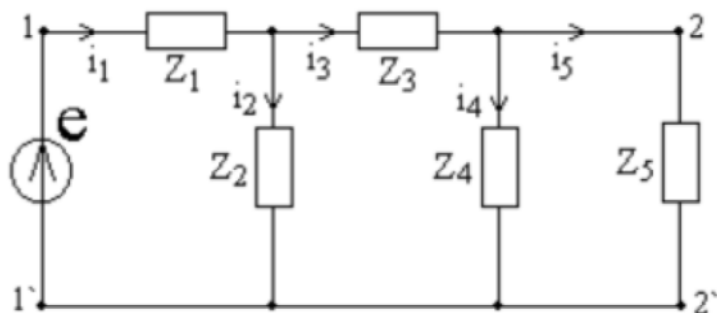


Параметри елементів схеми:

| | | |
|----------------------|--------------------|---------------------------------|
| $E := 240 \text{ V}$ | $XL1 := 35 \Omega$ | $j := \sqrt{-1}$ |
| $\psi := 50^\circ$ | $XL2 := 40 \Omega$ | |
| $R1 := 18 \Omega$ | $XL3 := 45 \Omega$ | $f := 60 \text{ Hz}$ |
| $R2 := 16 \Omega$ | $XC1 := 15 \Omega$ | |
| $R3 := 14 \Omega$ | $XC2 := 20 \Omega$ | $\omega := 2 \cdot \pi \cdot f$ |
| $R4 := 12 \Omega$ | $XC3 := 25 \Omega$ | |
| | $Xm := 23 \Omega$ | |

Розраховуємо струми комплексним методом



$$Z1 := R1 + j \cdot (XL1 - XC1) = (18 + 20j) \Omega$$

$$Z1 = (26.907 \angle 48.013^\circ) \Omega$$

$$Z2 := R2 + j \cdot XL2 = (16 + 40j) \Omega$$

$$Z2 = (43.081 \angle 68.199^\circ) \Omega$$

$$Z3 := j \cdot (XL3 - XC2) = 25j \Omega$$

$$Z3 = (25 \angle 90^\circ) \Omega$$

$$Z4 := R3 = 14 \Omega$$

$$Z4 = (14 \angle 0^\circ) \Omega$$

$$Z5 := R4 - j \cdot XC3 = (12 - 25j) \Omega$$

$$Z5 = (27.731 \angle -64.359^\circ) \Omega$$

$$Z := Z1 + \frac{\left(Z3 + \frac{Z4 \cdot Z5}{Z4 + Z5} \right) \cdot Z2}{Z3 + \frac{Z4 \cdot Z5}{Z4 + Z5} + Z2} = (24.22 + 33.885j) \Omega$$

$$Z = (41.651 \angle 54.444^\circ) \Omega$$

$$I1 := \frac{E \cdot e^{j \cdot \psi}}{Z} = (5.745 - 0.446j) \text{ A}$$

$$I1 = (5.762 \angle -4.444^\circ) \text{ A}$$

$$I3 := I1 \cdot \frac{Z2}{Z2 + Z3 + \frac{Z4 \cdot Z5}{Z4 + Z5}} = (3.724 - 0.206j) \text{ A}$$

$$I3 = (3.73 \angle -3.174^\circ) \text{ A}$$

$$I2 := I1 - I3 = (2.021 - 0.24j) \text{ A}$$

$$I2 = (2.035 \angle -6.773^\circ) \text{ A}$$

$$I4 := I3 \cdot \frac{Z5}{Z4 + Z5} = (2.627 - 1.151j) \text{ A}$$

$$I4 = (2.867 \angle -23.656^\circ) \text{ A}$$

$$I5 := I3 - I4 = (1.097 + 0.944j) \text{ A}$$

$$I5 = (1.448 \angle 40.703^\circ) \text{ A}$$

Перевірка за першим законом Кірхгофа:

$$I1 - I3 - I2 = 0 \text{ A}$$

$$I3 - I5 - I4 = 0 \text{ A}$$

Баланс активних і реактивних потужностей:

$$U := E \cdot e^{j \cdot \psi} = (154.269 + 183.851j) \text{ V}$$

$$U = (240 \angle 50^\circ) \text{ V}$$

$$S := U \cdot \overline{I_1} = (804.155 + 1.125j \cdot 10^3) \text{ W}$$

$$S = (1.383 \cdot 10^3 \angle 54.444^\circ) \text{ W}$$

$$P := |I_1|^2 \cdot R_1 + |I_2|^2 \cdot R_2 + |I_4|^2 \cdot R_3 + |I_5|^2 \cdot R_4 = 804.155 \text{ W}$$

$$Q := |I_2|^2 \cdot XL_2 + |I_1|^2 \cdot (XL_1 - XC_1) + |I_3|^2 \cdot (XL_3 - XC_2) - |I_5|^2 \cdot XC_3 = (1.125 \cdot 10^3) \text{ W}$$

Розрахуємо похибку обчислень:

$$\Delta P := \frac{P - \operatorname{Re}(S)}{P} \cdot 100\% = 0$$

$$\Delta Q := \frac{Q - \operatorname{Im}(S)}{S} \cdot 100\% = -9.561 \cdot 10^{-17} + 1.338j \cdot 10^{-16}$$

Розрахуємо покази вольтметра:

$$U_{\text{complex}} := -I_2 \cdot R_2 + I_3 \cdot (j \cdot XL_3) - I_5 \cdot (j \cdot XC_3) = (0.56 + 143.983j) \text{ V}$$

$$U_v := \sqrt{\operatorname{Re}(U_{\text{complex}})^2 + \operatorname{Im}(U_{\text{complex}})^2} = 143.984 \text{ V}$$

Суміщена діаграма струмів і напруг

Визначимо потенціали точок кола

$$U_a := 0 \text{ V}$$

$$U_b := U_a - I_1 \cdot (j \cdot XC_1) = (-6.697 - 86.172j) \text{ V}$$

$$U_c := U_b + I_1 \cdot R_1 = (96.709 - 94.208j) \text{ V}$$

$$U_d := U_c + I_3 \cdot (j \cdot XL_3) = (106 + 73.371j) \text{ V}$$

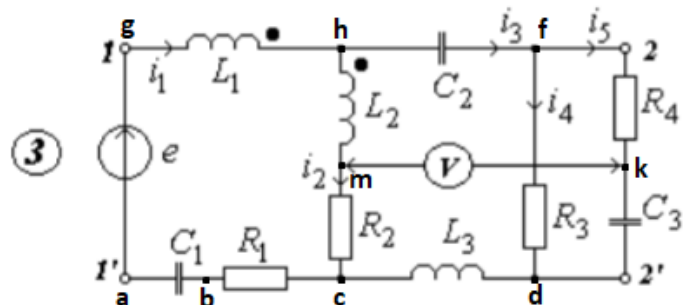
$$U_k := U_d - I_5 \cdot (j \cdot XC_3) = (129.602 + 45.934j) \text{ V}$$

