Fuzzy cellular automata: from theory to applications

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Fuzzy Cellular Automata: From Theory to Applications

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Abstract

In our paper we would like to present a fuzzified cellular automata structure called fuzzy cellular automata. We begin our paper with a fuzzified entity called fuzzy automaton, then we present basics of cellular automata and finally we define fuzzy cellular automata. At the end we present some simulation results from the field of fire spread in homogenuous nature environment.

Keywords: Fuzzy Logic, Fuzzy Automaton, Cellular Automata, Fuzzy Cellular Automata.

1. Introduction

The basics of theory of cellular automata (CA) have been established by Von Neumann in the middle of fifties in previous century as a result of a research with a goal to eliminate a well known Von Neumann's bottle neck in computer architecture theory field. After Von Neumann's work the research has stopped till the start of eighties when the research makes progress thanks to mathematicians and physicians. The main results are presented in [1, 2]. The main characteristics of the work from eighties is analytical approach to CA. It means that the CA structure or a model is built in advance and the behaviour of structure is than analyzed and compared to real world systems dynamics. On the other hand there are not a lot of results from synthesis approaches results. Their main goal is to build a CA structure or a model on the basis of real world system behaviour and knowledge about it. The simulation results of such model should be similar to the real behaviour. One of the main problems in synthesis approach is the lack of exact knowledge about system dynamics. Because of this reason we involve fuzzy logic into our work. The generalized structure is called *fuzzy cellular automata* (FCA).

In this paper we have changed the definition of fuzzy automaton and build a basic definition of fuzzy cellular automata. At the end we present a case of real world simulation on the basis of FCA. The application is a simulation of fire spread in the natural homogenous environment.

2. From ordinary to fuzzy automaton

The main entitity of CA is a cell, which could be interpreted as a finite automaton in an abstract sense. In [3] we can find a Moore's definition of a finite automaton, which can be used for a cell model:

Definition 1: Moore's automaton is defined as a quintuplet $A = \{X, Q, Y, \delta, \lambda\}$, where X, Q and Y are finite nonempty sets, which represent a set of possible inputs, internal states and output symbols, respectively. δ is a translating function which enables and defines the change of states $q(t+1) = \delta(q(t), x(t))$, and λ is a translating function which produces output symbols $y(t) = \lambda(q(t))$.

The weak point of such automaton is its unability to process on uncertain inputs and its unability to process with uncertain rules (fuzzy translating function). We have analyzed the existing definitions of fuzzy automata in [4, 5, 6, 7, 8], but they don't satisfy our needs. For this reason we have supplemented previous definitions to a new one presented in [9, 10]:

Definition 2: Fuzzy automaton is defined as a sextuplet $FA = \left\{\widehat{\mathbf{x}}, \widehat{\mathbf{q}}, Y, \widetilde{\delta}, \widetilde{\lambda}, \mu_{\widehat{q}}(t_0)\right\}$, where $\widehat{\mathbf{x}}$ and $\widehat{\mathbf{q}}$ are finite vectors of non-empty fuzzy sets of inputs and internal states, respectively. Y is an ordinary non-empty and finite set of possible output symbols. $\widetilde{\delta}$ is a stationary set of fuzzy rules, which lead to a new state of automaton and $\widetilde{\lambda}$ is a stationary set of fuzzy rules which produce new output symbol on the basis of global fuzzy state of automaton $\widehat{q}(t)$. $\mu_{\widehat{q}}(t_0)$ represents a vector of memberships to individual fuzzy states of automaton and have the meaning of a starting state in the context of $\widehat{q}(t_0)$.

