Unit 7. Discrete Mathematics

Lesson 19

Whole-Class Activity

Task 1. Pre-Assessment

You are going to read questions about Discrete Mathematics. Use your background knowledge to answer them. You may turn to Activity Pack if you need any scaffolds. You have 5 minutes to complete this task.

RATIONAL CONCERN

1. What topics of Discrete Math do you believe to be most useful?
2. What do you think is the hardest topic of Discrete Math?
3. How can knowledge of Discrete Math be applied in everyday life?

Analytical Concern

Practical Concern

1. What practical application does Discrete Math have?
2. What tasks does Discrete Math solve?
3. What spheres of activity need Discrete Math most?

1. What is Discrete Mathematics?
2. What is the difference between Discrete Math and Algebra?
3. How can Discrete Mathematics help in business?

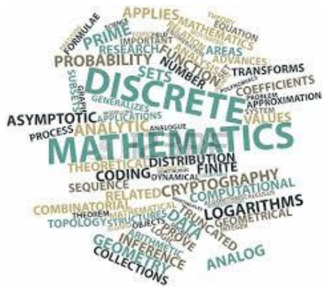
Creative Concern

1. What is the most interesting topic of Discrete Math?
2. What associations do you have with Discrete Mathematics?
3. How can Discrete Mathematics help to create art objects?

Task 2. Reading

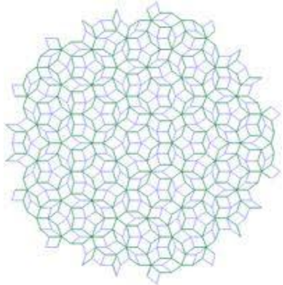
Now, read the text about Discrete Mathematics and answer the quiz questions. You have 20 minutes for this activity.

Discrete Mathematics



Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly", the objects studied in discrete mathematics – such as integers, graphs, and statements in logic – do not vary smoothly in this way, but have distinct, separated values. Discrete mathematics therefore excludes topics in "continuous mathematics" such as calculus and analysis.

Discrete objects can often be enumerated by integers. More formally, discrete mathematics has been characterized as the branch of mathematics dealing with countable sets (sets that have the same cardinality as subsets of the natural numbers, including rational numbers but not real numbers). However, there is no exact, universally agreed, definition of the term “discrete mathematics.” Indeed, discrete mathematics is described less by what is included than by what is excluded: continuously varying quantities and related notions.



The set of objects studied in discrete mathematics can be finite or infinite. The term finite mathematics is sometimes applied to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business.

The study of how discrete objects combine with one another and the probabilities of various outcomes is known as combinatorics. Other fields of mathematics that are considered to be part of discrete mathematics include graph theory and the theory of computation. Topics in number theory such as congruences and recurrence relations are also considered part of discrete mathematics.

The study of topics in discrete mathematics usually includes the study of algorithms, their implementations, and efficiencies. Discrete mathematics is the mathematical language of computer science, and as such, its importance has increased dramatically in recent decades.

Research in discrete mathematics increased in the latter half of the twentieth century partly due to the development of digital computers which operate in discrete steps and store data in discrete bits. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, cryptography, automated theorem proving, and software development. Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in operations research.

Although the main objects of study in discrete mathematics are discrete objects, analytic methods from continuous mathematics are often employed as well.

(The text is borrowed and modified from http://mathworld.wolfram.com/DiscreteMathematics.html and http://en.wikipedia.org/wiki/Discrete_mathematics as of 9th January, 2014)

1. What does Discrete Mathematics study?
    A. Any mathematical structures.
    B. Continuous structures.
    C. Integers, graphs etc.
    D. Astrological predictions.

2. What topics are excluded from Discrete Mathematics` study?
    A. Calculus and analysis.
    B. Logic.
    C. Combinatorics.
    D. Probability.
3. Combinatorics studies…
    A. words and sentences.
    B. computations.
    C. any numbers.
    D. how discrete objects combine.

