.

**1.**
**2.**
**3.**
**4.**
**5.**
**6.**
**7.**
**8.**
**9.**

( )

,

.

$G(V,E)$      –

$V$ ( )      $E$

$V$ ($E$ – ).

$G(V,E)$, $V \neq \varnothing$, $E \subset V \times V$, $E = E^{-1}$.

$G$      $p$,     –     $q$.

$$p = p(G) = |V|, \; q = q(G) = |E|$$

$E$ ,

( ).

$V$ ,      $E$ – .

$E$ ,

$G(V,E)$ .

$E$ , ,

. . $V$ , $E$

, .

$F : V \to M$ / $F : E \to M$ , $M$

,

.

$G = (V, E)$ ,

. $n$ $K_n$ .

. :

a) $K_2$ , b) $K_3$ , c) $K_4$ d) $K_5$ , e) $K_6$ .



a b c d e
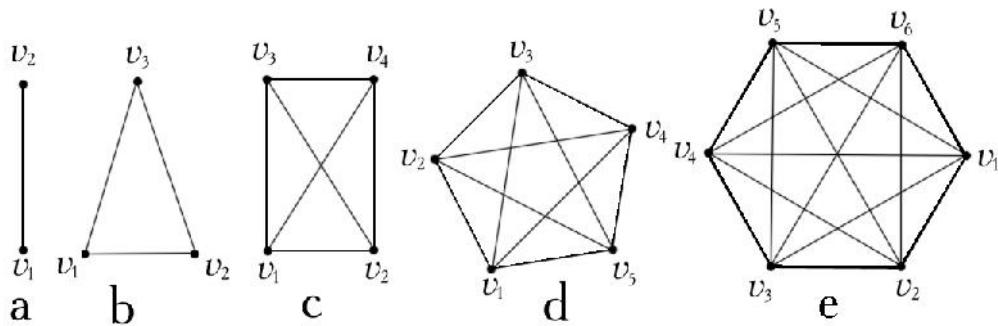
$G = (V, E)$ , $V$

, , $V = A \cup B$ ,

$\{a, b\}$ , $a \in A$ $b \in B$ . ,

$A$ $B$ , $A$

$B$ .

$K_{m,n}$ , $A$

$m$ , $B$ $n$ $a \in A$, $b \in B$

$\{a, b\} \in E$ . , $a \in A$ $b \in B$

. $K_{1,2}$ , $K_{2,3}$ , $K_{2,2}$ ,

$K_{3,3}$ .

$v_1 \in V$ $v_2 \in V -$ ,

$e = (v_1, v_2) -$ , $v_1$ $v_2$ , $e \in E$ .

$v_1$ $e$ .



$v_2$ $e$

.

,                                            ,                                          .

,                                            ,                                          .

$e_1$ ⎯ $v$ ⎯ $e_2$          $v_1$ ⎯⎯⎯ $e$ ⎯⎯⎯ $v_2$

,                          $v$,

$v$                    $\Gamma(v)$.
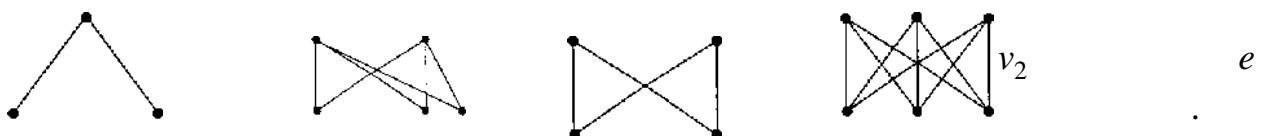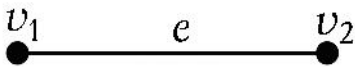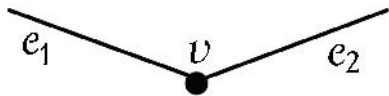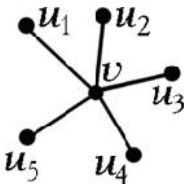
$u_1$  $u_2$
$v$
$u_3$
$u_5$  $u_4$

