A NOVEL SPANNING-TREE MATRIX APPROACH TO MODEL AND OPTIMIZE LARGE-SCALE TREE-SHAPED WATER DISTRIBUTION NETWORKS

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ABSTRACT

There exist many criteria for the optimal design of water distribution networks (WDNs). One of the most common criteria is to design the optimal cost WDN while satisfying the hydraulic design constraints. This study was carried out to derive a novel Spanning-Tree Matrix Approach (STMA) that can model large-scale tree-shaped WDNs. A case study was tested to demonstrate the use of the STMA model embedded in the Honey-Bee Mating Optimization (HBMO) algorithm to find the combination of pipe diameters that minimizes the cost of the network. The results show that the STMA is successful in modeling a tree-shaped WDN of any size. Moreover, proposed STMA has the flexibility to adapt to any desirable governing equation or design criteria being imposed, and the element of simplicity to be easily embedded into a modern stochastic optimization algorithm (i.e., Genetic Algorithm - GA, Simulated Annealing - SA, Honey-Bee Mating Optimization – HBMO, etc.) for the hydraulic modeling purpose. Authors recommend further research on employing STMA for other types of spanning-tree hydraulic networks, such as river networks, given the adaptability of STMA for river hydraulics.

Keywords: Water Distribution Networks, Hydraulic Modeling, Spanning-Trees, Stochastic Optimization Algorithms

INTRODUCTION

The recent past has witnessed uncountably many developments in computational methods in water resources management, with particular interest in WDNs modeling and optimization. The influence of modern mathematics upon these developments of WDNs is indispensable. Rapid urbanization, scarcity of natural resources, and upscale-costs involved are identified to be some of the reasons for this continuous research interest on WDNs. Among many criteria for the design of WDNs, one of the widely-used is finding the combination of pipe diameters that satisfies the minimum-nodal-hydraulic-head and maximum-pipe-head-loss requirements while minimizing the cost involved. Hence, this is regarded as a combinatorial optimization problem. Due to the environmental conditions, the non-linear nature of the information associated with the hydraulic constrains, and the type of architecture being adopted, the modeling and optimization for a WDN has, most of the time, governing models specific to its own. When existing hydraulic modeling packages do not facilitate the ability to model flexible governing equations, the development of problem specific new computational methods by engineers and analysts becomes crucial.

LITERATURE REVIEW

A large body of research shows that researchers have been using numerous mathematical methods to model and optimize water systems (Alporevits and Shamir, 1997; Lin *et al*., 1997; Cunha and Sousa, 1999; Hsu and Cheng, 2002; Dijk et al., 2008; Mohan and Babu, 2009; Mohan and Babu, 2010; Suribabu, 2012; Corte and Sörensen, 2013; Cong and Zhao, 2015; Awe et al., 2020; Senavirathna et al., 2022,).

As far as modeling of WDN is concerned, it is very noticeable that use of a commercially available hydraulic modeling software package is still very common practice among significant number of researchers, case in point, the use of EPANET by Mohan and Babu (2010), Suribabu (2012), Awe et al. (2020). It is true that commercially available hydraulic modeling software packages of this sort provide users considerable convenience when it comes to modeling a WDN. However, oversimplified hydraulic governing equations in those packages may give off undesired, or rather impractical, results upon simulation. Therefore, it has been an apparent need to keep researching for novel computational methods that are highly adaptable to specific governing equations of the WDN problem being concerned, however complex it might be.

Incorporation of graph theory to tackle WDN has been noticeable in the recent past. The work done by Cong and Zhao (2015) has focused on applying a minimum spanning-tree algorithm, the Kruskal's algorithm, to find the minimum spanning-tree for a WDN that has fixed nodal positioning. Kruskal's algorithm, however, finds the minimum-weight spanning-tree by choosing edges with minimum weights while avoiding potential loops, whereas Prim's algorithm searches the minimum-weight spanning-tree by continuously seeking the least-weight edge emanating from the parent node. Their study has assigned constant weights to the edges, or rather pipes, ignoring the possibility of using commercially available pipes of different sizes. It also needs to be understood that designing and optimizing WDNs should be done taking hydraulic constraints into consideration, as final goal is, of course, consumer satisfaction.