4. What was the reason of discrete mathematics research increase?
    A. Economical value.
    B. Development of digital computers.
    C. Social changes.
    D. Development of space exploration.
5. Where can concepts of discrete mathematics be useful?
    A. Geography.
    B. Linguistics.
    C. Computer Science.
    D. Physical Education.

Task 3. Vocabulary Practice

Match the spheres of Discrete Mathematics` study with their descriptions. You have 5 minutes for this task.

1. Coding theory	a. is concerned with identifying the values, uncertainties and other issues relevant in a given decision, its rationality, and the resulting optimal decision
2. Logic	b. deals with situations where success depends on the choices of others, which makes choosing the best course of action more complex
3. Decision theory	c. is the branch of mathematical logic that studies sets, which are collections of objects
4. Combinatorics	d. the study of the properties of codes and their fitness for a specific application
5. Game theory	e. is concerned with the properties of numbers in general, particularly integers
6. Probability theory	f. the study of the principles of valid reasoning and inference
7. Number theory	g. studies the way in which discrete structures can be combined or arranged
8. Set theory	h. deals with events that occur in countable sample spaces

Task 4. Vocabulary Practice

Read the following extract about social choice theory, which lies in the scope of study of Discrete Mathematics. Suggest your antonyms to the adjectives in bold. Underline suffixes and prefixes which are used in adjectives. You have 3 minutes for this task.

Social choice theory or social choice is a (1) **practical** framework for analysis of combining individual opinions to reach a (2) **individual** decision or social welfare. Individual preferences are collected to produce a social welfare function – essentially a preference ranking of the scenarios that are (3) **impossible** to society. Social choice theory is the philosophical and mathematical study of the type of conclusions that can be determined through (4) **same** aggregation methods.

Using (5) **similar** factors such as interests, values and welfare, social choice theory aims to determine the (6) **worst** rules of structuring a (7) **unfair** voting framework. Social choice theory is a (8) **failing** discipline started by Kenth Arrow who originated the study following his introduction of the impossibility theorem in

1951. This theory is (9) **inapplicable** to group decision making, negotiations and (10) **same** economic processes.

(The text is borrowed and modified from http://en.wikipedia.org/wiki/Social_choice_theory as of 9th January, 2014)

Task 5. Language in Use

Conditionals are often used in mathematical discourse with the aim of explaining hypothesis-conclusion relations. They are often used in proofs, equations solving, explanations etc. Study the table in which the use and functions of conditionals are explained. After doing so, match the parts of the table to create conditionals. You have 10 minutes for this task.

Conditionals

Conditionals consist of two parts: if-clause (hypothesis) and main clause (result). There are four main types of conditional sentences:

Type 0 – expresses something which is always true.
If+present simple, main clause – present simple

e.g. If we add two positive numbers, it makes positive number.

Type 1 – expresses real or very probable actions in the present/future.
If+present simple, main clause – future/imperative/modal verb

e.g. If you solve this equation, the next one will be easy to understand.

Type 2 – expresses imaginary situations which are unlikely to happen in the present of future.
If+ past simple/past continuous, main clause- would/could +pres. Infinitive

e.g. If I were you, I would use another method.

If the numbers were even, it would be easier to solve equation.

Type 3 – expresses imaginary situations that are contrary to facts in the past.
If+past perf./past perf. continuous, main clause -would/could+perf.infinitive

e.g. If we had used another method, the proof would have been shorter.

If the population of men is 45%,	we could solve the equation.
If x=2,	we receive even number.
If you score 95% at the lesson,	the conclusion would be true.
If the total number was given,	the population of women is 55%.
If he had known all the variants,	you will find y and z.
If you find x,	then 3-x=1.
If we add two odd numbers,	he would have chosen the other one.

If the hypothesis was true,	you will receive A.

Differentiated Activity

Task 6. Group activity

Work out the topical content of the extracts and report your findings to the class. You have 10 minutes for this task.

Group 1. Read the following extract and get ready to explain the algorithm by drawing an example.

