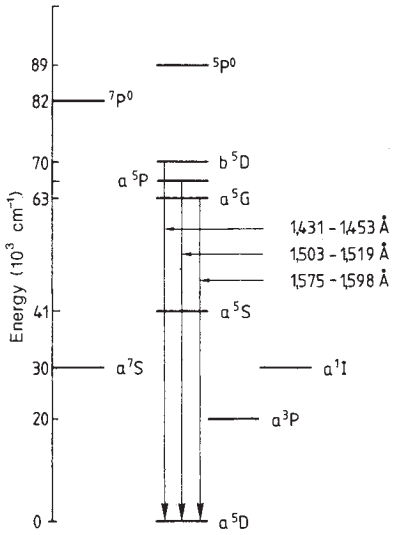

Fig. 2 A partial term scheme for Fe III, showing the observed transitions, some lower-lying terms of the same even parity and the first two terms of odd parity. Term energies are given in cm^{-1}.

abilities found by Garstang[8] for lower-lying levels are in each case ~1. Thus it is likely that the upper levels have populations close to their Boltzmann values with collisional de-excitation rates exceeding radiative decay rates. Then the ratio of the [Fe III] line fluxes to those of permitted transitions would be sensitive to the electron density. Detailed calculations of the atomic data are required to establish the collisional–radiative regime for the individual lines. The upper levels may attain only a pseudo-Boltzmann population if collisional excitation to higher states of odd parity exceeds the rate for collisional de-excitation to lower levels. In any case the new [Fe III] identifications will provide more information on the structure of the solar chromosphere–corona transition region.

If the a^{5}G, a^{5}P and b^{5}D levels are collisionally de-excited in the solar atmosphere, they could become stronger, relative to permitted transitions of species of similar excitation, in astrophysical sources of lower electron density. For this reason their presence is being investigated in such sources, including the Seyfert galaxy NGC 4151, which has unidentified emission features around 1,575, 1,581 and 1,518 Å (refs 13 and 14). [Fe III] emission is observed in the optical spectrum of NGC 4151[15,16]. The relative intensities of the quintet transitions in NGC 4151 and other sources cannot be predicted until collision cross-sections and transition probabilities are known, and these are urgently required to establish whether or not the [Fe III] lines are of wider astrophysical significance.

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A hierarchical $O(N \log N)$ force-calculation algorithm

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Until recently the gravitational N-body problem has been modelled numerically either by direct integration, in which the computation needed increases as N^2, or by an iterative potential method in which the number of operations grows as N log N. Here we describe a novel method of directly calculating the force on N bodies that grows only as N log N. The technique uses a tree-structured hierarchical subdivision of space into cubic cells, each of which is recursively divided into eight subcells whenever more than one particle is found to occupy the same cell. This tree is constructed anew at every time step, avoiding ambiguity and tangling. Advantages over potential-solving codes are: accurate local interactions; freedom from geometrical assumptions and restrictions; and applicability to a wide class of systems, including (proto-)planetary, stellar, galactic and cosmological ones. Advantages over previous hierarchical tree-codes include simplicity and the possibility of rigorous analysis of error. Although we concentrate here on stellar dynamical applications, our techniques of efficiently handling a large number of long-range interactions and concentrating computational effort where most needed have potential applications in other areas of astrophysics as well.

Until recently, the dynamics of a system of self-gravitating bodies (the gravitational N-body problem) has been modelled numerically in two fundamentally different ways. The first one, direct N-body integration, involves the computation of all $\frac{1}{2}N(N-1)$ forces between all pairs of particles. This allows an accurate description of the dynamical evolution but at a price that grows rapidly for increasing N[1]. The second way involves a two-step approach: after fitting the global potential field to a special model with a number of free parameters, each particle is propagated in this background field for a short time before the same procedure is reiterated. The potential method involves a number of operations that grow only as N log N. Thus calculations can be performed more quickly, but with a loss of accuracy and generality. The special nature of each potential-solving code is caused by the need to use some technique that is tuned to the geometry of the problem being considered (such as Fourier transforms or spherical or bispherical harmonics[2]).

Recently, some of the advantages of both approaches have been combined by using direct integrations of force while grouping together increasingly large groups of particles at increasingly large distances. This corresponds to the way humans interact with neighbouring individuals, further villages and increasingly further and larger states and countries—driven by increasing cost and decreasing need to deal with more removed groups on an individual basis. The first implementation of such a hierarchical grouping of interactions was given by Appel[3], who used a tree structure to represent an N-body system, with the particles stored in the leaves of the tree. An independent implementation by Jernigan[4] and Porter[5] incorporated regularization of close encounters. However, in both codes the logarithmic-growth gain in efficiency comes at the price of introducing additional errors that are hard to analyse because of the arbitrary structure of the tree. Nearby particles may be grouped as leaves of nearby branches, but the phase-space flow of realistic self-gravitating systems demands a continuous updating of the tree structure to avoid tangling and unphysical grouping, requiring complicated book-keeping. It is not at all clear how to understand and estimate the errors caused by the process of approximating lumps of particles together as single pseudo-particles, because individual lumps can take more or less arbitrary shapes and sizes.