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- ,

$P(x)$, A , x ,

$A = \{x | P(x)\}.$

, 2

$A = \{x | x = 2^n, n \in N\}.$ 1.

a A , $a \in A.$ a

A , $a \notin A.$ $5 \in \{1,3,5,7\},$

$4 \notin \{1,3,5,7\}.$ $A = \{x |$ $\}_{21},$

\notin .

A ()

$B (A \subseteq B),$ A $B,$

$x \in A,$ $x \in B.$ $A \subseteq B$ $A \neq B,$ A

$A \subset B.$

$(A = B),$

$A \quad B$, $A \subseteq B$ $B \subseteq A.$

A

A $|A|.$

U ,

!

$A = \{1,3,6,13\},$ $3 \in A, 6 \in A,$ $\{3,6\} \notin A,$

$\{3,6\} \subseteq A.$

$A,$ $P(A).$

$A = \{a, b, c, d\}.$ $A.$

$P(A)?$

$P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\},$
 $\{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}.$

$|P(A)| = 16.$

1.2.

, . , $A \quad B$, A $B.$,

$A \cup B = \{x | x \in A \vee x \in B\}.$ $A \cup B.$:

_____. $A = \{2, 3, 5, 6, 7\}$, $B = \{1, 2, 3, 7, 9\}$. $A \cup B$.
 , : $A \cup B = \{1, 2, 3, 5, 6, 7, 9\}$.

_____.

$A \quad B$

_____ , _____ A _____ B . _____ A _____ :

$B \quad A \cap B$

$A \cap B = \{x | x \in A \text{ and } x \in B\}$.

_____. $A = \{2, 3, 5, 6, 7\}$, $B = \{1, 2, 3, 7, 9\}$. $A \cap B$.
 , : $A \cap B = \{2, 3, 7\}$.

_____.

(\quad)

_____ A _____ , _____ \bar{A} .
 : $\bar{A} = U - A = \{x | x \in U \text{ and } x \notin A\}$.

_____.

$A \quad B \quad (\quad)$

_____ , _____ A , _____ B

$A \quad B \quad A - B$

: $A - B = \{x | x \in A \text{ and } x \notin B\}$.

_____. $A = \{2, 3, 5, 6, 7\}$, $B = \{1, 2, 3, 7, 9\}$. $A - B$.
 , : $A - B = \{5, 6\}$.

_____.

$A \quad B$

_____ , _____ , _____ A _____ A .

$B \quad A + B$

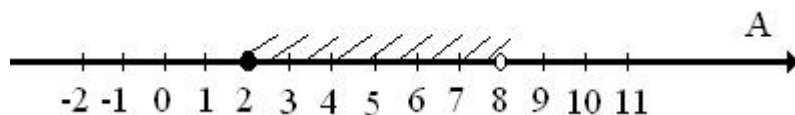
: $A \Delta B = (A - B) \cup (B - A)$.

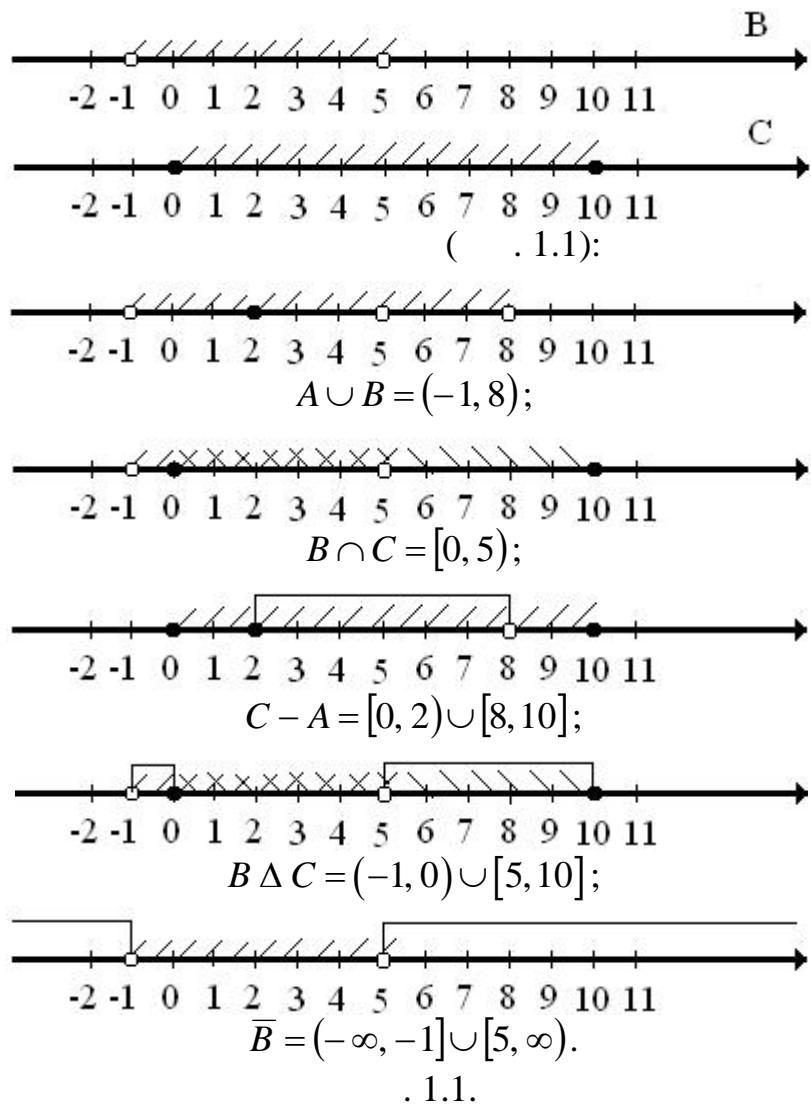
_____.

_____ , _____ , _____ , _____ , _____ , _____ .

_____. $A = \{2, 3, 5, 6, 7\}$, $B = \{1, 2, 3, 7, 9\}$. $A + B$.
 , : $A \Delta B = \{1, 5, 6, 9\}$.

_____. $A = [2, 8)$, $B = (-1, 5)$; $C = [0, 10]$. $A \cup B$, $B \cap C$, $C - A$,
 $B \Delta C$, \bar{B} .
 , :



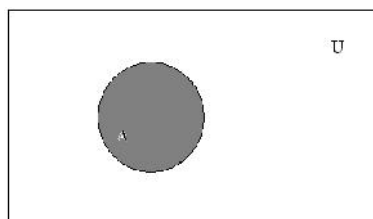


1.3.

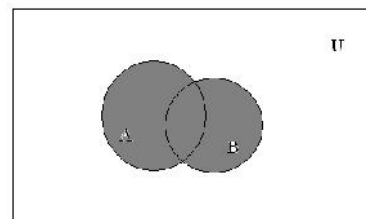
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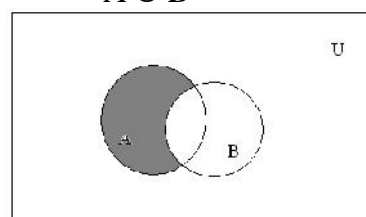
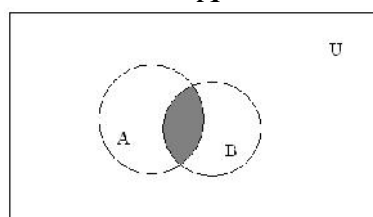
. 1.2

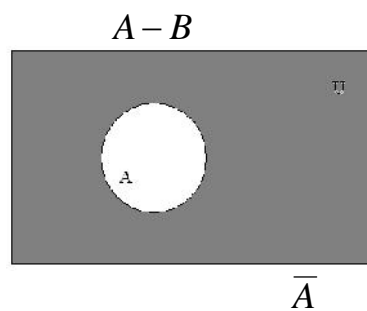
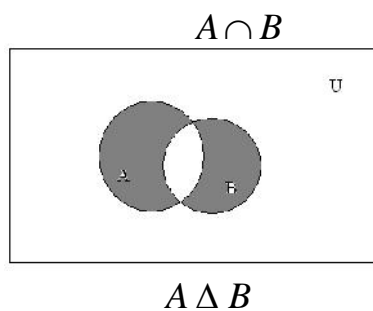


A



$A \cup B$





. 1.2.

- A, B, C U
:

1.4.

1. $X \cup Y = Y \cup X$	1. $X \cap Y = Y \cap X$
2. $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$	2. $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$
3. $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$	3. $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$
4. $X \cup \emptyset = X$ $X \cup \bar{X} = U$ $X \cup U = U$	4. $X \cap U = X$ $X \cap \bar{X} = \emptyset$ $X \cap \emptyset = \emptyset$
5. $X \cup X = X$	5. $X \cap X = X$
6. $\overline{X \cup Y} = \bar{X} \cap \bar{Y}$	6. $\overline{X \cap Y} = \bar{X} \cup \bar{Y}$
7. $X \cup (X \cap Y) = X$	7. $X \cap (X \cup Y) = X$
8. $(X \cap Y) \cup (X \cap \bar{Y}) = X$	8. $(X \cup Y) \cap (X \cup \bar{Y}) = X$
9.	9.

$X \cup (\bar{X} \cap Y) = X \cup Y$	$X \cap (\bar{X} \cup Y) = X \cap Y$
10.	$\overline{\overline{X}} = X$

1.5.

1.

Lazarus.

2.

LAB1_Project,

3.

4.

OperForm

5.

OperForm

,

1.

2.

3.

4.

5.

6.

7.

8.

1.6.

1.

?

2.

3.

4.

5.

B={2300,25,1}?

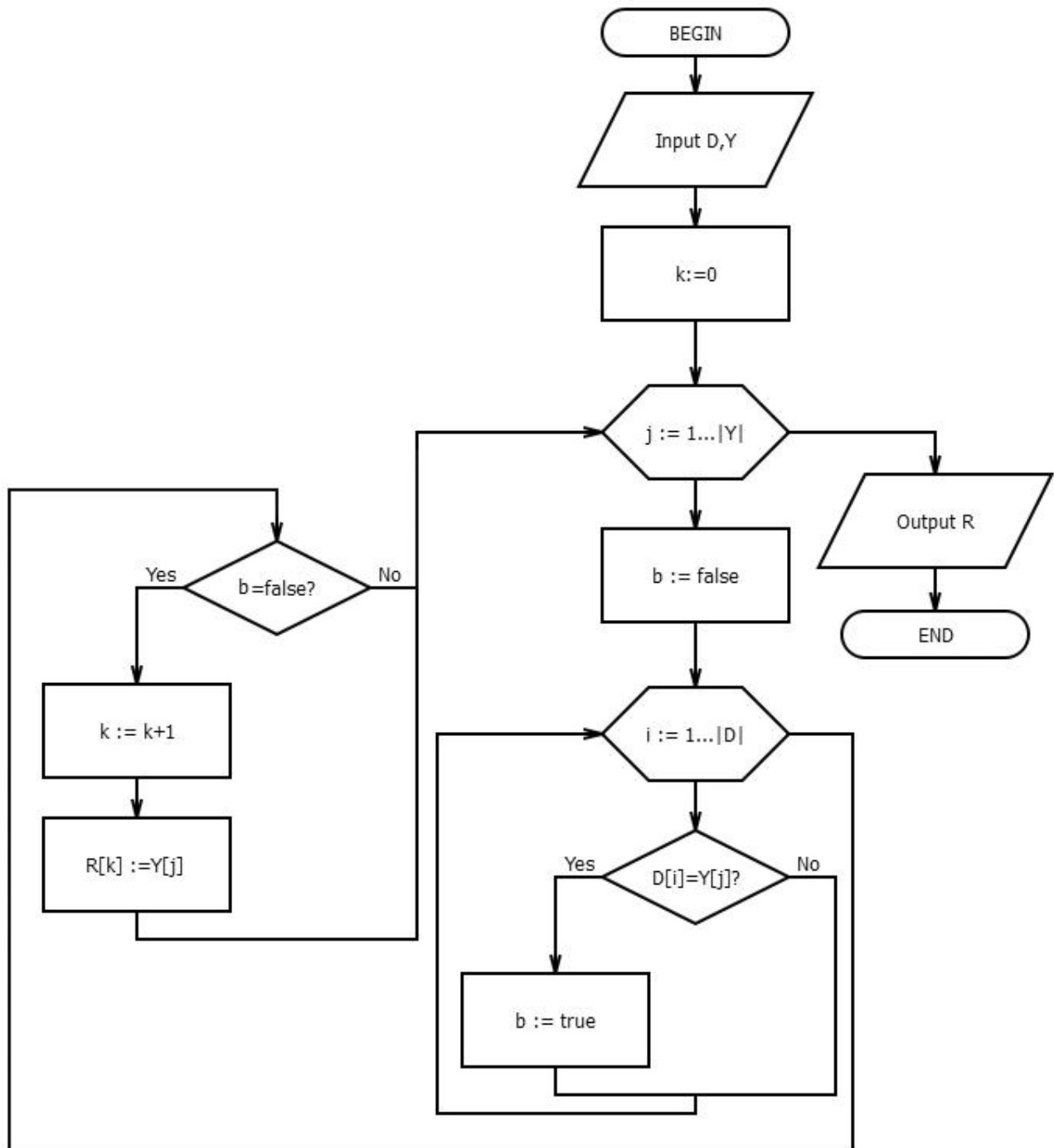
6.

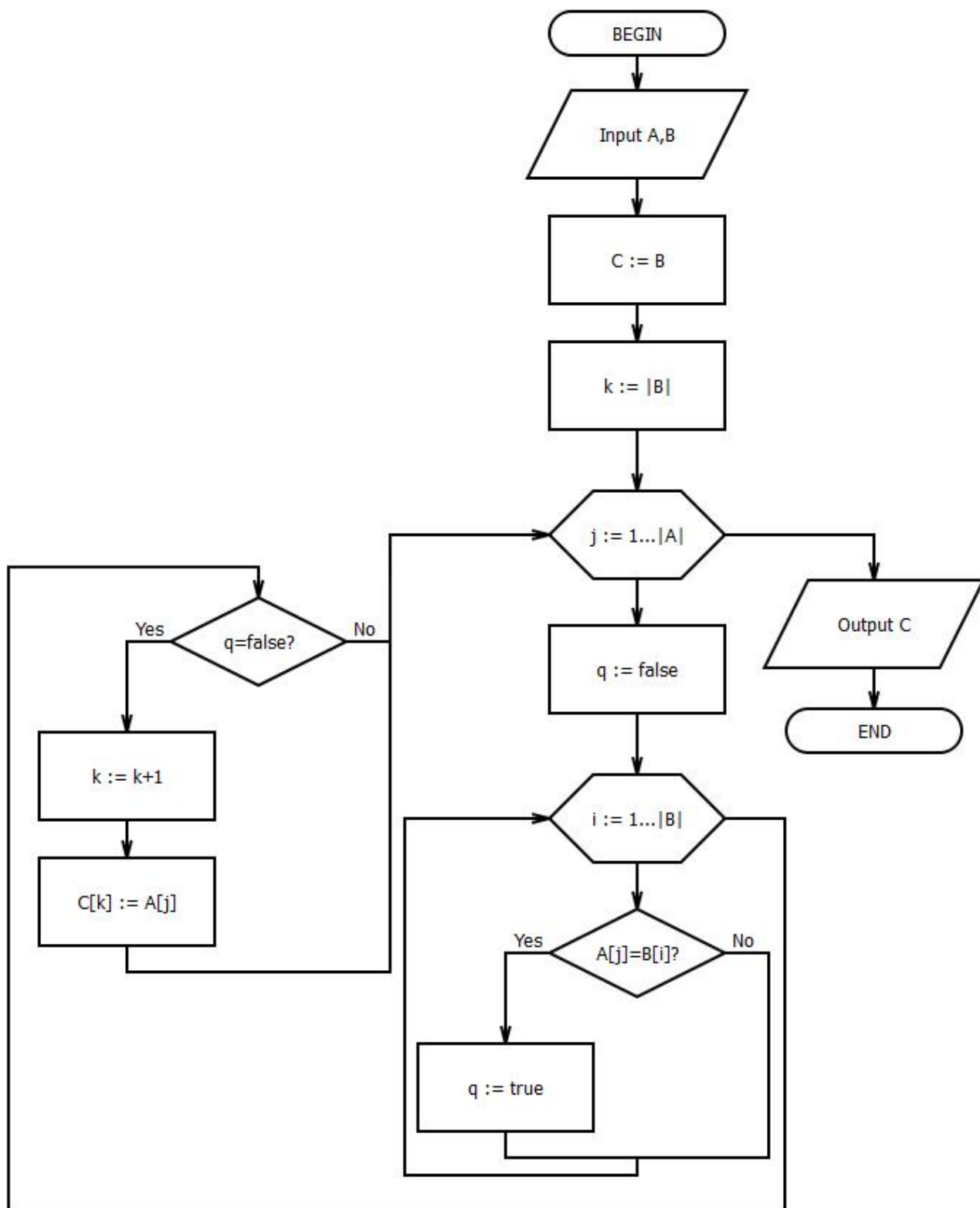
7.

8.

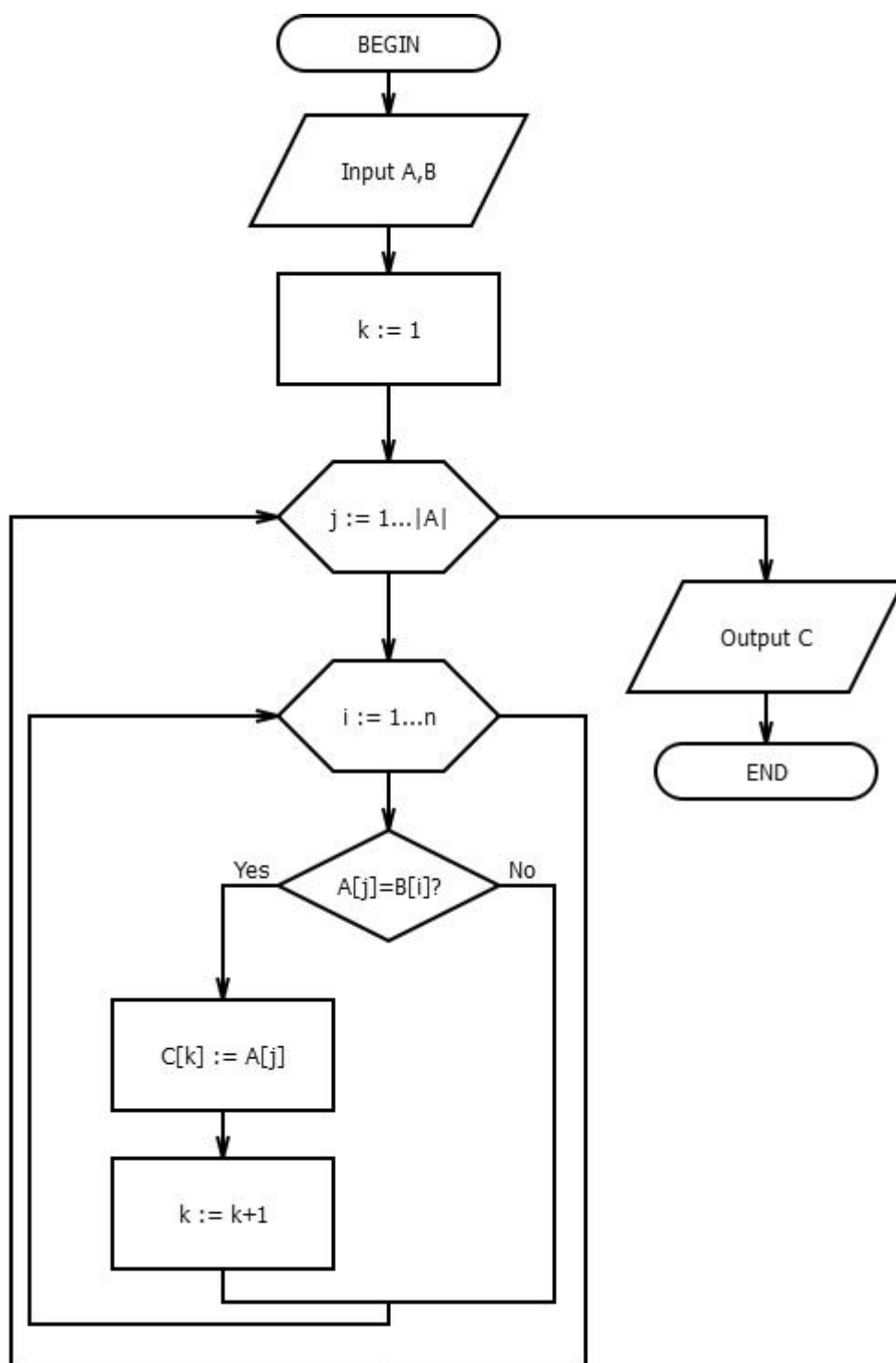
$A = \{1, 54, 12, 45, 11, 34\}$ $B = \{2, 11, 12, 13, 45, 54, 34\}$
: $C = \{1, 2, 13\}$.

