# 网络与信息安全第3次作业:实现一个RSA算法

#### 1. 算法思想

- 使用快速幂算法实现大素数模n乘法
- 使用Fermat素性测试生成两个指定位数的随机大素数p,q。
- 计算n=pq,phi(n)=(p-1)(q-1)
- 取一个与phi(n)互素的数e (直接取e=65537)
- 使用拓展欧几里得算法求e模phi(n)的模反元素d
- 给出密钥对{e,n}, {d,n}

### 2. 程序

- 详见文件夹中附带代码或后附代码
- src文件下为实现的代码,各个函数声明处有充分注释
- test文件夹下为单元测试
- main.cpp是一个样例
- 使用cmake构建
- 运行:

cmake -S . -B build
cmake --build build
(optional) ./build/unit\_test (运行单元测试)
./build/main

## 3.结果

使用1024位整数进行计算,中间结果和结果输出如下

```
tang@LAPTOP-G3IKMRA5:~/workspace/myrsh$ ./build/unit test
Running main() from /home/tang/googletest/googletest/src/gtest main.cc
    =====] Running 7 tests from 4 test suites.
        --- Global test environment set-up.
         - 2 tests from TestExtendedEuclidean
       TestExtendedEuclidean.TestExtendedEuclidean1
       OK ] TestExtendedEuclidean.TestExtendedEuclidean1 (0 ms)
         TestExtendedEuclidean.TestExtendedEuclidean2
 RUN
       OK ] TestExtendedEuclidean.TestExtendedEuclidean2 (2 ms)
      ----] 2 tests from TestExtendedEuclidean (3 ms total)
    ----- 2 tests from TestMyRsaKey
 RUN
       TestMyRsaKey.TestGenerateKeyPair
      OK ] TestMyRsaKey.TestGenerateKeyPair (66 ms)
 RUN
          TestMyRsaKey.TestEncodeAndDecode
      OK ] TestMyRsaKey.TestEncodeAndDecode (1781 ms)
     ----] 2 tests from TestMyRsaKey (1848 ms total)
    ----- 2 tests from TestPowerMod
 RUN
         TestPowerMod.TestPowerMod1
      OK ] TestPowerMod.TestPowerMod1 (0 ms)
         TestPowerMod.TestPowerMod2
 RUN
       OK ] TestPowerMod.TestPowerMod2 (0 ms)
      ----] 2 tests from TestPowerMod (0 ms total)
    ----- 1 test from TestPowerPrime
 RUN
       TestPowerPrime.TestPowerPrime1
       OK ] TestPowerPrime.TestPowerPrime1 (57 ms)
      -----] 1 test from TestPowerPrime (58 ms total)
    ----- Global test environment tear-down
       ===] 7 tests from 4 test suites ran. (1910 ms total)
  PASSED 1 7 tests.
```

## 4. 后附代码

src/util.h

```
#ifndef __UTIL_H_
#define __UTIL_H_
#include<vector>
#include <boost/multiprecision/cpp_int.hpp>
/**

* @brief 生成一个指定位数的大整数,保证最高位不是0

* @param bits 位数

*/
boost::multiprecision::cpp_int generateRandomBigNumberWithBits(int bits);

/**

* @brief 使用 Fermat素性测试对n是否为素数进行测试

*/
bool primeTest(boost::multiprecision::cpp_int n);

/**

* @brief 使用快速幂算法计算a的n次幂对modbase取模

* @param a 底数
```

```
* @param power 指数
 * @param modBase 模数
 * @return a^n mod modbase
*/
boost::multiprecision::cpp_int powermod(boost::multiprecision::cpp_int
a,boost::multiprecision::cpp_int power,boost::multiprecision::cpp_int modBase);
/**
 * @brief 生成一个指定位数的大素数,保证最高位不是0
 * @param bits 位数
*/
boost::multiprecision::cpp_int generateRandomPrimeNumberWithBits(int bits);
 * @brief 拓展欧几里得算法,求解不定方程ax+by=gcd(a,b)的一个特解
 * @param a 入参,即不定方程中的a
 * @param b 入参,即不定方程中的b
 * @param x 出参,引用类型,函数返回时会被修改为不定方程里x的值
 * @param x 出参,引用类型,函数返回时会被修改为不定方程里y的值
 * @return gcd(a,b)
*/
boost::multiprecision::cpp_int extendedEuclidean(
    boost::multiprecision::cpp_int a,
    boost::multiprecision::cpp_int b,
    boost::multiprecision::cpp_int &x,
    boost::multiprecision::cpp_int &y
);
#endif
```

