Digital Image Processing: Assignment 6

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1 Introduction

This report focuses on three essential image processing techniques: gradient filtering, histogram equalization, and histogram matching. These methods are crucial for enhancing and transforming images in various computer vision tasks.

2 Gradient Filters

Gradient filters detect changes in intensity, which helps in edge detection. Mathematically, the gradient of an image is the directional change in the intensity or color. The gradient filters covered here include the Sobel filter for first-order gradients and the Laplacian filter for second-order gradients.

2.1 First-Order Gradient (Sobel Filter)

The Sobel filter is used to compute the first-order derivative of the image intensity in both the horizontal and vertical directions. The Sobel operator is defined as two kernels:

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, \quad G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

The gradient magnitude is calculated as:

$$G = \sqrt{G_x^2 + G_y^2}$$

This gradient approximates the first-order derivative in the X and Y directions, highlighting edges in the image.

2.2 Second-Order Gradient (Laplacian Filter)

The Laplacian filter computes the second-order derivative, identifying regions where the intensity changes most rapidly. The Laplacian operator is defined as:

$$L = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

This filter uses a kernel such as:

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

This operator captures regions of rapid intensity change, corresponding to the image's edges.

(a) Original Image

(b) Applying sobel filter

(c) Applying Laplacian filter

Figure 1: Results of applying Sobel and Laplacian filters on the image.

3 Histogram Equalization

Histogram equalization is a contrast adjustment technique that redistributes intensity values across an image. Mathematically, the process aims to flatten and spread out the most frequent intensity values.

Let the probability of an intensity level i in the image be:

$$p(i) = \frac{\text{number of pixels with intensity } i}{\text{total number of pixels}}$$

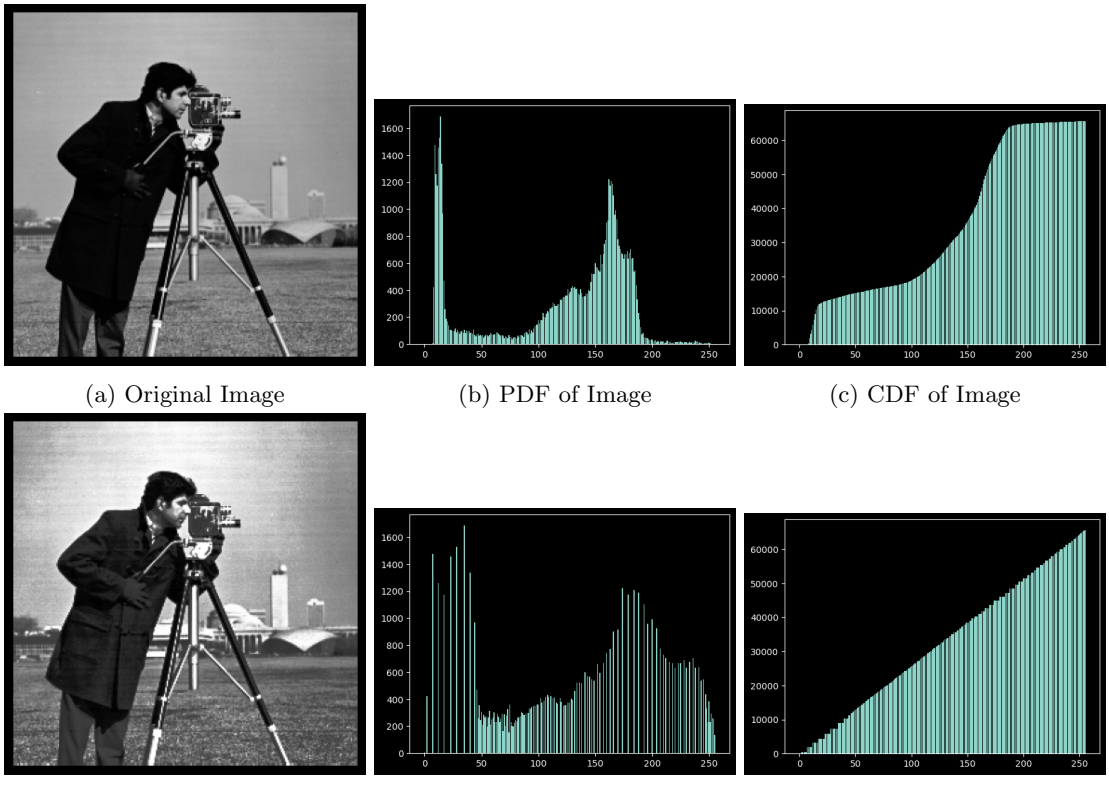
The cumulative distribution function (CDF) is computed as:

$$\text{CDF}(i) = \sum_{j=0}^{i} p(j)$$

The transformation function for histogram equalization is then:

$$T(i) = (\text{CDF}(i) - \text{CDF}_{\min}) \times \frac{L-1}{N}$$

Where: - L is the number of intensity levels, - N is the total number of pixels in the image. This function redistributes the intensities to enhance the contrast in the image.



(a) Original Image (b) PDF of Image (c) CDF of Image

(d) Equalized Image (e) PDF of equalized Image (f) CDF of equalized Image

Figure 2: Result of applying histogram equalization on the image.