1.7. -

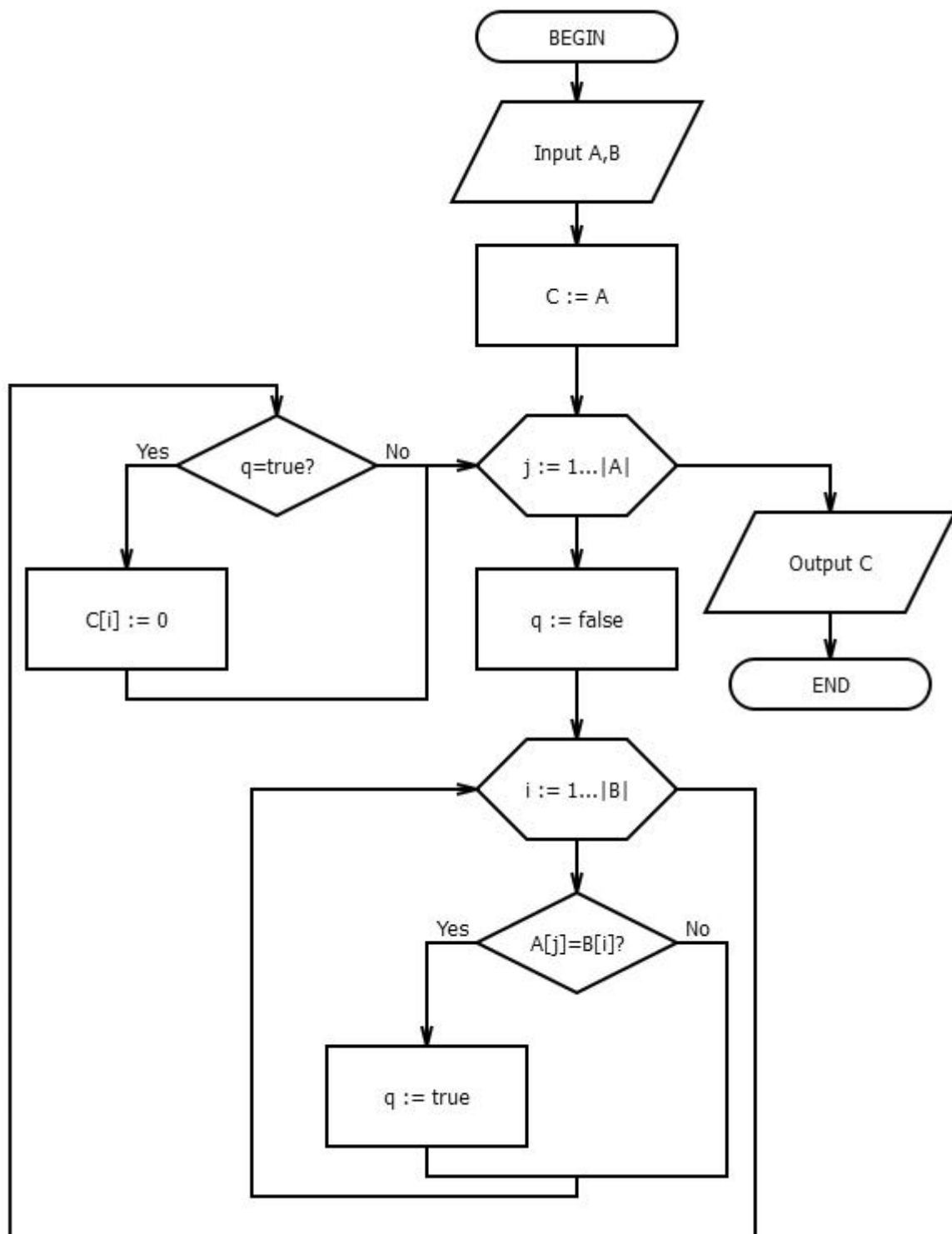




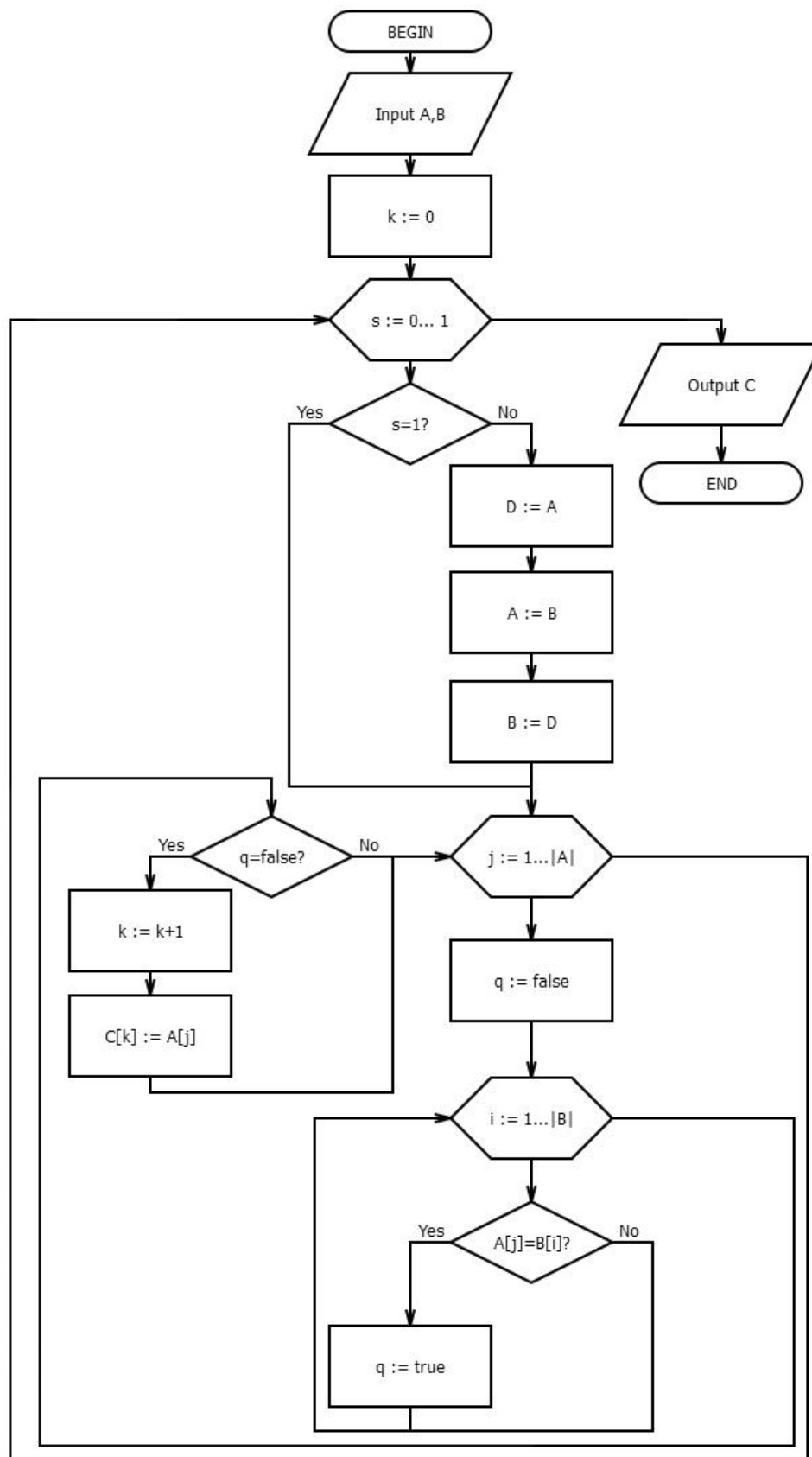
.1.4.



. 1.5. -



. 1.6. -



. 1.7. -

1.8.

) - .1.3, D
 - Y. Y R.
 R.
) - . 1.4
 - , A B.
 .
) - .1.5, A .
 - , .
 .
) - .1.6, .
 - , .
 .
) - .1.7,
 - , .
 .
) .
 .

NZK – I I = NZK mod 10,

:

0. ASCII

1. 0...255

2.

3.

4.

5. 0...1024

6. 0...1024

7. 0...1024

8. 0...1024, 5

9. 0...1024, 3

_____ : « _____ , _____ ».

_____ :

_____ :

_____ :

2.1.

_____ — _____ , _____ .

_____ , _____ :

_____) — _____ x _____ y _____ , _____ ,

$\langle x, y \rangle$, _____ ;

_____) $\langle x, y \rangle$ $\langle u, v \rangle$ — _____ , $\langle x, y \rangle = \langle u, v \rangle$

_____ , $x = u$, $y = v$.

_____ x _____ , _____ y —

_____ $\langle x, y \rangle$.

_____ (_____) _____ R

_____ , _____ , _____ $\langle x, y \rangle \in R$ _____ xRy .

_____ — _____ $\langle x, y \rangle$, _____ x

_____ X , _____ y —

_____ Y .

_____ $X \times Y$ _____ X _____ Y _____ .

$\{\langle x, y \rangle | x \in X, y \in Y\}$.

_____ X _____ R ,

_____ Y — _____ :

$D(R) = \{x | \langle x, y \rangle \in R\}$; $E(R) = \{y | \langle x, y \rangle \in R\}$

_____ R _____ $\langle x, y \rangle \in R$

_____ $X \times Y$, _____ $R \subseteq X \times Y$.

_____ , _____ , _____ :

1. (\quad) ,
2. $\quad - \quad R \subseteq X \times X, \quad X = \{x_1; x_2; \dots; x_n\}$
 $n, \quad a_{ij} \quad 1,$
 $x_i \quad x_j \quad R, \quad 0, \quad :$

$$a_{ij} = \begin{cases} 1, & x_i R x_j, \\ 0, & \end{cases}.$$

$$A = \{1, 2, 3\}, \quad B = \{2, 3, 4\}.$$

$$A \times B \quad B \times A. \quad (A \times B) - (B \times A), \quad (A \times B) \cap (B \times A), \quad (A \times B) + (B \times A).$$

$$: A \times B = \{\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 2, 4 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 3, 4 \rangle\};$$

$$B \times A = \{\langle 2, 1 \rangle, \langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 1 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle, \langle 4, 1 \rangle, \langle 4, 2 \rangle, \langle 4, 3 \rangle\};$$

$$(A \times B) - (B \times A) = \{\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 4 \rangle, \langle 3, 4 \rangle\};$$

$$(A \times B) \cap (B \times A) = \{\langle 2, 2 \rangle, \langle 2, 3 \rangle, \langle 3, 2 \rangle, \langle 3, 3 \rangle\};$$

$$(A \times B) + (B \times A) = \{\langle 1, 2 \rangle, \langle 1, 3 \rangle, \langle 1, 4 \rangle, \langle 2, 4 \rangle, \langle 3, 4 \rangle, \langle 2, 1 \rangle, \langle 3, 1 \rangle, \langle 4, 1 \rangle, \langle 4, 2 \rangle, \langle 4, 3 \rangle\}.$$

2.2.

1. $R \quad A \times A$,
 $\langle a, a \rangle \in R \quad a \in A.$
2. $R \quad A \times A$,
 $a \in A \quad \langle a, a \rangle \in R, \quad \langle a, b \rangle \in R, \quad a \neq b.$
3. $R \quad A \times A$,
 $a, b \in R \quad \langle a, b \rangle \in R, \quad \langle b, a \rangle \in R.$
 $c_{ij} = c_{ji} \quad i \quad j.$
4. $R \quad A \times A$,
 $a, b \in R, \quad \langle a, b \rangle \in R \quad \langle b, a \rangle \in R, \quad a = b,$
 $a \quad b, \quad (a \neq b),$
 $\langle a, b \rangle \in R \quad \langle b, a \rangle \in R.$
5. $R \quad A \times A$,
 $a, b, c \quad \langle a, b \rangle \in R \quad \langle b, c \rangle \in R \quad \langle a, c \rangle \in R.$
 \vdots
 $j-$, $c_{ij} = 1,$ $j-$
 $k-$ $(c_{jk} = 1)$ $i-$
 $k-$, $c_{ik} = 1$ (, ,).

6.

2.3.

$R \subseteq A \times B$,

1. $R_1 \cap R_2 = \{ \langle a, b \rangle \mid \langle a, b \rangle \in R_1 \text{ and } \langle a, b \rangle \in R_2 \}$.

2. $R_1 \cup R_2 = \{ \langle a, b \rangle \mid \langle a, b \rangle \in R_1 \text{ or } \langle a, b \rangle \in R_2 \}$.

3. $R_1 - R_2 = \{ \langle a, b \rangle \mid \langle a, b \rangle \in R_1 \text{ and } \langle a, b \rangle \notin R_2 \}$.

4. $\bar{R} = U - R, \quad U = A \times B$.

5. R^{-1} .

$\langle a, b \rangle \in R \iff \langle b, a \rangle \in R^{-1}$.

$R^{-1} = \{ \langle b, a \rangle \mid \langle a, b \rangle \in R \}$.

“ R – “ $\langle a, b \rangle \in R$ ””, R^{-1} – “ $\langle b, a \rangle \in R$ ””;

$R = \{ \langle a, b \rangle \mid b \in A, a \in B \}$ $R = \{ \langle x, y \rangle \mid x^2 + y^2 = 1 \}$.

$R = R^{-1}$.

$R \subseteq A \times B \iff A \times B, \quad S \subseteq B \times C \iff B \times C$.

$T = \{ \langle a, c \rangle \mid \langle a, b \rangle \in R, \langle b, c \rangle \in S \}$.

$T = S \circ R$.

$R \circ R = R^{(2)} = \{ \langle a, c \rangle \mid \langle a, b \rangle, \langle b, c \rangle \in R \}$.

R^0 – “ $\langle a, b \rangle \in R^0$ ”

$n+2 \ (n \geq 0)$ $A: a, c_1, c_2, \dots, c_n, b$,

$R: \langle a, c_1 \rangle \in R, \langle c_1, c_2 \rangle \in R, \dots, \langle c_n, b \rangle \in R$,

$R^0 = \{ \langle a, b \rangle \mid \langle a, c_1 \rangle, \langle c_1, c_2 \rangle, \dots, \langle c_n, b \rangle \in R \}$.

$R \circ R = R^{(2)}$ – “ $R \circ R = R^{(3)}$ ”

$R^0 = R$.

$$A, \quad R \quad R^0 \quad R \quad R^0$$

$R \in A \times A$:

- 1) $R_1 \leftarrow R$;
- 2) $R_1 \cup R_1^{(2)} = R_1 \cup R_1, \quad R_2 \leftarrow R_1^{(2)}$;
- 3) $R_1 \quad R_2. \quad R_1 = R_2, \quad 4, \quad R_1 \neq R_2,$
 $R_1 \leftarrow R_2 \quad 2;$
- 4) $R_1 = R_2 = R^0.$
- 8.

$$E = \{\langle a, a \rangle \mid a \in A\}.$$

$$R^* = R^0 \cup E.$$

$$R, \quad R^* = R.$$

$$A, \quad R^* \quad R.$$

2.4.

$$) \quad S \cup R, \quad S \times R, \quad R^{-1}, \quad S \times R^{-1}, \quad ($$

$$, \quad , \quad , \quad)$$

:

1.
Lazarus.

2. LAB2_Project,

3.

4. OperForm

5. OperForm ,

6. S R .

7.

8. ,

A B.

- 1.
- 2.
- 3.
- 4.
- 4.
- 5.
- 6.

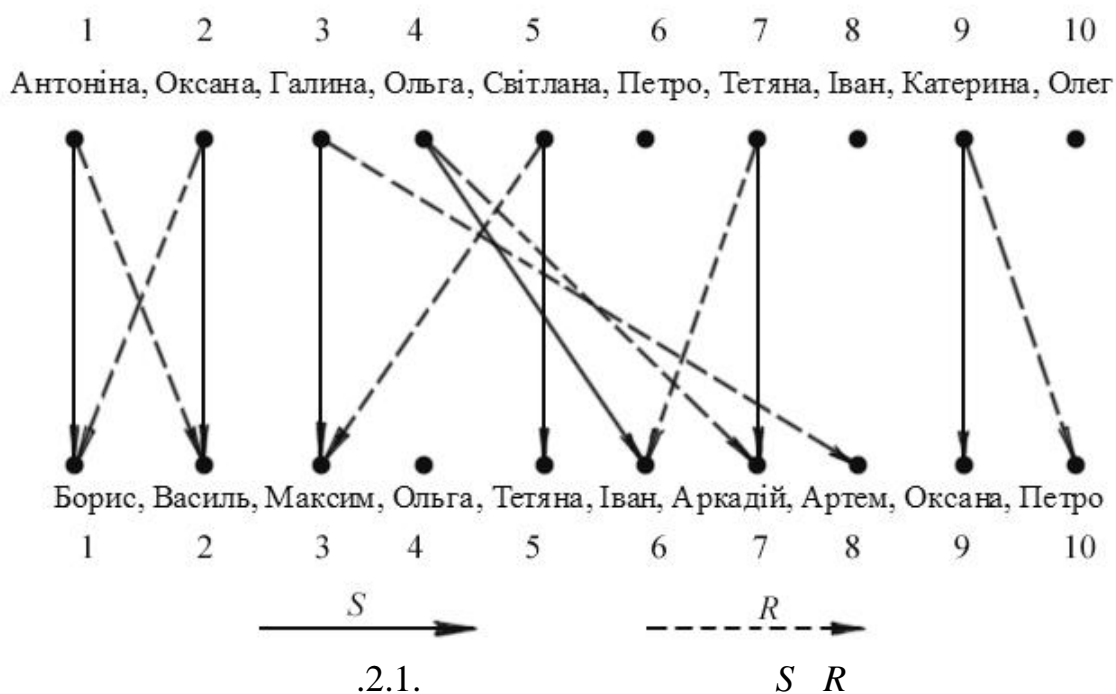
- 1.
- 2.
- 3.
- 4.

1.

2. $A \cap B = \emptyset$, $A \cup B = S$, $A = \{1, 2, 3, 4, 5\}$, $B = \{6, 7, 8, 9, 10\}$.

3. $A \cap B = \{1, 2, 3\}$, $A \cup B = S$, $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3, 6, 7, 8, 9, 10\}$.

aSb, a b. aRb, a b



5.

$$\begin{aligned} S &= \{\langle 1,1 \rangle, \langle 2,2 \rangle, \langle 3,3 \rangle, \langle 4,6 \rangle, \langle 5,5 \rangle, \langle 7,7 \rangle, \langle 9,9 \rangle\} - \\ R &= \{\langle 1,2 \rangle, \langle 2,1 \rangle, \langle 3,8 \rangle, \langle 4,7 \rangle, \langle 5,3 \rangle, \langle 7,6 \rangle, \langle 9,10 \rangle\} - \end{aligned}$$

3.1.

n ,
 ,
 n , n — ,
 .
 , , .
 : , , , CAB , CBA , (6).

$$n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n, \quad 0! = 1$$
$$m_n = n(n-1)(n-2)...(n-m+1), \quad 0 \leq m \leq n; \quad m, n \in N.$$
$$\text{ , } \quad {}^0_n = I \text{ .}$$

• M , , D .
 , : , AC , AD , , BD , ,
 , CD , DA , DB , DC .

•

$$\binom{n}{m} = \frac{n!}{(n-m)!}.$$

:

$$1) \quad \frac{m+1}{n} = A_n^m \cdot (n-m);$$

$$2) \quad \frac{n}{n} = P_n = n!.$$

$$n \qquad m,$$

$$(m, n \in N \quad n \geq m).$$

$$\frac{m}{n} = \frac{A_n^m}{P_m} \cdot \qquad n \qquad m, \qquad m \qquad m \qquad :$$

$$\frac{m}{n} = \frac{m!}{(m-n)!} \quad P_n = n!,$$

$$: \quad C_n^m = \frac{n!}{(n-m)! \, m!}.$$

$$: C_n^{n-m} = \frac{P_n}{P_{n-m} \cdot P_m} = \frac{n!}{(n-m)! \, m!}; \quad C_n^m = C_n^{n-m}.$$

$$\cdot \qquad \qquad \qquad , \quad , \quad , \quad D.$$

$$\qquad \qquad \qquad , \qquad \qquad \qquad .$$

$$, \quad AC, \quad AD, \qquad , \quad BD, \quad CD. \qquad \qquad ,$$

$$6. \qquad \qquad \qquad : \quad \frac{2}{4} = 6.$$

$$n \qquad k$$

$$, \quad / \quad /=n.$$

$$\cdot \qquad \qquad \qquad , \qquad \qquad \qquad = \{A, \quad \}$$

$$: \quad , \quad , \quad , \quad , \quad , \quad ,$$

$$, \quad , \quad . \qquad n = 2, \quad k = 3. \qquad \qquad k-$$

$$k. \qquad \qquad \qquad (\qquad \qquad \qquad)$$

$$,$$

$$\cdot$$

$$\overline{k}_n = \underbrace{n \cdot n \dots n}_k = n^k$$

$$n! \quad ,$$

$$P[i], \quad i=1,2,\dots,n$$

vrem –

”

”

X.

,

X.

,

•

()

()	()
1 2 3	1 2 3
1 3 2	2 1 3
2 1 3	1 3 2
2 3 1	3 1 2
3 1 2	2 3 1
3 2 1	3 2 1

3.2.1.