Expressions (1) and (2) explain the meaning of $\widehat{q}(t)$ and $\mu_{\widehat{q}}(t)$ where k represents the number of fuzzy states and $\widetilde{Q}_i$ is an ordinary fuzzy set.

$$\widehat{q}(t) = \left[(\mu_{\widetilde{Q}_i}(t), \widetilde{Q}_i)\right],\ 1 \leq i \leq k, \tag{1}$$

$$\mu_{\widehat{q}}(t) = \left[(\mu_{\widetilde{Q}_i}(t))\right],\ 1 \leq i \leq k. \tag{2}$$

Definition 3: Fuzzy automaton for the needs of FCA is defined as in Definition 2, with additional relations. First relation is $\widehat{\mathbf{x}} = \widehat{\mathbf{q}}^n \cdot \widehat{\mathbf{x}}_c$. It means that an *"input"* vector of fuzzy sets is defined as concatenation of vectors of fuzzy states of n neighbouring cells $\widehat{\mathbf{q}}^n$ in cellular structure and of vector $\widehat{\mathbf{x}}_c$, which represents a vector of non-empty fuzzy sets. The last item has a function of delivering some global variable's value to a cell with an influence to state changing function. $\widetilde{\delta}'$ is an extended version of $\widetilde{\delta}$ and it operates on a finite string of input fuzzy symbols. The definition is extended to the following expressions:

$$\widetilde{\delta} : \widehat{\mathbf{q}} \times \widehat{\mathbf{x}} \times \widehat{\mathbf{q}} \rightarrow [0,1] = \widehat{\mathbf{q}} \times \widehat{\mathbf{q}}^n \times \widehat{\mathbf{x}}_c \times \widehat{\mathbf{q}} \rightarrow [0,1], \quad (3)$$

$$\widetilde{\lambda} : \widehat{\mathbf{q}} \times Y \rightarrow [0,1], \quad (4)$$

$$D^1\widehat{q} = \widehat{q}(t+1) = \widetilde{\delta}(\widehat{x}, \widehat{q}), \ \widehat{x} \in \widehat{\mathbf{x}} \quad (5)$$

$$\forall \widehat{x}_1 \in \widehat{\mathbf{x}}, \forall \widehat{x}_2 \in \widehat{\mathbf{x}}^* : \widetilde{\delta}'(\widehat{q}_0, \widehat{x}_1\widehat{x}_2, \widehat{q}_n) =$$

$$\max_{\forall \widehat{q}_l \in \widehat{\mathbf{q}}} \min \left[\widetilde{\delta}(\widehat{q}_0, \widehat{x}_1, \widehat{q}_l), \widetilde{\delta}'(\widehat{q}_l, \widehat{x}_2, \widehat{q}_n) \right]. \quad (6)$$

In vector $\widehat{\mathbf{q}}$ every fuzzy set represents a single fuzzy state while in vector $\widehat{\mathbf{x}}$ every fuzzy set represents a single fuzzy input symbol. The processing on the base of such automaton and the Mamdani's type of processing is presented in[11]. The above treatment and approach to the fuzziness of global automaton state is taken as the basis on which we form the notion of fuzzy automaton state as applied in the field of FCA.

3. Towards fuzzy cellular automata

Before we define fuzzy cellular automata let us present basic definition of CA summarized from [1, 12, 13]:

Definition 4: Let $P = \mathcal{Z}^m$ and let Q be a finite non-empty set of all possible states of an individual cell. F is a function which forms a new state of c_c in time t (10), N is a definition of the neighborhood set of observed cell c_c from expression (9) and T a set of discrete time points. Then we call a quadrouple $CA =< P, Q, N, F >$ a cellular automaton (CA).

$$E : P \times T \rightarrow Q, \quad (7)$$

$$\forall t \in T, \ \forall c_c \in P, \ \exists!, \ q \in Q : q = E\ (c_c, t), \quad (8)$$

$$N : \ P \rightarrow \mathcal{P}(P), \quad (9)$$

$$F : \ Q^{k+1} \rightarrow Q, \quad (10)$$

$$\forall c_c \in P : N(c_c) = \{c_1, ..., c_k\} \Rightarrow d(c_c - c_j) \leq R,$$

$$j = 1, ..., k; \ R, \ k \in \mathcal{N}, \quad (11)$$

$$k = (2 * R + 1)^m - 1. \quad (12)$$

Time, space and states of cells are discrete. Parameter R in Expressions (11) and (12) represents the radius for the neighborhood criteria, k is the number of neighbors which influence the formation of the state of the observed cell c_c (this includes the cell c_c itself) and m is the dimension of cell space P. The general dynamical characteristic of the CA structure is also determined by the state transition function which is, in most of the applications [1, 14], given by the set of rules F:

$$\forall c_c \in P : E(c_c, t+1) = F(E(c_c, t), E(N(c_c, t))). \quad (13)$$

We see that the cell's c_c next-state (in time step $t+1$) is formed on the basis of the states of cell c_c and neighbouring cells in time step t. According to the above definition we can conclude that parameters Q and F correspond to vector $\widehat{\mathbf{q}}$ and transition function $\widetilde{\delta}$, respectively. The only direct item of the definition which is eventually left to be fuzzified is the neighborhood criteria N. The discussion of its fuzzification is given in [15]. Let us now provide a definition of a fuzzy cellular automaton on the basis of the above treatments.

Definition 5: Fuzzy cellular automaton (FCA) is a dynamical structure defined as a septet $FCA =< P, \widehat{\mathbf{q}}, \widehat{N}, \widetilde{F}, \widehat{T}, \widetilde{\lambda}, \widehat{\mathbf{x}} >$, where P is an m-dimensional space of cells (individual automata), $\widehat{\mathbf{q}}$ is a vector of finite non-empty fuzzy sets of all possible states of an individual cell, $\widehat{N}$ is fuzzy definition of the neighborhood set and $\widetilde{F}$ is a finite set of fuzzy stationary rules which define state transitions for individual cells. Item $\widehat{T}$ denotes an eventual fuzziness also in the sense of temporal processing, while $\widetilde{\lambda}$ provides a stationary mapping function, which forms a new output symbol based on the global fuzzy state of an automaton (cell) $\widehat{q}$. Item $\widehat{\mathbf{x}}$ represents a set of fuzzy variables (pseudo-neighbours) which are input to the inference process as a parameter of some global situation which is equal by value for all cells. The space of cells P is discrete.

The cell space P of the FCA structure remains unaltered compared to the classical CA structure. The fuzzy definition of the neighborhood $\widehat{N}$ depends on the actual application. The mapping function $\widetilde{\lambda}$ is, as opposed to CA, necessary for the formation of a crisp value which can be used in the external presentation of the cell state (for example, in the computer screen plot).

4. Results

Within a research project with a Ministry of Defence of Republic Slovenia, we selected five of the largest nature fires in last ten years from the Karst area of Slovenia which is one of the most fire threatened area in the Europe. All areas were digitized in the grid of 50x50m squares (granulation level). The knowledge base of fuzzy rules of fire spread were developed with local fireman experts for all five areas. It is represented with a set of 288 rules which covers the following items:

- the direction and the speed of wind,
- the absolute altitude of basic entities (squares), to which the area is granulated,
- the fire inflammability of the basic entitites (squares),
- the fuel supply of the basic entities and
- the weather conditions.

4.1. Simple example of FCA use

The basic theory of fire spread has been built in the middle of previous century by Fons [16],[17]. The experiments which have been done by him are still used today in a well known method of two semielipses. The only parameter which is used is the speed of the wind. Our first goal was to build an FCA model which will give us similar results as semielipses modelling does. We have used only three parameters in rule tables: fire state in a cell, inflammability of a cell and the speed of wind which is the same for all cells. The rules are built from the observed cell's point of view. We use annotation $\widehat{\mathbf{q}}_1$ for a vector of three fuzzy sets, which describes from left to right the states from zero fire in a cell ($\tilde{Q}_1$) to the maximum fire in the cell ($\tilde{Q}_3$). They could be interpeted with linguistical terms *Zero, Medium, Max.* With sign $\widehat{\mathbf{q}}_2$ we represent two possible states of inflammability of area.

$$\widehat{\mathbf{q}}=\widehat{\mathbf{q}}_1 \times \widehat{\mathbf{q}}_2,\ \widehat{\mathbf{q}}_1 = \left[\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3\right],\ \widehat{\mathbf{q}}_2 = \left[\tilde{Q}_4, \tilde{Q}_5\right]. \quad (14)$$

If we suppose the constant speed (*max*) and direction of wind and also the homogenous inflammability (max), we get rule tables (15 ... 18).

