\mathbf{d}) - b = 0$$

solving for $t$, we get

$$\mathbf{n}^\top(\mathbf{o} + t\mathbf{d}) - b = 0$$
$$\mathbf{n}^\top\mathbf{o} + t\mathbf{n}^\top\mathbf{d} - b = 0$$
$$\implies t\mathbf{n}^\top\mathbf{d} = b - \mathbf{n}^\top\mathbf{o}$$
$$\implies t = \frac{b - \mathbf{n}^\top\mathbf{o}}{\mathbf{n}^\top\mathbf{d}}$$

# 2 Ray-cylinder intersection

The implicit equation for a cylinder centered on $z$ with radius $r$ is

$$x^2 + y^2 = r^2$$

where

$$x = r\cos\phi$$
$$y = r\sin\phi$$

Now we can write

$$\|\mathbf{q} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{q} - \mathbf{p}_a))\mathbf{v}_a\|^2 - r^2 = 0$$

- $r$: radius of a cylinder
- $\mathbf{p}_a$: cylinder center
- $\mathbf{v}_a$: cylinder axis
- $\mathbf{q}$: point on the cylinder

to find intersection points with a ray $\mathbf{p} + t\mathbf{v}$, substitute $\mathbf{q} = \mathbf{p} + t\mathbf{v}$

$$\|\mathbf{p} + t\mathbf{v} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} + t\mathbf{v} - \mathbf{p}_a))\mathbf{v}_a\|^2 - r^2 = 0$$
$$\|\mathbf{p} + t\mathbf{v} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a) + \mathbf{v}_a \cdot t\mathbf{v}))\mathbf{v}_a\|^2 - r^2 = 0$$
$$\|\mathbf{p} + t\mathbf{v} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a - (\mathbf{v}_a \cdot t\mathbf{v}))\mathbf{v}_a\|^2 - r^2 = 0$$
$$\|t\underbrace{(\mathbf{v} - (\mathbf{v}_a \cdot \mathbf{v}))\mathbf{v}_a)}_{\mathbf{F}} + \underbrace{\mathbf{p} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a}_{\mathbf{G}}\|^2 - r^2 = 0$$
$$\|t\mathbf{F} + \mathbf{G}\|^2 - r^2 = 0$$
$$t^2\underbrace{\mathbf{F}^\top\mathbf{F}}_{a} + t\underbrace{2\mathbf{F}^\top\mathbf{G}}_{b} + \underbrace{\mathbf{H}^\top\mathbf{H} - r^2}_{c} = 0$$

This is a quadratic equation with the following coefficients:

- $a = \|\mathbf{v} - (\mathbf{v}_a \cdot \mathbf{v}))\mathbf{v}_a\|^2$
- $b = 2\left(\mathbf{v} - (\mathbf{v}_a \cdot \mathbf{v}))\mathbf{v}_a\right)^\top\left(\mathbf{p} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a\right)$
- $c = \|\mathbf{p} - \mathbf{p}_a - (\mathbf{v}_a \cdot (\mathbf{p} - \mathbf{p}_a))\mathbf{v}_a\|^2 - r^2$

# 3 Intersection Normal Vector

- **c2**: projection of intersection point to cylinder axis
- **n**: normal vector on intersection

$$\mathbf{c2} = \mathbf{p}_a + (\mathbf{q} - \mathbf{p}_a \cdot \mathbf{v}_a)\mathbf{v}_a$$
$$\mathbf{n} = \frac{\mathbf{c2} - \mathbf{q}}{r}$$

# References

[1] Denis Zorin, https://mrl.nyu.edu/~dzorin/rend05/lecture2.pdf