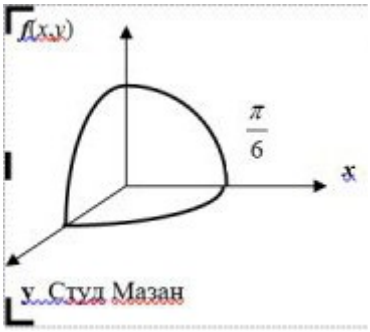
Варіант:




Вважатиму висоту рівною π/6. Тоді радіус основи:

$$\frac{\frac{4}{3}\pi h r^2}{8}=1,\ h=\frac{\pi}{6}$$

$$r=\frac{6}{\pi}$$

f(x,y):

$$\frac{x^2}{r^2}+\frac{y^2}{r^2}+\frac{f^2(x,y)}{h^2}=1$$

$$f(x,y)=h\sqrt{1-\frac{x^2}{r^2}-\frac{y^2}{r^2}}=\frac{h}{r}\sqrt{r^2-x^2-y^2}=\frac{\pi^2}{36}\sqrt{\frac{36}{\pi^2}-x^2-y^2}\ \text{при}\ x^2+y^2\le\frac{36}{\pi^2},\text{x>0,y>0}$$

Функції часткового розподілу:

$$\varphi(x)=\int_{-\infty}^{\infty}f(x,y)\,dy=\frac{\pi^2}{36}\int_0^{\sqrt{\frac{36}{\pi^2}-x^2}}\sqrt{\frac{36}{\pi^2}-x^2-y^2}\,dy=\left|\begin{array}{c} y=\sqrt{\frac{36}{\pi^2}-x^2}\sin t \\ dy=\sqrt{\frac{36}{\pi^2}-x^2}\cos t dt \\ t=arc\sin\frac{y}{\sqrt{\frac{36}{\pi^2}-x^2}} \\ y:\ 0,\ \sqrt{\frac{36}{\pi^2}-x^2} \\ t:\ 0,\ \frac{\pi}{2} \end{array}\right|=$$

$$=\frac{\pi^2}{36}\int_0^{\frac{\pi}{2}}\left(\frac{36}{\pi^2}-x^2\right)\cos^2 t dt=\frac{\pi^2}{4\cdot 36}\left(\frac{36}{\pi^2}-x^2\right)\int_0^{\frac{\pi}{2}}(1+\cos 2t)\,d2t=\frac{1}{4}\left(1-\frac{\pi^2}{36}x^2\right)(2t+\sin 2t)\Big|^{\pi/2}{}_0$$

$$=\frac{\pi}{4}\left(1-\frac{\pi^2}{36}x^2\right)$$

$$\psi(y)=\int_{-\infty}^{\infty} f(x,y)\,dx=\frac{\pi^2}{36}\int_0^{\sqrt{\frac{36}{\pi^2}-y^2}}\sqrt{\frac{36}{\pi^2}-x^2-y^2}\,dx=\left|\begin{array}{c}\textit{інтегрування відбувається}\\ \textit{аналогічно}\end{array}\right|=$$
$$=\frac{\pi}{4}\left(1-\frac{\pi^2}{36}y^2\right)$$

$$f_y(y|x)=\frac{f(x,y)}{\varphi(x)}=\frac{\frac{\pi^2}{36}\sqrt{\frac{36}{\pi^2}-x^2-y^2}}{\frac{\pi}{4}\left(1-\frac{\pi^2}{36}x^2\right)}=\frac{\pi}{9}\frac{\sqrt{\frac{36}{\pi^2}-x^2-y^2}}{\left(1-\frac{\pi^2}{36}x^2\right)}$$

