N4

$$\int_0^\infty \frac{x^2dx}{(x^2+N^2)(x^2+4N^2)}$$

$$\int_0^\infty \frac{z^2dz}{(z^2+15^2)(z^2+30^2)} = \frac{1}{2}\int_{-\infty}^{\infty} \frac{z^2dz}{(z^2+15^2)(z^2+30^2)} =$$

$$= \pi i \cdot \sum_{k=1}^{m} \underset{z_k}{Res}\, f(z)$$

$z_1 = 15i$ - простий полюс
так як $Im(z) > 0$
$z_2 = 30i$ - простий полюс

$$\underset{z_1}{Res}\, f(z) = \lim_{z\to 15i} \frac{z^2\cdot(z-15i)}{(z+15i)(z-15i)(z^2+30^2)} =$$

$$= \left[\frac{(15i)^2}{(15i+15i)((15i)^2+30^2)}\right] = \frac{-225}{30i\cdot(900-225)} =$$

$$= -\frac{1}{90i}$$

$$\underset{z_2}{Res}\, f(z) = \lim_{z\to 30i} \frac{z^2(z-30i)}{(z^2+15^2)(z+30i)(z-30i)} =$$

$$= \left[\frac{(30i)^2}{((30i)^2+15^2)(30i+30i)}\right] = \frac{-900}{(225-900)\cdot 60i} = \frac{1}{45i}$$

$$\int_0^\infty \frac{z^2dz}{(z^2+15^2)(z^2+30^2)} = \pi i\cdot\left(\frac{1}{45i} - \frac{1}{90i}\right) = \frac{\pi}{90}$$

Відповідь: $\int_0^\infty \frac{x^2dx}{(x^2+15^2)(x^2+4\cdot 15^2)} = \frac{\pi}{90}$

$p^2Y(p) - p - 1 - 2pY(p) + 2 - 3Y(p) = \frac{2}{p^2}$

$Y(p)\cdot(p^2 - 2p - 3) = \frac{2}{p^2} + p - 1$

$Y(p) = \frac{2 + p^3 - p^2}{p^2(p^2-2p-3)} = \frac{A}{p} + \frac{B}{p^2} + \frac{C}{p-3} + \frac{D}{p+1}$

$Ap(p-3)(p+1) + B(p-3)(p+1) + Cp^2(p+1) +$
$+ Dp^2(p-3) = p^3 - p^2 + 2$

$p = 3 \quad ; \quad 36C = 20$
$C = \frac{5}{9}$

$p = -1 \quad ; \quad -4D = 0$
$D = 0$

$p = 0 \quad ; \quad -3B = 2$
$B = -\frac{2}{3}$

$p = 1 \quad ; \quad -4A - 4B + 2C - 2D = 2$
$-4A = 2 - 4\cdot\frac{2}{3} - 2\cdot\frac{5}{9}$
$-4A = -\frac{16}{9}$
$A = \frac{4}{9}$

$Y(p) = \frac{4}{9p} - \frac{2}{3p^2} + \frac{5}{9(p-3)}$

$y(t) = \frac{4}{9} - \frac{2}{3}t + \frac{5}{9}e^{3t}$

Відповідь: $y(t) = \frac{4}{9} - \frac{2}{3}t + \frac{5}{9}e^{3t}$

$$I_2 = \int_1^2 e^{-pt}\,dt = -\frac{e^{-pt}}{p}\Big|_1^2 = \frac{e^{-p} - e^{-2p}}{p} = \frac{e^p - 1}{p \cdot e^{2p}}$$

$$I_3 = \int_2^3 (3-t)e^{-pt}\,dt = \left| \begin{matrix} u = 3-t \\ dv = e^{-pt}dt \\ du = -dt \\ v = -\frac{e^{-pt}}{p} \end{matrix} \right| =$$

$$= (3-t)\left(-\frac{e^{-pt}}{p}\right)\Big|_2^3 - \int \frac{e^{-pt}}{p}\,dt = -\frac{e^{-2p}}{p} + \frac{1}{p^2} \cdot e^{-pt}\Big|_2^3 =$$

$$= -\frac{e^{-2p}}{p} + \frac{1 - e^p}{p^2 \cdot e^{3p}}$$

$$F(p) = I_1 + I_2 + I_3 = \frac{1}{p}\left(1 + \frac{e^p - 1}{p \cdot e^p}\right) + \frac{e^p - 1}{p \cdot e^{2p}} - \frac{e^{-2p}}{p} +$$

$$+ \frac{1 - e^p}{p^2 \cdot e^{3p}} = \frac{1}{p}\left(1 + \frac{e^p - 1}{p \cdot e^p} + \frac{e^p - 2}{e^{2p}} + \frac{1 - e^p}{p \cdot e^{3p}}\right)$$