#### src/util.cpp

```
#include "util.h"
using namespace std;
using boost::multiprecision::cpp_int;
// primes smaller than 100
vector<int> primes = {
                      11, 13, 17, 19, 23, 29, 31, 37, 41,
   2, 3, 5, 7,
 43, 47, 53, 59, 61, 67,
   71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131,
137, 139, 149, 151, 157, 163,
   167, 173, 179, 181, 191, 193, 197, 199, 211, 223, 227, 229, 233,
239, 241, 251, 257, 263, 269,
   271, 277, 281, 283, 293, 307, 311, 313, 317, 331, 337, 347, 349,
353, 359, 367, 373, 379, 383,
   389, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 457, 461,
463, 467, 479, 487, 491, 499,
   503, 509, 521, 523, 541, 547, 557, 563, 569, 571, 577, 587, 593,
599, 601, 607, 613, 617, 619,
   631, 641, 643, 647, 653, 659, 661, 673, 677, 683, 691, 701, 709,
719, 727, 733, 739, 743, 751,
   757, 761, 769, 773, 787, 797, 809, 811, 821, 823, 827, 829, 839,
853, 857, 859, 863, 877, 881,
   883, 887, 907, 911, 919, 929, 937, 941, 947, 953, 967, 971, 977,
983, 991, 997, 1009, 1013, 1019,
```

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1973, 1979, 1987, 1993, 1997, 1999, 2003,
    2011, 2017, 2027, 2029, 2039, 2053, 2063, 2069, 2081, 2083, 2087, 2089,
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    3539, 3541, 3547, 3557, 3559, 3571, 3581, 3583, 3593, 3607, 3613, 3617,
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5419, 5431, 5437, 5441, 5443, 5449, 5471,
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8969, 8971, 8999, 9001, 9007, 9011, 9013,
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    9203, 9209, 9221, 9227, 9239, 9241, 9257, 9277, 9281, 9283, 9293, 9311,
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9463, 9467, 9473, 9479, 9491, 9497, 9511,
    9521, 9533, 9539, 9547, 9551, 9587, 9601, 9613, 9619, 9623, 9629, 9631,
9643, 9649, 9661, 9677, 9679, 9689, 9697,
    9719, 9721, 9733, 9739, 9743, 9749, 9767, 9769, 9781, 9787, 9791, 9803,
9811, 9817, 9829, 9833, 9839, 9851, 9857,
    9859, 9871, 9883, 9887, 9901, 9907, 9923, 9929, 9931, 9941, 9949, 9967,
9973};
cpp_int generateRandomBigNumberWithBits(int bits) {
    cpp_int mask = 1;
    cpp_int res = 0;
    int times = bits / 32;
```

```
if (bits % 32 != 0) {
        times++;
    uint32_t lastMask=0xffffffff;
    if(bits%32!=0){
        lastMask=1;
        for(int i=1;i<bits%32;i++){</pre>
            lastMask=lastMask&(lastMask<<1);</pre>
        }
    }
    for (int i = 0; i < times; i++) {
        cpp_int tmp = rand();
        if(i==times-1){
            tmp=tmp&lastMask;
        }
        res |= (tmp << (i * 32));
    if (!(res & (mask << (bits - 1)))) {
        // ensure that the 1st bit is not zero
        res |= (mask << (bits - 1));
    }
    return res;
}
bool primeTest(cpp_int n) {
   //首先排除掉偶数和10000以内的质数,节省时间
    if (n % 2 == 0) {
        return false;
    }
    for (auto i : primes) {
        if(n\%i==0){
            return false;
        }
    }
    //费马素性测试
    for(int i=0;i<5;i++){
        cpp_int tmp=((rand())\%100)+3;
        cpp_int f=powermod(tmp,n-1,n);
        if(f!=1){
           return false;
        }
    return true;
}
cpp_int powermod(cpp_int a,cpp_int power,cpp_int modBase){
    cpp_int mask=1;
    cpp_int current=a;
    cpp_int res=1;
    if(power==0){return 1;}
    //快速幂算法
    while(1){
        if(mask>power){
            break;
        }
        if(mask&power){
            res=((res%modBase)*(current%modBase))%modBase;
```

```
mask=(mask<<1);
        current=((current%modBase)*(current%modBase))%modBase;
    }
    return res%modBase;
}
cpp_int generateRandomPrimeNumberWithBits(int bits){
    cpp_int tmp=generateRandomBigNumberWithBits(bits);
    if(tmp%2==0){
        tmp+=1;
    }
    while(1){
        if(primeTest(tmp)){
            return tmp;
        }else{
            tmp+=2;
        }
   }
}
cpp_int extendedEuclidean(cpp_int a,cpp_int b,cpp_int &x,cpp_int &y){
    if(b==0){
       x=1;
        y=0;
        return a;
    }
    cpp_int gcd=extendedEuclidean(b,a%b,y,x);
    y=a/b*x;
    return gcd;
}
```

#### src/MyRsaKey.h

```
#ifndef __MY_RSA_H_
#define __MY_RSA_H_
#include<map>
#include <boost/multiprecision/cpp_int.hpp>
#include"util.h"
class MyRsaKey{
   public:
   /**
    * @brief 构造函数
    * @param _e 对应公钥{e,n}里的e或者私钥{d,n}里的d(反正公钥私钥地位等同)
    * @param _n 对应公钥{e,n}里的n或者私钥{d,n}里的n(反正公钥私钥地位等同)
   MyRsaKey(boost::multiprecision::cpp_int _e,boost::multiprecision::cpp_int
_n):n(_n),e(_e){}
   /**
    * @brief 复制构造函数
   */
   MyRsaKey(const MyRsaKey& )=default;
```