Математичні очікування, середньоквадратичні відхилення та дисперсії:

$$m_x=\int_0^r dx\int_0^{\sqrt{r^2-x^2}} xf(x,y)\,dy=\int_0^r x\varphi(x)\,dx=\frac{\pi}{4}\int_0^{\frac{6}{\pi}}\left(x-\frac{\pi^2}{36}x^3\right)dx=\frac{\pi}{4}\left(\frac{x^2}{2}-\frac{\pi^2}{144}x^4\right)\Bigg|_0^{\frac{6}{\pi}}=$$
$$=\frac{9}{4\pi}\approx 0.716$$
$$m_y=\int_0^r y\psi(y)\,dy=\frac{\pi}{4}\int_0^{\frac{6}{\pi}}\left(y-\frac{\pi^2}{36}y^3\right)dy=\left|\begin{array}{c}\textit{інтегрування}\\ \textit{ідентичне}\end{array}\right|=\frac{9}{4\pi}\approx 0.716$$
$$D_x=\int_0^r x^2\varphi(x)\,dx-m^2{}_x=\frac{36}{5\pi^2}-\frac{81}{16\pi^2}=\frac{171}{80\pi^2}\approx 0.217$$

$$\int_0^r x^2\varphi(x)\,dx=\frac{\pi}{4}\int_0^{\frac{6}{\pi}}\left(x^2-\frac{\pi^2}{36}x^4\right)dx=\frac{\pi}{4}\left(\frac{x^3}{3}-\frac{\pi^2}{180}x^5\right)\Bigg|_0^{\frac{6}{\pi}}=\frac{36}{5\pi^2}$$
$$D_y=\int_0^r y^2\psi(y)\,dy-m^2{}_y=\frac{36}{5\pi^2}-\frac{81}{16\pi^2}=\frac{171}{80\pi^2}\approx 0.217$$

$$\int_0^r y^2\psi(y)\,dy=\frac{\pi}{4}\int_0^{\frac{6}{\pi}}\left(y^2-\frac{\pi^2}{36}y^4\right)dx=\frac{\pi}{4}\left(\frac{y^3}{3}-\frac{\pi^2}{180}y^5\right)\Bigg|_0^{\frac{6}{\pi}}=\frac{36}{5\pi^2}$$
$$\sigma_x=\sigma_y\approx 0.465$$

Коефіцієнт коваріації:

$$cov=\iint_S xyf(x,y)\,dxdy-m_xm_y=\left|\begin{array}{c}S:x^2+y^2\le r^2,x,y\ge 0\\ \textit{Перехід до ПСК:}\\ x=\rho\cos\varphi,y=\rho\sin\varphi,|J|=\rho\\ \rho\in[0,r]\\ \varphi\in\left[0,\frac{\pi}{2}\right]\\ f(x,y)=\frac{h}{r}\sqrt{r^2-\rho^2}\end{array}\right|=\frac{h}{r}\int_0^{\frac{\pi}{2}}\mathrm{d}\varphi\int_0^r\rho^3\sin\varphi\cos\varphi\sqrt{r^2-\rho^2}\,\mathrm{d}\rho-m_xm_y$$

$$\frac{h}{r}\int_0^{\frac{\pi}{2}}\sin\varphi\cos\varphi d\varphi\cdot\int_0^{r}\rho^3\left(r^2-\rho^2\right)^{0.5}\mathrm{d}\rho=\left|\begin{array}{c} r^2-\rho^2=t^2,\rho=\sqrt{r^2-t^2} \\ \mathrm{d}\rho=\dfrac{-t\,\mathrm{d}t}{\sqrt{r^2-t^2}} \\ \rho\in[0,r],t\in[r,0] \end{array}\right|=-\frac{h}{r}\int_0^{\frac{\pi}{2}}\sin\varphi\cos\varphi d\varphi\cdot$$

$$\cdot\left(-\int_0^{r}\left(r^2-t^2\right)t^2\mathrm{d}t\right)$$

$$\left(\frac{r^2t^3}{3}-\frac{t^5}{5}\right)\Bigg|_0^r=\frac{2}{15}r^5$$

$$\frac{h}{r}\cdot\frac{2}{15}r^5\int_0^{\frac{\pi}{2}}\sin\varphi\cos\varphi\,\mathrm{d}\varphi=\frac{hr^4}{30}\int_0^{\frac{\pi}{2}}\sin2\varphi d(2\varphi)=\frac{hr^4}{15},h=\frac{\pi}{6},r=\frac{6}{\pi}$$

$$\frac{hr^4}{15}-\mathrm{m}_x^2\mathrm{m}_y^2=\frac{72}{5\pi^3}-\frac{81}{16\pi^2}=\frac{1152-405\pi}{80\pi^3}\approx-0,0485$$