- , $(1, 2, \dots, n-1, n)$.
- $(x_1, x_2, \dots, x_{n-1}, x_n)$
- (y_1, y_2, \dots, y_n)
- $(n, n-1, \dots, 2, 1)$.
- $(y_1, y_2, \dots, y_{n-1}, y_n)$.
- (1, 2, 3)** **I**
11. $x = (x_1, x_2, \dots, x_{n-1}, x_n)$
- i , $x_i < x_{i+1}$.
- $x_1 > x_2 > \dots > x_n$, $x = (n, n-1, \dots, 1)$.
- .
- $x = (1, 2, 3)$. $x_1 = 1$, $x_2 = 2$, $x_3 = 3$.
- $x_2 < x_3$ $2 < 3$.
- $x_i < x_{i+1}$ $i = 2$
12. i , $x_i < x_{i+1} > x_{i+2} > \dots > x_n$.
- $x = (1, 2, 3)$ $i = 2$ $2 < 3 > \emptyset$ 3
- .
13. j
- n i , $x_i < x_j$. $i < j$.
- $x = (1, 2, 3)$. $j = 3$ $x_2 < x_3$. $2 < 3$.
14. x_i x_j
- $x' = (x'_1, x'_2, \dots, x'_n)$.
- $x = (1, 2, 3)$. x_2 x_3 .
- $x = (1, 3, 2)$
15. $x_{i+1}, \dots, x_{n-1}, x_n$,
- .

$$y = (y_1, y_2, \dots, y_n).$$

$$. \quad x = (1, 3, 2).$$

$$(x_3),$$

$$x = (1, 3, 2).$$

$$2$$

$$(1, 3, 2) \quad 2.$$

$$21.$$

$$x = (x_1, x_2, \dots, x_{n-1}, x_n)$$

$$i, \quad x_i < x_{i+1}.$$

$$x_1 > x_2 > \dots > x_n, \quad x = (n, n-1, \dots, 1).$$

$$. \quad x = (1, 3, 2). \quad x_1 = 1, \quad x_2 = 3, \quad x_3 = 2.$$

$$x_1 < x_2 \quad 1 < 3.$$

$$x_i < x_{i+1} \quad i = 1$$

$$22.$$

$$i, \quad x_i < x_{i+1} > x_{i+2} > \dots > x_n.$$

$$. \quad x = (1, 2, 4, 3) \quad i = 1 \quad 1 < 3 > 2.$$

$$23.$$

$$j$$

$$n \quad i, \quad x_i < x_j. \quad i < j.$$

$$. \quad x = (1, 3, 2). \quad j = 3 \quad x_1 < x_3. \quad 1 < 2.$$

$$24.$$

$$x_i \quad x_j$$

$$x' = (x'_1, x'_2, \dots, x'_n).$$

$$. \quad x = (1, 3, 2).$$

$$x_1 \quad x_3.$$

$$x = (2, 3, 1)$$

$$25.$$

$$x_{i+1}, \dots, x_{n-1}, x_n,$$

$$y = (y_1, y_2, \dots, y_n).$$

$$. \quad x = (2, 3, 1).$$

$$(x_2, x_3).$$

$$x = (2, 1, 3).$$

$$3$$

$$(2, 1, 3) \quad 3.$$

$$31.$$

$$x = (x_1, x_2, \dots, x_{n-1}, x_n)$$

$$i, \quad x_i < x_{i+1}.$$

$$x_1 > x_2 > \dots > x_n, \quad x = (n, n-1, \dots, 1).$$

$$\cdot$$

$$\cdot x = (2, 1, 3). \quad x_1 = 2, \quad x_2 = 1, \quad x_3 = 3.$$

$$) \quad x_2 < x_3 \Rightarrow 1 < 3. \quad i = 2. \quad C, \\ 1 < 3.$$

$$32. \quad i, \quad x_i < x_{i+1} > x_{i+2} > \dots > x_n. \\ \cdot x = (2, 1, 3) \quad i = 2 \quad 1 < 3 > \emptyset.$$

$$33. \quad j \\ n \quad i, \quad x_i < x_j. \quad i < j. \\ \cdot x = (2, 1, 3). \quad j = 3 \quad x_2 < x_3. \quad 1 < 3.$$

$$34. \quad x_i \quad x_j \\ x' = (x'_1, x'_2, \dots, x'_n). \\ \cdot x = (2, 1, 3). \quad x_2 \quad x_3. \\ x = (2, 3, 1)$$

$$35. \quad x_{i+1}, \dots, x_{n-1}, x_n, \\ y = (y_1, y_2, \dots, y_n). \\ \cdot \\ \cdot x = (2, 3, 1). \\ (x_3). \quad x = (2, 3, 1). \\ 4.$$

$$(2, 3, 1) \quad 4.$$

$$41. \quad x = (x_1, x_2, \dots, x_{n-1}, x_n)$$

$$i, \quad x_i < x_{i+1}. \\ x_1 > x_2 > \dots > x_n, \quad x = (n, n-1, \dots, 1).$$

$$\cdot$$

$$\cdot x = (2, 3, 1). \quad x_1 = 2, \quad x_2 = 3, \quad x_3 = 1.$$

$$) \quad x_2 < x_3 \Rightarrow 3 > 1. \quad , \quad 3 > 1.$$

$$) \quad x_1 < x_2 \Rightarrow 2 < 3. \quad , \quad 2 < 3.$$

$$i = 1.$$

$$42. \quad i, \quad x_i < x_{i+1} > x_{i+2} > \dots > x_n. \\ \cdot x = (2, 3, 1) \quad i = 1 \quad 2 < 3 > 1.$$

$$43. \quad \begin{matrix} n & i & , & x_i < x_j . & i < j . & j \\ . & x = (2, 3, 1) . & j = 2 & x_1 < x_2 . & 2 < 3 . \end{matrix}$$

$$44. \quad \begin{matrix} x_i & x_j \\ x' = (x'_1, x'_2, \dots, x'_n) . \\ . & x = (2, 3, 1) . \\ x = (3, 2, 1) & x_2 & x_3 . \end{matrix}$$

$$45. \quad \begin{matrix} x_{i+1}, \dots, x_{n-1}, x_n , \\ . \\ y = (y_1, y_2, \dots, y_n) . \\ . \\ . & x = (3, 2, 1) . \\ (x_2, x_3) . & x = (3, 1, 2) . \\ 5 & . \end{matrix}$$

$$\begin{matrix} (3, 1, 2) & 5. \\ 51. & x = (x_1, x_2, \dots, x_{n-1}, x_n) \\ & i , \quad x_i < x_{i+1} . \\ x_1 > x_2 > \dots > x_n , & x = (n, n-1, \dots, 1) . \\ . \\ . & x = (3, 1, 2) . \quad x_1 = 3, \quad x_2 = 1, \quad x_3 = 2 . \\) & x_2 < x_3 \Rightarrow 1 < 2.C , \quad 1 < 2 . \\ i = 2 . \end{matrix}$$

$$52. \quad \begin{matrix} i , \quad x_i < x_{i+1} > x_{i+2} > \dots > x_n . \\ . & x = (3, 1, 2) \quad i = 2 \quad 1 < 2 > \emptyset . \end{matrix}$$

$$53. \quad \begin{matrix} j \\ n & i , \quad x_i < x_j . & i < j . \\ . & x = (3, 1, 2) . & j = 3 \quad x_2 < x_3 . \quad 1 < 2 . \end{matrix}$$

$$54. \quad \begin{matrix} x_i & x_j \\ x' = (x'_1, x'_2, \dots, x'_n) . \\ . & x = (3, 1, 2) . \\ x = (3, 2, 1) & x_2 & x_3 . \end{matrix}$$

$$55. \quad \begin{matrix} x_{i+1}, \dots, x_{n-1}, x_n , \\ . \end{matrix}$$

$$y = (y_1, y_2, \dots, y_n).$$

$$x = (3, 2, 1).$$

$$(x_3).$$

$$x = (3, 2, 1).$$

6

(3, 2, 1) 6.

61.

$$x = (x_1, x_2, \dots, x_{n-1}, x_n)$$

$$i, \quad x_i < x_{i+1}.$$

$$x_1 > x_2 > \dots > x_n, \quad x = (n, n-1, \dots, 1).$$

$$x = (3, 2, 1). \quad x_1 = 3, \quad x_2 = 2, \quad x_3 = 1.$$

$$x_i < x_{i+1}.$$

3.2.2.

1.

$n -$

$s -$

$P -$

2-3.

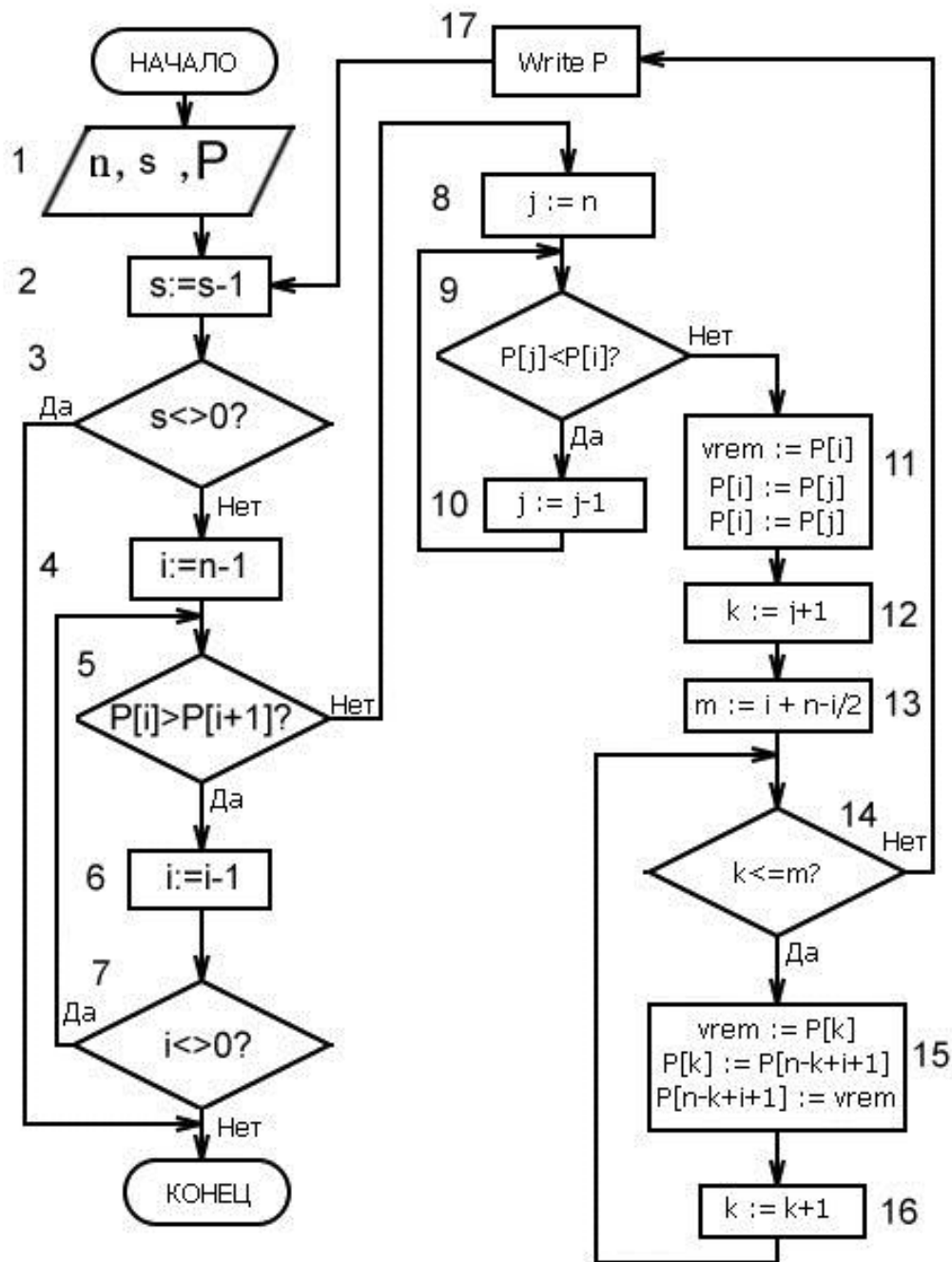
4-7.

8-10.

11.

12-17.

$$x_{i+1}, \dots, x_{n-1}, x_n$$



3.1. -

3.2.3.

n .

$$b = (b_{n-1}, b_{n-1}, \dots, b_1, b_0)$$

n

$b[n], b[n-1], \dots, b[1], b[0],$ $b[n] := 0.$
 $b = (0, 0, 0).$ $n = 2$ $b[2] = 0, b[1] = 0, b[0] = 0$

(0,0,0)

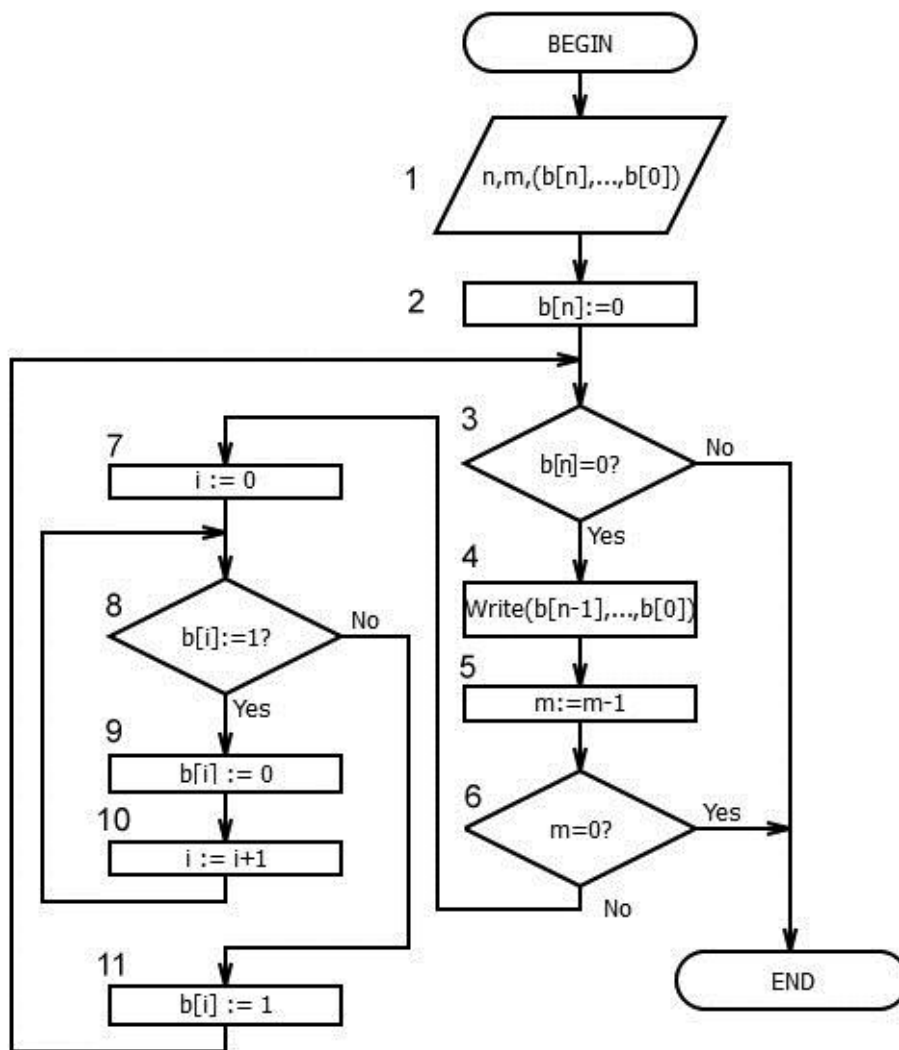
1.

1.1.

$b[i] = 0.$

$b[i]$

$\cdot b = (0,0,0), i = 0, b[0] := 0$
 1.2. $b[i] := 1, \quad b[j], j < i, \quad b[i],$
 $0.$
 $\cdot b[0] := 1. \quad i = 0, \quad \cdot$
(0,0,1) 2.
 2.1. $\quad b[i] \quad ,$
 $b[i] = 0.$
 $\cdot b = (0,0,1), i = 1, b[1] = 0.$
 2.2. $b[i] := 1, \quad b[j], j < i, \quad b[i],$
 $0.$
 $\cdot b[1] := 1, b[0] := 0.$
(0,1,0) 3.
 1.1. $\quad b[i] \quad ,$
 $b[i] = 0.$
 $\cdot b = (0,1,0), i = 0, b[0] := 0$
 1.2. $b[i] := 1, \quad b[j], j < i, \quad b[i],$
 $0.$
 $\cdot b[0] := 1. \quad i = 0, \quad \cdot$
(0,1,1) 4.
 2.1. $\quad b[i] \quad ,$
 $b[i] = 0.$
 $\cdot b = (0,1,1), i = 1, b[2] = 0.$
 2.2. $b[i] := 1, \quad b[j], j < i, \quad b[i],$
 $0.$
 $(1,1,\dots,1), i = n. \quad b[n] \quad b[n] = 1, \quad \cdot$
 $\cdot b[2] := 1. \quad n = 2, \quad b[n] = 1$
 \cdot
3.2.4. - $n.$
1.
 $n -$
 $m -$
 $(b[n-1], b[n-2], \dots, b[0]) -$
2. $b[n] := 0.$
 \cdot
3.
 \cdot



.3.2.

n.

4.

5.

6.

7.

8,9,10.

11.

1

11

3.2.5.

$$A = \{a_0, a_1, \dots, a_i, \dots, a_{n-1}\}.$$

$$a_n \in A.$$

$$B^i$$

$$A = \{a_0, a_1, \dots, a_i, \dots, a_{n-1}, a_n\}.$$

$$B^i$$

$$B^{i-1}$$

$$\cdot A = \{a_0, a_1, a_2\} \quad n = 2$$

$$B^0 = \emptyset.$$

$$\emptyset$$

$$1.$$

$$1.1.$$

$$i$$

$$\cdot a_0 \notin B^0.$$

$$1.2.$$

$$\cdot B^1 := B^0 \cup a_0.$$

$$B^0 -$$

$$B^1 = \{a_0\}$$

$$2.$$

$$2.1.$$

$$i$$

$$\cdot a_1 \notin B^1.$$

$$2.2.$$

$$\cdot a_1: B^2 := B^1 \cup a_1.$$

$$a_0: B^2 := B^2 \setminus \{a_0\}.$$

$$B^2 = \{a_1\}$$

$$3.$$

$$2.1.$$

$$i$$

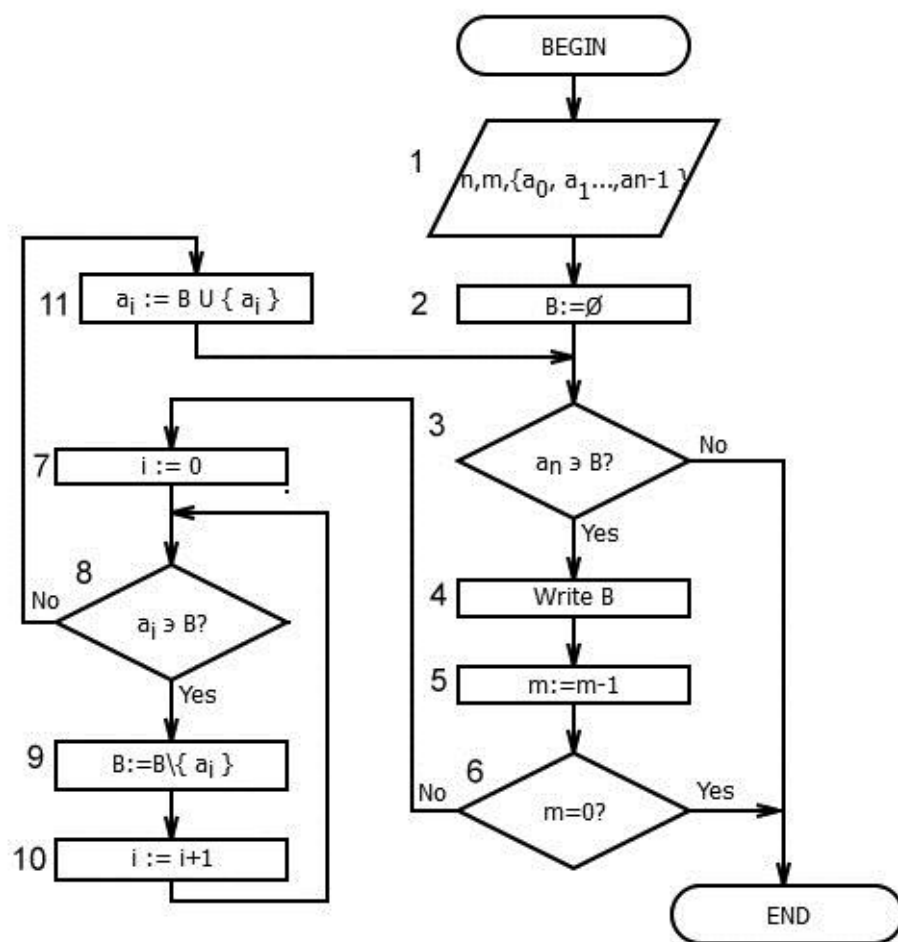
$$\cdot a_0 \notin B^2.$$

$$2.2.$$

$$\cdot a_0: B^3 := B^2 \cup a_0.$$

$$i = 0$$

$$B^3$$



3.3.

3.2.7.

$b_1 b_2 \dots b_n$ –

$(n - \quad)$,

$b_1 b_2 b_3 \dots b_{n-1} b_n$

$\oplus b_1 b_2 b_3 \dots b_{n-1} b_n$

$c_1 c_2 c_3 \dots c_{n-1} c_n$

$c_i = b_i \oplus b_{i-1}$,

$b_0 = 0$.

i			
0	000	$000 \oplus 00 = 000$	000
1	001	$001 \oplus 00 = 001$	001
2	010	$010 \oplus 01 = 011$	011
3	011	$011 \oplus 01 = 010$	010
4	100	$100 \oplus 10 = 110$	110
5	101	$101 \oplus 10 = 111$	111
6	110	$110 \oplus 11 = 101$	101
7	111	$111 \oplus 11 = 100$	100

$$(b_1 b_2 b_3) = 000.$$

(000) 1.

1.1.

0

$$(g_0 g_1 g_2) := 000 \text{ shr } 1. \quad (g_0 g_1 g_2) = 000.$$

1.2.

$2 : (b_1 b_2 b_3)$

$$(g_0 g_1 g_2) : (g_1 g_2 g_3) := (b_1 b_2 b_3) \text{ xor } (g_0 g_1 g_2) \quad (g_1 g_2 g_3) := 000$$

(001) 2.

2.1.

0

$$(g_0 g_1 g_2) := 001 \text{ shr } 1. \quad (g_0 g_1 g_2) = 000.$$

2.2.

$2 : (b_1 b_2 b_n)$

$$(g_0 g_1 g_2) : (g_1 g_2 g_3) := (b_1 b_2 b_3) \text{ xor } (g_0 g_1 g_2) \quad (g_1 g_2 g_3) := 001$$

(010) 3.

3.1.

0

$$(g_0 g_1 g_2) := 010 \text{ shr } 1. \quad (g_0 g_1 g_2) = 001.$$

3.2.

$2 : (b_1 b_2 b_n)$

$$(g_0 g_1 g_2) : (g_1 g_2 g_3) := (b_1 b_2 b_3) \text{ xor } (g_0 g_1 g_2) \quad 011 := 010 \text{ xor } 001 \\ (g_1 g_2 g_3) := 011$$

(011) 4.

3.1.

0

$$(g_0 g_1 g_2) := 011 \text{ shr } 1. \quad (g_0 g_1 g_2) = 001.$$

3.2.

$2 : (b_1 b_2 b_n)$

$$(g_0 g_1 g_2) : (g_1 g_2 g_3) := (b_1 b_2 b_3) \text{ xor } (g_0 g_1 g_2) \quad 010 := 011 \text{ xor } 001 \\ (g_1 g_2 g_3) := 010$$

(100) 5.

3.1.

0

$$(g_0 g_1 g_2) := 100 \text{ shr } 1. \quad (g_0 g_1 g_2) = 010.$$

3.2.