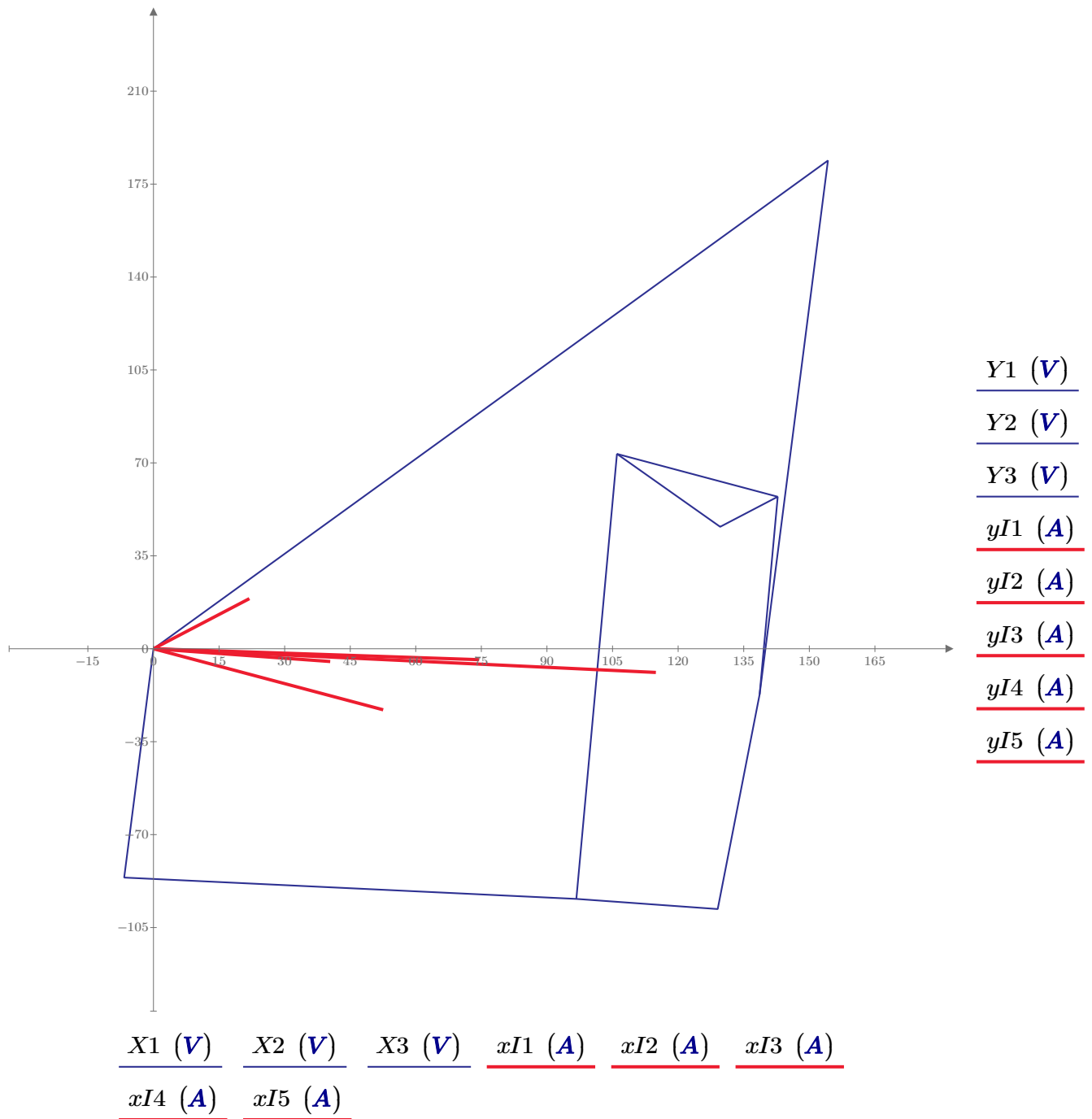
$$U_f := U_k + I_5 \cdot R_4 = (142.771 + 57.263j) \text{ V}$$

$$U_m := U_c + I_2 \cdot R_2 = (129.042 - 98.049j) \text{ V}$$

$$U_h := U_m + I_2 \cdot (j \cdot XL_2) = (138.642 - 17.217j) \text{ V}$$

$$U_g := U_h + I_1 \cdot (j \cdot XL_1) = (154.269 + 183.851j) \text{ V}$$





Приймаємо активний опір другої вітки рівним нулю і за умови резонансу струмів розраховуємо реактивний опір цієї вітки.

$$Y := \frac{1}{Z3 + \frac{Z4 \cdot Z5}{Z4 + Z5}} = (0.018 - 0.038j) \text{ S}$$

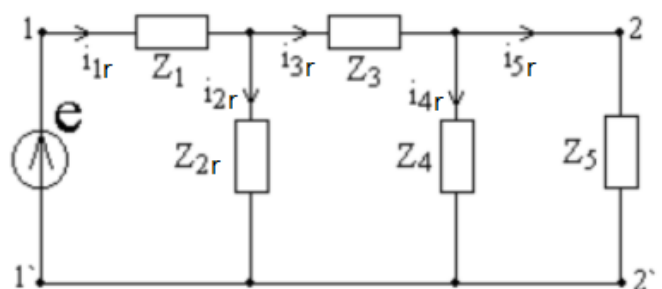
-> за умовою резонансу.

$$XL2r := \frac{1}{\text{Im}(Y)} = -26.022 \text{ } \Omega$$

-> опір котушки L2.

$$Z2r := j \cdot XL2r = -26.022j \text{ } \Omega$$

-> опір вітки 2.