Відповідь: $F(p) = \frac{1}{p}\left(1 + \frac{e^p - 2}{e^{2p}} + \frac{e^{3p} - e^{2p} - e^p + 1}{p \cdot e^{3p}}\right)$

N

Операційним методом розв'язати задачу Коші:

$y'' - 2y' - 3y = 2t \qquad y(0) = y'(0) = 1$

$y(t) \risingdotseq Y(p)$

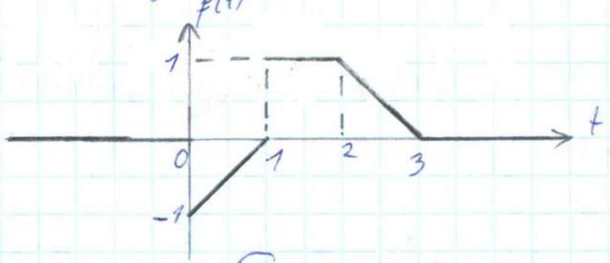
$y'(t) \risingdotseq pY(p) - 1$

$y''(t) \risingdotseq p^2Y(p) - p - 1$

$2t \risingdotseq \frac{2}{p^2}$

№3

Знайти зображення за поданим графіком оригіналу



$$f(t)=\begin{cases} t-1 & ; \quad t\in[0;1] \\ 1 & ; \quad t\in[1;2] \\ 3-t & ; \quad t\in[2;3] \\ 0 & ; \quad t\in(-\infty;0)\cup(3;+\infty) \end{cases}$$

$$F(p)=\int_0^1 (t-1)e^{-pt}dt+\int_1^2 e^{-pt}dt+$$

$$+\int_2^3 (3-t)e^{-pt}dt=I_1+I_2+I_3$$

$$I_1=\int_0^1 (t-1)e^{-pt}dt=\left|\begin{matrix} u=t-1 \\ dv=e^{-pt}dt \\ du=dt \\ v=-\frac{e^{-pt}}{p} \end{matrix}\right|=$$

$$=(t-1)\cdot\left(-\frac{e^{-pt}}{p}\right)\Big|_0^1+\int_0^1\frac{e^{-pt}}{p}dt=\frac{1}{p}+\frac{1}{p}\cdot\left(\frac{e^{-pt}}{-p}\right)\Big|_0^1=$$

$$=\frac{1}{p}\left(1+\frac{1-e^{-p}}{p}\right)=\frac{1}{p}\left(1+\frac{e^{p}-1}{p\cdot e^{p}}\right)$$

ст. 64 № 1

Знайти зображення

$$f(t) = \frac{1-\cos t}{t} \cdot e^t$$

За властивістю інтегрування зображення

$$\frac{(1-\cos t)\cdot e^t}{t} \doteq \int_p^\infty F(p)\,dp \quad , \text{ де } F(p) \doteq e^t(1-\cos t)$$

$$e^t(1-\cos t) = e^t - e^t\cos t \doteq \frac{1}{p-1} - \frac{p-1}{(p-1)^2+1^2}$$

$$\frac{(1-\cos t)\cdot e^t}{t} \doteq \int_p^\infty \left(\frac{1}{p-1} - \frac{p-1}{(p-1)^2+1}\right)dp =$$

$$= \int_p^\infty \frac{d(p-1)}{p-1} - \frac{1}{2}\int_p^\infty \frac{d((p-1)^2+1)}{(p-1)^2+1} = \left(\ln|p-1| - \frac{1}{2}\ln|(p-1)^2+1|\right)\Big|_p^\infty =$$

$$= \ln\left|\frac{p-1}{\sqrt{(p-1)^2+1}}\right|\Big|_p^\infty = \ln\left|\frac{1}{\sqrt{1+\frac{1}{(p-1)^2}}}\right|\Big|_p^\infty =$$

$$= \ln\sqrt{1+\left(\frac{1}{p-1}\right)^2}$$

Відповідь: $f(t) = \frac{1-\cos t}{t} \cdot e^t \doteq \ln\sqrt{1+\frac{1}{(p-1)^2}}$