$2 : (b_1 b_2 b_n)$

$$(g_0 g_1 g_2) : (g_1 g_2 g_3) := (b_1 b_2 b_3) \text{ xor } (g_0 g_1 g_2) \quad 110 := 100 \text{ xor } 010$$

$$(g_1 g_2 g_3) := 110$$

$$2^3 = 8$$

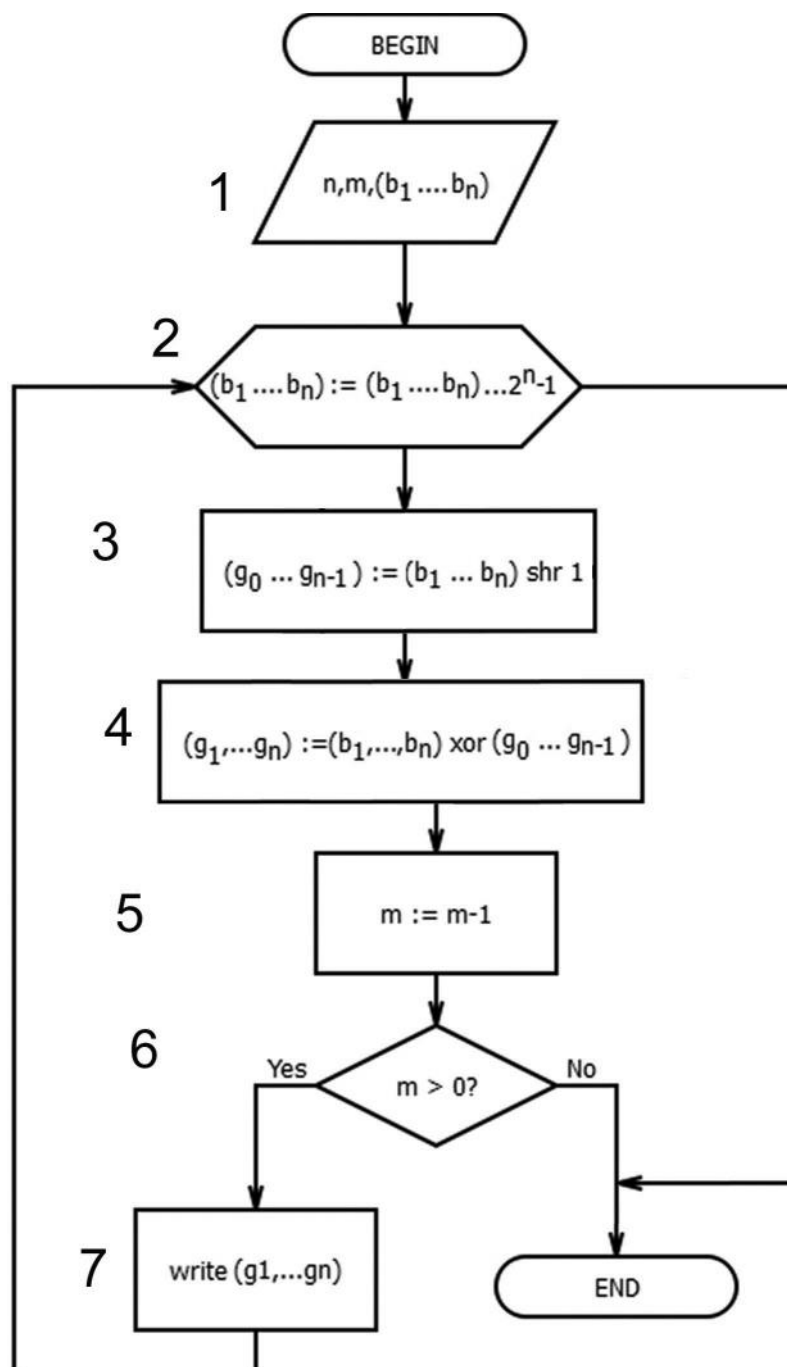
3.2.8.

1.

$n -$

$m -$

$(b_1 b_2 \dots b_n)$



.3.4.

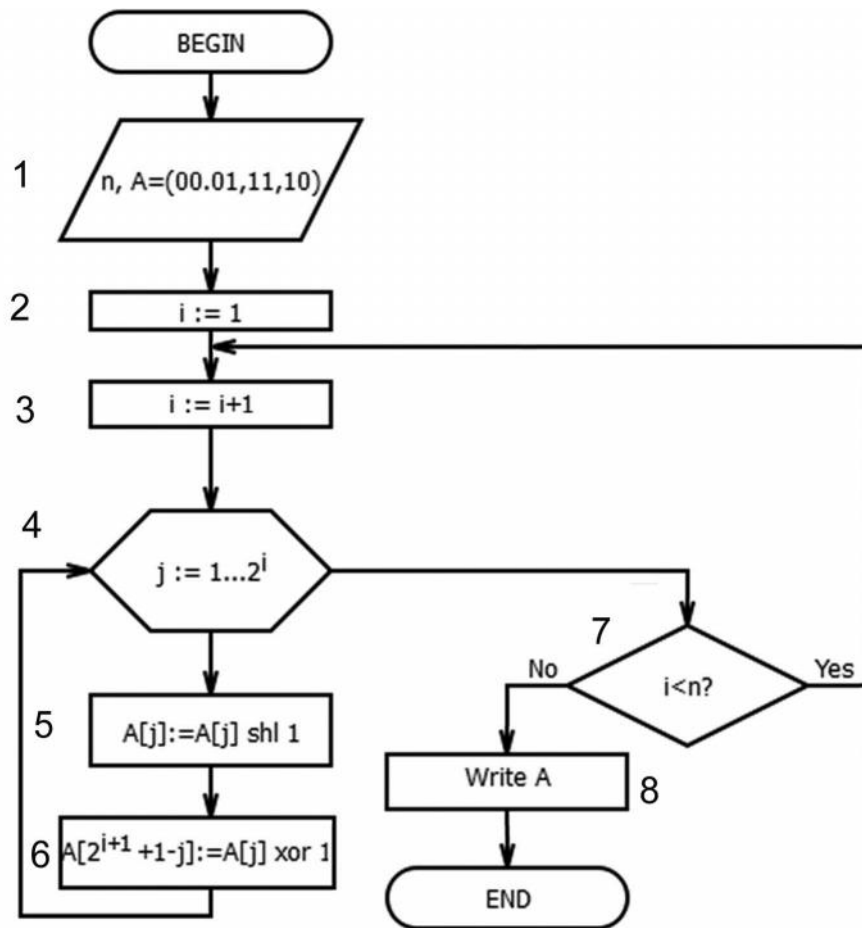
2. $(b_1 b_2 \dots b_n)$
- $2^n - 1,$
- $n -$
3. $(b_1 \dots b_n).$
- $(g_0 g_1 \dots g_{n-1}).$
4. $(b_1 \dots b_n)$
- $(g_1 g_2 \dots g_n),$
- $(g_0 g_1 \dots g_{n-1}).$
- $(b_1 b_2 \dots b_n)$
- 5.
- 6.
- 7.

3.2.9.

1. $: 00,01,11,10.$
2. $:$
- 2 . 00,01,11,10 0: 000,010,110,100.
- 2 . 00,01,11,10 : 10,11,01,00.
- 2 . 10,11,01,00 1: 101,111,011,001.
- 2 . , .2 .2 :
- 000, 010,110,100,101,111,011,001.
3. .1 ,
- , . 2 .
4. $n - 2$, $n -$
- .
- ,
- .

3.2.10.

- 1.
- $n -$
- $A = (00,01,11,10) -$
- 2.
- 3.
- 2.
4. $(i + 1) -$
- $i -$



3.5. -

5.

: $0100 * 0010 = 1000$ $0100 \text{ shl } 1 = 1000$ 2.

$(i+1) -$

6.

$\frac{1}{(i+1) -}$

$(i+1) -$

7.

n

8.

$n -$

3.2.11.

$A = \{a_1, a_2, a_3\}$

,

$$a_i$$

.

$$A$$

$$A$$

$$0.$$

:

i	$b_1b_2b_3$	$g_1g_2g_3$	B_i
0	000	000	\emptyset
1	001	001	a_3
2	010	011	a_2, a_3
3	011	010	a_2
4	100	110	a_1, a_2
5	101	111	a_1, a_2, a_3
6	110	101	a_1, a_3
7	111	100	a_1

.

3.2.12. -

1.

$$n -$$

$$m -$$

$$(b_1b_2...b_n) -$$

$$\{a_1, a_2, ..., a_n\} -$$

2.

3.

$$(g_0g_1...g_{n-1}).$$

4.

$$(g_0g_1...g_{n-1}).$$

5.

6.

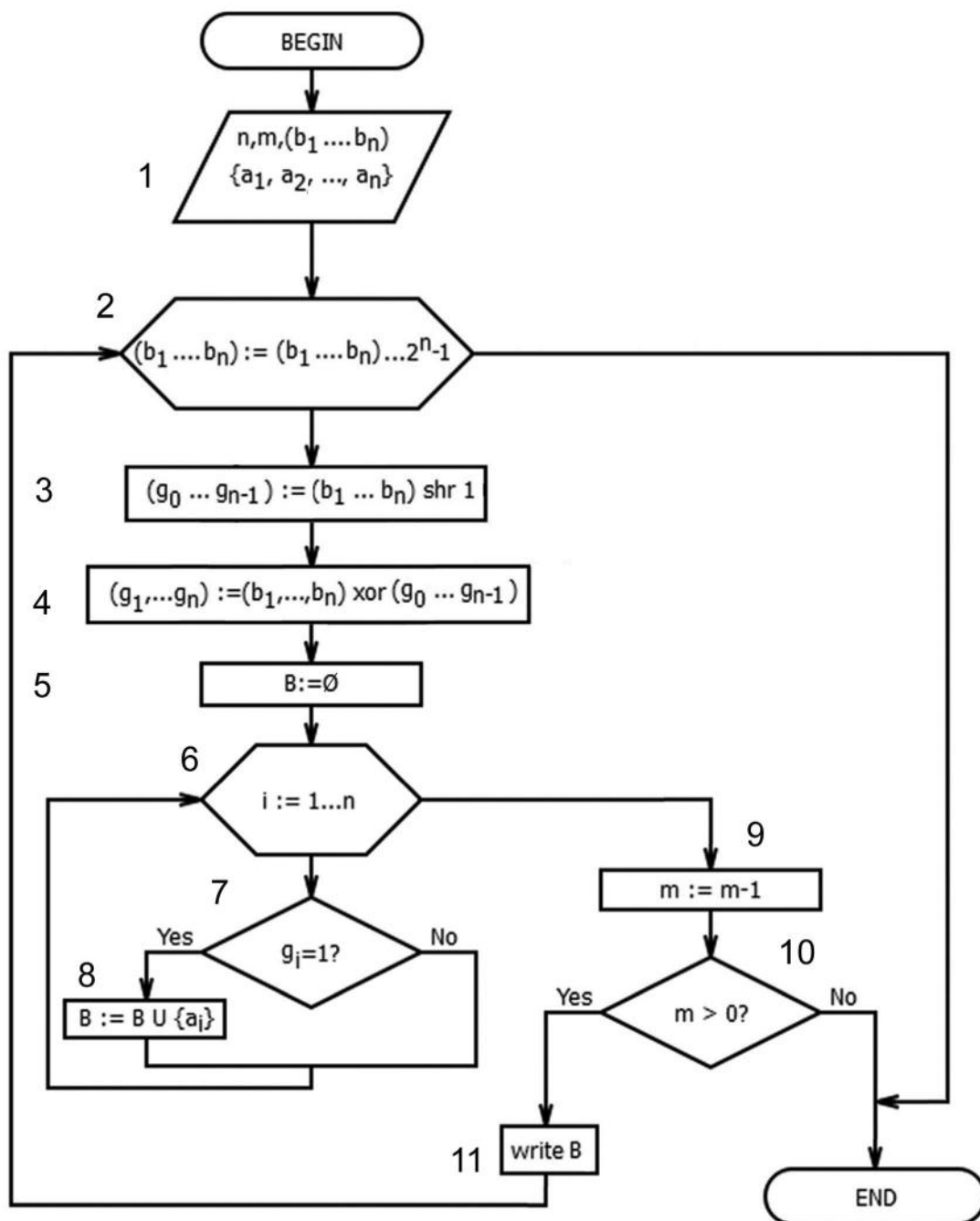
$$(g_1g_2...g_n)$$

7.

8.

$$1.$$

a_i
 $\{a_1, a_2, \dots, a_n\}$.



.3.6. -

9.
10.
11.
 3.2.13.

B .

$X = \{x_1, x_2, x_3\}$

x_i X
 (g_1, \dots, g_n) \cdot
 X
 B X ,
 1.

3.2.14. -

1.
 $n -$,
 $A = (00, 01, 11, 10)$ -
 $\{x_1, x_2, \dots, x_n\}$ -

2. ,

3. 2.

4. $(i+1) -$
 $i -$

5.
 2.
 $: 0100 * 0010 = 1000 \quad 0100 \text{shl}1 = 1000$
 $(i+1) -$

6. , $\frac{1}{(i+1)} -$

$(i+1) -$
7. n

8.
 $(g_1 g_2 \dots g_n)$

9.

10. ,
 1.

x_k $\{x_1, x_2, \dots, x_k, \dots, x_n\}$.
11. $n -$

, $(1, 2, \dots, k)$, -

$(n - k + 1, n - k, \dots, n - 1, n)$.

1. (a_1, a_2, \dots, a_k) .

2. $:$

$(b_1, b_2, \dots, b_k) = (a_1, \dots, a_{p-1}, a_p + 1, a_p + 2, \dots, a_p + k - p + 1)$,

$p = \max \{i | a_i < n - k + 1\}$

3. $,$ (b_1, b_2, \dots, b_k) ,

$:$

$(c_1, \dots, c_k) = (b_1, \dots, b_{p'-1}, b_{p'} + 1, b_{p'} + 2, \dots, b_{p'} + k - p' + 1)$,

$p' = \begin{cases} p - 1, & b_k = n, \\ k, & b_k < n \end{cases}$

. $A = (a_1, a_2, a_3, a_4, a_5) = (1, 2, 3, 4, 5)$.

$n = 5 \quad k = 3$

$: (a_1, a_2, a_3) = (1, 2, 3)$.

$: (a_3, a_4, a_5) = (3, 4, 5)$.

C_n^k , $(1, 2, 3, \dots, k)$.

. $(1, 2, 3, 4, 5)$ C_5^3

$(1, 2, 3)$.

$(i + 1, i + 2, \dots, i + k)$,

$(i + 1, i + 2, \dots, i + k)$

$(n - k + 1, n - k + 2, \dots, n - 1, n)$.

. $(1, 2, 3, 4, 5)$ C_5^3

$(2, 3, 5)$.

$(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 4), (1, 3, 5), (1, 4, 5), (2, 3, 4), (2, 3, 5), (2, 4, 5), (3, 4, 5)$

$: (2, 3, 5), (2, 4, 5), (3, 4, 5)$.

.

. $(1, 2, 3, 4, 5)$ C_5^3

$(1, 2, 5)$.

$(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 4), (1, 3, 5), (1, 4, 5), (2, 3, 4), (2, 3, 5), (2, 4, 5), (3, 4, 5)$

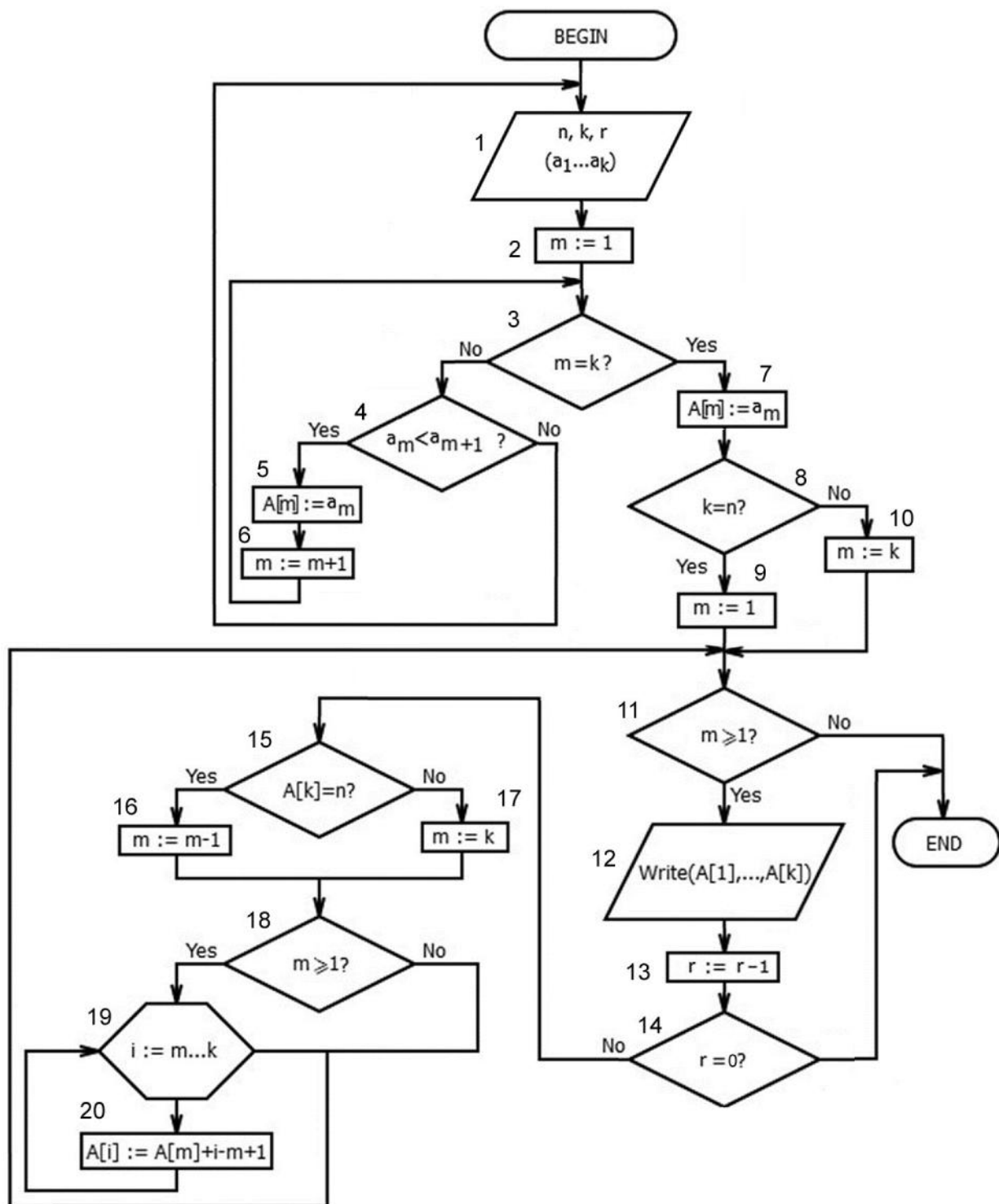
$: (1, 2, 5), (1, 3, 4), (1, 3, 5)$.

3.2.16. - $n \quad k$

1. $:$

$n -$
 $k -$
 $r -$
 $(a_1, a_2, \dots, a_k) -$

$a_1 < a_2 < \dots < a_k$



.3.8.

$n \quad k$

2.

$m := 1.$

3.

4.

a_m

5.

a_m

$m -$

$A[m] := a_m$

6.

$m := m + 1.$

7.

m

$m = k,$

$: A[m] := a_m.$

!

m

$n \quad k.$

8.

9.

$n = k,$

m

1.

10.

$k < n$

$m := k.$

11.

$m.$

$m \geq 1,$

12.

$A.$

13.

14.

$r > C_n^k.$

$r = 0,$

15-20.

3.2.17.

$n \quad k$

$R = \{r_1, r_2, \dots, r_n\}.$

$n \quad k$

$R.$

R .

$$R = \{r_1, r_2, r_3, r_4, r_5\}.$$

$$A = (1, 2, 3, 4, 5)$$

$$n = 5 \quad k = 3.$$

$$(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 3, 4), (1, 3, 5), (1, 4, 5), (2, 3, 4), (2, 3, 5), (2, 4, 5), (3, 4, 5)$$

$$R,$$

$$(r_1, r_2, r_3), (r_1, r_2, r_4), (r_1, r_2, r_5), (r_1, r_3, r_4), (r_1, r_3, r_5),$$

$$(r_1, r_4, r_5), (r_2, r_3, r_4), (r_2, r_3, r_5), (r_2, r_4, r_5), (r_3, r_4, r_5)$$

3.2.18.

1.

$$n - R = \{r_1, r_2, \dots, r_n\}.$$

$k -$

$$\{r_1, r_2, \dots, r_n\} - R.$$

2, 3.

R .

4, 5, 6.

$m,$

7.

$$m \geq 1,$$

$$m < 1,$$

8.

$$R,$$

$$A.$$

9.

$$A[k]$$

$n.$

10.

$$A[k] = n,$$

$$m : m := m - 1$$

11.

$$A[k] \neq n,$$

$$m \quad m := k.$$

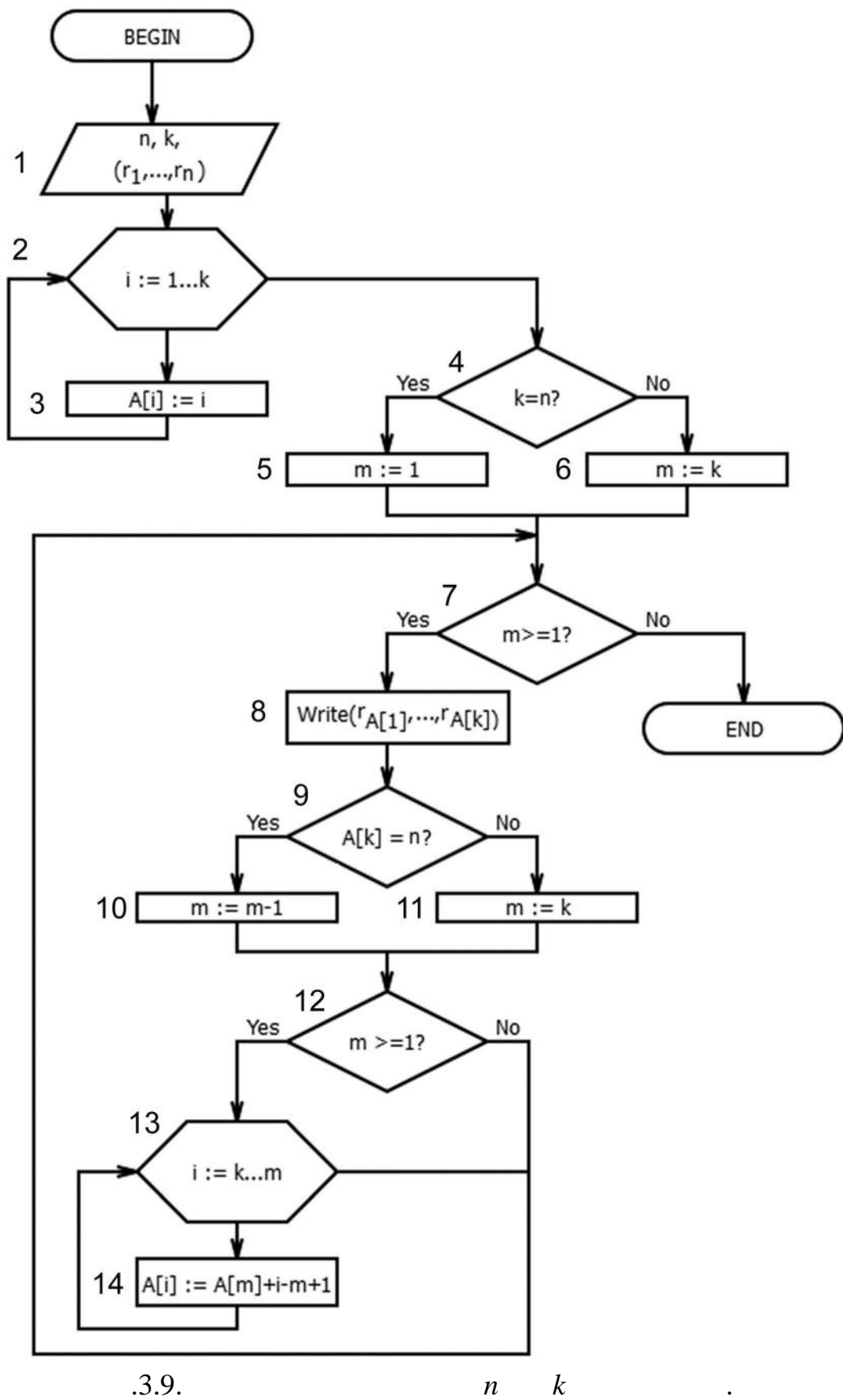
12.

$$m \geq 1,$$

$$m < 1,$$

13, 14.