Обчислимо струми віток за умови резонансу:

$$Z_r := Z_1 + \frac{\left(Z_3 + \frac{Z_4 \cdot Z_5}{Z_4 + Z_5} \right) \cdot Z_{2r}}{Z_3 + \frac{Z_4 \cdot Z_5}{Z_4 + Z_5} + Z_{2r}} = (72.799 + 20j) \, \Omega$$

$$Z_r = (75.496 \angle 15.362^\circ) \, \Omega$$

$$I_{1r} := \frac{E \cdot e^{j \cdot \psi}}{Z_r} = (2.616 + 1.807j) \, A$$

$$I_{1r} = (3.179 \angle 34.638^\circ) \, A$$

$$I_{3r} := I_{1r} \cdot \frac{Z_{2r}}{Z_{2r} + Z_3 + \frac{Z_4 \cdot Z_5}{Z_4 + Z_5}} = (6.421 - 3.701j) \, A$$

$$I_{3r} = (7.411 \angle -29.961^\circ) \, A$$

$$I_{2r} := I_{1r} - I_{3r} = (-3.805 + 5.508j) \, A$$

$$I_{2r} = (6.695 \angle 124.638^\circ) \, A$$

$$I_{4r} := I_{3r} \cdot \frac{Z_5}{Z_4 + Z_5} = (3.629 - 4.393j) \, A$$

$$I_{4r} = (5.698 \angle -50.443^\circ) \, A$$

$$I_{5r} := I_{3r} - I_{4r} = (2.792 + 0.692j) \, A$$

$$I_{5r} = (2.877 \angle 13.916^\circ) \, A$$

Баланс активних і реактивних потужностей:

$$U := E \cdot e^{j \cdot \psi} = (154.269 + 183.851j) \, V$$

$$U = (240 \angle 50^\circ) \, V$$

$$S_r := U \cdot \overline{I_{1r}} = (735.695 + 202.118j) \, W$$

$$S_r = (762.954 \angle 15.362^\circ) \, W$$

$$P_r := |I_{1r}|^2 \cdot R_1 + |I_{4r}|^2 \cdot R_3 + |I_{5r}|^2 \cdot R_4 = 735.695 \, W$$

$$Q_r := |I_{2r}|^2 \cdot X_{L2r} + |I_{1r}|^2 \cdot (X_{L1} - X_{C1}) + |I_{3r}|^2 \cdot (X_{L3} - X_{C2}) - |I_{5r}|^2 \cdot X_{C3} = 202.118 \, W$$

Розрахуємо покази вольтметра:

$$U_{complex} := I_{3r} \cdot (j \cdot X_{L3}) - I_{5r} \cdot (j \cdot X_{C3}) = (183.845 + 219.127j) \, V$$

$$U_v := \sqrt{\text{Re}(U_{complex})^2 + \text{Im}(U_{complex})^2} = 286.034 \, V$$

Суміщена діаграма струмів і напруг

Визначимо потенціали точок кола

$$U_{ar} := 0 \, V$$

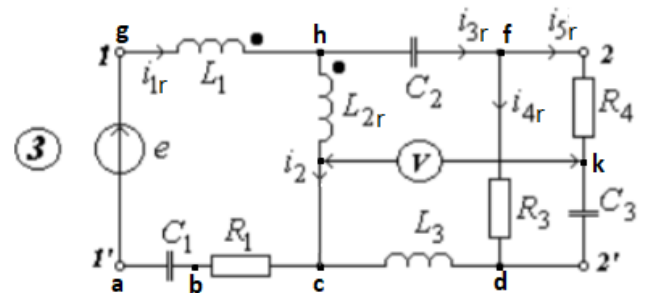
$$U_{br} := U_{ar} - I_{1r} \cdot (j \cdot X_{C1}) = (27.104 - 39.233j) \, V$$

