б) $R_2 = H$

$$\frac{(v_0 \cos\alpha)^2}{2g} = \frac{(v_0 \sin\alpha)^2}{g}$$

$$tg\,\alpha = \sqrt{2}, \quad \alpha = 54{,}8^\circ$$

1.34

$$v_x = \alpha y$$

а) Оскільки по осі $y$ шар рухається рівномірно із швидк. $v_0$, то координата шара $y$ змінюється від часу за законом $y = v_0 t$.

$$v_x = \alpha y = \frac{dx}{dt}$$

$$\alpha v_0 t = \frac{dx}{dt}$$

$$\int_0^t v_0 \alpha t\, dt = \int_0^x dx$$

$$\frac{\alpha v_0 t^2}{2} = x$$

$$x(y) = \frac{\alpha v_0}{2}\frac{y^2}{v_0^2} = \frac{\alpha y^2}{2v_0}$$

б) $a = \sqrt{a_x^2 + a_y^2}$

$$a_y = \frac{d^2y}{dt} = 0, \quad a_x = \frac{d^2x}{dt} = \alpha v_0$$

$$a = \alpha v_0$$

$$a_\tau = \frac{dv}{dt}$$

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = \frac{dx}{dt} = \alpha v_0 t$$

$$v_y = \frac{dy}{dt} = v_0$$

$$v = v_0 \sqrt{\alpha^2 t^2 + 1}$$

$$a_\tau(t) = \frac{2 v_0 \alpha^2 t}{2\sqrt{\alpha^2 t^2 + 1}} = \frac{v_0 \alpha^2 t}{\sqrt{\alpha^2 t^2 + 1}}$$

$$a_\tau(y) = \frac{v_0 \alpha^2 y}{v_0 \sqrt{\frac{\alpha^2 y^2}{v_0^2} + 1}} = \frac{v_0 \alpha^2 y}{\sqrt{\alpha^2 y^2 + v_0^2}}$$

$$a_n = \sqrt{a^2 - a_\tau^2} = \sqrt{(\alpha v_0)^2 - \left(\frac{\alpha^2 v_0 y}{\sqrt{\alpha^2 y^2 + v_0^2}}\right)^2} =$$

$$= \frac{\alpha v_0^2}{\sqrt{\alpha^2 y^2 + v_0^2}}$$

1. 35

$$v = \alpha \vec{i} + \beta x \vec{j}, \quad x = y = 0$$

a) $\frac{dx}{dt} = \alpha$, $\int_0^x dx = \alpha \int_0^t dt$, $x = \alpha t$

$\frac{dy}{dt} = \beta x$, $\int_0^y dy = \int_0^t \alpha \beta t \, dt$, $y = \alpha \beta \frac{t^2}{2}$

$$y(x) = \frac{\alpha \beta}{2} \frac{x^2}{\alpha^2} = \frac{\beta}{2\alpha} x^2$$

б) $R = \frac{v^2}{a_n}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\alpha^2 + \beta^2 x^2}$$

$$a_n = \sqrt{a^2 - a_\tau^2}$$

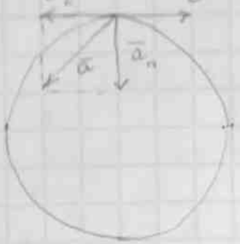
$$\bar{a} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j} = \alpha\beta\vec{j}$$

$$a_\tau = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{\alpha\cdot\beta^2 x}{\sqrt{\alpha^2+\beta^2x^2}}$$

$$R = \frac{\alpha^2+\beta^2x^2}{\sqrt{(\alpha\beta)^2 - \frac{\alpha^2\beta^4x^2}{\alpha^2+\beta^2x^2}}} = \frac{(\alpha^2+\beta^2x^2)^{3/2}}{\sqrt{\alpha^4\beta^2+\alpha^2\beta^4x^2-\alpha^2\beta^4x^2}} =$$

$$= \frac{\alpha}{\beta}\left(1+\left(\frac{x\beta}{\alpha}\right)^2\right)^{3/2}$$

1.38




$$|\bar{a}_n| = |\bar{a}_\tau|$$

$$t = 0 \ - \ v = v_0$$

a) $$\frac{v^2}{R} = -\frac{dv}{dt}$$

$$-\int_{v_0}^{v}\frac{dv}{v^2} = \int_0^t \frac{dt}{R}$$

$$\frac{1}{v}\Big|_{v_0}^{v} = \frac{t}{R}$$

$$\frac{1}{v} - \frac{1}{v_0} = \frac{t}{R}$$

$$v(t) = \frac{1}{\frac{t}{R}+\frac{1}{v_0}} = \frac{v_0 R}{v_0 t + R}$$

$$s(t) = \int_0^t v\,dt = v_0 R\int_0^t \frac{dt}{v_0 t + R} = R\ln|R+v_0t|\Big|_0^t =$$