$A.$



3.2.19.

n

\leq

(
 $a = (a_1, \dots, a_q) \quad b = (b_1, \dots, b_s) -$,

.

. $(a_1, \dots, a_q) \leq (b_1, \dots, b_s)$, :

$q \leq s \quad a_i = b_i \quad - \quad i \leq q$,
 $p \leq \min(s, q)$, $a_p < b_p \quad a_i = b_i \quad i < p$.
 \leq . n

.

. $a, b -$ n ,

$a < b \Leftrightarrow \exists p \leq \min(s, q) (a_p < b_p \& \forall i < p (a_i = b_i))$.

n

(
 $(1, 1, \dots, 1)$ n , (n) -
 (n) ,

$a = (a_1, \dots, a_q)$.
 $p < q$, a_p 1,

.

$\left(\sum_{i=p+1}^q a_i - 1 \right)$.

b

a .

a

b

a

n .

$a = (a_1, \dots, a_q)$.

$(a_{i_1}, \dots, a_{i_k}) \quad (a_1, \dots, a_q) \quad a_{i_1} > \dots > a_{i_k}$.

$m_j -$

a_{i_j}

$a \quad m = (m_1, \dots, m_k)$,

a

n

$(a_{i_1}, \dots, a_{i_k})$

(m_1, \dots, m_k) .

a

n

$a = (a_1 \cdot m_1, \dots, a_k \cdot m_k)$,

$$a_1 > a_2 > \dots > a_k > 0, \quad m_i > 0, \quad i = 1, 2, \dots, k, \quad 1 \leq k \leq n, \quad m = \sum_{i=1}^k m_i a_i.$$

$$m_k = n, \quad m_i \cdot a_i, \quad a_i, a_i, a_i, \dots, a_i, \quad a, \quad k = 1, \quad n, \quad p$$

$$b, \quad a$$

$$a = (a_1 \cdot m_1, \dots, a_k \cdot m_k) - a, \quad n, \quad b$$

$$1. \quad m_k = 1, \quad k \geq 2, \quad b = (m_1 \cdot a_1, \dots, m_{k-2} \cdot a_{k-2}, 1 \cdot (a_{k-1} + 1), S' \cdot 1).$$

$$2. \quad m_k \geq 2, k \geq 2, \quad a_{k-1} = a_k + 1, \quad b = (m_1 \cdot a_1, \dots, m_{k-2}, a_{k-2}, (m_{k-1} + 1) \cdot a_{k-1}, S \cdot 1).$$

$$3. \quad m_k \geq 2, k \geq 2, \quad a_{k-1} \neq a_k + 1, \quad b = (m_1 \cdot a_1, \dots, m_{k-1}, a_{k-1}, 1 \cdot (a_k + 1), S \cdot 1).$$

$$4. \quad k = 1, \quad b = (1 \cdot (a_k + 1), S \cdot 1).$$

$$S' = m_k a_k + m_{k-1} a_{k-1} - (a_{k-1} + 1), \quad S = m_k a_k - (a_k + 1).$$

$$a, \quad a, \quad b, \quad 1, \quad x, \quad ,$$

$$n. \quad 1. \quad m_k = 1. \quad a_k, \quad k \geq 2, \quad a_{k-1} + a_k \geq a_k + 1, \quad x = a_{k-1}, \quad x, \quad (k-1), \quad 1, \quad S' = m_k a_k + m_{k-1} a_{k-1} - (a_{k-1} + 1),$$

$$2. \quad m_k \geq 2. \quad m_k a_k \geq a_k + 1, \quad x = a_k, \quad x, \quad 1, \quad ,$$

$$S = m_k a_k - (a_k + 1)$$

$$\begin{array}{c} \cdot \\ a_i \quad m_i \quad , \quad \cdot \\ \cdot \end{array}$$

$$(4) \quad \begin{array}{c} \cdot \\ a_0 \quad , \quad a_0 \neq a_1 + 1, \\ k \geq 2 \quad (2), (3), \end{array}$$

$$\begin{array}{c} \cdot \\ \cdot \end{array} \quad 7 \quad :$$

$$(7 \cdot 1) = (1, 1, 1, 1, 1, 1, 1),$$

$$(1 \cdot 2, 5 \cdot 1) = (2, 1, 1, 1, 1, 1),$$

$$(2 \cdot 2, 3 \cdot 1) = (2, 2, 1, 1, 1),$$

$$(3 \cdot 2, 1 \cdot 1) = (2, 2, 2, 1),$$

$$(1 \cdot 3, 4 \cdot 1) = (3, 1, 1, 1, 1),$$

$$(1 \cdot 3, 1 \cdot 2, 2 \cdot 1) = (3, 2, 1, 1),$$

$$(1 \cdot 3, 2 \cdot 2) = (3, 2, 2),$$

$$(2 \cdot 3, 1 \cdot 1) = (3, 3, 1),$$

$$(1 \cdot 4, 3 \cdot 1) = (4, 1, 1, 1),$$

$$(1 \cdot 4, 1 \cdot 2, 1 \cdot 1) = (4, 2, 1),$$

$$(1 \cdot 4, 1 \cdot 3) = (4, 3),$$

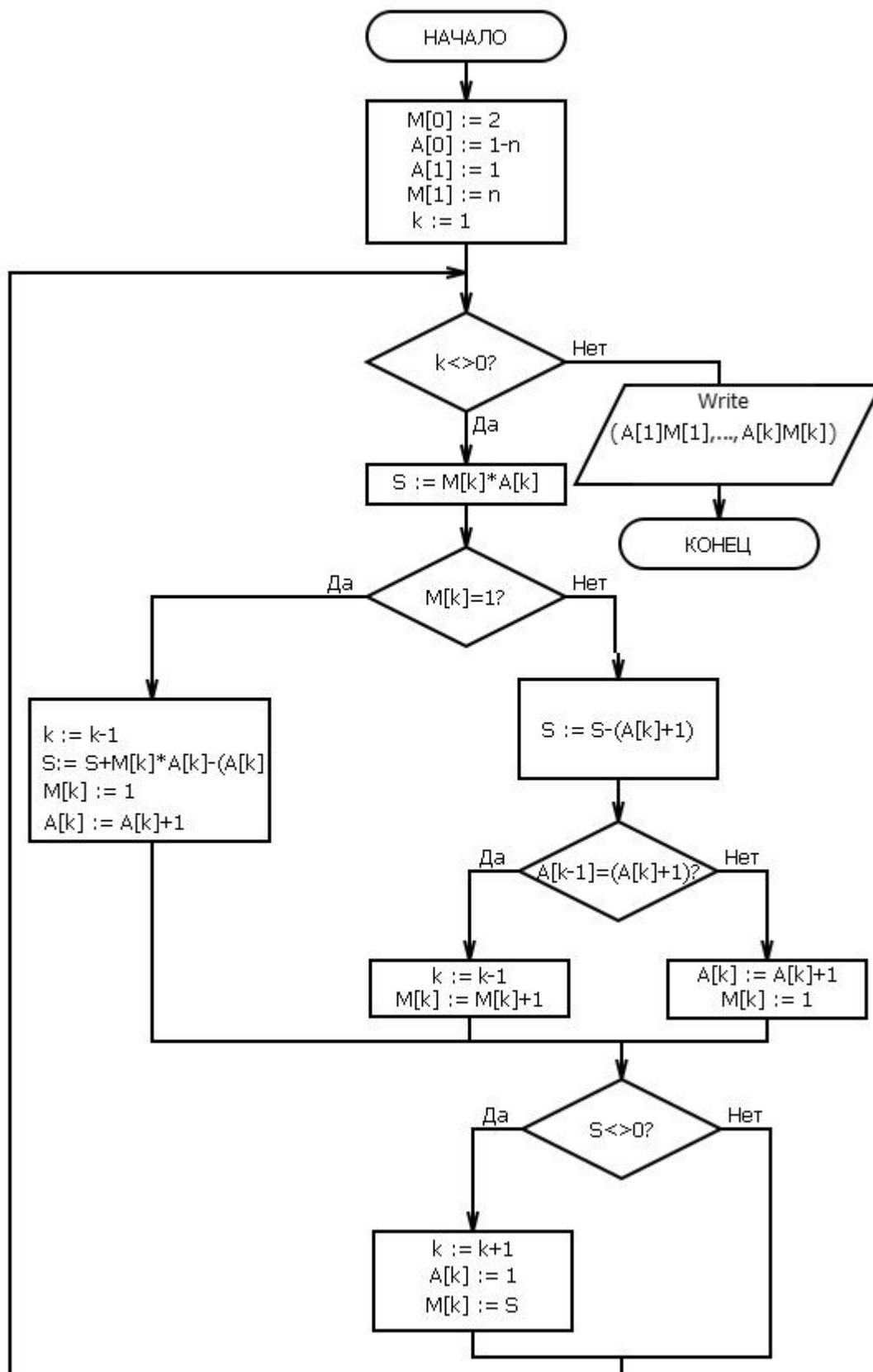
$$(1 \cdot 5, 2 \cdot 1) = (5, 1, 1),$$

$$(1 \cdot 5, 1 \cdot 2) = (5, 2),$$

$$(1 \cdot 6, 1 \cdot 1) = (6, 1),$$

$$(1 \cdot 7) = (7).$$

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8. n k
9. n .

NZK – I I = NZK mod 14+1,

1	<p>1 .</p> <p>2. s n (10+NZK mod 11).</p> <p>3. P ,</p>
2	<p>1. n (10+NZK mod 11).</p> <p>2. s 1 n</p>

	3. P ,
3	<p>2 ,</p> <p>:</p> <p>1. n (NZK).</p> <p>2. m</p> <p>3. $(b[n], \dots, b[0])$,</p>
4	<p>2 ,</p> <p>:</p> <p>1. n (NZK).</p> <p>2. m</p> <p>3. $(b[n], \dots, b[0])$,</p>
5	<p>3 ,</p> <p>:</p> <p>1. $\{a_0, a_1, \dots, a_{n-1}\}$,</p> <p>2. n .</p> <p>3. m .. 1 n</p>
6	<p>3 ,</p> <p>:</p> <p>1. $\{a_0, a_1, \dots, a_{n-1}\}$</p> <p>20.</p> <p>2. n $n \geq 20$.</p> <p>3. m 1 n</p>
7	<p>4 ,</p> <p>:</p> <p>1. $(b_1 b_2 \dots b_n)$</p> <p>12 1998 101110001111011110001110.</p> <p>12121998, $12121998_{10} = 101110001111011110001110_2$</p> <p>2. n 101110001111011110001110 $n = 24$</p> <p>3. m 1 n</p>
8	<p>5 .</p> <p>1. $(b_1 b_2 \dots b_n)$</p>

	<p>1999 , , 01 .</p> <p>19990101,</p> <p>1001100010000011001010101.</p> <p>19990101₁₀ = 1001100010000011001010101₂</p> <p>2. n .</p> <p>1001100010000011001010101</p> <p>$n = 26$</p> <p>3. m 1 n</p>
9	<p>6 ,</p> <p>:</p> <p>1. $\{a_1, a_2, \dots, a_n\}$ 20.</p> <p>2. $n - \{a_1, a_2, \dots, a_n\}$.</p> <p>3 $m -$, ,</p> <p>1 n .</p> <p>4. $(b_1 b_2 \dots b_n) -$</p> <p>n .</p>
10	<p>7 ,</p> <p>:</p> <p>1. $A = (00, 01, 11, 10) -$,</p> <p>.</p> <p>2. $\{x_1, x_2, \dots, x_n\} -$</p> <p>20 .</p> <p>3. $n - \{x_1, x_2, \dots, x_n\}$.</p> <p>4. $(b_1 b_2 \dots b_n) -$</p> <p>n .</p>
11	<p>8 ,</p> <p>:</p> <p>1. $(1, 2, 3, \dots, n)$, $n \geq 32$.</p> <p>2. $(a_1, a_2, \dots, a_k) -$, , k</p> <p>1 n .</p> <p>3. $r -$, .</p> <p>1 C_n^k .</p> <p>$a_1 < a_2 < \dots < a_k$ - .</p>
12	<p>9 ,</p> <p>:</p> <p>1. R ,</p>

	<p> $\cdot r_1 = \cdot$, $r_2 =$, $r_3 =$ $\cdot \cdot$ 2. $n \geq 16$. 3. $k -$ n. </p>	1
13	<p> 9 : 1. R, \cdot $\cdot r_1 =$, $r_2 =$, $r_3 =$ $\cdot \cdot$ 2. $n \geq 16$. 3. $k -$ n. 4. </p>	1
14	<p> 10 , 1 100. </p>	

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G
 (X)

x_1, x_2, \dots, x_n (

a_1, a_2, \dots, a_m (

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G

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(X, Y).

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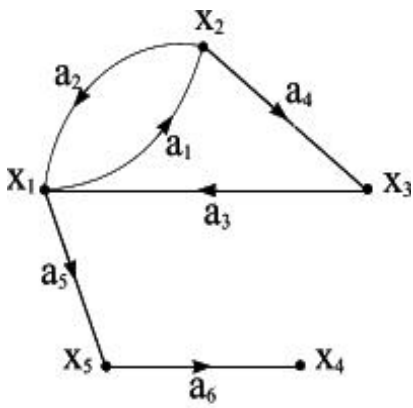
, $G = (X, Y)$

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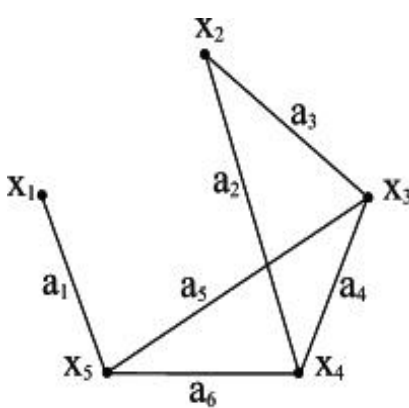
G ,

$G = (X, Y)$.

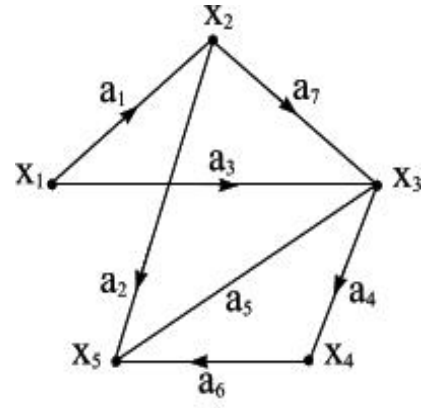


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2. , 4.1 () (x_l, x_2) $a_l, (x_2, x_l) -$

, , G ,

. $G = (X,)$.

4.1 () $(x_l) = \{x_2, x_5\}$, $x_2 \quad x_5$
 x_l .

$(x_2) = \{x_l, x_3\}$, $(x_3) = \{x_l\}$, $(x_4) = \emptyset -$, $(x_5) = \{x_4\}$

(, , , 4.1 () 4.1 ()),

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, , 4.1 (), $(x_5) =$
 $\{x_l, x_3, x_4\}$, $(x_l) = \{x_5\}$.

$x_j \in X$, G (x_i, x_j) , $^{-1}(x_i)$
 x_k , G $(, x_j)$.

4.1(),

$^{-1}(x_l) = \{x_2, x_3\}$, $^{-1}(x_2) = \{x_l\}$. .

, $^{-1}(x_i) = (x_i)$ $x_i \in X$.

$X_q = \{x_l, x_2, \dots, x_q\}$, (X_q) , $(x_l) \cup (x_2) \cup \dots \cup (x_q)$,
 (X_q) $x_j \in X$,
 $(x_i, x_j) \in G$, $x_i \in X_q$. , 4.1(),

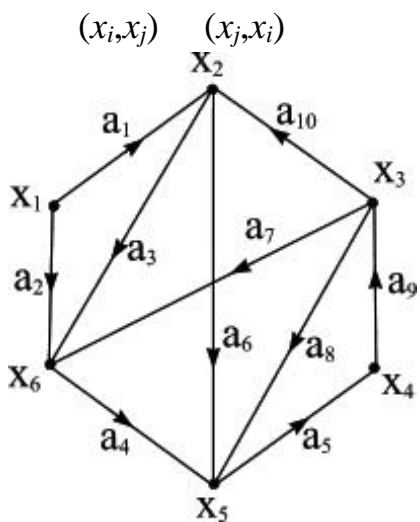
$(\{x_2, x_5\}) = \{x_l, x_3, x_4\}$ $(\{x_l, x_3\}) = \{x_2, x_5, x_l\}$.

((x_i)) $^2(x_i)$. " "
 $(((x_i)))$ $^3(x_i)$. . ,
4.1(), :

$^2(x_l) = ((x_l)) = (\{x_2, x_5\}) = \{x_l, x_3, x_4\}$;

$^3(x_l) = (^2(x_l)) = (\{x_l, x_3, x_4\}) = \{x_2, x_5, x_l\}$.
 $^{-2}(x_i)$, $^{-3}(x_i)$. .

$a = (x_i, x_j)$, $x_i \leftrightarrow x_j$, ,
 $x_i \quad x_j$, -



.4.2.

G ,

$G=(X, \quad)$

$$deg(x_i)=| \quad (x_i)|=| \quad ^{-1}(x_i)|.$$

.4.3,

$a_3 \quad a_{10}$

G

G_p ,

$G=(X,A) -$

n

$(\quad) \quad G.$

G_p ,

$G_p=(X_p,A_p) \subseteq G$,

$X_p = X, A_p \subseteq A$

(4.2)

1)

n ,

$n-1$

$(|X_p|=|A_p|-1);$

2)

$\forall x_i, x_j \in X_p (i \neq j) \rightarrow \exists ! \quad \sim(x_i, x_j).$

$G -$,
 $G.$

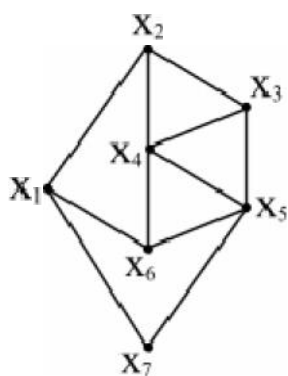
.4.3(a),

.4.3(,)

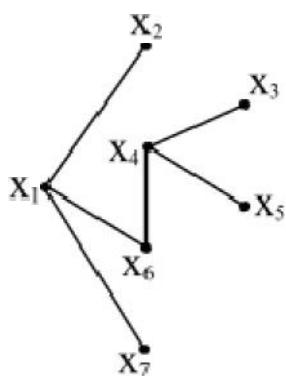
G

$G.$

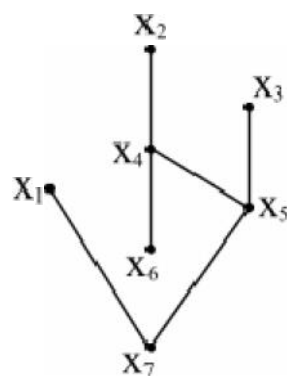
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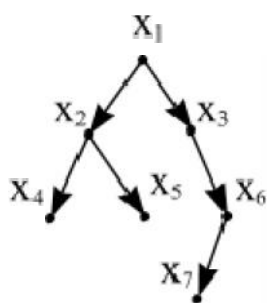
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X_1 .
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. 4.5

$\mu_1=\{a_6, a_5, a_9, a_8, a_4\}$, $\mu_2=\{a_1, a_6, a_5, a_9\}$, $\mu_3=\{a_1, a_6, a_5, a_9, a_{10}, a_6, a_4\}$

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μ_1 μ_2

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μ_3

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a_6

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4.6 $\mu_4=\{a_2, a_4, a_8, a_{10}\}, \mu_5=\{a_2, a_7, a_8, a_4, a_3\}$
 $\mu_6=\{a_{10}, a_7, a_4, a_8, a_7, a_2\}$
 () (),

A diagram of a hexagon with vertices labeled $X_1, X_2, X_3, X_4, X_5, X_6$ and edges labeled $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$. The vertices are arranged in a regular hexagonal pattern. The edges are labeled as follows: a_1 (top-left), a_2 (top), a_3 (top-right), a_4 (right), a_5 (bottom), a_6 (bottom-right), a_7 (bottom-left), and a_8 (left).

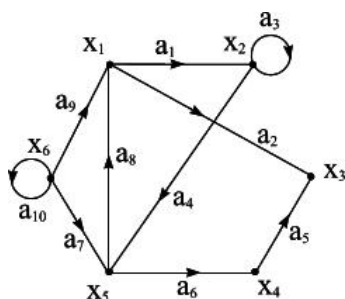
$$\mu_6, \mu_4, \mu_5 - \mu_6, \mu_1 : \mu_1 = \{x_2, x_5, x_4, x_3, x_5, x_6\}$$
$$\begin{aligned} & \left(\frac{\partial}{\partial x_i} - G_{ij} \right) \psi_j = 0 \\ & \text{where } G_{ij} = \frac{1}{2} (\delta_{ij} + \epsilon_{ijk} A_k) \end{aligned}$$

$$L(\sim) = \sum_{(x_i, x_j) \in \mu} c_{ij} \quad (4.3)$$

" " " " " "

4.2.

4.2.1.



G ,
 $A=[a_{ij}]$
 $:$
 $a_{ij}=1$, G (x_i, x_j) ,
 $a_{ij}=0$, G (x_i, x_j) .