The nearest neighbour algorithm was one of the first algorithms used to determine a solution to the travelling salesman problem. In it, the salesman starts at a random city and repeatedly visits the nearest city until all have been visited. It quickly yields a short tour, but usually not the optimal one.

These are the steps of the algorithm:

1. stand on an arbitrary vertex as current vertex.
2. find out the shortest edge connecting current vertex and an unvisited vertex V.
3. set current vertex to V.
4. mark V as visited.
5. if all the vertices in domain are visited, then terminate.
6. Go to step 2.

Group 2. Read the following extract and get ready to explain the strengths and drawbacks of the algorithm.

The nearest neighbour algorithm is easy to implement and executes quickly, but it can sometimes miss shorter routes which are easily noticed with human insight, due to its "greedy" nature. As a general guide, if the last few stages of the tour are comparable in length to the first stages, then the tour is reasonable; if they are much greater, then it is likely that there are much better tours. Another check is to use an algorithm such as the lower bound algorithm to estimate if this tour is good enough.

In the worst case, the algorithm results in a tour that is much longer than the optimal tour. To be precise, for every constant r there is an instance of the travelling salesman problem such that the length of the tour computed by the nearest neighbour algorithm is greater than r times the length of the optimal tour. Moreover, for each number of cities there is an assignment of distances between the cities for which the nearest neighbour heuristic produces the unique worst possible tour. The nearest neighbour algorithm may not find a feasible tour at all, even when one exists.

(The text is borrowed and modified from http://en.wikipedia.org/wiki/Nearest_neighbour_algorithm as of 10th January, 2014)

Group 3. Read the extraxts from the tasks of Group 1 and 2. Inside your group, discuss what practical applications the algorithm may have and where the "travelling salesman problem" may appear.

Task 7. Tiered Task

You are going to watch a video with Robert Sedgewik talking about the course in Analytic Combinatorics. Choose whatever part of the task you feel confident to complete or do them all. You have 10 minutes for the task. Use the following link to watch the video: http://www.youtube.com/watch?v=Zrq-8qZks3U

Part 1. Decide whether the following statements are true (T) of false (F). Justify your answer.

1. According to D. Knuth, people who analyze algorithms can appreciate either elegant mathematical patterns or elegant computational procedures. □
2. The theoretical payoff of algorithm study is getting the job done more quickly and more economically. □
3. The study of algorithms on a scientific basis started over 50 years ago. □
4. Donald Knuth put the study of algorithms on a scientific basis. □
5. The course begins with the introduction to the Symbolic Method. □
6. The Symbolic Method for defining Generating Functions is described in the context of applications and the analysis of algorithms. □
7. Permutations, Trees, Streams, Words and Mappings belong to Basic Structures. □

Part 2. Fill in the gaps:

"People who analyse (1) ………… have double happiness. First of all, they (2) ………… the sheer beauty of elegant mathematical (3) ………… that surround elegant (4) …………… procedures. Then they receive a practical (5) ………… when their theories make it (6) ………… to get other jobs done more quickly and more(7) ………… ." – that`s how Donald Knuth, who put the study of algorithms on a scientific (8) ………… over 50 years ago, described the (9) ………… behind the analysis of algorithms. This course begins with the introduction to (10) ………… Functions and Real Asymptotics and then introduces the (11) ………….. Method for defining Generating Functions in the context of applications and the (12) ………. of algorithms, as well as basic structures, such as (13) ………… , Trees, Streams, Words and (14) ………… . As such, it is an introduction to Analytic (15) ………… in a context of the analysis of algorithms.

Part 3. Answer the following questions:

1. What does D. Knuth mean by saying that “People who analyse algorithms have double happiness”?
2. What practical effect may be achieved through the study of algorithms?
3. What topics are included in the Basic Structures?
4. What other topics are introduced in the course of Analytic Combinatorics?

Task 8. Pair work

Work in pairs. Using the help box from Task 5, ask each other questions about what would happen:

…if you were given the Nobel Prize?

…if there were no calculating machines?

…if you had to choose only one science to study?

…if every person was a mathematical prodigy?

…if you could analyse all the probabilities of any event?