$$= R \ln \frac{R + v_0 t}{R}$$

$$e^{\frac{S}{R}} = 1 + \frac{v_0}{R} t = \frac{v_0 t + R}{R} = \frac{v_0}{v}$$

$$v(S) = v_0 e^{-\frac{S}{R}}$$

$$a(t) = \cancel{\frac{dv}{dt}} \sqrt{a_\tau^2 + a_n^2} = \sqrt{2} |a_n| \quad (a_\tau = a_n)$$

$$a_n = \frac{dv}{dt} \cancel{t}, \quad v = \frac{dS}{dt}$$

$$dt = \frac{dv}{a_n}, \quad dt = \frac{dS}{v}$$

$$\frac{dv}{a_n} = \frac{dS}{v}, \quad \frac{dv}{dS} = \frac{a_n}{v}$$

$$\frac{dv}{dS} = -\frac{v_0}{R} e^{-\frac{S}{R}}$$

$$a_n = -\frac{v_0^2}{R} e^{-\frac{2S}{R}}$$

$$a(S) = \sqrt{2} \frac{v_0^2}{R} e^{-\frac{2S}{R}}$$

$$a(v) = \frac{\sqrt{2}}{R} v^2$$

1.39

$v = \alpha\sqrt{S}$


$$tg\,\varphi = \frac{a_n}{a_\tau}$$

$$a_n = \frac{v^2}{R}$$

$$a_\tau = \frac{dv}{dt}$$

$$a_n = \frac{\alpha^2 S}{R}$$

$$a_\tau = \frac{dv}{dt} = \frac{dv}{dS}\frac{dS}{dt} = \frac{dv}{dS}v = \frac{\alpha}{2\sqrt{S}}\alpha\sqrt{S} = \frac{\alpha^2}{2}$$

$$tg\,\varphi = \frac{2S}{R}$$

1.42

a) $x = 0$, $y = \alpha x^2$

$$\frac{dy}{dt} = 2\alpha x\frac{dx}{dt}$$

$$\frac{d^2y}{dt^2} = 2\alpha\Big(\underbrace{\frac{dx}{dt}\frac{dx}{dt}}_{v_x} + x\frac{d^2x}{dt}\Big)$$

$$a = \frac{d^2y}{dt^2}\Big|_{x=0} = 2\alpha v^2 = a_n$$ (оскільки частинка рухається рівномірно)

$$a_n = 2\alpha v^2 = \frac{v^2}{R}, \quad R = \frac{1}{2\alpha}$$

1.46

$$\varphi = at - bt^3$$


$$\omega_z = \frac{d\varphi}{dt}, \quad \beta_z = \frac{d\omega_z}{dt}$$

$$\omega_z = a - 3bt^2$$

$$\beta_z = -6bt, \quad \beta = |\beta_z|$$

Тіло зупиниться, коли $\omega_z = 0$:

а)

$$a = 3bt^2, \quad t_0 = \sqrt{\frac{a}{3b}}$$

$$\langle \omega \rangle = \frac{\int_0^{t_0} \omega_z dt}{\int_0^{t_0} dt} = \frac{a\sqrt{\frac{a}{3b}} - b\left(\frac{a}{3b}\right)^{3/2}}{\sqrt{\frac{a}{3b}}} = a - \frac{a}{3} = \frac{2a}{3}$$

$$\langle \beta \rangle = \frac{\int_0^{t_0} |\beta_z| dt}{\int_0^{t_0} dt} = \frac{6b\left(\sqrt{\frac{a}{3b}}\right)^2}{2\sqrt{\frac{a}{3b}}} = \sqrt{3ab}$$

б)

$$\beta_z \Big|_{t=\sqrt{\frac{a}{3b}}} = -6b\sqrt{\frac{a}{3b}} = -2\sqrt{3ab}$$

$$\beta = \left| \beta_z \Big|_{t=t_0} \right| = 2\sqrt{3ab}$$

1.47

$\beta = \alpha t \quad , \quad \varphi = 60^\circ$

$tg\,\varphi = \frac{a_n}{a_\tau}$

$a_\tau = \beta R = \alpha R t$

$a_\tau = \frac{dv}{dt}$

$\frac{dv}{dt} = \alpha R t, \qquad \int_0^v dv = \int_0^t \alpha R t\, dt$

$v = \frac{\alpha R t^2}{2}$

$a_n = \frac{v^2}{R} = \frac{\alpha^2 t^4 R}{4}$

$tg\,\varphi = \frac{\alpha^2 t^4 R}{4 \alpha R t} = \frac{\alpha t^3}{4}$

$t = \sqrt[3]{\frac{4\, tg\,\varphi}{\alpha}}$

1.48

$\beta = \alpha \sqrt{\omega} \quad , \quad t = 0 - \omega = \omega_0$

$\beta_z < 0$

$\beta = -\frac{d\omega}{dt}$

$-\frac{d\omega}{dt} = \alpha \sqrt{\omega}$

$$-\int_{\omega_0}^{\omega} \frac{d\omega}{\sqrt{\omega}} = \alpha \int_0^t dt$$

$$-2\sqrt{\omega} + 2\sqrt{\omega_0} = \alpha t$$

$$\sqrt{\omega} = \sqrt{\omega_0} - \frac{\alpha t}{2}$$

При $\omega = 0$ , $t_0 = \frac{2\sqrt{\omega_0}}{\alpha}$

$$\langle \omega \rangle = \frac{\int \omega\, dt}{\int dt} = \frac{\int_0^{t_0} \left(\omega_0 - \alpha t\sqrt{\omega_0} + \frac{\alpha^2 t^2}{4}\right) dt}{\int_0^{t_0} dt}$$