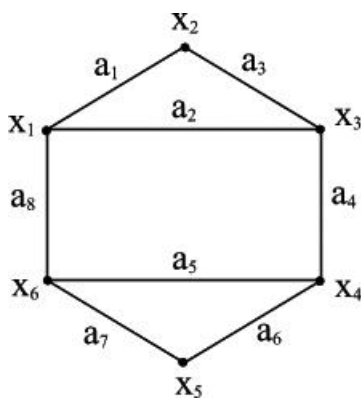
4.7,

. 4.7.

$$A = \begin{array}{c|cccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \hline x_1 & 0 & 1 & 1 & 0 & 0 & 0 \\ x_2 & 0 & 1 & 0 & 0 & 1 & 0 \\ x_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ x_4 & 0 & 0 & 1 & 0 & 0 & 0 \\ x_5 & 1 & 0 & 0 & 1 & 0 & 0 \\ x_6 & 1 & 0 & 0 & 0 & 1 & 1 \end{array}$$

x_i
 $x_i -$
 1 x_i (x_i) , 1
 x_i , $^{-1}(x_i)$.
 a_{22}, a_{66} , 1 . 4.7.

(.4.8).



. 4.8

$$A = \begin{array}{c|cccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \hline x_1 & 0 & 1 & 1 & 0 & 0 & 1 \\ x_2 & 1 & 0 & 1 & 0 & 0 & 0 \\ x_3 & 1 & 1 & 0 & 1 & 0 & 0 \\ x_4 & 0 & 0 & 1 & 0 & 1 & 1 \\ x_5 & 0 & 0 & 0 & 1 & 0 & 1 \\ x_6 & 1 & 0 & 0 & 1 & 1 & 0 \end{array}$$

4.2.2.

G — n — m .
 G : $\mathbf{B}=[b_{ij}]$ $n \times m$,
 $b_{ij}=1$, x_i a_j ;
 $b_{ij}=-1$, x_i a_j ;
 $b_{ij}=0$, x_i a_j .

, 4.7, :

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
x_1	1	1	0	0	0	0	0	-1	-1	0
x_2	-1	0	1	1	0	0	0	0	0	0
x_3	0	-1	0	0	0	0	0	0	0	0
x_4	0	0	0	0	-1	-1	0	0	0	0
x_5	0	0	0	-1	1	1	-1	1	0	0
x_6	0	0	0	0	0	0	1	0	1	1

,
 1, — -1.
 ($b_{ij}=1$).
 G (4.8),

:
 $b_{ij}=1$, x_i a_j ;
 $b_{ij}=0$, x_i a_j .

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
x_1	1	1	0	0	0	0	0	1
x_2	1	0	1	0	0	0	0	0
x_3	0	1	1	1	0	0	0	0
x_4	0	0	0	1	1	1	0	0
x_5	0	0	0	0	0	1	1	0
x_6	0	0	0	0	1	0	1	1

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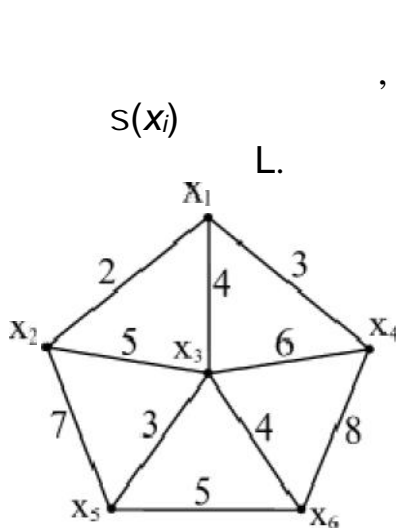
4.3.

().

3. : $X_p = X_p \cup \{x_j^*\}$; $A_p = A_p \cup (x_i, x_j^*)$. $S(x_j^*) = 1..$
 4. $|X_p| = n$, A_p
 . $|X_p| < n$, 2.

4.5.

. 4.9



$S(x_i)$

L.

$$1: X_p = \{x_1\}; A_p = \emptyset; {}^*(X_p) = \{x_2, x_3, x_4\};$$

$$c(x_1, x_2^*) = 2; B = \{1, 1, 0, 0, 0, 0\}; L = 2.$$

$$2: X_p = \{x_1, x_2\}; A_p = \{(x_1, x_2)\};$$

$${}^*(X_p) = \{x_3, x_4, x_5\}; c(x_1, x_4^*) = 3;$$

$$B = \{1, 1, 0, 10, 0\}; L = 2 + 3 = 5.$$

$$3: X_p = \{x_1, x_2, x_4\}; A_p = \{(x_1, x_2); (x_1, x_4)\};$$

$${}^*(X_p) = \{x_3, x_5, x_6\}; c(x_1, x_3^*) = 4;$$

$$B = \{1, 1, 1, 1, 0, 0\}; L = 5 + 4.$$

. 4.9.

$$4: X_p = \{x_1, x_2, x_3, x_4\}; A_p = \{(x_1, x_2); (x_1, x_4); (x_1, x_3)\};$$

$${}^*(X_p) = \{x_5, x_6\}; c(x_3, x_5^*) = 2; B = \{1, 1, 1, 1, 1, 0\}; L = 9 + 3 = 12.$$

$$5: X_p = \{x_1, x_2, x_3, x_4, x_5\}; A_p = \{(x_1, x_2); (x_1, x_4); (x_1, x_3); (x_3, x_5)\};$$

$${}^*(X_p) = \{x_6\}; c(x_3, x_6^*) = 4; B = \{1, 1, 1, 1, 1, 1\}; L = 12 + 4 = 16.$$

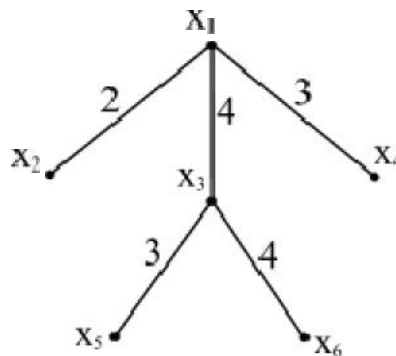
X_p

A_p

$$X_p = \{x_1, x_2, x_3, x_4, x_5, x_6\}; A_p = \{(x_1, x_2); (x_1, x_4); (x_1, x_3); (x_3, x_5); (x_3, x_6)\};$$

$$L = 16$$

. 4.10.



. 4.10.

4.6.

$$G=(X, \quad), \quad (\quad),$$

$$=[ij].$$

() s

$\mu(s,t) = L(\mu) \rightarrow \min, s, t \in R(s), R(s) -$

$$ij$$

G

G

$$(s-t) - \sum_{i=1}^n x_i (\forall x_i \in \mathbb{R}). \quad (1)$$
$$ij \leq ik + kj \quad i, j, k \in G \quad (x_i, x_j)$$
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$
$$(\quad, \quad ij \quad),$$

4.6.1.

$$c_{ij} \geq 0.$$
$$1. \quad l(v_i) =$$
 $v_i.$

2. S

5. S

Крок 1. $l(s) := 0$ і $l(v_i) := \infty$ $v_i \neq s$ $p = s$.

Крок 2. $v_i \in \Gamma(p)$,
 $l(v_i) \leftarrow \min[l(v_i), l(p) + c(p, v_i)]$

Крок 3. $l(v_i^*) = \min l(v_i), v_i \in \Gamma(p)$

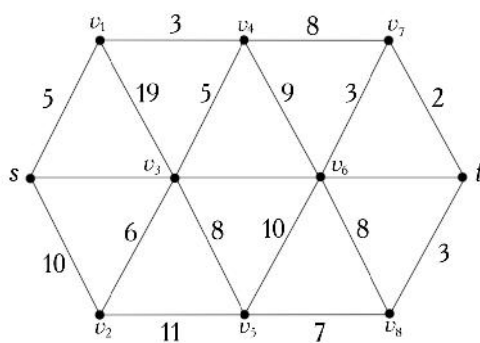
Крок 4. $l(v_i^*)$ $p = v_i^*$.

Крок 5. Коли треба знайти шлях від s до t $p = t$, $l(p)$

Крок 6. $p \neq t$, 2.

Крок 7. Якщо s .

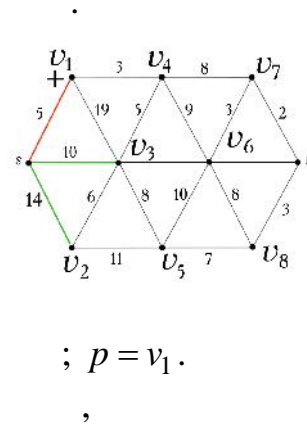
Крок 8. 2.



1. $l(s) = 0^+, l(v_i) = \infty, i = 1, \dots, 8, p = s$.

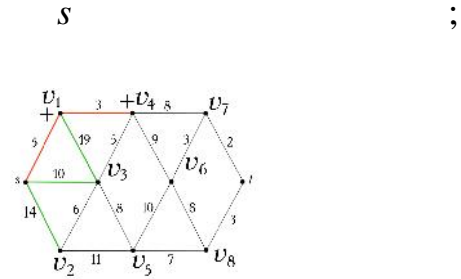
$$\begin{aligned}
2. \quad & \Gamma(s) = \{v_1, v_2, v_3\} - \\
& l(v_1) = \min[\infty, 0^+ + 5] = 5, \\
& l(v_2) = \min[\infty, 0^+ + 14] = 14, \\
& l(v_3) = \min[\infty, 0^+ + 10] = 10. \\
3. \quad & l(v_1) = \min_{i=1,2,3} l(v_i) = 5. \\
4. \quad & l(v_1) = 5^+ - v_1 \\
5. \quad &
\end{aligned}$$

2.



$$; p = v_1.$$

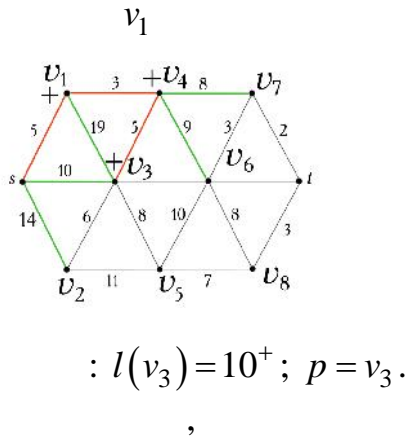
$$\begin{aligned}
2. \quad & \Gamma(p) = \Gamma(v_1) = \{s, v_3, v_4\}. \\
& l(v_3) = \min[10, 5^+ + 19] = 10, \\
& l(v_4) = \min[\infty, 5^+ + 3] = 8. \\
3. \quad & l(v_4) = \min_{i=3,4} l(v_i).
\end{aligned}$$



$$: l(v_4) = 8^+; p = v_4.$$

2.

$$\begin{aligned}
2. \quad & \Gamma(p) = \Gamma(v_4) = \{v_1, v_3, v_6, v_7\}. \\
& l(v_3) = \min[10, 8^+ + 5] = 10, \\
& l(v_7) = \min[\infty, 8^+ + 8] = 16, \\
& l(v_6) = \min[\infty, 8^+ + 9] = 17. \\
3. \quad & l(v_3) = \min_{i=3,6,7} l(v_i) = 10. \\
4. \quad & v_3 \\
5. \quad &
\end{aligned}$$



$$: l(v_3) = 10^+; p = v_3.$$

2.

$$2. \Gamma(p) = \Gamma(v_3) = \{s, v_1, v_2, v_4, v_5, v_6\}.$$

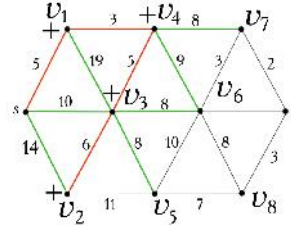
$$s, v_1, v_4$$

;

$$l(v_2) = \min[14, 10^+ + 6] = 14,$$

$$l(v_5) = \min[\infty, 10^+ + 8] = 18,$$

$$l(v_6) = \min[17, 10^+ + 8] = 17.$$



$$3. l(v_2) = \min_{i=2,5,6} l(v_i) = 14.$$

$$4. v_2$$

$$: l(v_2) = 14^+; p = v_2.$$

$$5.$$

2.

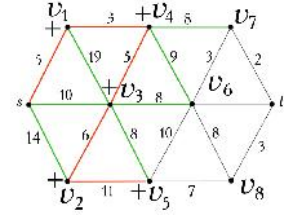
,

$$2. \Gamma(p) = \Gamma(v_2) = \{s, v_3, v_5\}.$$

$$s, v_3$$

;

$$l(v_5) = \min[18, 14^+ + 11] = 18.$$



$$3. l(v_5) = \min_{i=5} l(v_i) = 18.$$

$$4. v_5$$

$$: l(v_5) = 18^+; p = v_5.$$

$$5.$$

2.

$$2. \Gamma(p) = \Gamma(v_5) = \{v_2, v_3, v_6, v_8\}.$$

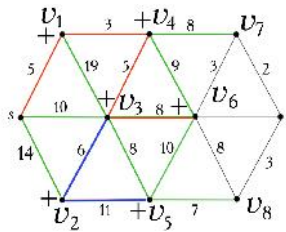
$$v_2, v_3$$

;

$$l(v_6) = \min[17, 18^+ + 10] = 17,$$

$$l(v_8) = \min[\infty, 18^+ + 7] = 25.$$

$$3. l(v_6) = \min_{i=6,8} l(v_i) = 17.$$



$$4. v_6$$

$$: l(v_6) = 17^+; p = v_6.$$

$$5.$$

2.

$$2. \Gamma(p) = \Gamma(v_6) = \{v_3, v_4, v_5, v_7, v_8, t\}.$$

$$v_3, v_4, v_5$$

;

$$l(v_7) = \min[16, 17^+ + 3] = 16,$$

$$l(v_6) = \min[25, 17^+ + 8] = 25,$$

$$l(t) = \min[\infty, 17^+ + 14] = 31.$$

$$3. l(v_7) = \min_{i=7,8,t} l(v_i) = 16.$$

$$4. \quad v_7$$

$$5.$$

2.

.

$$2. \quad \Gamma(p) = \Gamma(v_7) = \{v_4, v_6, t\}.$$

v_4, v_6

;

$$l(t) = \min[31, 16^+ + 2] = 18.$$

$$3. l(v_t) = \min_{i=t} l(v_i) = 18.$$

$$4. \quad t$$

$$5.$$

2.

,

.

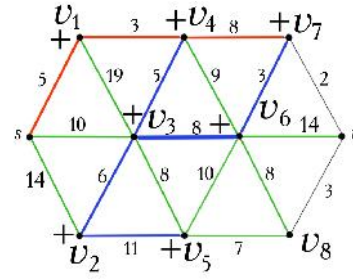
$$2. \quad \Gamma(p) = \Gamma(t) = \{v_6, v_7, v_8\}.$$

$$l(v_8) = \min[25, 18^+ + 3] = 21.$$

$$3. l(v_8) = \min_{i=8} l(v_i) = 21.$$

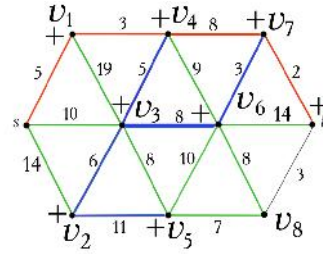
$$4. \quad v_8$$

$$5.$$



$$: l(v_7) = 16^+; \quad p = v_7.$$

,

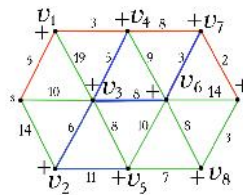


$$: l(t) = 18^+; \quad p = t.$$

,

v_6, v_7

;



$$: l(v_8) = 21^+; \quad p = v_8.$$

.

.

.

,
 s - ,
 ,

$$l(v_i') + c(v_i', v_i) = l(v_i).$$

$c(v_i', v_i)$ - , , v_i' v_i .
 v_i v_i'
 s v_i .

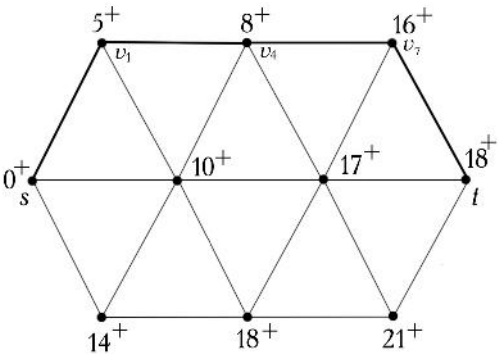
s t

:

$$\begin{aligned} l(t) &= l(v_6) + c(v_6, t), \\ l(v_6) &= l(v_4) + c(v_4, v_7), \\ l(v_4) &= l(v_1) + c(v_1, v_4), \\ l(v_1) &= l(s) + c(s, v_1), \end{aligned}$$

7- , 2- 1- .

s, v_1, v_4, v_7, t .



n . Visited
 : False () True
 (); Len
 ; C

$-k$ -

C

k - .

Matrix -

:


```

1 (      ).      1      n      False      Visited;
      i      C (i -      );      i-
      Matrix      Len;

Visited[i]:=True; C[i]:=0;

2 (      ).      (      k ,
Visited[k]=False);      j,      Len[j] ≤ Len[k];

      :
Visited[i]:=True;
      Len[k]>Len[j]+Matrix[j, k],      (Len[k]:=Len[j]+Matrix[j, k]; C[k]:=j)
{      Visited[k]      ,      vi      vk      C[k].
      ,      }.

3 (      ). {      vi      vk
      :}

3.1 z:=C[k];
3.2      z
3.3 z:=C[z].      z =0,      ,
      3.2.

```

Program Deikstra;

Uses Crt;

Const Maxsize=10;

Infinity=1000;

Var Mattr: array [1..Maxsize, 1..Maxsize] of integer;

Visited: array [1..Maxsize] of boolean;

Len, Path: array [1..Maxsize] of integer;

n, Start, Finish, k, i: integer;

Procedure Init;

Var f: text;

i, j: integer;

begin

Assign(f, 'INPUT.MTR');

Reset(f);

Readln(f, n);

For i:=1 **to** n **do**

begin

For j:=1 **to** n **do Read**(f, mattr[i,j]);

Readln(f)

end;

```

Write('                                : '); Readln(Start);
For i:=1 to n do
begin
  Visited[i]:=False;
  Len[i]:=Mattr[Start, i];
  Path[i]:=Start;
end;
Path[Start]:=0;
Visited[Start]:=True;
end;

```

```

Function Possible: Boolean;
Var i: integer;
begin
  Possible:=True;
  For i:=1 to n do If not Visited[i] then Exit;
  Possible:=False;
end;

```

```

Function Min: Integer;
Var i, minvalue, currentmin: integer;
begin
  Minvalue:=Infinity;
  For i:=1 to n do
  If not Visited[i] then
  If Len[i]<minvalue then
  begin
    currentmin:=i;
    minvalue:=Len[i]
  end;
  min:=currentmin;
end;

```

```

begin
  Clrscr;
  Init;
  While Possible do
  begin
    k:=min;
    Visited[k]:=True;
    For i:=1 to n do
    If Len[i]>Len[k]+Mattr[i, k] then
    begin

```

```

    Len[i]:=Len[k]+Matr[i, k];
    Path[i]:=k;
end;
end;
Write('          : '); Readln(Finish);
Write(Finish);
Finish:=Path[Finish];
While Finish<>0 do
begin
    Write('<- ', Finish);
    Finish:=Path[Finish];
end;
Readkey;
end.

```

4.6.2.

,
 .
 1. (—)
 , $\lambda_i(k), \quad i = 1, 2, \dots, n \quad (n$
 — $); k = 1, 2, \dots, n - 1.$
 $i \quad k \quad \lambda_i(k)$,
 $v_1 \quad v_i$
 , k .
 1. n
 $C = |c_{ij}|.$
 2. $k = 0.$ $\lambda_i(0) = \infty$, $v_1;$
 $\lambda_1(0) = 0.$
 3. $k, k = 1, 2, \dots, n - 1,$ $v_i \quad k -$
 $\lambda_i(k)$:
 $\lambda_i(k) = \min_{1 \leq j \leq n} \{ \lambda_j(k-1) + c_{ji} \}$ (1)
 , $v_1,$ $\lambda_1(k) = 0.$
 $\lambda_i(k), i = 1, 2, \dots, n; k = 0, 1, 2, \dots, n - 1.$
 $\lambda_i(k)$
 $i - ,$, k .

$$\begin{aligned}\lambda_2(1) &= \min\{\lambda_1(0) + c_{12}; \lambda_2(0) + c_{22}; \lambda_3(0) + c_{32}; \lambda_4(0) + c_{42}; \lambda_5(0) + c_{52}\} = \\ &= \min\{0 + 1; \infty + \infty; \infty + \infty; \infty + \infty; \infty + \infty\} = 1.\end{aligned}$$

$$\begin{aligned} \lambda_3(1) &= \min\{\lambda_1(0) + c_{13}; \lambda_2(0) + c_{23}; \lambda_3(0) + c_{33}; \lambda_4(0) + c_{43}; \lambda_5(0) + c_{53}\} = \\ &= \min\{0 + \infty; \infty + 8; \infty + \infty; \infty + 2; \infty + \infty\} = \infty. \end{aligned}$$

$$\begin{aligned}\lambda_4(1) &= \min\{\lambda_1(0) + c_{14}; \lambda_2(0) + c_{24}; \lambda_3(0) + c_{34}; \lambda_4(0) + c_{44}; \lambda_5(0) + c_{54}\} = \\ &= \min\{0 + \infty; \infty + 7; \infty + 1; \infty + \infty; \infty + 4\} = \infty.\end{aligned}$$

$$\begin{aligned}\lambda_5(1) &= \min\{\lambda_1(0) + c_{15}; \lambda_2(0) + c_{25}; \lambda_3(0) + c_{35}; \lambda_4(0) + c_{45}; \lambda_5(0) + c_{55}\} = \\ &= \min\{0 + 3; \infty + 1; \infty - 5; \infty + \infty; \infty + \infty\} = 3.\end{aligned}$$

$$\lambda_i(1)$$

$$k = 2. \lambda_1(2) = 0.$$

(1) $k = 2$:

$$\lambda_i(2) = \min_{1 \leq j \leq 5} \{ \lambda_j(1) + c_{ji} \}$$

$$\lambda_2(2) = \min\{0 + 1; 1 + \infty; \infty + \infty; \infty + \infty; 3 + \infty\} = 1.$$

$$\lambda_3(2) = \min\{0 + \infty; 1 + 8; \infty + \infty; \infty + 2; 3 + \infty\} = 9.$$

$$\lambda_4(2) = \min \{0 + \infty; 1 + 7; \infty + 1; \infty + \infty; 3 + 4\} = 7.$$

$$\lambda_5(2) = \min\{0 + 3; 1 + 1; \infty - 5; \infty + \infty; 3 + \infty\} = 2.$$

$$\lambda_i(2) \qquad \qquad \qquad \lambda_i(2)$$

$$k = 3. \lambda_1(3) = 0.$$

(1) $k = 3$:

$$\lambda_i(3) = \min_{1 \leq j \leq 5} \{ \lambda_j(2) + c_{ji} \}$$

$$\lambda_2(3) = \min\{0 + 1; 1 + \infty; 9 + \infty; 7 + \infty; 2 + \infty\} = 1.$$

$$\lambda_3(3) = \min\{0 + \infty; 1 + 8; 9 + \infty; 7 + 2; 2 + \infty\} = 9.$$

$$\lambda_4(3) = \min\{0 + \infty; 1 + 7; 9 + 1; 7 + \infty; 2 + 4\} = 6.$$

$$\lambda_5(3) = \min\{0 + 3; 1 + 1; 9 - 5; 7 + \infty; 2 + \infty\} = 2.$$

$$(3) \quad \lambda_i(3) \quad , \quad i=1, \dots, 5.$$

$$k=4. \lambda_1(4)=0.$$

$$(1) \quad k=4 \quad :$$

$$\lambda_i(4)=\min_{1 \leq j \leq 5}\left\{\lambda_j(3)+c_{ji}\right\}$$

$$\lambda_2(4)=\min \left\{0+1; 1+\infty; 9+\infty; 6+\infty; 2+\infty\right\}=1.$$

$$\lambda_3(4)=\min \left\{0+\infty; 1+8; 9+\infty; 6+2; 2+\infty\right\}=8.$$

$$\lambda_4(4)=\min \left\{0+\infty; 1+7; 9+1; 6+\infty; 2+4\right\}=6.$$

$$\lambda_5(4)=\min \left\{0+3; 1+1; 9-5; 6+\infty; 2+\infty\right\}=2$$

$$\lambda_i(4) \quad , \quad i=1, \dots, 5.$$

i ()	$\lambda_i(0)$	$\lambda_i(1)$	$\lambda_i(2)$	$\lambda_i(3)$	$\lambda_i(4)$
1	0	0	0	0	0
2	∞	1	1	1	1
3	∞	∞	9	9	8
4	∞	∞	7	6	6
5	∞	3	2	2	2