Коефіцієнт кореляції: $\rho=\dfrac{cov}{\sigma_x\sigma_y}\approx-0.2243$

Код програми:

```
import math
from random import random
from matplotlib.backends.backend_tkagg import FigureCanvasTkAgg
from matplotlib.figure import Figure
from Tkinter import *
from matplotlib import use
use("TkAgg")
h = math.pi/6
r = 6/math.pi
generated_nums = 1000
def f_xy(x,y):
    return h/r*math.sqrt(math.pow(r,2)-math.pow(x,2)-math.pow(y,2))
def phi_x(x):
    return math.pi/4*(1-math.pow(h*x,2))
def psi_y(y):
    return phi_x(y)
def f_y_while_x(x,y):
    return f_xy(x,y)/phi_x(x)
def generate_x():
    r = random()
    step = 1.0/10**4
    searched_x = 0.0
    sum = 0.0
    while r > sum:
        sum += phi_x(searched_x)*step
        searched_x += step
    return searched_x
def generate_y(x):
    r = random()
    step = 1.0 / 10 ** 4
    searched_y = 0.0
    sum = 0.0
    while r > sum:
        sum += f_y_while_x(x,searched_y) * step
        searched_y += step
    return searched_y
```

```
def math_expectation(arr):
    return sum(arr)/generated_nums
def deviation(arr, math_expectation):
    return sum(list(map(lambda x: (x-math_expectation)**2/generated_nums,arr)))
def covariation(arr_a,arr_b,expected_a,expected_b):
    return sum(list(map(lambda a,b: (a-expected_a)*(b-expected_b)/generated_nums,
                        arr_a,arr_b)))
def standard_deviation(deviation):
    return math.sqrt(deviation)
def correlation(cov,stand_dev_a,stand_dev_b):
    return cov/(stand_dev_a*stand_dev_b)
generated_x = []
generated_y = []
for i in range(generated_nums):
    x = generate_x()
    y = generate_y(x)
    generated_x.append(x)
    generated_y.append(y)
exp_x = math_expectation(generated_x)
exp_y = math_expectation(generated_y)
deviation_x = deviation(generated_x,exp_x)
deviation_y = deviation(generated_y,exp_y)
standard_x_deviation = standard_deviation(deviation_x)
standard_y_deviation = standard_deviation(deviation_y)
cov = covariation(generated_x,generated_y,exp_x,exp_y)
corr = correlation(cov,standard_x_deviation,standard_y_deviation)
root = Tk()
root.title("Lab work #3")
root.geometry("500x680")
frame = Frame(root,bg = "white")
frame.grid(row = 1, column = 1)
f = Figure(figsize = (5,5), dpi = 100)
canvas = FigureCanvasTkAgg(f,frame)
a = f.add_subplot(111)
a.scatter(generated_x,generated_y,s=0.3)
canvas._tkcanvas.grid(row = 1,column = 1)
canvas.draw()
text = "Generated distribution with:\nexpected x: " + str(exp_x) + "\nexpected y: " +
str(exp_y) + "\nx deviation: "+ str(deviation_x) +"\ny deviation: " + str(deviation_y) +
"\nstandard x deviation: "+str(standard_x_deviation)+ "\nstandard y deviation:
"+str(standard_y_deviation)+ "\ncovariation: " + str(cov) + "\ncorrelation: " +
str(corr)
information = Label(root, text = text)
information.grid(row = 2,column = 1)
root.mainloop()
```