$$\Gamma(v)=\left\{u_i \in V \,\middle|\, (u_i,v)\in E,\ 0\le i\le p-1\right\}, \qquad p=|V|$$
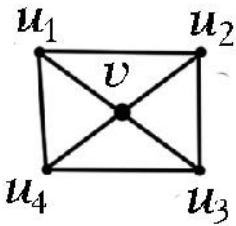
$u_1$          $u_2$        $\Gamma(v)=\left\{u_i \in V \,\middle|\, (u_i,v)\in E,\ i=1,...,4\right\}$
$v$
$u_4$          $u_3$        $V=\left\{v,u_1,u_2,u_3,u_4\right\}$

$p=|V|=5$, $q=|E|=8$

$$E=\left\{(u_1,v),(u_2,v),(u_3,v),(u_4,v),(u_1,u_2),(u_1,u_4),(u_3,u_4),(u_3,u_2)\right\}$$

$v$                                                    ,

.

$\deg(v)$        $d(v)$,

$\forall v \in V\ 0\le \deg(v)\le p-1, \qquad p=|V|.$

:

$\deg(v)=|\Gamma(v)|.$

$G$               $u(G),$

–          $\Delta(G).$

$$u(G(V,E))=\min_{v\in V}\deg(v)$$
$$\Delta(G(V,E))=\max_{v\in V}\deg(v)$$

$$k,$$
$$k. \qquad\qquad k\text{-}$$
$$\text{и}(G)=\Delta(G)=k.$$

$v$,     $\deg(v)=0$         .

$v$,     $\deg(v)=1$           .

,                $v$,

$\Gamma^{+}(v),$

,            $v -$

$\Gamma^{-}(v)\,.$

.               .

.

2-

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  2.     ,

     ,             2, . .,    .

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$S=2 \qquad S=4 \qquad\quad S=6 \qquad\quad S=8$

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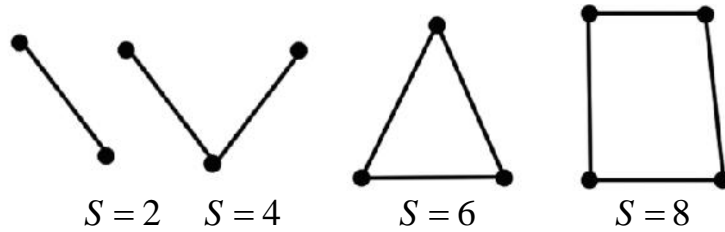
  1.          ,               ,

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  2.                 ,

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3. , ,

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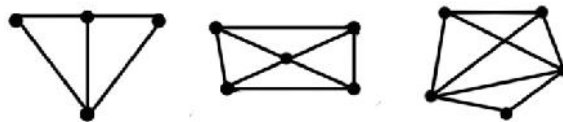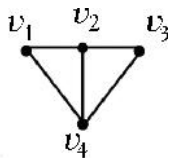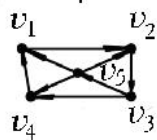
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:

$$\sum_{v\in V}\deg(v)=2q \ -$$ ,

$$\sum_{v\in V}d^-(v)+\sum_{v\in V}d^+(v)=2q-$$ ,
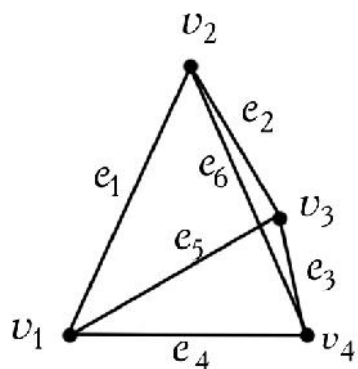
$$q=|E| \ -$$ .

.

:

.



$$\sum_{i=1}^{3}\deg(v_i)=10,\ q=|E|=5,$$

$$\sum_{i=1}^{5}\deg^-(v_i)=8,\ \sum_{i=1}^{5}\deg^+(v_i)=8,\ q=8.$$

(               )                $r(G)$.

3.