$$U_{cr} := U_{br} + I_{1r} \cdot R_1 = (74.183 - 6.709j) \, V$$

$$U_{dr} := U_{cr} + I_{3r} \cdot (j \cdot X_{L3}) = (240.733 + 282.221j) \, V$$

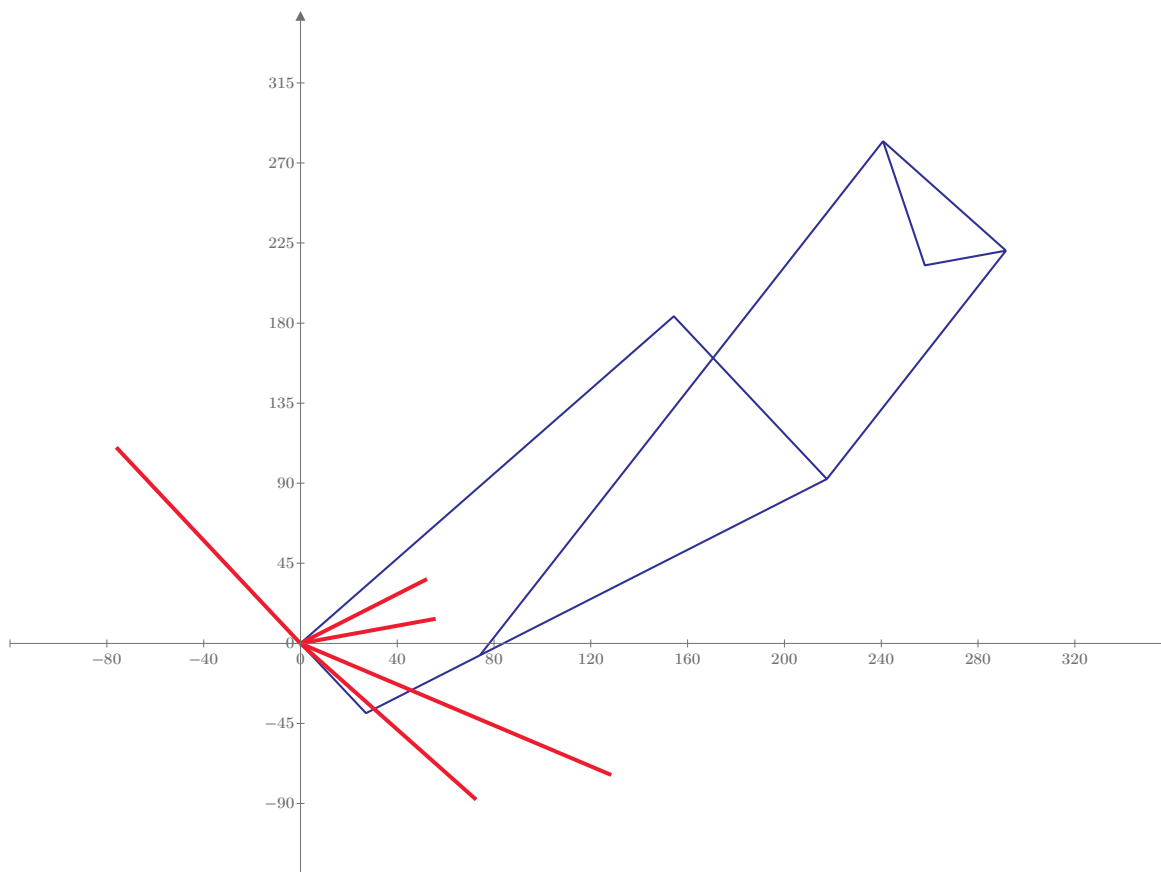
$$U_{kr} := U_{dr} - I_{5r} \cdot (j \cdot X_{C3}) = (258.028 + 212.419j) \, V$$

$$U_{fr} := U_{kr} + I_{5r} \cdot R_4 = (291.533 + 220.72j) \, V$$



$$U_{hr} := U_{cr} + I_{2r} \cdot (j \cdot X_{L2r}) = (217.511 + 92.307j) \, V$$

$$U_{gr} := U_{hr} + I_{1r} \cdot (j \cdot X_{L1}) = (154.269 + 183.851j) \, V$$



$Y1r$ (V)

$Y2r$ (V)

$Y3r$ (V)

$yI1r$ (A)

$yI2r$ (A)

$yI3r$ (A)

$yI4r$ (A)

$yI5r$ (A)

$X1r$ (V)

$X2r$ (V)