$$= \frac{\left(\omega_0 t - \frac{\alpha}{2}\sqrt{\omega_0}\, t^2 + \frac{\alpha^2 t^3}{12}\right)\Big|_0^{\frac{2\sqrt{\omega_0}}{\alpha}}}{\frac{2\sqrt{\omega_0}}{\alpha}} =$$

$$= \frac{\frac{2\omega_0^{3/2}}{\alpha} - \frac{\alpha\, \omega_0^{3/2}\, 4}{2\alpha^2} + \frac{\alpha^2\, 8\, \omega_0^{3/2}}{12\alpha^3}}{\frac{2\sqrt{\omega_0}}{\alpha}} = \frac{\omega_0}{3}$$

1.49

$\omega = \omega_0 - a\varphi$ , $t = 0$ — $\varphi = 0$

a) $\omega = \frac{d\varphi}{dt}$

$$\omega_0 - a\varphi = \frac{d\varphi}{dt}$$

$$\int_0^{\varphi(t)} \frac{d\varphi}{\omega_0 - a\varphi} = \int_0^t dt$$

$$-\frac{1}{a} \ln|\omega_0 - a\varphi|\Big|_0^{\varphi(t)} = t$$

$$\ln \left| \frac{w_0 - a\varphi(t)}{w_0} \right| = -ta$$

$$1 - \frac{a}{w_0}\varphi(t) = e^{-ta}$$

$$\varphi(t) = \frac{w_0}{a}\left(-e^{-at} + 1\right)$$

δ) $\beta = \frac{dw}{dt}$ $\quad w = \frac{d\varphi}{dt} = w_0 e^{-at}$

1.50

$$\bar{\beta} = \bar{\beta}_0 \cos\varphi$$

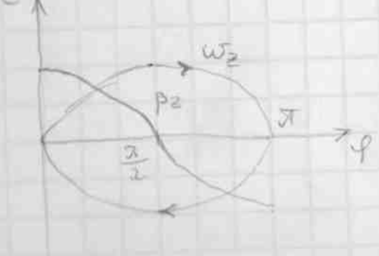
$$\frac{dw_z}{dt} = \beta_z \quad , \quad \frac{d\varphi}{dt} = w_z$$

$$w_z \frac{dw_z}{d\varphi} = \beta_0 \cos\varphi$$

$$\int_0^{w(\varphi)} w_z \, dw_z = \int_0^{\varphi} \beta_0 \cos\varphi \, d\varphi$$

$$\frac{w(\varphi)^2}{2} = \beta_0 \sin\varphi$$

$$w(\varphi) = \pm\sqrt{2\beta_0 \sin\varphi}$$



1.56

$\vec{\omega} = a t \vec{i} + b t^2 \vec{j}$

$\omega = |\vec{\omega}| = \sqrt{a^2 t^2 + b^2 t^4}$

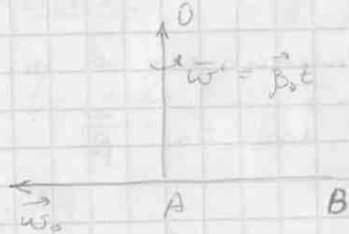
$\vec{\beta} = \frac{d\vec{\omega}}{dt} = a \vec{i} + 2 b t \vec{j}$

$\beta = |\vec{\beta}| = \sqrt{a^2 + 4 b^2 t^2}$

1.58

$\omega' = \vec{\beta}_0 t$

$\vec{\omega} = \vec{\omega}' + \vec{\omega}_0$




$\vec{\omega}' \perp \vec{\omega}_0$

$\omega = \sqrt{\omega'^2 + \omega_0^2} =$

$= \sqrt{\omega_0^2 + \beta_0^2 t^2}$

$\vec{\beta} = \frac{d\vec{\omega}}{dt} = \frac{d(\vec{\omega}' + \vec{\omega}_0)}{dt} = \frac{d\vec{\omega}'}{dt} + \frac{d\vec{\omega}_0}{dt}$

$\frac{d\vec{\omega}_0}{dt} = \vec{\omega}' \times \vec{\omega}_0 \; , \; \frac{d\vec{\omega}'}{dt} = \vec{\beta}_0$ X

$\vec{\beta} = (\vec{\beta}_0 t + \vec{\omega}_0) + \vec{\beta}_0$

$\vec{\beta}_0 \perp \vec{\omega}_0 \; ; \; \beta = \sqrt{(\omega_0 \beta_0 t)^2 + \beta_0^2}$