You have 5 minutes to complete this task.

Task 9. Team Work: Brain Ring: Revanche

You are going to be divided into three teams. Choose your team name and the captain. The Brain Ring competition consists of two parts:

1. You have to solve and explain the solution of one of the suggested puzzles.

2. You have to prepare the puzzle to the rival team and solve theirs.

The team which does both parts of the Brain Ring correctly, wins.

Puzzle 1.
How many words of length 8 can you form, where the first letter is the same as the last letter (there are 26 letters in English alphabet)?

Puzzle 2.
How many ways can 5 people sit in a row if 2 of them insist on not sitting together?

Puzzle 3.

The code to open a combination lock is an ordered sequence of four digits chosen from the set {1, 2, 3, 4, 5, 6}. How many codes are possible if repetition

(1) is allowed

(2) is not allowed?

Home Assignment

Do Tasks 1-3 from Workbook section.

Optional Activity

Task 10. Facilitated Task

Read the extract from the text about Discrete Mathematics. Put the words in brackets into the correct form.

Discrete mathematics **(be)** the study of mathematical structures that **(be)** discrete rather than continuous. In contrast to real numbers, discrete mathematics **(study)** objects such as integers, graphs, and statements in logic. These objects do not vary smoothly, but have distinct, separated values. Discrete mathematics therefore **(exclude)** topics such as calculus and analysis. Discrete objects can often **(count)** using integers.

Mathematicians say that this is the branch of mathematics **(deal)** with countable sets (sets that have the same cardinality as subsets of the natural numbers, including rational numbers but not real numbers). The set of objects **(study)** in discrete mathematics can be finite or infinite. The term finite mathematics is sometimes **(apply)** to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business.

Research in discrete mathematics **(increase)** in the latter half of the twentieth century partly due to the development of digital computers which **(operate)** in discrete steps and store data in discrete bits. Concepts discrete mathematics **(be)** useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, cryptography and software development. In turn, computer implementations **(be)** significant in applying ideas from discrete mathematics to real-world problems, such as in operations research.

(The text is borrowed and modified from http://simple.wikipedia.org/wiki/Discrete_mathematics as of 10th January, 2014)

Task 11. Complex Task

Read the article about one of the trends of future programming. Answer the following questions:

1. What does the “Platonish position on Math” mean?
2. What are the earliest known Mathematics applications?
3. What definition of Math does the author give?
4. What do the words “construct useful fictions” mean in the context of the text?
5. What may happen if we encounter other systems of intelligence, according to the author?

Does Math Really Exist?

By [Alex Knapp]

In the midst of a rather interesting discussion of the notion of Aristotle's Unmoved Mover, Leah Libresco went on a mild digression about the philosophy of mathematics that I couldn't let go of, and feel compelled to respond to. She says:

"I take what is apparently a very Platonist position on math. I don't treat it as the relationships that humans make between concepts we abstract from day to day life. I don't think I get the concept of 'two-ness' from seeing two apples, and then two people, and then two houses and abstracting away from the objects to see what they have in common.

I think of mathematical truths existing prior to human cognition and abstraction. The relationship goes the other way. The apples and the people and the houses are all similar insofar as they share in the form of two-ness, which exists independently of material things to exist in pairs or human minds to think about them."

The notion that there's something special about math – that it has some sort of metaphysical significance – only makes sense if you ignore the history of how we uncovered math to begin with. It was, despite Leah's protestations, *exactly* just the abstraction of pairs and triplets and quartets, etc. The earliest known mathematics appear to be attempts to quantify time and make calendars, with other early efforts directed towards accounting, astronomy, and engineering.

Mathematics is nothing more and nothing less a tool that's useful for humans in solving particular problems. Math can be used to describe reality or **construct useful fictions**. For example, we know now that the spacetime we live in is non-Euclidian. But that doesn't make Euclidian geometry useless for everyday life. Quite the contrary – it's used every day. You can use mathematics to build models of reality that may not actually have any bearing on what's real. For example, the complicated math used to describe how the planets moved in the Ptolemaic model of the solar system – where everything orbited in circles around the Earth – actually produced very accurate predictions. But it was also wrong. There aren't actually trillions of physical dollars circulating in the economy – there are just symbols for them floating around.