Corte and Sörensen (2013), in their critical review, have shown that not only modeling, but also optimization of WDNs is much needed to reach the goals of delivering reliable water supply at optimum cost. In literature, there can be seen two main categories for WDN optimization algorithms, namely, direct-heuristic and stochastic-heuristic algorithms.

Direct-heuristic algorithms provide methods incorporating gradient and implicit information associated with the WDN to reach an optimal solution. The main advantage of algorithms of this kind is that, it takes a very smaller number of iterations to reach the optimal solution compared to stochastic-heuristic algorithms. Moreover, direct-heuristic algorithms cannot guarantee if the solution so obtained is the global optimum solution. The studies done by Lin *et al*. (1997), Hsu and Cheng (2002), Mohan and Babu (2009), , Suribabu (2012), and Awe et al. (2020), provide criteria for employing direct-heuristic algorithms to reach near optimal solutions for WDNs.

On the other hand, stochastic-heuristic, or sometimes called modern-stochastic-heuristic, methods neither take gradient nor implicit information associated with the WDNs. These kinds of algorithms evaluate the objective function at randomly taken different regions of the solution space to seek the feasible solution that minimizes the cost function. The main advantage of this method, as described earlier, is that it investigates the whole solution space to probe the global optimum solution. On the contrary to direct-heuristic methods, stochastic-heuristic methods perform extremely large number of objective function evaluations to reach this global optimum. Some of the notable studies that utilize these optimization algorithms for WDNs include Simulated Annealing (SA) Approach done by Cunha and Sousa (1999), Genetic Algorithm (GA) by Dijk et al. (2008), and Honey-Bee Mating Optimization (HBMO) by Mohan and Babu (2010). Later, Senavirathna et al. (2022) have adapted the HBMO to facilitate a design criterion slightly different to what had been used by Mohan and Babu (2010), in order to address their case study.

This paper, however, brings about a simple but very effective hydraulic modeling technique, called Spanning-Tree Matrix Approach (STMA), to model tree-shaped WDNs. Proposed STMA also can function, similar to dynamic programming problems, as an adaptable sub-system in an optimization algorithm for WDNs that have tree-shaped architecture. A case study has also been presented to demonstrate the applicability of STMA in a modern stochastic optimization algorithm, the version of the HBMO algorithm adopted from Senavirathna et al. (2022).

FORMULATION OF THE OPTIMIZATION MODEL

Optimal WDN design problem focused in this study can be written as,

$$Min\ Z = \sum_{i=1}^{N} C_i(D, L) \tag{1}$$

subjects to the constraints,

$$H_{R\ j} \geq\ H_{R\ j}^{min}\quad j = 1,2,3, \dots, nd \tag{2}$$

$$g_{FF\ i}\ \leq\ g_{FF\ i}^{max}\quad i = 1,2,3, \dots, np \tag{3}$$

where,

Z	=	total cost involved with the WDN;
N	=	number of pipes in the water distribution network;
$C_i(D, L)$	=	cost of the i^{th} pipe having diameter D and length L;
$H_{R\ j}$	=	residual water head available at the j^{th} node;
$H_{R\ j}^{min}$	=	minimum residual water head required at the j^{th} node;
$g_{FF\ \ i}$	=	friction and fitting loss gradient in the i^{th} pipe;
$g_{FF\ \ i}^{max}$	=	maximum allowable friction and fitting loss gradient in the i^{th} pipe;
nd	=	number of demand nodes; and
np	=	number of pipes.

The flowchart shown in Figure 1, depicts the algorithm which accommodates the above optimization model. It is typically required the WDN data, such as reservoir elevation, nodal demands, pipe lengths, nodal elevations, and the pipe connectivity layout. In addition to WDN data, commercially available pipe diameters and their unit costs are needed as the fulfillment of the data requirement.