$X3r$ (V)

$xI1r$ (A)

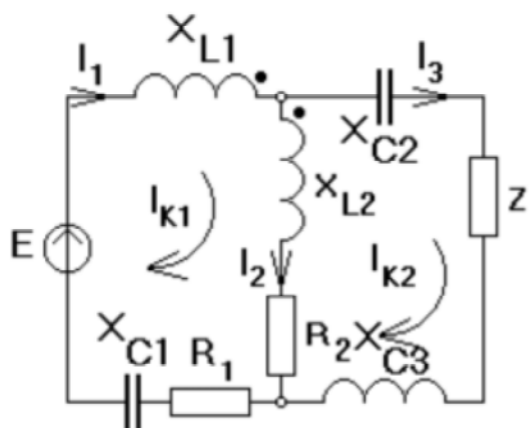
$xI2r$ (A)

$xI3r$ (A)

$xI4r$ (A)

$xI5r$ (A)

Розрахунок струмів за наявності магнітного зв'язку між L1 та L2



$$Z_e := \frac{R3 \cdot (R4 - j \cdot XC3)}{R3 + R4 - j \cdot XC3} = (10.083 - 3.766j) \, \Omega$$

Обчислюємо опори:

$$Z11 := R1 + R2 + j \cdot (XL1 - XC1 + XL2 - 2 \cdot Xm) = (34 + 14j) \, \Omega$$

$$Z22 := R2 + j \cdot (XL2 - XC2 + XL3) + Z_e = (26.083 + 61.234j) \, \Omega$$

$$Z12 := R2 + j \cdot (XL2 - Xm) = (16 + 17j) \, \Omega$$

$$Z21 := Z12 = (16 + 17j) \, \Omega$$

$$\begin{cases} Ek1 = Ik1 \cdot Z11 - Ik2 \cdot Z12 \\ Ek2 = -Ik1 \cdot Z21 + Ik2 \cdot Z22 \end{cases}$$

Контурні ЕРС:

$$Ek1 := U = (154.269 + 183.851j) \, V$$

$$Ek2 := 0 \, V$$

$$\Delta := \left\| \begin{bmatrix} Z_{11} & -Z_{12} \\ -Z_{21} & Z_{22} \end{bmatrix} \right\| = (62.551 + 1.903j \cdot 10^3) \Omega^2$$

$$\begin{aligned} \Delta_1 &:= \left\| \begin{bmatrix} Ek_1 & -Z_{12} \\ Ek_2 & Z_{22} \end{bmatrix} \right\| = (-7.234 \cdot 10^3 + 1.424j \cdot 10^4) \frac{kg^2 \cdot m^4}{s^6 \cdot A^3} \\ \Delta_2 &:= \left\| \begin{bmatrix} Z_{11} & Ek_1 \\ -Z_{21} & Ek_2 \end{bmatrix} \right\| = (-657.157 + 5.564j \cdot 10^3) \frac{kg^2 \cdot m^4}{s^6 \cdot A^3} \end{aligned} \Rightarrow \begin{aligned} Ik_1 &:= \frac{\Delta_1}{\Delta} = (7.351 + 4.043j) A \\ Ik_2 &:= \frac{\Delta_2}{\Delta} = (2.909 + 0.441j) A \end{aligned}$$

$$I_{1m} := Ik_1 = (7.351 + 4.043j) A$$

$$I_{2m} := Ik_1 - Ik_2 = (4.441 + 3.602j) A$$

$$I_{3m} := Ik_2 = (2.909 + 0.441j) A$$

$$I_{4m} := I_{3m} \cdot \frac{R_4 - j \cdot XC_3}{R_3 + R_4 - j \cdot XC_3} = (2.214 - 0.465j) A$$

$$I_{5m} := I_{3m} \cdot \frac{R_3}{R_3 + R_4 - j \cdot XC_3} = (0.695 + 0.906j) A$$