The bottom line is that human beings have brains capable of counting to high numbers and manipulating them, so we use mathematics as a useful tool to describe the world around us. But numbers and math themselves are no more real than the color blue – 'blue' is just what we tag a certain wavelength of light because of the way we perceive that wavelength. An alien intelligence that is blind has no use for the color blue. It might learn about light and the wavelengths of light and translate those concepts completely differently than we do.

In the same way, since the only truly good mathematicians among the animals are ourselves, we assume that if we encounter other systems of intelligence that they'll have the same concepts of math that we do. But there's no evidence to base that assumption on. For all we know, there are much *easier* ways to describe physics than through complicated systems of equations, but our minds may not be capable of symbolically interpreting the world in a way that allows us to use those tools, any more than we're capable of a tool that requires the use of a prehensile tail.

Math is a useful descriptor of both real and fictional concepts. It's very fun to play around with and its essential for understanding a lot of subjects. But it's just a tool. Not a set of mystical entities.

(The text is borrowed and modified from http://www.forbes.com/sites/alexknapp/2012/01/21/does-math-really-exist/ as of 10th January, 2014)

WORKBOOK

Task 1. Tiered Task

Part 1. Replace the words in bold with their synonyms.

1. Game theory is the study of mathematical **models** of conflict and cooperation between intelligent rational decision-makers.
2. An **alternative** term suggested "as a more descriptive name for the discipline" is interactive decision theory.
3. Game theory is **mainly** used in economics, political science, and psychology, as well as logic and biology.
4. Today, however, game theory applies to a wide **range** of behavioral relations.
5. Modern game theory began with the idea **regarding** the existence of mixed-strategy equilibria in two-person zero-sum games and its proof by John von Neumann.
6. His paper was followed by his 1944 book which **considered** cooperative games of several players.
7. The second edition of this book **provided** an axiomatic theory of expected utility.
8. This theory was developed **extensively** in the 1950s by many scholars.

(The text is borrowed and modified from http://en.wikipedia.org/wiki/Game_theory as of 13th January, 2014)

Part 2. Complete the collocations with the words in italics. Make up sentences using these collocations.

sets	*science*	*values*	*integers*	*numbers*
field	*structures*	*mathematics*	*notions*	*theory*

mathematical ……………..
distinct ……………………
countable…………………..
rational……………………
related……………………
graph………………………
computer…………………
discrete……………………
be enumerated by…………
be applied to the …………

Part 3. Fill in the gaps with the derivatives from the words in capitals.

The term "discrete mathematics" is used in contrast with "(1) .... mathematics," which is the branch of mathematics (2) .... with objects that can vary smoothly (and which includes, for example, (3) ....). Whereas discrete objects can often be (4) .... by integers, continuous objects require real numbers.	CONTINUE DEAL CALCULATE CHARACTER
The study of how discrete objects combine with one another and the (5) .... of various outcomes is known as (6) .... Other fields of mathematics that are considered to be part of discrete mathematics include graph theory and the theory of (7) .... Topics in number theory such as congruences and recurrence (8) .... are also considered part of discrete mathematics.	PROBABLY COMBINE COMPUTE RELATE
The study of topics in discrete mathematics usually includes such topics as the study of algorithms, their (9) .... , and (10) .... Discrete mathematics is the mathematical language of computer science, and as such, its (11) .... has increased dramatically in recent decades.	IMPLEMENT IMPORTANT

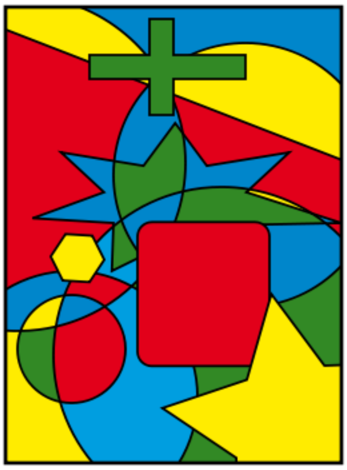
(The text is borrowed and modified from http://mathworld.wolfram.com/DiscreteMathematics.html as of 13th January, 2014)

Task 2. Tiered Task

Read the text and do at least one part of the task after reading.