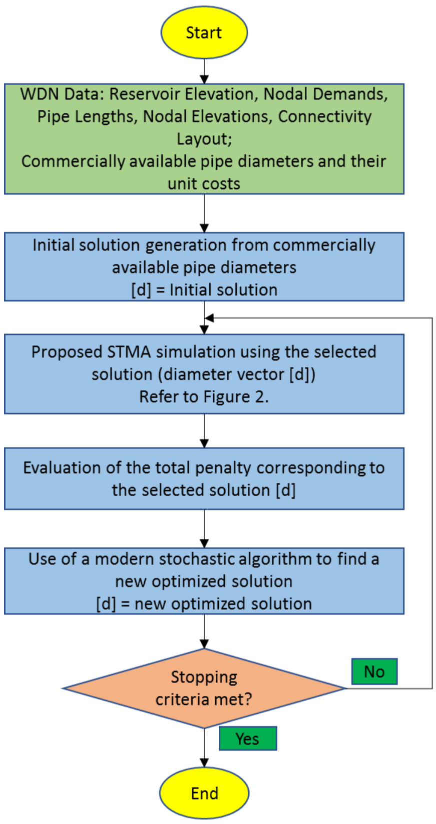

Figure 1: Flowchart of the optimization algorithm considered in this study

At the beginning, an initial solution [d], a combination of pipe diameters, is generated from the commercially available pipe diameters. The WDN is then simulated for [d] according to the proposed STMA. If the selected [d] is violating the design constraints above, the solution [d] is penalized by adding a penalty value to discourage its fitness for application in the design. This study uses a modern stochastic algorithm to implement the optimization algorithm described in the Figure 1, embedding the STMA for hydraulic modeling. The algorithm then iteratively finds a new optimized solution from the commercially available pipe diameters until a stopping criterion is met. The solution thus obtained can be regarded as the global optimum solution, or rather the optimal combination of pipe diameters that minimizes the cost of the WDN to the maximum extent.

Next, it can be discussed how STMA can be used to simulate a given tree-shaped WDN of any size.

SPANNING-TREE MATRIX APPROACH (STMA)

[Using the WDN data + selected pipe diameter vector]
Perform the calculations of,

1. Demand-to-flow transformation matrix $[T_D^Q]$
2. Flow vector $[Q]$
3. Friction and fitting loss gradient vector $[g_{FF}]$
4. Pipeline-head-loss vector $[F]$
5. Nodal-head vector $[H]$
6. Nodal-residual-head vector $[H_R]$

Finally, export $[H_R]$ and $[g_{FF}]$ for penalty evaluations

Figure 2: Basic steps involved with the version of the STMA used in the study

The goal of using STMA is to obtain the nodal-residual-head vector $[H_R]$ and the friction and fitting loss gradient vector $[g_{FF}]$ for the selected diameter vector [d] abiding to the algorithm shown in the Figure 1. To accomplish this task, the basic steps that can be used are listed in the Figure 2.

In order for the employed mathematical concepts in the computations to be clearly understood, it is convenient to consider a fairly small, tree-shaped, dummy WDN that has six consumer demand nodes. As the flow occurs in one direction only, the spanning-tree of WDN here is a directed graph rooted at the reservoir node(see the Figure 3).