Перевірка за I законом Кірхгофа

$$I_{1m} - I_{2m} - I_{3m} = 0 A$$

$$I_{3m} - I_{4m} - I_{5m} = (-4.441 \cdot 10^{-16} - 1.11j \cdot 10^{-16}) A$$

$$I_{2m} + I_{5m} + I_{4m} - I_{1m} = 0 A$$

Перевірка за II законом Кірхгофа

$$I_{1m} \cdot (R_1 + j \cdot (XL_1 - XC_1 - X_m)) - Ek_1 + I_{2m} \cdot (R_2 + j \cdot (XL_2 - X_m)) = 0 V$$

$$I_{2m} \cdot (R_2 + j \cdot (XL_2 - X_m)) - I_{4m} \cdot R_3 - I_{3m} \cdot j \cdot (-XC_2 + XL_3 + X_m) = 0 V$$

$$I_{4m} \cdot R_3 - I_{5m} \cdot (R_4 - j \cdot XC_3) = 0 V$$

Рахуємо потужності магнітного зв'язку

$$SM_1 := -I_{1m} \cdot \overline{I_{2m}} \cdot j \cdot X_m = (-195.968 - 1.086j \cdot 10^3) W$$

$$SM_2 := -I_{2m} \cdot \overline{I_{1m}} \cdot j \cdot X_m = (195.968 - 1.086j \cdot 10^3) W$$

Перевіряємо за балансом потужностей

$$S_m := U \cdot \overline{I_{1m}} = (1.877 \cdot 10^3 + 727.735j) W$$

$$P_m := |I_{1m}|^2 \cdot R_1 + |I_{2m}|^2 \cdot R_2 + |I_{4m}|^2 \cdot R_3 + |I_{5m}|^2 \cdot R_4 = (1.877 \cdot 10^3) W$$

$$Q_m := |I_{1m}|^2 \cdot (XL_1 - XC_1) + |I_{2m}|^2 \cdot XL_2 + |I_{3m}|^2 \cdot (XL_3 - XC_2) + |I_{5m}|^2 \cdot (-XC_3) + \text{Im}(SM_1 + SM_2)$$

$$Q_m = 727.735 W$$

Розрахуємо покази вольтметра:

$$U_{complexm} := -I_{2m} \cdot R_2 + I_{3m} \cdot (j \cdot XL_3) - I_{5m} \cdot (j \cdot XC_3) = (-68.253 + 55.903j) V$$

$$U_{vm} := \sqrt{\text{Re}(U_{complexm})^2 + \text{Im}(U_{complexm})^2} = 88.225 V$$

Суміщена діаграма струмів і напруг

Визначимо потенціали точок кола

$$U_{am} := 0 \text{ V}$$

$$U_{bm} := U_{am} - I_{1m} \cdot (j \cdot X_{C1}) = (60.642 - 110.259j) \text{ V}$$

$$U_{cm} := U_{bm} + I_{1m} \cdot R_1 = (192.952 - 37.489j) \text{ V}$$

$$U_{dm} := U_{cm} + I_{3m} \cdot (j \cdot X_{L3}) = (173.111 + 93.427j) \text{ V}$$

