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GA-ILP Method for Optimization of Water Distribution Networks

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Abstract Optimization of water distribution networks has been of central importance for recent decades. Genetic Algorithms (GA) are the most famous metaheuristics widely used for this purpose with great success. However, the fact that GA basically requires a large number of computations, has led to investigate for faster solvers. In this research, a new approach is proposed in which a simple GA is linked with the Integer-Linear Programming (ILP) method resulting in a hybrid optimization scheme. Using the mathematical method of ILP, the search space is significantly reduced thereby a limited number of evaluations are required to achieve a good solution. The approach is applied to two benchmark pipe-networks in order to show its ability in terms of accuracy and speed. The results are then compared with the previous works. The obtained results indicate that the proposed model is computationally efficient, like classic methods, while is still very promising in finding the global optimum like the nature-inspired metaheuristics.

Keywords Optimization · Water distribution networks · Genetic algorithm · Integer linear programming

1 Introduction

Optimal design of water distribution networks is to determine the pipe sizes and the energy facilities so that the total cost is minimized while the pipe flow velocity and nodal pressure constraints are satisfied. For looped networks, this problem may

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be very difficult to handle because, the pipe sizes are discrete in nature and the hydraulic constraints are nonlinear. In addition, the problem often includes multiple local optima that is also a major concern with the optimization of pipe-networks. Dealing with these issues, makes the problem challenging and developing. Being computationally efficient, able in finding the global design and capable in solving large problems are the marked aims in developing new models. In general, the optimization methods applied to design of pipe-networks are classified as mathematical and natural-inspired evolutionary methods where the last classification has been recently more focused. In the following, a brief review on the previous researches is presented:

The problem was first investigated using linear programming (LP) (Dantzig 1963; Gupta 1969; Gupta and Hassan 1972; Alperovits and Shamir 1977; Quindry et al. 1981; Kessler and Shamir 1989; and Bhave and Sonak 1992). Other researchers then used nonlinear programming (NLP) to solve the problem (Watanatada 1973; Morgan and Goulter 1985; Chiplunkar et al. 1986; Walski 1987; Lansey and Mays 1989; Taher and Labadie 1996; and Samani and Naeeni 1996). Samani and Mottaghi (2006) also used the Integer Linear Programming (ILP) for obtaining the optimum pipe sizes and reservoir elevations in water supply networks. They linearized the loop equations and utilized the benefits of the ILP as a global fast solver. Samani and Zangeneh (2010) then developed the previous ILP to design more complex systems including reservoir and pump optimization.

All the cited investigations go under the title of classic or mathematical methods, which generally require some simplifications to overcome the discreteness and nonlinearity of the problem. The mathematical methods are computationally very efficient with fast convergence but oftentimes result in local optimums. Furthermore, mathematically planning and implementing such methods, including the network equations and constraints are generally complex.

In recent years, metaheuristics mostly inspired by nature have been highly developed and widely applied to complicated engineering problems. As the most popular method in this field, Genetic Algorithms (GA) are developed based on the principles of the natural-random selection and evolution of a crowd. This method has been also applied to optimization of pipe-networks (Simpson et al. 1994; Savic and Walters 1997; Vairavamoorthy and Ali 2000, 2005; Wu and Walski 2005; Reca et al. 2008; Kadu et al. 2008). It is concluded from these works that the random-evolutionary search mechanism in GA provides the potentiality of global optimization. Other evolutionary methods were also used by other researchers. Cunha and Sousa (1999) employed the Simulated Annealing (SA) approach for optimization of water distribution networks. Maier et al. (2003) employed the Ant Colony (AC) optimization method, Eusuff and Lansey (2003) utilized the Shuffled Frog Leaping (SFL) algorithm and Liong and Atiquzzaman (2004) used the Shuffled Complex Evolution (SCE) method. Among them, GAs proved to be more successful in locating global optimum or near-global optimum solutions (Dandy et al. 1996; Simpson et al. 1994; Savic and Walters 1997; Prasad and Park 2004). However, due to the relatively large number of simulations required in GAs, many researchers tried to reduce the computational time by reducing the search space (Lippai et al. 1999; Vairavamoorthy and Ali 2000). Recently, Kadu et al. (2008) proposed a modified GA based on critical path method (Bhave 1978) to reduce the search space. In that method, the looped network is converted to a tree one thereby primary pipe diameters are obtained, on which the initial population is generated based. This approach has been considerably able in reducing the search space and consequently, the time of computations.