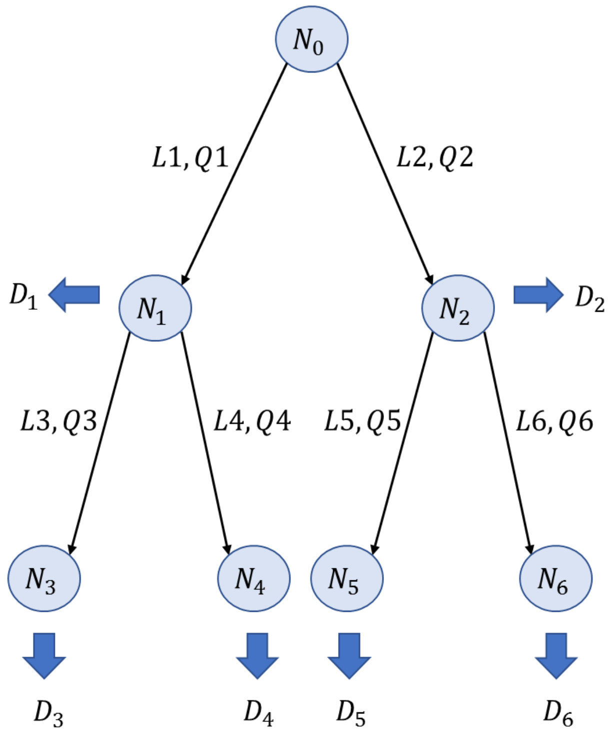

Figure 3: The dummy WDN for the demonstration of the functionality of STMA

The reservoir, which is the root of the spanning-tree, is denoted by N_0. However, N_is ($i =$ 1,2,3,4,5,6) are the consumer demand nodes of the WDN; L_is ($i =$ 1,2,3,4,5,6) are the lengths of pipes; Q_is ($i =$ 1,2,3,4,5,6) are the flows in pipes in the indicated direction; and D_js ($j =$ 1,2,3,4,5,6) are the nodal water demands.

In matrix notation, the nodal water demands and pipeline flows can be written as (4) and (5), respectively.

$$[D] = [D_j] = [D_1\ D_2\ D_3\ D_4\ D_5\ D_6]^T \qquad (4)$$
$$j = 1,2,3,4,5,6$$

$$[Q] = [Q_i] = [Q_1\ Q_2\ Q_3\ Q_4\ Q_5\ Q_6]^T \qquad (5)$$
$$i = 1,2,3,4,5,6$$

Then the flow vector $[Q]$, computed to facilitate the continuity equation, is given by,

$$[Q] = [Q_i] = \left[T_D^Q\right].[D] \tag{6}$$

where, $\left[T_D^Q\right]$ is a form of adjacency matrix that not only illustrates the connectivity of pipes through nodes, but also a linear-transformation matrix that transforms nodal demands in to pipeline flows in a bottom-up approach. The matrix $\left[T_D^Q\right]$ for the WDN corresponding to the Figure 3 can be considered as (7). Intuitively, for example, if the pipe number 1 has a flow of Q_1, then Q_1 is equal to the summation of the demands of all the downstream nodes, resulting 1s at the corresponding columns and 0s for all the other entries of the first row of $\left[T_D^Q\right]$. Similarly, all the entries of all the other rows of $\left[T_D^Q\right]$ can be determined.

$$\left[T_D^Q\right] = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

The matrix form, or rather the vector form, of the selected solution can be seen as given in (8). There, the d_i is the diameter of the i^{th} pipe.

$$[d] = [d_i] = [d_1\ d_2\ d_3\ d_4\ d_5\ d_6]^T \tag{8}$$

$$i = 1,2,3,4,5,6$$

Friction-loss-gradient of the i^{th} pipe, $h_{f\ i}$, is then calculated according to the Hazen-Williams Equation shown in (9).

$$h_{f\ i} = \frac{10.666 * Q_i^{1.85}}{C_{HW}^{1.85} * d_i^{4.87}} \tag{9}$$

$$i = 1,2,3,4,5,6$$

The matrix form of the set of friction-loss gradients for all the pipes can be identified as (10).

$$[h_f] = \left[h_{f\ i}\right] = \frac{10.666}{C_{HW}^{1.85}}.\,[[Q]^{o\,(1.85)}\ ./\ [d]^{o\,(4.87)}\,] \tag{10}$$

where, $[Q]^{o\,(1.85)}$ is the notation for the vector obtained by the element-wise exponentiation of the elements in $[Q]$ to the power 1.85; and "./" is the operator "Hadamard Division" for denoting the element wise division between two vectors $[Q]^{o\,(1.85)}$ and $[d]^{o\,(4.87)}$.

The equations (6) and (10) can be combined to obtain the equation (11) as shown below.

$$[h_f] = [h_{f\ i}] = \frac{10.666}{C_{HW}^{1.85}} \cdot [([T_D^Q].[D])^{o\,(1.85)} \,./\, [d]^{o\,(4.87)}] \tag{11}$$

To account for the fitting losses that occur in each pipe, friction-loss gradient vector is multiplied by the fitting-loss coefficient C_{ft} to obtain the $[g_{FF}]$, the gradient vector for both friction and fitting losses.

$$[g_{FF}] = C_{ft}.[h_f] \tag{12}$$

The pipe-length vector $[L]$ of the WDN shown in Figure 3, can be regarded equal to the expression (13).