5. .

$$(2), \quad v_3 \quad v_r \quad s=3:$$

$$\lambda_r(3)+c_{r3}=\lambda_3(4), \quad v_r \in G^{-1}\left(v_3\right), \quad (3)$$

$$G^{-1}\left(v_3\right)-v_3.$$

$$(3) \quad G^{-1}\left(v_3\right)=\left\{v_2, v_4\right\} . \quad r=2 \quad r=4, \quad , \quad r$$

:

$$\lambda_2(3)+c_{23}=1+8 \neq \lambda_3(4)=8,$$

$$\lambda_4(3)+c_{43}=6+2=\lambda_3(4)=8,$$

$$v_3, \quad v_4 \quad v_r \quad (2) \quad s=4:$$

$$\lambda_r(2) + c_{r4} = \lambda_4(3), v_r \in G^{-1}(v_4), \quad (4)$$

$$G^{-1}(v_4) - v_4. \\ G^{-1}(x4) = \{x2, x3, x5\}. \\ (4) \quad r = 2, r = 3 \quad r = 5, \quad , \quad r$$

$$\vdots \\ \lambda_2(2) + c_{24} = 1 + 7 \neq \lambda_4(3) = 6, \\ \lambda_3(2) + c_{34} = 1 + 1 \neq \lambda_4(3) = 6, \\ \lambda_5(2) + c_{54} = 2 + 4 = \lambda_4(3) = 6 \\ , \quad , \quad v_4, \quad v_5. \\ v_5 \quad v_r \quad (2),$$

$$s = 5: \\ \lambda_r(1) + c_{r5} = \lambda_5(2), v_r \in G^{-1}(v_5), \quad (5)$$

$$G^{-1}(v_5) - v_5. \\ G^{-1}(v_5) = \{v_1, v_2\}. \\ (5) \quad r = 1 \quad r = 2, \quad , \quad r$$

$$\vdots \\ \lambda_1(1) + c_{15} = 0 + 3 \neq \lambda_5(2) = 2, \\ \lambda_2(1) + c_{25} = 1 + 1 = \lambda_5(2) = 2. \\ , \quad , \quad v_5, \quad v_2. \\ v_2 \quad v_r \quad (2), \\ s = 2.$$

$$\lambda_r(0) + c_{r2} = \lambda_2(1), v_r \in G^{-1}(v_2), \quad (6) \\ G^{-1}(v_2) - v_2. \\ G^{-1}(v_2) = \{v_1\}. \\ (6) \quad r = 1, \quad , \quad :$$

$$\lambda_1(0) + c_{12} = 0 + 1 = \lambda_2(1) = 1 \\ , \quad , \quad v_2, \quad v_1. \\ , \quad -v_1, v_2, v_5, v_4, v_3, \quad 8.$$

-

(* - *)
Program Ford;
var a : array [1..20,1..20] of word;(* *)

```

c, pred, fl, d : array [1..20] of word;
(*c –
pred –
fl –
d – *)

i, j, k, n, first, last : byte;
f : text;(* in.txt*)
(* – *)
Procedure Dfs(x : word);
var i : byte; (* *)
begin
  if x=last then (* *)
  begin
    write(first,' ');
    for i:=1 to j do (* *)
    write(d[i],' ');
    writeln;
    exit; (* *)
  end;
  fl[x]:=1; (* , *)
  for i:=1 to n do
  if (fl[i]=0)and(a[x,i]<>32767) then
  begin
    inc(j);
    d[j]:=i; (* *)
    dfs(i); (* i- *)
    dec(j);
  end;
  fl[x]:=0; (* , *)
end;
(* *)
begin
  assign(f,'in.txt'); (* *)
  reset(f);
  readln(f, n); (* *)
  for i := 1 to n do
  for j := 1 to n do
  read(f, a[i,j]); (* *)
  writeln('Matrix:');
  for i:=1 to n do (* *)
  for j:=1 to n do
  if j=n then writeln(a[i,j]) else write(a[i,j],' ');
  for i:=1 to n do (* *)
  for j:=1 to n do

```



```

if a[i,j]=0 then a[i,j]:=32767;
writeln('                1');
readln(first);
writeln('                2');
readln(last);
close(f); (*                file in.txt*)
for j := 1 to n do
begin
    c[j] := a[first,j]; (*                *)
    if a[first,j] < 32767 then
        pred[j] := first;
end;
for i := 3 to n do
for j := 1 to n do
if j <> first then
for k := 1 to n do (*                *)
if (c[k] < 32767) and (c[k] + a[k,j] < c[j]) then
begin
    c[j] := c[k] + a[k,j];(*                *)
    pred[j] := k;{                }
end;
if c[last] = 32767 then writeln('                ') else
begin
    writeln;
    writeln('                :');
    write(first, ' ');
    i := last;
    k := 1;
    while i <> first do (*                *)
    begin
        d[k] := i;(*                *)
        k := k + 1;
        i := pred[i];
    end;
    for i:= k-1 downto 1 do (*                *)
        write(d[i], ' ');
    writeln;
    writeln('                :');
    j:=0;
    Dfs(first);(*                *)
end;
readln; readln; (*                *)
end.

```

4.6.3.

1962

$$\left(\begin{array}{c} 1 \\ \vdots \end{array} \right)$$

$$A \quad n \times n, \\ A[i, j] \quad (i, j),$$

$$A[i, k] + A[k, j] < A[i, j], \\ i \rightarrow j \quad i \rightarrow k \rightarrow j.$$

$$0. \quad A_0$$

$$S_0. \\ 0, \quad k = 1.$$

$$k. \quad k \quad k \\ A[i, j] \quad A_{k-1}.$$

$$A[i, k] + A[k, j] < A[i, j], \quad (i \neq k, j \neq k, i \neq j),$$

:

$$\begin{array}{lll} 1. & A_k & A_{k-1} \\ A[i, j] & A[i, k] + A[k, j]; & \\ 2. & S_k & S_{k-1} \\ S[j, j] \quad k. & k = k + 1 & k. \end{array}$$

Program Floyd_Uorsh 1;

Uses Crt;

Const

PP=50;

Type

Graph = array[1..pp,1..pp] of integer;

Var

p:integer;

t,c,h:graph;

i,j: integer;

Procedure Floyd (var t:graph; c:graph; var h:graph);

var i,j,k:integer;

GM:real;

begin

GM:=10000;

for i:=1 **to** p **do**

for j:=1 **to** p **do** t[i,j]:=c[i,j];

if c[i,j]=GM **then** H[i,j]:=0 **else**

begin

H[i,j]:=j;

end;

for i:=1 **to** p **do**

for j:=1 **to** p **do**

for k:=1 **to** p **do**

if (i<>j)and(T[j,i]<>GM)and(i<>k)and(T[i,k]<>GM)and(T[j,k]=GM) or
(T[j,k]>T[j,i]+T[i,k]) **then**

begin

H[j,k]:=H[j,i];

T[j,k]:=T[j,i]+T[i,k]

end;

end;

Procedure Readfilegraph (var T:graph);

var

i,j:integer;

f: text;

begin

Writeln ('Reading from the text file');

```

Assign (f,'nell.txt');
reset(f);
Readln(f,P);
for i:=1 to p do for j:=1 to p do
  read(f,t[i,j]); close(f);
end;

begin
  Clrscr;
  Readfilegraph(c);
  floyd(t,c,h);
  writeln('-----');
  for i:=1 to p do
    begin
      for j:=1 to p do write (t[i,j]:3);
      writeln
    end;
    writeln('-----');
    for i:=1 to p do
      begin
        for j:=1 to p do write (h[i,j]:3);
        writeln
      end;
      readln;
    end.

```

4.6.4

(x_i, x_j)
 x_i x_j
 s 1 , $t - n$.
 $(x_i) -$ x_i , $t -$ $()$.
 1 x_i , $s -$ $()$, $t -$ $()$.
 $1.$ $(s)=0$, $(x_i) = \infty$ $x_i \in X/s$; $i=1$.
 $2.$ $i=i+1$. x_j (x_j) ,
 1 x_j ,
 $(x_j) = \min_{x_i \in {}^{-1}(x_j)} [(x_i) + c_{ij}]$ (6.2)
 $3.$ $. 2.$, n
 (t) .
 $x_i \in {}^{-1}(x_j)$, x_j , (x_i) ,
 $x_i < x_j$, x_i .

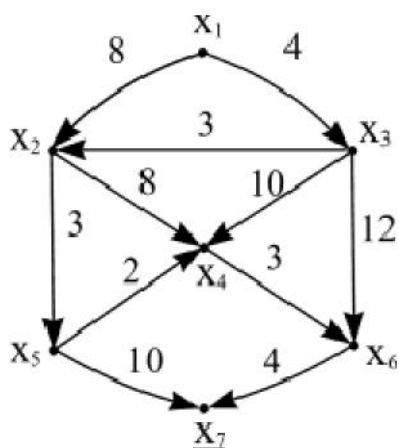
(t) s t ,
 (x_i, x_j) , (6.2),
 $(x_j) = (x_i) + c_{ij}$, (6.3)
 t , n ,
 x_j (, x_j^*),
 (6.3) ,
 $(x_j^* s)$.
 (x_j) x_j
 μ s x_j .

4.6.5.

(x_j, x_i) , x_j x_i ,
 $(j > i)$.

1. $i = n$, $n -$ G .
 2. x_k ,
 $| (x_k) | = \emptyset$ (, ,) . x_k
 i ()
 $i = i - 1$.

3. 2. , :
 1) $i = 1$ - .
 2) ,
 $| (x_k) | = \emptyset$..



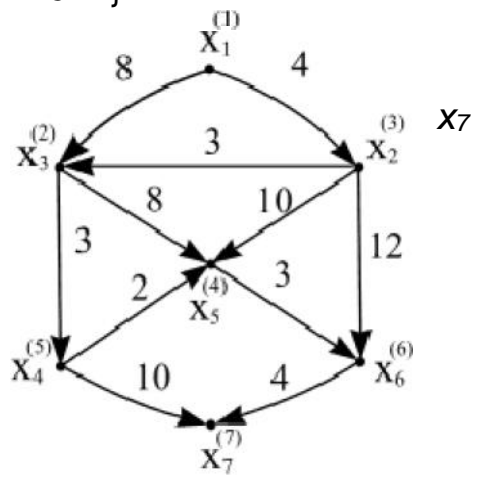
4.6.6.

. 6.3, x_1 x_7 ,

$(x_3, x_2) \quad (x_5, x_4),$
 $(x_1, x_2),$
 $x_7, x_6,$
 $x_4, x_5, x_2, x_3, x_1.$

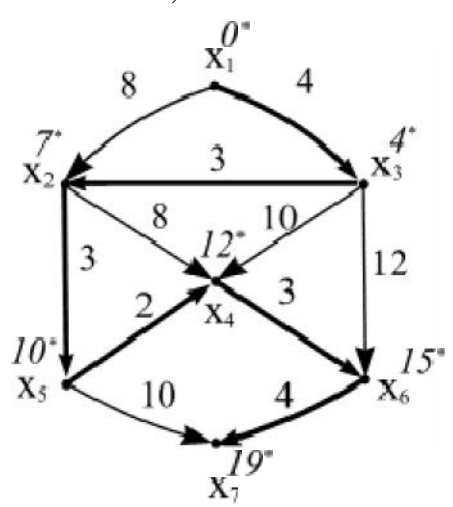
6.4 ($x_1=0$).
 6.3.

$x_2.$ $^{-1}(x_2)$ $x_1.$
 $(x_2)=\min\{0+4\}=4.$ $x_2,$ $(6.2),$
 $^1(x_3)=\{x_1, x_2\}.$ $x_3.$ x_3
 $: (x_3)=\min\{0+8, 4+3\}=7.$ x_4
 $: (x_4)=\min\{7+3\}=10.$ x_5
 $(x_2, x_5), (x_3, x_5), (x_4, x_5).$
 $x_5: (x_5)=\min\{4+10, 7+8,$
 $10+2\}=12.$



x_6
 $(x_6)=\min\{4+12, 12+3\}=15,$
 $(x_7)=\min\{10+10, 15+4\}=19.$
 x_7 $\mu(x_1, x_7)$
 $L(\mu)=19.$
 (6.3),

$x_7,$
 (6.3)
 $\ll \gg: x_6, x_5, x_4, x_3, x_2, x_1,,$
 6.4.



$(x_7)= (x_6)+ c_{67}, \quad (19=15+4);$
 $(x_6)= (x_5)+ c_{56}, \quad (15=12+3);$
 $(x_5)= (x_4)+ c_{45}, (12=10+2);$
 $(x_4)= (x_3)+ c_{34}, (10=7+3);$
 $(x_3)= (x_2)+ c_{23}, (7=4+3);$
 $(x_2)= (x_1)+ c_{12}, (4=0+4);$

$\mu(x_1, x_7)=\{x_1, x_3, x_2, x_5, x_4, x_6, x_7\}.$

6.5

. 6.5.

« » (x_i) .

4.7.

1.

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2.

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3.

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4.

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5.

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6.

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7.

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8.

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1.

Lazarus.

2.

LAB4_Project,

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3.

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4.

OperForm

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5.

OperForm

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6.

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- 1. ;
- 2. ;
- 3. ;
- 4. - ;
- 5. ;
- 6. .
- 7. .

- 1. .
- 2. .
- 3. ?
- 4. ?
- 5. ?
- 6. ?
- 7. ; .
- 8. ?
- 9. . .
- 10. ?
- 11. ?
- 12. ?
- 13. .
- 14. .
- 15. -
- 16. . , , , .
- 17. ?
- 18. ?
- 19. .
- 20. .
- 21. .
- 22. ?
- 23. .
- 24. .

25.

26.

NZK – I I = NZK mod 8+1,

1) 1 .) , , .) , ,
2) 2 .) , ,) , , .
3) 3 .) , , .) , , , -
4) 4 .) , , $C = [c_{i,j}]$.) , C ,
5) 5 .) , , $C = [c_{i,j}]$) , C , -
6) 6 .) , , $C = [c_{i,j}]$) , C ,

7	<p>) 7 .</p> <p>) , , $C = [c_{i,j}]$</p> <p>) , C ,</p>
8	<p>) 8 .</p> <p>) , , $C = [c_{i,j}]$</p> <p>) , C ,</p>

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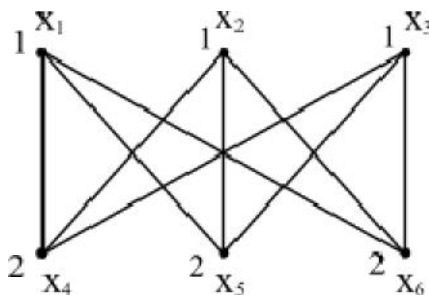
•

9

 \rangle, m

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5.1.

«1» «2» , .

$$r(G),$$

$$G,$$

, :

$$\chi(G) \geq \left\lceil \frac{n}{r(G)} \right\rceil, \tag{5.1}$$

n -

$$G, \quad \lceil x \rceil,$$

x .

$$\chi(G) :$$

$$\chi(G) \geq \frac{n^2}{n^2 - 2m}. \tag{5.2}$$

:

$$\chi(G) \leq 1 + \max_{x_j \in X} [d(x_j) + 1]. \tag{5.3}$$

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, , .
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 , , .
 , , .
 , -
 , G - , $\chi(G) \leq 5$.
 ().

$$\chi(G) \leq 4.$$

1852 .

« » ,

, K_n n ,

.

5.2.

.

```

Const Nmax=100; {*
Type V=0..Nmax;
    TS=Set of V;
    TColArr = Array (1..Nmax) of V;
    TA = Array (1..Nmax, 1..Nmax) of Integer;

Var ColArr: TColArr; {*
    A:TA; {*

Function Color (i): Integer;
{
Var W:TS;
    j:Byte;
Begin
    W:=[];
    For j=1 to i-1 do if A[j,i]=1 then W:=W+[ColArr[j]];
{
    j:=0; {
    Repeat
        Inc(j);
    Until NOT (j In W);
    Color:=j;
End;
Begin
    <
    {
    For i=1 to Nmax do ColArr[i]:=Color(i);
    <
End;

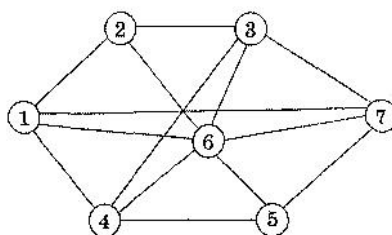
```

5.3.

3.

$G(V, E)$,

. 5.2.



.5.2

$$V = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

1. 1, Color(1)

2. 1. 2. 1 Color(2) W 2

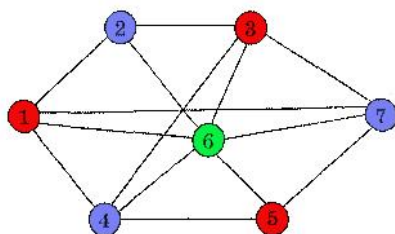
3. 3 2. Color(3) 2. 1

4. 4 :1 3. 1. Color(4) 2

5. 5 2. Color(5) 4. 1

6. 6 :1,3,4 5. :1 2. Color(6) 3 W

7. 7 :1,3, 5 6. :1 2. Color(7) 1 2 3. W



. 5.3.


```

Type TArr = Array (1..Nmax) of Byte;
      TA = Array (1..Nmax, 1..Nmax) of Byte;

Var ColArr: TArr; {*                                     *}
      DegArr: TArr {*                                     *}
      SortArr: TArr; {*                                     *}
      A: TA; {*                                     *}
      CurCol: Byte; {*                                     *}
      n: Byte;

Procedure DegForming; {*                                     *}
Var i: Byte;
Begin
  For i:=1 to Nmax do
    begin
      DegArr[i]:=0; ColArr[i]:=0;
      For j:=1 to Nmax do
        DegArr[i]:= DegArr[i]+A[i,j];
      end;
    End;
Procedure SortNodes; {*                                     *}
Var max,c,k,i: Byte;
Begin
  For k:=1 to Nmax-1 do
    begin
      max:=DegArr[k]; c:=k;
      For i:=k+1 to N do
        If DegArr[i] > max then
          begin
            max:= DegArr[ i];
            c:=i;
          end;
      DegArr[c]:= DegArr[ k];
      DegArr[k]:=max;
      SortArr[k]:=c;
    end;
  End;
Procedure Color (i: Byte);
  {*                                     *}
Var j: Byte;
Begin
  For j=1 to Nmax do if A[j,i]=0 then
    begin
      If ColArr[j]=0 then ColArr[j]:=CurCol;
    end;

```



```

End;
Begin
  CurCol:=1;
  <                                     >
  DegForming; {*                       *}
  SortNodes;  {*                       SortArr*}
  For n:=1 to Nmax do
  begin
    If ColArr[SortArr[n]]=0 then
    begin
      ColArr[SortArr[n]]:=CurCol;
      Color(SortArr[n]);
      Inc(CurCol);
    end;
  end;
  <                                     >
end;

```

5.5.

G ,

5.4.

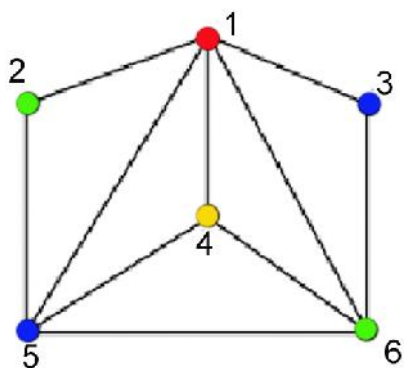
$\text{SortArr} = (1, 5, 6, 4, 2, 3)$

$D = (5, 4, 4, 3, 2, 2)$

SortArr , —

$\text{ColArr}[\text{SortArr}]$.

, $\chi(G) = 4$.



. 5.4.

SortArr	x_1	x_5	x_6	x_4	x_2	x_3
DegArr	5	4	4	3	2	2
CurCol = 1	1	-	-	-	-	-
CurCol = 2	1	2	-	-	-	2
CurCol = 3	1	2	3	-	3	2
CurCol = 4	1	2	3	4	3	2

5.6.

1.

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2.

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1.