Grand challenges, past and present

The history of discrete mathematics has involved a number of challenging problems which have focused attention within areas of the field. In graph theory, much research was motivated by attempts to prove the four colour theorem, first stated in 1852, but not proved until 1976 (by Kenneth Appel and Wolfgang Haken, using substantial computer assistance).



In logic, the second problem on David Hilbert's list of open problems presented in 1900 was to prove that the axioms of arithmetic are consistent. Gödel's second incompleteness theorem, proved in 1931, showed that this was not possible – at least not within arithmetic itself. Hilbert's tenth problem was to determine whether a given polynomial Diophantine equation with integer coefficients has an integer solution. In 1970, Yuri Matiyasevich proved that this could not be done.

The need to break German codes in World War II led to advances in cryptography and theoretical computer science, with the first programmable digital electronic computer being developed at England's Bletchley Park. At the same time, military requirements motivated advances in operations research. The Cold War meant that cryptography remained important, with fundamental advances such as public-key cryptography being developed in the following decades. Operations research remained important as a tool in business and project management, with the critical path method being developed in the 1950s. The telecommunication industry has also motivated advances in discrete mathematics, particularly in graph theory and information theory. Formal verification of statements in logic has been necessary for software development of safety-critical systems, and advances in automated theorem proving have been driven by this need.

Computational geometry has been an important part of the computer graphics incorporated into modern video games and computer-aided design tools.
Several fields of discrete mathematics, particularly theoretical computer science, graph theory, and combinatorics, are important in addressing the challenging bioinformatics problems associated with understanding the tree of life.

Currently, one of the most famous open problems in theoretical computer science is the P = NP problem, which involves the relationship between the complexity classes P and NP. The Clay Mathematics Institute has offered a $1 million USD prize for the first correct proof, along with prizes for six other mathematical problems

(The text is borrowed and modified from http://en.wikipedia.org/wiki/Discrete_mathematics as of 12th January, 2014)

Part 1. Which spheres the following problems belong to?

1. Four colour theorem.
2. Hypothesis that the axioms of arithmetic are consistent.
3. Hypothesis that a given polynomial Diophantine equation with integer coefficients has an integer solution.
4. Public-key cryptography.
5. Critical path method.
6. P = NP problem.

Part 2. Match the concepts related to Discrete Mathematics history with their descriptions.

1.	Four colour theorem	a.	is not possible within arithmetic.
2.	Proof that the axioms of arithmetic are consistent	b.	is important as a tool in business and project management.
3.	Advances in cryptography and theoretical computer science	c.	was proved by Kenneth Appel and Wolfgang Haken, using substantial computer assistance.
4.	Operations research	d.	were stimulated by the need to break German codes.
5.	Critical path method	e.	is used in modern video games design.

6.	Advances in automated theorem proving	f.	is one of the most famous open problems in theoretical computer science.
7.	Computational geometry	g.	was developed in the 1950s.
8.	P = NP problem	h.	have been driven by the need to develop safety-critical systems.

Part 3. Fill in the table with the information from the text.

#	Problem	Sphere	Description	Date of proof /development
1	Four colour theorem	*graph theory*	*proved by Kenneth Appel and Wolfgang Haken, using substantial computer assistance*	*stated 1852, proved 1976*
2	Hypothesis that the axioms of arithmetic are consistent			
3	Hypothesis that a given polynomial Diophantine equation with integer coefficients has an integer solution			
4	Public-key cryptography			
5	Critical path method			
6.	P = NP problem			

Task 3. Internet Search.

Use the Internet to prepare a report on the new or developing topics of Discrete Mathematics. Mention scientists who worked with this topic, stages of its development, results and proofs.