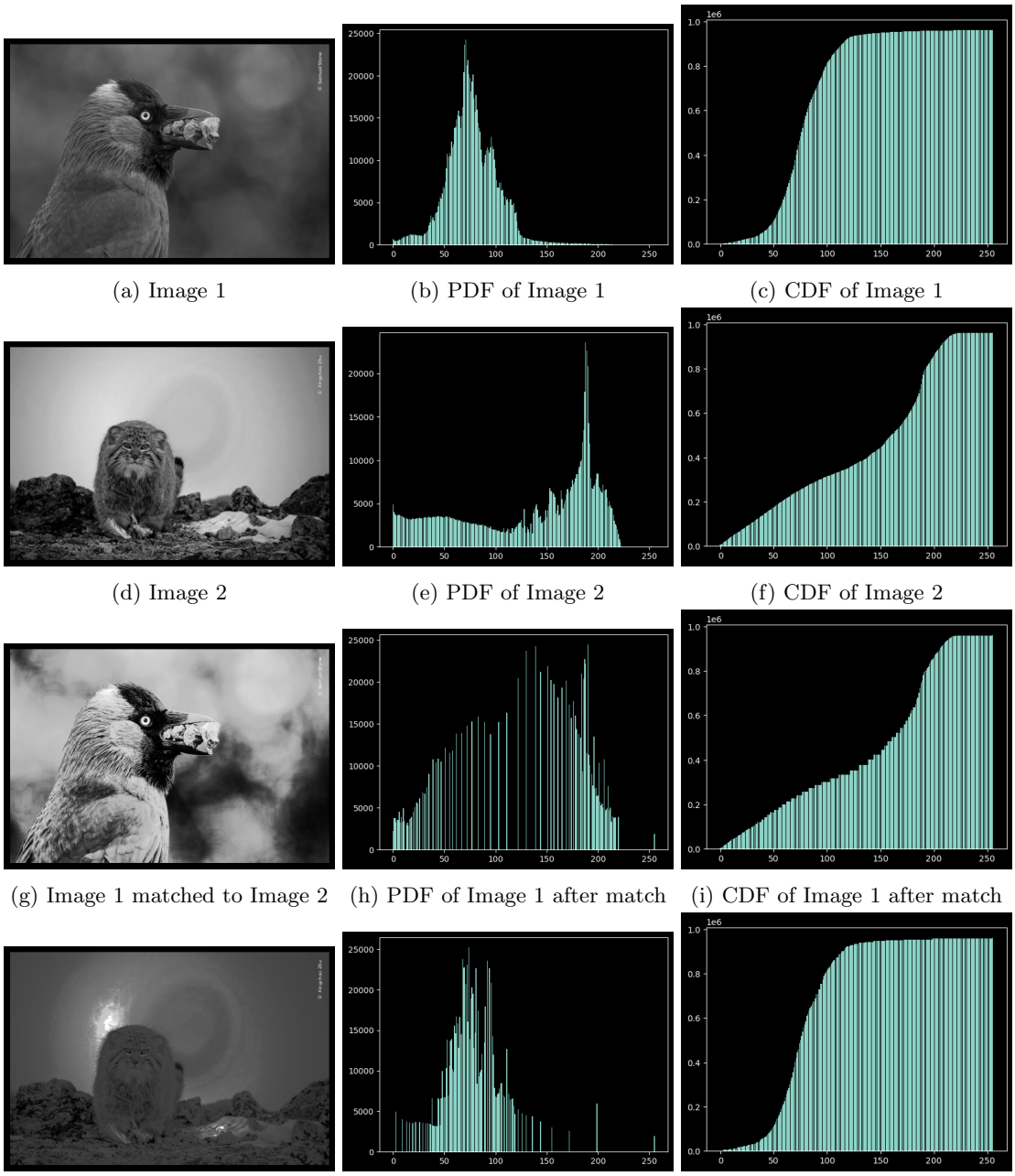
4 Histogram Matching

Histogram matching is the process of adjusting the pixel values of an image so that its histogram matches that of a reference image. The goal is to map the cumulative distribution function (CDF) of the original image to the CDF of the reference image.

Given the CDFs of the original image I and the reference image R, the transformation function can be defined as:

$$T(I(i)) = R^{-1}(\text{CDF}_R(\text{CDF}_I^{-1}(I(i))))$$

This function maps the pixel values of image I to match the histogram of image R.



(a) Image 1 (b) PDF of Image 1 (c) CDF of Image 1

(d) Image 2 (e) PDF of Image 2 (f) CDF of Image 2

(g) Image 1 matched to Image 2 (h) PDF of Image 1 after match (i) CDF of Image 1 after match

(j) Image 2 matched to Image 1 (k) PDF of Image 2 after match (l) CDF of Image 2 after match

Figure 3: Results of applying histogram matching between two images.

5 Conclusion

In this report, we applied gradient filters for edge detection, performed histogram equalization to enhance image contrast, and implemented histogram matching to align the intensity distributions of two images. We explored the mathematical formulations behind these operations to understand their significance in image processing.

Code Availability

All the code used in this project is available in a public GitHub repository. You can access it at: https://github.com/Computer-Science-Practicum/DIP-Lab-Assignment.