$$[L] = [L_i] = [L_1\ L_2\ L_3\ L_4\ L_5\ L_6]^T \tag{13}$$

$$i = 1,2,3,4,5,6$$

The pipeline-head-loss vector $[F]$, is the vector whose i^{th} element is the product of i^{th} elements of $[L]$ and $[g_{FF}]$. The matrix notation of this element-wise multiplication of two vectors, $[L]$ and $[g_{FF}]$, is given by the "Hadamard Product ⊙" as shown in (14).

$$[F] = [F_i] = [L] \odot [g_{FF}] \tag{14}$$

The equations (12) and (14) can be merged to obtain the matrix equation (15).

$$[F] = [L] \odot C_{ft}.[h_f] \tag{15}$$

As the hydraulic-head-loss along the flow path is a continuous function, total algebraic summation of the head-loss around a closed path, is equal to zero. The mathematical notation of this law of conservation of energy is given by,

$$\sum_{i=1}^{np_L} F_i = 0 \tag{16}$$

where, npL is the number of pipes in a closed path.

As the focus here in this study is directed only towards the tree-shaped WDNs, the adaptation of the above (16) is shown in (17) as a matrix form.

$$[H] = E_0[1] - [T_H].[F] \tag{17}$$

where, $[H]$ is the nodal-head vector; E_0 is the elevation of the reservoir node N_0 measured from the mean sea level (MSL); [1] is the vector in which each element is equal to 1; $[T_H]$ is the transformation matrix that transforms pipeline-head-loss values $[F]$ into total head-loss values through a bottom-up approach at all the nodes, and total head-loss values are given by $[T_H].[F]$. The matrix expression of $[T_H]$, however, is given by (18).

$$[T_H] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

It can be noticed, if N_4 is considered for example, head-loss occurs only through pipe number 1 and 4, hence first and fourth column entries of row number four are equal to 1, resulting all the other entries in that row being equal to zero. Likewise, entries of all the other rows of $[T_H]$ can be determined by mere layout of the WDN concerned.

By considering expressions (17) and (15) together, the following expression (19) can be derived for $[H]$.

$$[H] = E_0[1] - [T_H].([L] \odot C_{ft}.[h_f]) \tag{19}$$

The nodal-elevation vector $[E]$ has entries E_js where E_j is the nodal-elevation of the j^{th} node measured from MSL. $[E]$ is given by expression (20).

$$[E] = [E_j] = [E_1\ E_2\ E_3\ E_4\ E_5\ E_6]^T \tag{20}$$

$$j = 1,2,3,4,5,6$$

As the residual water head H_R at each node is equal to the difference between the nodal-head and nodal-elevation, the matrix form is given by (21).

$$[H_R] = [H] - [E] \tag{21}$$

By merging the expressions (19) and (21) together, below expression (22) can be obtained.

$$[H_R] = E_0[1] - [T_H].([L] \odot C_{ft}.[h_f]) - [E] \tag{22}$$

As a summary, from the proposed STMA, $[g_{FF}]$ and $[H_R]$ are now computed for selected $[d]$. Hence, it is now possible to proceed for the penalty evaluations in order to continue on with the algorithm being described under Figure 1.

CASE EXAMPLE

Warapitiya Service Zone, Sri Lanka

The general optimization algorithm that includes proposed STMA simulation described under Figure 1, was implemented on Python, a High-Level Programming Language, for the WDN scheme planned for Warapitiya Service Zone, Sri Lanka, aiming for obtaining the combination of pipe diameters that minimizes the cost function while satisfying the design constraints, as per the optimization model formulation described. In this study the modern stochastic optimization algorithm Senavirathna et al. (2022) have used, i. e., Honey-Bee Mating Optimization Algorithm (HBMO), was updated with the STMA component to facilitate the hydraulic simulations for the tree-shaped WDN. The minimum-allowable-nodal-hydraulic-head value H_R^{min} and the maximum-allowable-gradient for friction-and-fitting losses g_{FF}^{max} were considered to be 10 m and 0.005 m/m, respectively. The Hazen-Williams coefficient and the fitting loss coefficient were taken to be 130 and 1.15, respectively, for all the pipes.

Here, in fact, are two main attributes of Warapitiya WDN to be an ideal choice for STMA simulation. On the one hand, WDN of Waraptiya Service Zone has the geometric configuration of a spanning-tree, and, on the other hand, it has so many demand nodes and pipes that it can be considered as a relatively "large-scale" WDN.