```
* @brief 生成一个公钥-私钥对
    * @param bits 位数
    * @return pair<MyRsaKey,MyRsaKey>,两个分别是公钥和私钥
   */
   static std::pair<MyRsaKey,MyRsaKey>generateKeyPair(int bits);
   /**
    * @brief 在已经生成了两个大素数情况下,用给定大素数生成一个公钥-私钥对
    * @param p 大素数
    * @param q 大素数
    * @return pair<MyRsaKey,MyRsaKey>,两个分别是公钥和私钥
   */
std::pair<MyRsaKey,MyRsaKey>generateKeyPair(boost::multiprecision::cpp_int
p,boost::multiprecision::cpp_int q);
   /**
    * @brief 使用密钥加密
    * @param content 明文
    * @return 密文
   */
   boost::multiprecision::cpp_int encode(boost::multiprecision::cpp_int
content);
   /**
    * @brief 使用密钥解密
    * @param content 密文
    * @return 明文
   boost::multiprecision::cpp_int decode(boost::multiprecision::cpp_int
content);
   boost::multiprecision::cpp_int getE(){return e;}
   boost::multiprecision::cpp_int getN(){return n;}
   private:
   //对应公钥\{e,n\}里的e或者私钥\{d,n\}里的d(反正公钥私钥地位等同)
   boost::multiprecision::cpp_int n;
   //对应公钥{e,n}里的n或者私钥{d,n}里的n(反正公钥私钥地位等同)
   boost::multiprecision::cpp_int e;
};
#endif
```

#### src/MyRsaKey.cpp

```
#include "MyRsaKey.h"
using namespace std;
using boost::multiprecision::cpp_int;
std::pair<MyRsaKey, MyRsaKey> MyRsaKey::generateKeyPair(int bits) {
    while (1) {
        //生成两个大素数
```

```
cpp_int p = generateRandomPrimeNumberWithBits(bits);
       cpp_int q = generateRandomPrimeNumberWithBits(bits);
       //欧拉函数值
       cpp_{int} phi = (p - 1) * (q - 1);
       //排除掉phi不和65537互素的情况(因为通常选取65537这个质数做密钥/公钥)
       if ((phi % 65537) == 0) {
           continue;
       return generateKeyPair(p, q);
   }
}
std::pair<MyRsaKey, MyRsaKey>
MyRsaKey::generateKeyPair(boost::multiprecision::cpp_int p,
boost::multiprecision::cpp_int q) {
    cpp_int n = p * q;
   cpp_{int} phi = (p - 1) * (q - 1);
   cpp_int e, d;
   cpp_int x, y;
   e = 65537;
   //使用拓展欧几里得算法求解模反元素特解
   auto gcd = extendedEuclidean(e, phi, x, y);
   //找出一个确保为正的模反元素
   if (x < 0) {
       d = x + ((-x) / (phi / gcd) + 1) * (phi / gcd);
   } else {
       d = x;
   }
   MyRsaKey k1(e, n);
   MyRsaKey k2(d, n);
   return {k1, k2};
}
cpp_int MyRsaKey::encode(cpp_int content){
   return powermod(content,e,n);
}
cpp_int MyRsaKey::decode(cpp_int content){
   return powermod(content,e,n);
}
```

main.cpp

```
#include <iostream>

#include "MyRsaKey.h"
using namespace std;
using boost::multiprecision::cpp_int;
int main() {
    cpp_int p, q, phi;
    while (1) {
        p = generateRandomPrimeNumberWithBits(1024);
        q = generateRandomPrimeNumberWithBits(1024);
}
```

```
phi = (p - 1) * (q - 1);
            if ((phi % 65537) != 0) {
                 break;
            }
        }
    cout<<"p="<<p<<end1;</pre>
    cout<<"q="<<q<<end1;</pre>
    cout<<"n="<<p*q<<endl;</pre>
    cout<<"phi(n)="<<phi<<endl;</pre>
    auto tmp = MyRsaKey::generateKeyPair(p,q);
    MyRsaKey key1 = tmp.first;
   MyRsaKey key2 = tmp.second;
    cout<<"key1={"<<key1.getE()<<","<<key1.getN()<<"}"<<endl;</pre>
    cout<<"key2={"<<key2.getN()<<"}"<<end1;</pre>
    auto raw=generateRandomBigNumberWithBits(1024);
    cout<<"raw content: "<<raw<<endl;</pre>
    auto encoded=key1.encode(raw);
    cout<<"encrypted content "<<encoded<<endl;</pre>
    auto decoded=key2.decode(encoded);
    cout<<"decrypted content "<<decoded<<endl;</pre>
}
```