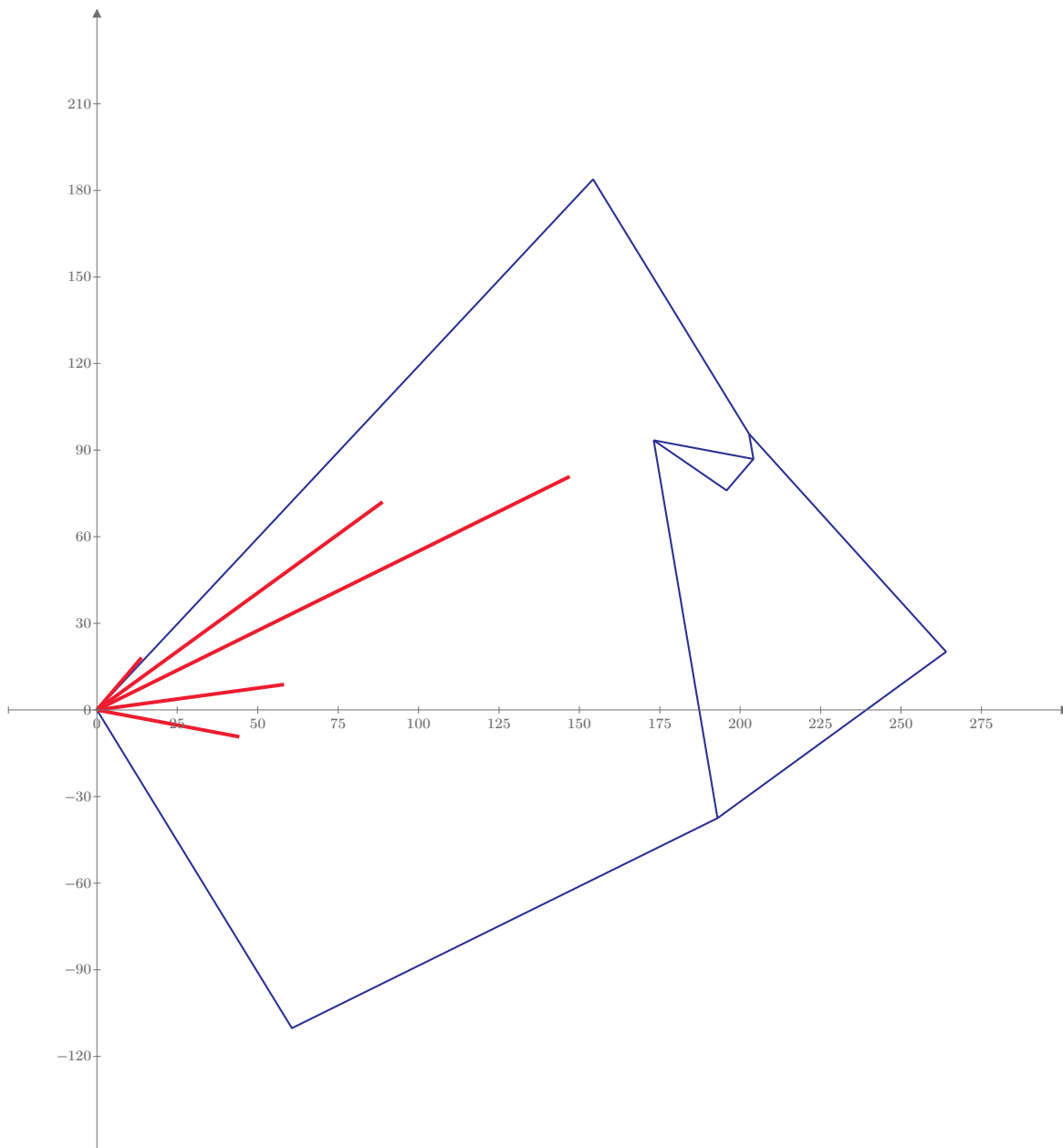
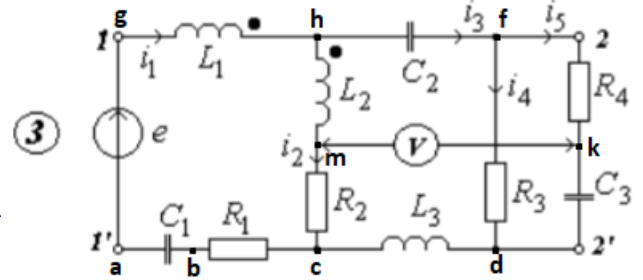
$$U_{km} := U_{dm} - I_{5m} \cdot (j \cdot X_{C3}) = (195.761 + 76.044j) \text{ V}$$

$$U_{fm} := U_{km} + I_{5m} \cdot R_4 = (204.105 + 86.916j) \text{ V}$$

$$U_{mm} := U_{cm} + I_{2m} \cdot R_2 = (264.014 + 20.141j) \text{ V}$$

$$U_{hm} := U_{mm} + I_{2m} \cdot (j \cdot (X_{L2} - X_m)) = (202.782 + 95.644j) \text{ V}$$

$$U_{gm} := U_{hm} + I_{1m} \cdot (j \cdot (X_{L1} - X_m)) = (154.269 + 183.851j) \text{ V}$$



Y1m (V)

Y2m (V)

Y3m (V)

yI1m (A)

yI2m (A)

yI3m (A)

yI4m (A)

yI5m (A)

X1m (V)

X2m (V)

X3m (V)

xI1m (A)

xI2m (A)

xI3m (A)

xI4m (A)

xI5m (A)