:

$$\deg(x_i) \geq \deg(x_j), \forall x_i, x_j \in G.$$

$$\deg(x_i) = \deg(x_j), \forall x_i, x_j \in G$$

$$\Gamma(x_i) \cap \Gamma(x_j) = \emptyset.$$

:

$$[\deg(x_{i1}) + \deg(x_{i2}) + \dots + \deg(x_{ik})] \geq [\deg(x_{j1}) + \deg(x_{j2}) + \dots + \deg(x_{jn})],$$

$$x_{i1}, x_{i2}, \dots, x_{ik} \in \Gamma(x_i);$$

$$x_{j1}, x_{j2}, \dots, x_{jn} \in \Gamma(x_j);$$

$$p := 1, i := 1.$$

2.

$$\text{col}(x_i) := p; X = \{x_i\}.$$

3. $i := i + 1$.

$$x_i$$

$$: x_i \cap \Gamma(X) = \emptyset, \quad X \text{ -$$

,

$$p.$$

$$x_i$$

,

$$p : \text{col}(x_i) := p.$$

4.

3

$$(i = n).$$

5.

,

—

;

$$: p := p + 1; i := 1.$$

2.

SortNodes,

```

    A.
Const Nmax=100; {*
Type TArr = Array (1..Nmax) of Integer;
    TA = Array (1..Nmax, 1..Nmax) of Byte;

Var ColArr: TArr; {*
    DegArr: TArr {*
    SortArr:TArr; {*
    A:TA; {*
    CurCol: Byte; {*
    n:Byte;

Procedure DegForming; {*
Var k:Byte;
    Function DegCount(m:Byte):Integer;
    Var Deg:Integer;
    Begin
        Deg:=0;
        For k:=1 to Nmax do Deg:= Deg+A[k,m];
        DegCount:=Deg;
    End;
Begin
    For j:=1 to Nmax do
    begin
        ColArr[i]:=0;
        DegArr[j]:= DegCount(j)*100;
        For i:=1 to Nmax do
            If A[i,j]=1 then DegArr[i]:= DegArr[i]+DegCount(i);
        end;
    End;

Procedure SortNodes; {*
Var max,c,k,i:Byte;
Begin
    For k:=1 to Nmax-1 do
    begin
        max:=DegArr[k]; c:=k;
        For i:=k+1 to N do
            If DegArr[i] > max then
            begin
                max:= DegArr[ [i];

```

```

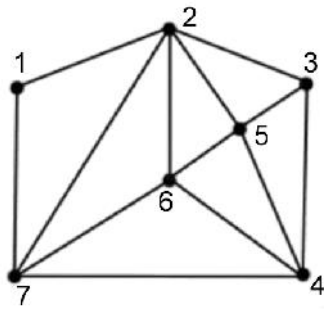
    c:=i;
  end;
  DegArr[c]:= DegArr[ [k];
  DegArr[k]:=max;
  SortArr[k]:=c;
end;
End;
Procedure Color (i:Byte);
{
*}
Var j:Byte;
Begin
  For j=1 to Nmax do if A[j,i]=0 then
  begin
    If ColArr[j]=0 then ColArr[j]:=CurCol;
  end;
End;
Begin
  CurCol:=1;
  <                                     >
  DegForming; {
  SortNodes; {
*}
For n:=1 to Nmax do
begin
  If ColArr[SortArr[n]]=0 then
  begin
    ColArr[SortArr[n]]:=CurCol;
    Color(SortArr[n]);
    Inc(CurCol);
  end;
end;
  <                                     >
end;

```

5.7.

G ,

5.3



. 5.5.

$$\text{SortArr} = (2, 6, 5, 4, 7, 3, 1)$$

$$D = (5, 4, 4, 4, 4, 3, 2)$$

$$\text{SortArr}, \quad - \quad D,$$

$$- \quad D^2.$$

$$D \quad D^2$$

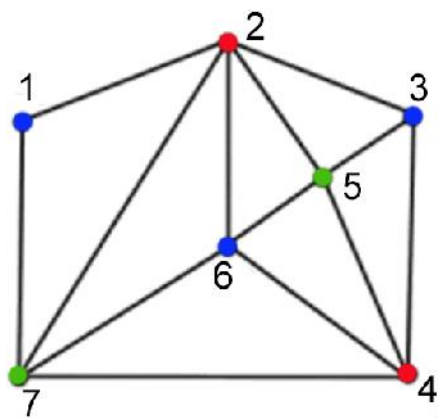
DegArr.

$$\text{col}(X^*).$$

$$\chi(G) = 3.$$

X^*	a_2	a_6	a_5	a_4	a_7	a_3	a_1
D	5	4	4	4	4	3	2
D^2		17	16	15	15		
DegArr	500	417	416	415	415	300	200
CurCol = 1	1	-	-	1	-	-	-
CurCol = 2	1	2	-	1	-	2	2
CurCol = 3	1	2	3	1	3	2	2

, . 5.6.



. 5.6.

5.8.

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(1931-1988 .),

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.

$v \in V$ $G(V, E)$

1-

$R_1(v)$.

v ,

2-

$R_2(v)$.

$G(V, E)$,

$v \in V$

$R_1(v)$

-

v .

r

v

$R_1(v) \ll$

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r .

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v_1 v_2

$R_2(v_1)$

$v_2 \in R_2(v_1)$.

,

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r

$v_2 \in R_2(v_1)$.

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r

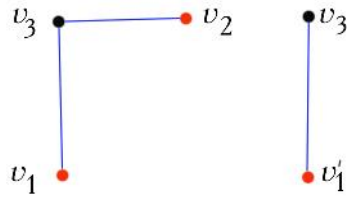
.

v_1 v_2

«

»

.



. 14.1.

$$: v'_1 := v_1 \cup v_2$$

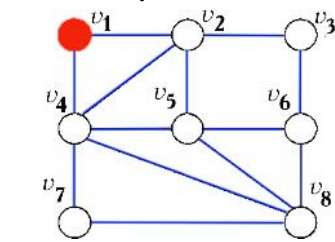
$$G,$$

1. $i := 0.$
2. G
3. $i := i + 1.$
4. v $i.$
5. i $G,$
6. $R_2(v),$ - $v.$
7. $G.$
8. $X(K_i) = i.$

5.9.

.14.2

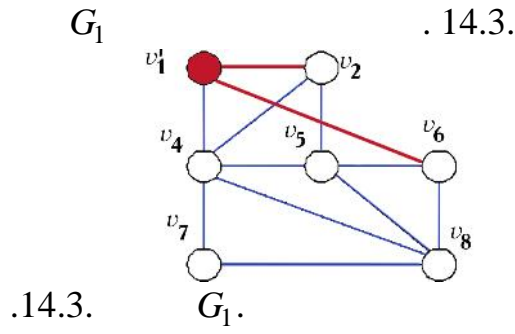
$G,$



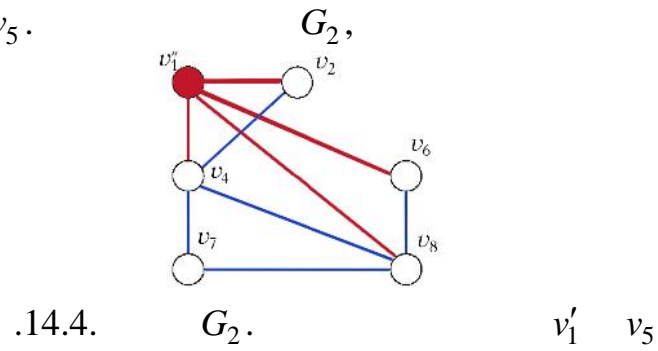
.14.2.

G

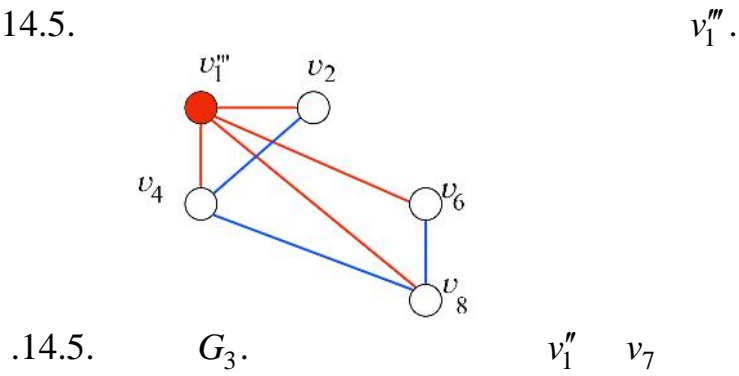
$R_2(v_1) = \{v_3, v_5, v_7, v_8\}$.
 $v'_1 = v_1 \cup v_3$.



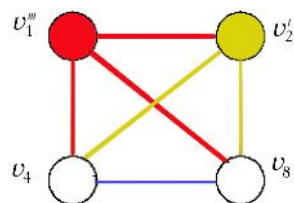
$R_2(v'_1) = \{v_5, v_7, v_8\}$.
 $v''_1 := v'_1 \cup v_5$.



$R_2(v''_1) = \{v_7\}$.
 G_3 ,



$R_2(v_2) = \{v_6, v_8\}$.
 $v'''_1 = v''_1 \cup v_6$.

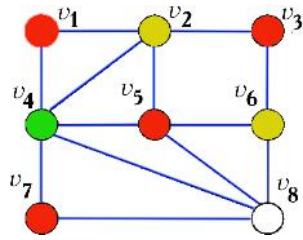


.14.6. G_4 K_4 . v_1'' v_6

G_4 K_4 . , G_4 .14.6

()
 v_1 : v_3, v_5 v_7 .
 v_2 v_4 () v_6 .
 v_8 .

.14.7.



.14.7. G ,

. . .

G , . 14.2.

$$A = \begin{pmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \\ v_1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ v_2 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_4 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ v_5 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ v_6 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ v_7 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ v_8 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

, , ,

.
 , 1 3.

v_1' , v_1 v_3 .

$$A' = \begin{pmatrix} & v'_1 & v_2 & v_4 & v_5 & v_6 & v_7 & v_8 \\ v'_1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ v_2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ v_5 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ v_6 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ v_7 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ v_8 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

$$A''' = \begin{pmatrix} & v'''_1 & v'_2 & v_4 & v_8 \\ v'''_1 & 0 & 1 & 1 & 1 \\ v'_2 & 1 & 0 & 1 & 1 \\ v_4 & 1 & 1 & 0 & 1 \\ v_8 & 1 & 1 & 1 & 0 \end{pmatrix},$$

K_4 .

```

Const n=10; {*
Type V=0..n;
    R2S=Set of V;
    TA = Array (1..n, 1..n) of Integer;

Var A:TA; {*
    MainNode:Byte;

Procedure Glue(master,slave:Byte);
{
    *
Begin
    {
        master      slave*}
    For i=1 to n do A[i,master]:= A[i,master] OR A[i,slave];
{
    {
        master      slave*}
    For j=1 to n do A[master,j]:= A[master,j] OR A[slave,j];
End;

Procedure Reduce(master,slave:Byte);
{
    *
Begin
    For i:=1 to n do
        For j:=1 to n-1 do
            {
                slave*}
            If (j≥slave) then A[i,j]:= A[i,j+1];

```

```

For j:=1 to n-1 do
  For i:=1 to n-1 do
    {*          slave*}
    If (i≥slave) then A[i,j]:= A[i+1,j];
  n:=n-1;
End;
Function Check_K:Byte;
{*          *}
Var Gh:Byte;
Begin
  {*          ,          ,          *}
  Ch:=0;
  For i:=1 to n do
    For j:=1 to n do if (i≠j) AND (A[i,j]=0) then Ch:=j;
  Check_K:=Ch;
End;

Procrdure R2(master:Byte);
{*          2-          *}
Begin
  For j:=1 to n do
    begin
      If (j≠master) AND (A[master,j]=1) then
        begin
          {*          master*}
          For i=1 to n do
            begin
              If (i≠master) AND (A[j,i]=1) then
                {*          2-          master*}
                R2S:=R2S+[i]; {*          *}
            end;
          end;
        end;
      end;
    End;

Begin
  MainNode:=1; {*          *}
  K_finded:=false; {*          *}
  While K_finded do
    begin
      R2S:=[]; {*          2-          *}
      R2(MainNode); {*          2-          MainNode*}
      For k=1 to n do {*          *}
        begin
          If k in R2S then
            begin {*          *}

```

```

    {*
      MainNode
    k *}
    Glue(MainNode,k);
    {*
      *
    Reduce(MainNode,k);
  end;
end;
MainNode:= Check_K(MainNode);
If MainNode=0 then K_finded:=true;
end;
End;

```

5.10.

1. .
2. , ,
3. ,

```

Const n=10;
      Cmax=10;

```

Type

```

TA = Array (1..n, 1..n) of Byte;
TArr = Array (1..n) of Byte;

```

```

Var i:Byte;
    color:TArr;
    A:TA;
    C:Byte;

```

```

procedure visit(i:Byte);

```

```

  Function Nicecolor:Boolean;

```

```

  {*
    *}

```

```

  Var CN:Boolean;

```

```

    j:integer;

```

```

  Begin

```

```

    CN:=true;

```

```

    For j=1 to n do

```

```

      If (A[j,i]=1) AND (color[j]=c) then CN:=false;

```

```

      {*
        c.
      *}

```

```

    End;

```

```

begin

```

```

  if i = n + 1 then Print else

```

```

  {*
    *}
  begin

```

```

If color[i]=0 then {*                                *}
begin
  for c:=color[i]+1 to Cmax do
    if Nicecolor then
      begin
        color[i]:=c;
        {*                                *}
        visit(i+1);
        {                                }
      end;
    end;
  end;
end;

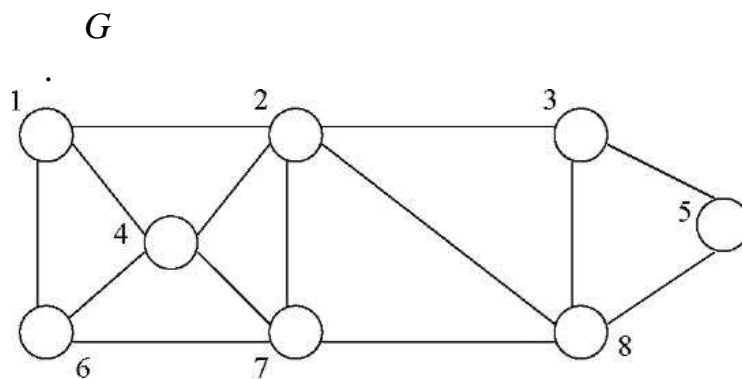
```

```

Begin
  i:=1;
  visit(i);
End;

```

5.11.

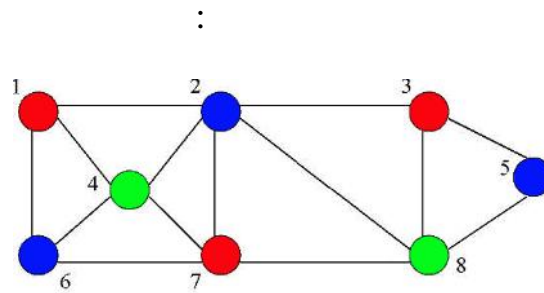


A

$$A = \begin{pmatrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 4 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 5 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 6 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 7 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 8 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Visit,

Visit(1)	+		
Visit(2)	-	+	
Visit(3)	+		
Visit(4)	-	-	+
Visit(5)	-	+	
Visit(6)	-	+	
Visit(7)	+		
Visit(8)	-	-	+



5.12. « »

, $G(V, E)$.

1. $monochrom := \emptyset$, , .

2. « » :

Procedure Greedy

For ($v \in V$) **do**

If v $monochrom$ **then**

begin

$color(v) :=$;

$monochrom := monochrom \cup \{v\}$

End;

Const $N=10$; { * }

Type $V=0..N$;

$TS = \text{Set of } V$;

$TColArr = \text{Array } (1..N) \text{ of } V$;

$TA = \text{Array } (1..N, 1..N) \text{ of Integer}$;

```

Var ColArr: TColArr; {*                                     *}
    A:TA; {*                                     *}
    Color:Byte;
    AllColored:Boolean;
    k:Byte;
Procedure Avid(i:Integer);
{*                                     i *}
Var W:TS;
    j:Byte;
function Check(i):Boolean; {*                                     *}
var Ch:Boolean;
begin
    Ch:=true;
    For j=1 to n do
    If (A[j,i]=1)then {*                                     j                                     ,                                     *}
    If (j in W)then Ch:=false;
    {*                                     j                                     *}
    Check:= Ch;
end;
Begin
    Inc(Color); {*                                     *}
    W:=[]; {*                                     *}
    ColArr[i]:=Color; {*                                     *}
    W:=W+[i]; {*                                     *}
{*                                     *}
    For k:=1 to n do if ColArr[k]=0 then
    If Check(k)then begin ColArr[k]:=Color; W:=W+[k];end;
End;
Begin {*                                     *}
    <                                     >
{*                                     *}
    Color:=0;
    AllColored:=false; {*                                     ,                                     *}
    While not AllColored do
    Begin
        AllColored:=true;
        For i=1 to N do If ColArr[i]=0 then
        begin
            {*                                     *}
            AllColored:=false;
            Avid(i); {*                                     *}
        end;
    End;
    <                                     >
End;

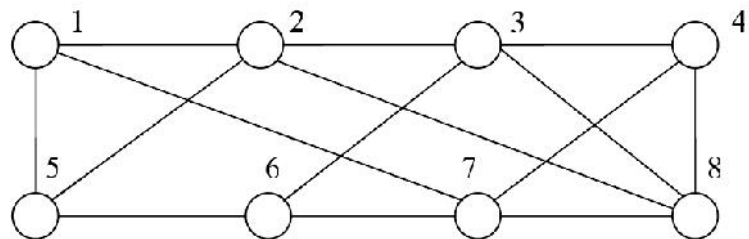
```

5.13.

« »

G

« »



A

$$A = \begin{pmatrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 5 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 6 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 7 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 8 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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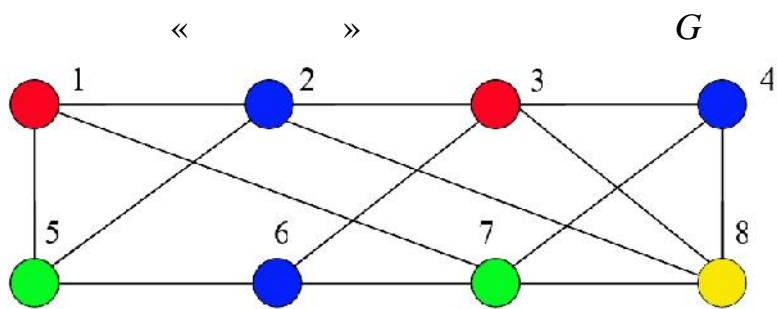
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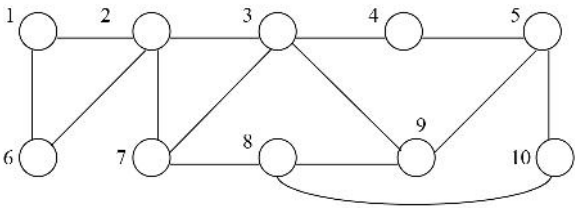
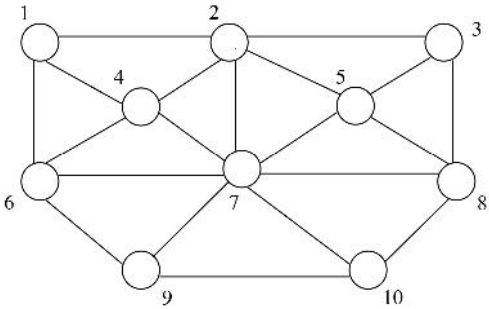
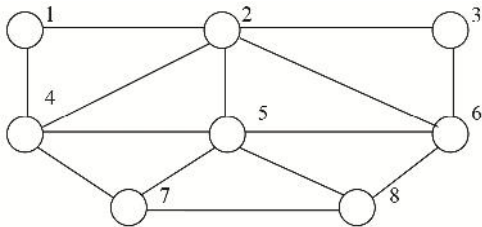
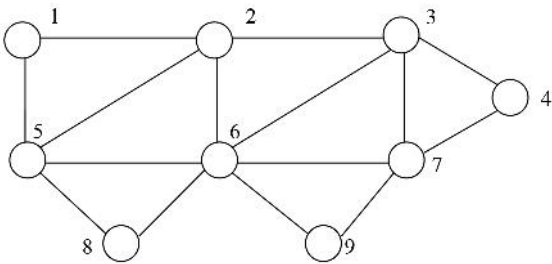
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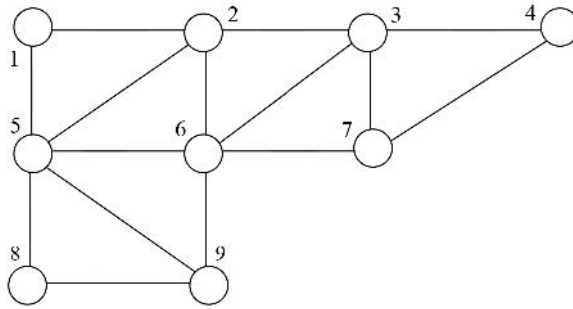
1	<p>) 1 .</p> <p>)</p> <p>)</p> <p><i>G</i></p> <p>,</p>

		.
2)))	2 . . . G  , .
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 G 

1.	1. «	:	
	,	».....	3
1.1.			3
1.2.			4
1.3.			6
1.4.			7
1.5.			8
1.6.			8
1.7.	-		9
1.8.		1	14
2.	2. «		
	,	»	15
2.1.			15
2.2.			16
2.3.			17
2.4.			18
3.	3. «	:	
		,	
		,	
	».....		22
3.1.			22
3.2.			24
3.2.1.			25
3.2.2.	-		29
3.2.3.	-	n	30
3.2.4.	-	n	31
3.2.5.			33
3.2.6.	-		34
3.2.7.			35
3.2.8.	-		37
3.2.9.			38
3.2.10.	-		38
3.2.11.			39
3.2.12.	-		40
3.2.13.			42
3.2.14.	-		42
3.2.15.		n k	43
3.2.16.	-	n k	44
3.2.17.		n k	46
3.2.18.	-	n k	47
3.2.19.		n	49

3.2.20.	-	<i>n</i>	52
4.		4. « . ».....	58
4.1.			58
4.2.			63
4.2.1.			63
4.2.2.			64
4.3.			64
4.4.	-		65
4.5.			66
4.6.			67
4.6.1.			67
4.6.2.	-		75
4.6.3.	-		82
4.6.4.			84
4.6.5.			85
4.7.			85
5.		5. « , »	91
5.1.			91
5.2.			92
5.3.			93
5.4.			95
5.5.			97
5.6.			98
5.7.			100
5.8.		. .	102
5.9.		. .	103
5.10.			108
5.11.			109
5.12.	« »		110
5.13.	« »		112
5.14.		5	113