Laboratorio de Métodos Computacionales

Taller 5

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Instrucciones de Entrega

La solución a este taller debe subirse por SICUA antes de las 12:59PM del jueves 8 de octubre de 2015. Debe entregarse un archivo llamado `NombreApellido_hw5.ipynb`. Este puede iniciar con `%pylab inline`

1. 50 pt **3 ecuaciones diferenciales de primer orden acopladas**

   Solve the following system of coupled ODEs

   $$\frac{dx}{dt} = \sigma(y - x)$$

   $$\frac{dy}{dt} = x(\rho - z) - y$$

   $$\frac{dz}{dt} = xy - \beta z$$

   with $\sigma = 10$, $\beta = 8/3$ and $\rho = 28$ with a 4th order Runge-Kutta in the time interval $0 < t < 3$. The intial conditions for x, y and z are random numbers between -10 and 10.

2. 50 pt **Oscilador Armónico Forzado y Amortiguado**

   Take an idealised pendulum: a weightless string of length ℓ, fixed at one end with a mass m at the other. The pendulum: - Is free to swing in a plane subject to gravity. - Has a friction proportional to its velocity v. - May be driven by an external periodic force $F_d \cos \omega_d t$.

   We want to consider angular displacement, so substitute $\dot{x} = \ell\dot{\theta}$ and $\ddot{x} = \ell\ddot{\theta}$, and now apply the driving force

   $$m\ell\ddot{\theta} + k\ell\dot{\theta} + mg\sin\theta = F_d \cos\omega_d t$$

   We can re-write this 2nd order ODEs as a set of coupled first-order ODEs:

   Let $y_0 = \theta$, $y_1 = \dot{\theta}$ and $y_2 = \ddot{\theta}$:

   $$y_0' = y_1 = \dot{\theta}$$

   $$y_1' = y_2 = \ddot{\theta} = -\frac{k}{m}\dot{\theta} - \frac{g}{\ell}\sin\theta + \frac{F_d}{m\ell} cos\omega_d t$$

   Using a simple change of variable: $\alpha = g/\ell$, $\beta = k/m$ and $\gamma = F/m\ell$

   $$y_0' = y_1$$

$$y_1' = -\alpha \sin y_0 - \beta y_1 + \gamma \cos \omega t$$

Using the Runge-Kutta(4) method described in ODE-exercises:

- Solve without friction and external force.
- Solve without external force.
- Solve without friction.
- Solve for all forces!