The data requirement, as identified for the optimization algorithm, included unit-costs of commercially available pipe sizes, reservoir water head, nodal-water-demands, pipe lengths, nodal-elevations, and the pipe-connectivity layout. The pipe layout of Warapitiya WDN can be seen from the Figure 4 as a directed-graph to identify the flow directions. It is to be noted that the symbology used under the Warapitiya Service Zone mean the same as that was utilized in STMA methodology explanation. Pipe length data are given in the Table 1. Nodal-water demand data and nodal-elevation data, including the datum of reservoir elevation, can be referred to as listed in the Table 2. All these data, excluding unit-costs of commercially available pipes, were collected from the National Water Supply and Drainage Board (NWS&DB), Sri Lanka. Unit-cost data for different pipe sizes were obtained from Mohan and Babu (2010), and shown in the Table 3. Corresponding unit-costs for commercially available pipe sizes used in Warapitiya WDN, however, were estimated by interpolating these data on Python programming language using Lagrange-Interpolation-Method.

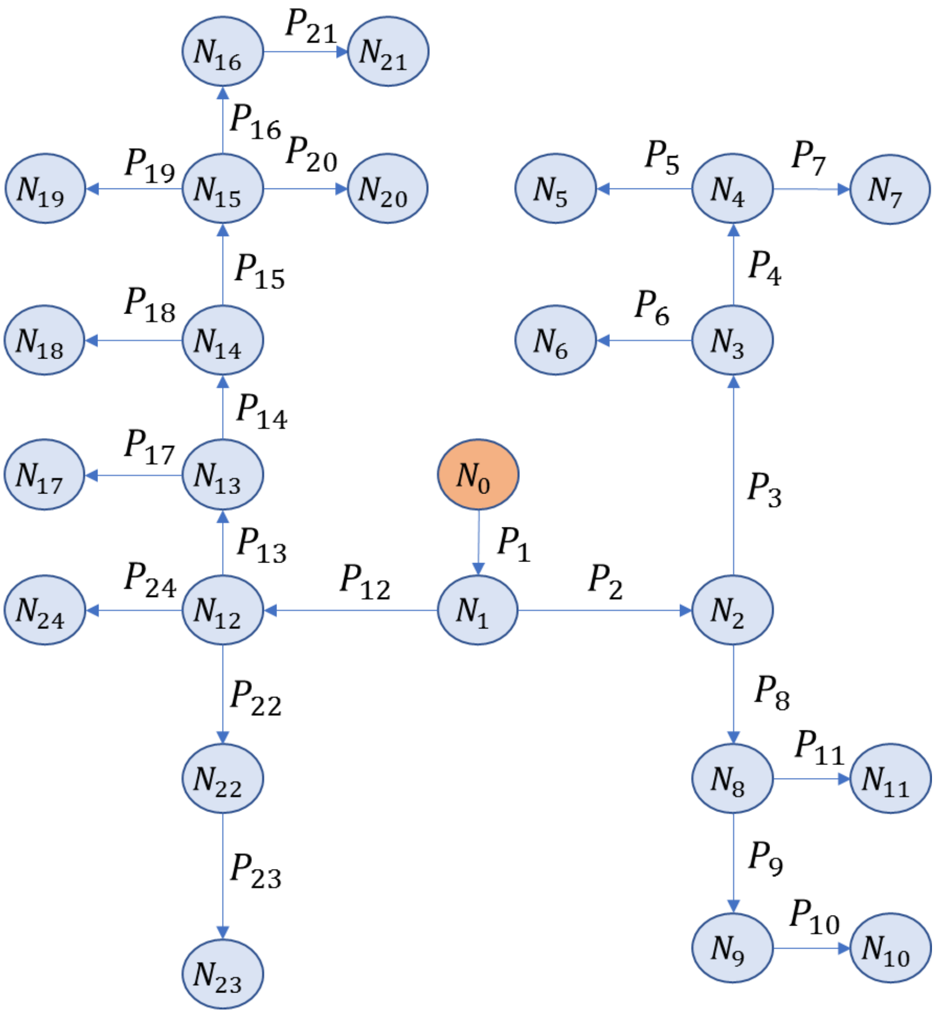

Figure 4: Pipe layout directed-graph for WDN of Warapitiya Service Zone, Sri Lanka