$G(V,E),$      $V=\{v_1,v_2,v_3\}$,   $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$.

$$r(G)=\deg(v_1)=\deg(v_2)=d\,\deg(v_3)=d\,\deg(v_4)=3$$

$,\quad \cdot\quad \cdot$

$r(G)$          .

   ) 0-           ,

b) 1-           ,

c) 2-

d) 3-          .
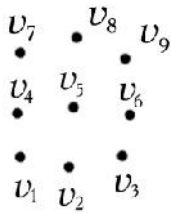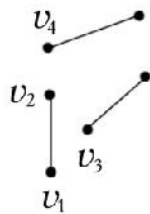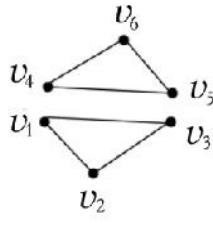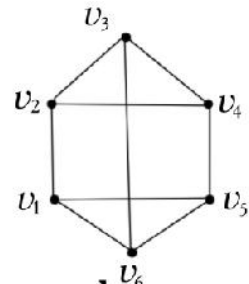


a          b          c          d

$G'(V',E')$          $G(V,E),$

$$G'(V',E')\preceq G(V,E)$$

$V'\subseteq V$    $E'\subseteq E.$          :

-          $G'$          $G,$

-       $G'$         $G.$

$V'=V$    $E'\subseteq E,$

$G'$          $G$          $G.$

$G'(V',E')$    $V'\subset V$          $G,$

$G'$          $G$ :

$$\forall u,v\in G'(u,v)\in E\Rightarrow(u,v)\in E'.$$

.     (**a**)       $G(V,E).$

(**b**).        $G_1(V_1,E_1)$     $G(V,E),$

$V_1\subset V$    $E_1\subset E.$

( ).  $G_2(V_2, E_2)$  $G(V, E)$,

$V_2 = V$  $E_2 \subset E$.

(**d**).  $G_3(V_3, E_3)$  $G(V, E)$,
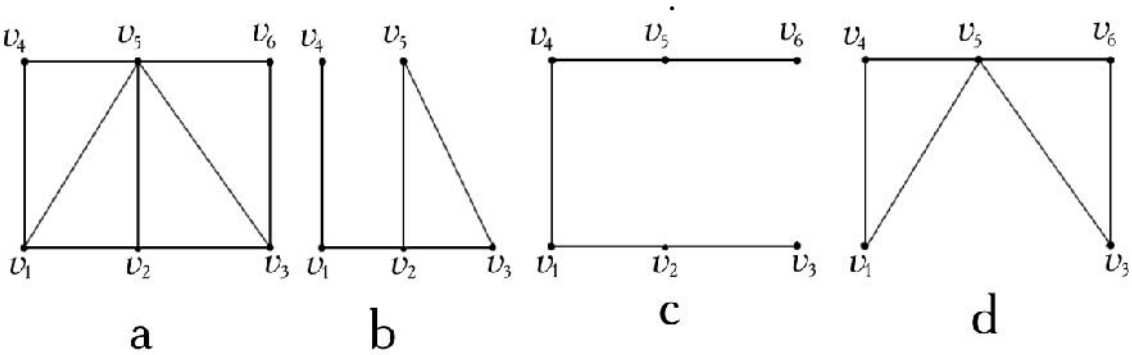


a   b   c   d

−  ,

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,

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ILLIAC-IV, MPP, Cray T3D.

.

,

.

$S$.

$s_1, s_2, ..., s_m, ..., s_k, n -$  ,  ,  :

$$1 \le s_1 < s_2 < ... < s_m < ... < s_k < n.$$

$$V = \{0, 1, 2, ..., n-1\}$$

,  :

$$E = \left\{(i, j) \big| (|i - j| \bmod n) = s_m, m = 1, 2, ..., k\right\}.$$

$n$  .

$k -$  .

