(1a)

$$p = p_0 e^{j(\omega t + \vec{k}\cdot\vec{r})}$$

p_0 ... max amplitude
ωt ..... oscillation
$\vec{k}\cdot\vec{r}$ ..... wave vibrance
↳ determines the phase

1. Assume a stationary solution: $t = 0$

$$\Rightarrow \boxed{p = p_0 e^{j(\vec{k}\cdot\vec{r})}}$$

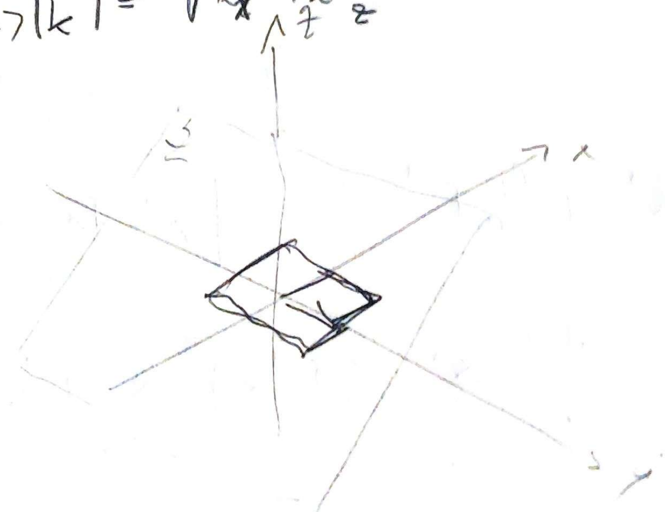
2. Wave propagation is parallel to xz-plane at angle θ.

$$\theta = \measuredangle(\vec{k}, \hat{e}_z).$$

$$k_x, k_y, k_z = ? \; // \; k = |\vec{k}|, \; \theta,$$

$$\vec{k}\cdot\vec{r} = ?$$

$$\text{xz-plane} \rightarrow \vec{k} = \begin{pmatrix} k_x \\ 0 \\ k_z \end{pmatrix} \Rightarrow |\vec{k}| = \sqrt{k_x^2 + k_z^2}$$



$$\cos\theta = \frac{\vec{k}\cdot\hat{e}_z}{|\vec{k}|} \Rightarrow$$

$$\cos\theta = \frac{k_z}{|\vec{k}|} \Rightarrow$$

$$\boxed{\begin{aligned} k_z &= k\cdot\cos\theta \\ k_x &= k\sin\theta \\ k_y &= 0 \end{aligned}} \Rightarrow \vec{k}\cdot\vec{r} = \begin{pmatrix} k\sin\theta \\ 0 \\ k\cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x k \sin\theta + z k \cos\theta$$

$$\boxed{= k(x\sin\theta + z\cos\theta)}$$

3. $$\boxed{p = p_0 e^{jk(x\sin\theta + z\cos\theta)}}$$

1b.) $F(p_0, k, \theta, d) = \int p(p_0, k, x, z, \theta)\, dy$

F

$$\Rightarrow \int_S p_0 e^{jkx\sin\theta} = \int_0^d p_0 e^{jk\sin\theta x}\, dx\, dy \quad // a = k\sin\theta$$

$$= \int_0^d p_0 \cdot \int_0^d e^{jax}\, dx\, dy =$$

$$= \int_0^d p_0 \cdot \left[\frac{e^{jax}}{ja} \right]_0^d dy = \int_0^d p_0 \cdot \frac{e^{jk\sin\theta d}}{j \cdot k\sin\theta}\, dy$$

$$\boxed{= \frac{p_0 \left(e^{jk\sin(\theta)d} - 1\right)}{jk\sin\theta}} \quad // kd\sin\theta = b$$

$$= \frac{p_0(e^{jb} - 1)}{jk\sin(\theta)} = \frac{p_0(e^{\frac{jb}{2}} e^{\frac{jb}{2}} - 1)}{jk\sin(\theta)} =$$

$$\boxed{\sin x = \frac{1}{2j}(e^{x} - e^{-x})}$$

$$= \frac{p_0 e^{\frac{jb}{2}} \left(e^{\frac{jb}{2}} - e^{-\frac{jb}{2}}\right)}{jk\sin(\theta)} =$$

$$= \frac{p_0 e^{\frac{jb}{2}} \cdot 2 \cdot \cancel{j} \cdot \sin\left(\frac{b}{2}\right)}{\cancel{j}k\sin(\theta)} = \boxed{\frac{2p_0 e^{\frac{jkd\sin(\theta)}{2}} \cdot \sin\left(\frac{kd\sin(\theta)}{2}\right)}{k\sin(\theta)}}$$

1c)

$$\lim_{\theta \to 0} F(p_0, k, \theta, d) = \lim_{\theta \to 0} \left(\frac{2 p_0 e^{\frac{jkd\sin(\theta)}{2}} \sin\left(\frac{kd\sin(\theta)}{2}\right)}{k \sin(\theta)} \right)$$

$$\lim_{\theta \to 0} \frac{\cancel{2} p_0 \overset{\to 1}{e^{\frac{jkd\theta}{2}}} \frac{\cancel{k} d \,\cancel{\sin(\theta)}}{\cancel{2}}}{\cancel{k}\,\cancel{\theta}} = \boxed{p_0 \cdot d}$$