Table 1: Lengths of pipes being deployed for Warapitiya WDN

Pipe (P_i)	Length (L_i) / (m)
P_1	250
P_2	160
P_3	740
P_4	80
P_5	290
P_6	350
P_7	130
P_8	330
P_9	160
P_{10}	180
P_{11}	410
P_{12}	30
P_{13}	260
P_{14}	630
P_{15}	300
P_{16}	530
P_{17}	850
P_{18}	520
P_{19}	290
P_{20}	620
P_{21}	140
P_{22}	300
P_{23}	1280
P_{24}	1160

Table 2: Nodal-water-demands and nodal-elevations of Warapitiya WDN

Node (N_j)	Demand (D_j) / (m^3/day)	Elevation (E_j) / (m)
N_0	-	506.0
N_1	3.75	485.0
N_2	381.56	475.0
N_3	292.50	455.0
N_4	148.13	460.0
N_5	48.75	455.0
N_6	11.25	450.0

N_7	91.88	460.0
N_8	101.25	480.0
N_9	45.00	480.0
N_{10}	18.75	455.0
N_{11}	43.13	476.0
N_{12}	71.25	482.0
N_{13}	133.13	479.0
N_{14}	105.00	472.0
N_{15}	116.25	462.5
N_{16}	61.88	445.0
N_{17}	61.88	450.0
N_{18}	28.13	474.0
N_{19}	33.75	450.0
N_{20}	28.13	452.0
N_{21}	20.63	450.0
N_{22}	76.88	475.0
N_{23}	71.25	450.0
N_{24}	58.13	450.0

Table 3: Unit-cost data for corresponding pipe sizes [obtained from Mohan and Babu (2010)]

Diameter / (mm)	Unit-cost
25.4	2
50.8	5
76.2	8
101.6	11
152.4	16
203.2	23
254.0	32
304.8	50
355.6	60
406.4	90
457.2	130
508.0	170
558.8	300
609.6	550

According to Lagrange-Interpolation-Method, as the number of data points given in Table 3 is 14 (=13+1), the Lagrange-interpolating-polynomial interpolates 14 data points by a polynomial of degree 13, $f_{13}(x)$, defined as,

$$f_{13}(x) = \sum_{k=0}^{13} L_k\,(x)\, f(x_k) \tag{23}$$

where,

$$L_k(x) = \prod_{\substack{m=0 \\ m \neq k}}^{13} \frac{(x - x_m)}{(x_k - x_m)} \tag{24}$$

It is apparent from the Figure 5 that, the interpolated unit-costs for commercially available pipe diameters listed in Table 4, are very much in-line with the actual data obtained from Mohan and Babu (2010). This also proves the applicability of the Lagrange-Interpolation-Method for the unit-cost interpolations.

Table 4: Commercially available pipe sizes and their corresponding interpolated unit-costs

Diameter / (mm)	Interpolated unit-cost
55	5.0259
79	8.4781
97	10.6801
140	14.0679
198	22.5046
246	29.6739

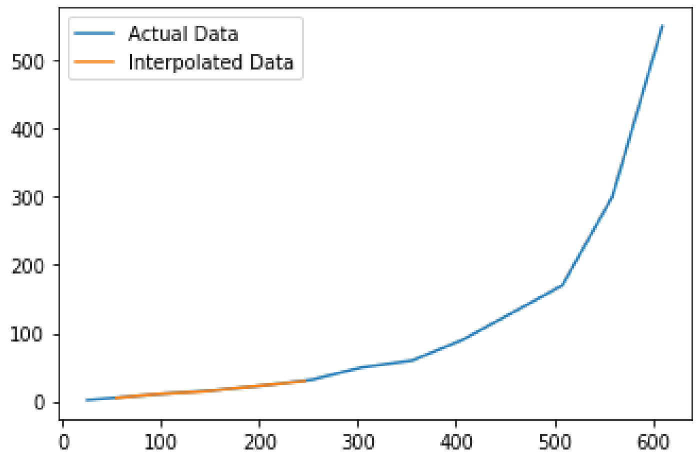

Figure 5: Comparison between the actual unit-cost data and the interpolated unit-cost data; x- axis: Diameter / (mm); y-axis: Unit-cost