$s_m \in S -$  (  ).

$$G(n;S) = G(n; s_1, s_2, \ldots, s_k),$$

,                                                    .

$$G(n; s_1, \ldots, s_k) \qquad :$$

- $2k$,            $s_k \neq \dfrac{n}{2}$

- $(2k-1)$,          ,        $n -$            $s_k = \dfrac{n}{2}.$

.



$$G(16; 1, 4)$$
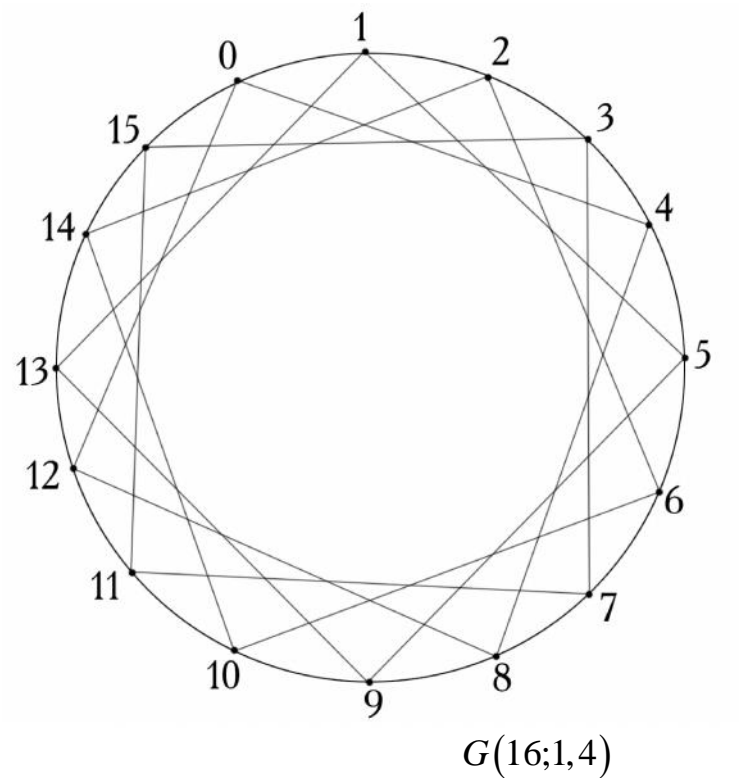
$$G(V, E)$$

$:$

$$v_0, e_1, v_1, \ldots, v_{t-1}, e_t, v_t,$$

$e_i = (v_{i-1}, v_i)$         $1 \leq i \leq t.$

$(v_0, v_t)$ -                                                               ,

$v_0$     $v_t$ ,                                                                              .

:

$$v_0 \xrightarrow{e_1} v_1 \xrightarrow{e_2} \ldots \xrightarrow{e_t} v_t .$$

,

.

(          )

.              ,                                                              ,                          ;

.

,                                                                              (        )

$v_0, v_1, \ldots, v_t$                                       .

$v_0 = v_t$       $(v_0, v_t)$ -                                                    .

(          )                                                                                        ,

,                                    .

,

(          ).

—                                                              .

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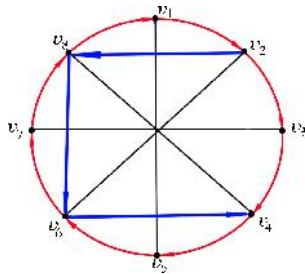
.                                                                          1.

2.

**3**                                                                          .



.                                                                          $u$          $v$

$(u,v)$ -                    ,                                            $(u,v)$ -            .

.

$(u,v)$-                                    .              ,

.

$w$,      ,

$w$

$w$,

$(u,v)$-              .



$G$                          ,                                                    $u$

$v$              $(u,v)$-              .
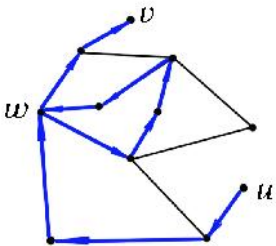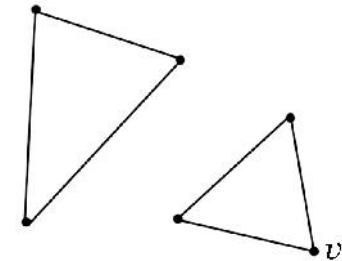


$G$                                          $u$      $v$,                              .