$c_{i,j}(t)$	$\rightarrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_3$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_3$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_3$

$c_{i,j}(t)$	$\searrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_3$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_3$

(15)

$c_{i,j}(t)$	$\nearrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_3$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_3$

$c_{i,j}(t)$	$\downarrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_3$

(16)

$c_{i,j}(t)$	$\uparrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_3$

$c_{i,j}(t)$	$\swarrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_3$

(17)

$c_{i,j}(t)$	$\leftarrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_2$

$c_{i,j}(t)$	$\nwarrow$	$c_{i,j}(t+1)$
$\tilde{Q}_1$	$\tilde{Q}_2$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_2$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_2$	$\tilde{Q}_3$
$\tilde{Q}_1$	$\tilde{Q}_3$	$\tilde{Q}_1$
$\tilde{Q}_2$	$\tilde{Q}_3$	$\tilde{Q}_2$
$\tilde{Q}_3$	$\tilde{Q}_3$	$\tilde{Q}_3$

(18)

All influences of fire transfer from neighbours is denominated with arrows. The only variables in rules are then the state of the cell and its neighbouring cell. If we run simulation for 50 simulation periods we get the result in Figure 1. The black point on the left shows us the ignition point and the scale on the right possible intensities of fire spread from *max* to *zero*. On Figure 2. we see a comparison between a real fire presented in [16] and the semielipses method and on Figure 3. we can see a comparison between FCA method and a real fire shape.

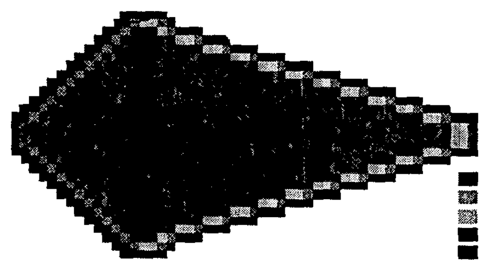

Figure 1. FCA fire spread simulation example.

From comparison figures we can get a conclusion that FCA approach of building models gives us relatively good results. If we also know that Fons method requires a lot of measurement on specific areas we can say that our method was also very cost efficiant. Building such a model in practice means only collecting some kind of approximate and uncertain knowledge from the field specialists.

4.2. Complex example of FCA use

Presented method has been also compared to real fires shapes. One of locations is presented in Figure 4. It predicts the fire covering with precision of approximately 75%. In this example, the relative altitudes between cells also used in rules.

5. Conclusion

In our paper we present generalized definition of fuzzy automaton and a new definition of fuzzy cellular automata.

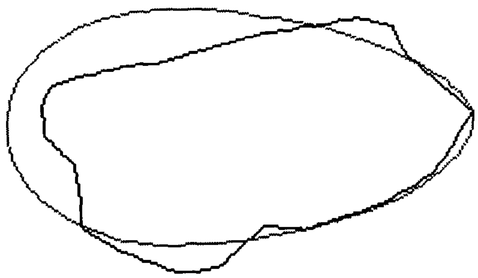

Figure 2. Comparison between semielipses shape and a nature fire shape.

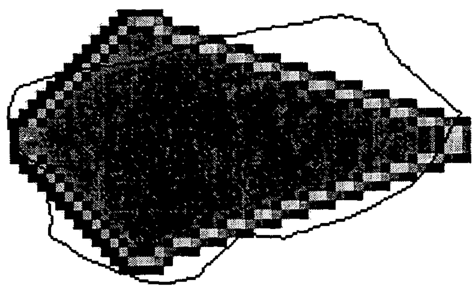

Figure 3. Comparison between FCA simulation spread and nature fire shape.

At the end we also show an example of FCA programming in the field of fire spread prediction in nature environment. The main concept of FCA enables programming to non-programming specialists from different fields of work, where local interaction and parallelism are present. By this we get a new approach for modelling real life systems.

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Figure 4. Fire spread model, Tolsti vrh, 1990.

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