RESULTS

As what is listed in the Table 5, the global optimal solution for [d] was identified from the execution of the HBMO algorithm together with the STMA simulation. The Table 5 also widens to tabulate the results, $[g_{FF}]$ and $[H_R]$, obtained at the STMA simulation stage, corresponding to the global optimal solution. In short, it can be noticed without effort that, HBMO algorithm together with the proposed STMA simulation was able to deliver the combination of pipe diameters that minimizes the cost of the given spanning-tree pipe configuration network while satisfying the design constraints being concerned to a maximum extent.

Table 5: Tabulation of the optimal solution given by HBMO [Senavirathna et al., 2022] algorithm and constraint satisfaction status obtained by STMA

Pipe P_i	Optimal Solution [d] by HBMO / (mm)	$g_{FF\ i}$ / (m/m)	Constraint $g_{FF\ i} \leq g_{FF\ i}^{max}$ satisfied?	Node N_j	$H_{R\ j}$ / (m)	Constraint $H_{R\ j} \geq H_{R\ j}^{min}$ satisfied?
P_1	246	0.0014	Yes	N_1	20.6556	Yes
P_2	198	0.0014	Yes	N_2	30.4269	Yes
P_3	198	0.0004	Yes	N_3	50.1323	Yes
P_4	140	0.0006	Yes	N_4	45.0867	Yes
P_5	79	0.0003	Yes	N_5	49.9869	Yes
P_6	55	0.0001	Yes	N_6	55.0857	Yes
P_7	79	0.0011	Yes	N_7	44.9422	Yes
P_8	140	0.0003	Yes	N_8	25.3243	Yes
P_9	79	0.0006	Yes	N_9	25.2339	Yes
P_{10}	55	0.0003	Yes	N_{10}	50.1722	Yes
P_{11}	79	0.0003	Yes	N_{11}	29.2119	Yes
P_{12}	198	0.0008	Yes	N_{12}	23.6315	Yes
P_{13}	198	0.0004	Yes	N_{13}	26.5292	Yes
P_{14}	140	0.0010	Yes	N_{14}	32.8920	Yes
P_{15}	140	0.0005	Yes	N_{15}	42.2506	Yes
P_{16}	79	0.0009	Yes	N_{16}	59.2678	Yes
P_{17}	79	0.0005	Yes	N_{17}	55.0745	Yes
P_{18}	55	0.0007	Yes	N_{18}	30.5146	Yes
P_{19}	55	0.0010	Yes	N_{19}	54.4558	Yes
P_{20}	55	0.0007	Yes	N_{20}	52.3006	Yes
P_{21}	55	0.0004	Yes	N_{21}	54.2106	Yes
P_{22}	97	0.0010	Yes	N_{22}	30.3346	Yes
P_{23}	79	0.0007	Yes	N_{23}	54.4459	Yes
P_{24}	79	0.0005	Yes	N_{24}	55.0788	Yes

CONCLUSIONS AND RECOMMENDATIONS

Aiming for building-up a hydraulic model that has flexible hydraulic governing equations, this study was carried out to derive a novel technique, namely, the Spanning-Tree Matrix Approach (STMA), that can model large-scale tree-shaped water distribution networks (WDNs). Taking the WDN of Warapitiya Service Zone in Sri Lanka as a case study, it was tested the use of the STMA model embedded in the Honey-Bee Mating Optimization (HBMO) algorithm to find the combination of pipe diameters that minimizes the cost of the network. The results show that the STMA is successful in modeling a tree-shaped WDN of any size, and proved its applicability as a hydraulic model in optimization algorithms. As a byproduct of this study, Lagrange-Interpolation-Method was found to be an effective means to interpolate unit-cost values corresponding to commercially available pipe sizes.

Furthermore, proposed STMA can be updated with desired governing equations or design criteria of interest. Authors recommend further research on employing STMA for other types of spanning-tree hydraulic networks, such as river networks, given the adaptability of STMA for river hydraulics.

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