,

$V$                                                                    $V_1$      $V_2$

,                                                                          .

$V$            $G$                                                    $\sim$

,

$$u \sim v \Leftrightarrow \qquad (u,v)\text{-} \qquad .$$

(                    ,

).

$$G_i = G(V_i) - \qquad ,$$

$V_i,\ \left(1 \le i \le k\right).$



$G_1, G_2, ..., G_k,$                                                          $G.$

,                                                    $G_i$

.
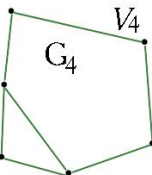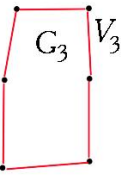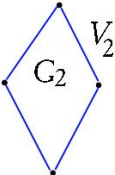
$$G = \{G_1, ..., G_k\} \quad -$$

,

.

,                                    :

.

1.                                                                        .
2.                                                            .
3.            ,                                    ,                .
4.                                            .
5.                                        ,                            .

,

$(p,q,k)$-       ,         $p$ –                                    , $q$ –
   ,   $k$ –                                    .

.



$G$                                              :

$$p = |V_1| + |V_2| + |V_3| + |V_4| = 3 + 4 + 6 + 6 = 19$$
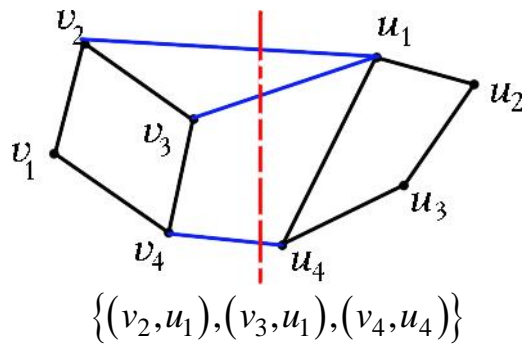$$q = |E_1| + |E_2| + |E_3| + |E_4| = 3 + 4 + 6 + 7 = 20$$
$$k = 4$$

, $G = G(19, 20, 4)$.

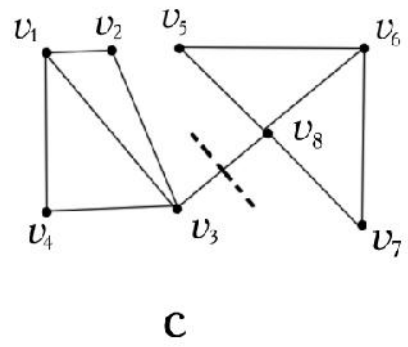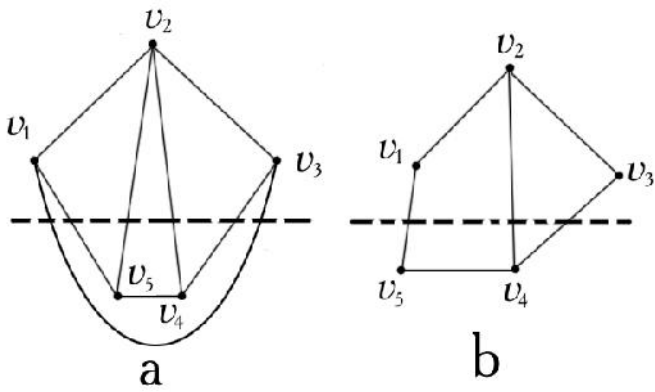,

,

.

$$\{(v_2,u_1),(v_3,u_1),(v_4,u_4)\}$$

.

–                    ,                                    .

:

a)                         ,
b)
c)        .



a



b



c

)                                                                               :

$$E_r = \{(v_1,v_3),(v_1,v_5),(v_4,v_3),(v_1,v_3)\}.$$

,                                            $(v_1,v_3)$.

**b)**                         ,                                    ,

: $E_r = \{(v_1,v_5),(v_2,v_4),(v_3,v_4)\}.$

**c)**                                                                               :

$$E_r = \{(v_3,v_8)\}.$$