Later on, developing hybrid optimization models was also taken into account in which a mathematical method is coupled with a metaheuristic. Using the hybrid schemes, the problem computationally becomes more efficient while the accompanying metaheuristic covers the mathematical method's drawbacks. On the other hand, the hybridization provides a fast solver that can also be successful in finding the global solution. Cisty (2010) combined a standard linear programming (LP) method and genetic algorithm as a hybrid model. Through that work, GA decomposes the looped network into a group of branched ones. Afterward, LP optimizes each branched system with known flow distributions. Solving several examples, it was concluded that their results are more stable in terms of closeness to the global optimum design.

These, researches encourage the authors to investigate for more combinational models. Improving the solution efficiency and speeding up the optimization by reducing the number of useless evaluations are still the targets in developing new evolutionary methods. This research intends to deal with this issue introducing a hybrid genetic algorithm.

Here, a simple GA and the Integer-Linear Programming (ILP) are combined as a GA-ILP model. The proposed scheme has the merits of the GA in which the tendency to become entrapped in local minima is reduced and the advantage of the ILP which is computationally very fast.

2 Integer-Linear Programming (ILP)

Integer-linear programming (ILP) belongs to a class of mathematical optimization able in solving the problems with linear objective function, linear equality and inequality constraints and also with integer decision variables. If the variables include only 0-1 values, ILP may also be called as binary linear programming (BLP). Samani and Mottaghi (2006) introduced an ILP approach to optimize looped networks. In that work, the nonlinearity of the objective function and constraints due to the loop equations were skillfully linearized. In the proposed ILP, zero-unity variables were adopted in the optimization analysis to convert the nonlinear system to integer linear one. In the following, the components of the standard ILP applied to optimization of looped networks are described step by step:

2.1 Objective Function

The total construction cost of a municipal water distribution network is mostly related to the cost of pipes' preparation and installation resulting in the following objective function in the ILP:

$$F(D_N) = \sum_{J=1}^{NPA} \sum_{N=1}^{NP} L_{NJ} \, CP_{NJ}(D_N) X_{NJ} \tag{1}$$

where NP = number of pipes in the network, NPA = number of commercially available pipe sizes, L_{NJ} = length of pipe number N in the network, CP_{NJ} = unit length cost of pipe whose length is L_{NJ} which is a function of pipe diameter D_N and X_{NJ} = the zero-unity variable related to the pipe diameters.

2.2 One Size Constraint

Equation 1 includes all the commercially available pipe sizes for every pipe in the network. The number of the terms of the pipe costs is therefore equal to the number of pipes of the network multiplied by the number of commercially available pipe sizes. The final solution will include only one size for every branch in the network. In other words, no more than one of the variables X_{NJ} for a given N can be equal to unity. Therefore, the following constraint is considered for the zero-unity variables for pipe N:

$$\sum_{J=1}^{NPA} X_{NJ} = 1 \tag{2}$$

2.3 Pressure Constraint

In order to define pressure constraints, a reference node should be selected first. The reference node may be one of the nodes representing the location of a reservoir in the network. On this basis, a specific path is considered for each node that eventually connects it to the reference point. The pipelines in these paths are called as involved pipes herein. Developing the energy equation between the reference point and node i using zero-unity variables leads to the following constraint:

$$\frac{P_{\min}}{\gamma} \leq H_R - \Delta Z_{R-i} - \sum_{J=1}^{NPA} \sum_{I=NHR}^{L_{R-i}} h_{fI} X_{IJ} \leq \frac{P_{\max}}{\gamma} \tag{3}$$

where $P_{\max}$ and $P_{\min}$ = respectively, the upper and lower pressure limit, R indicates the reference point, H_R = head at the reference point, ΔZ_{R-i} = elevation difference of the reference point and node i, NHR = number of the pipes being in the path connecting node i to the reference point, and L_{R-i} = set of all pipes in the path connecting R to i, and $\sum_{I=NHR}^{L_{R-i}} h_{fI}$ represents the summation of headlosses of the path starts from the reference point and ends at node i. When Eq. 3 was developed for all nodes, one pipe of each loop in the network is inevitably missed in the paths. These pipes are called as ignored pipes in this research.

2.4 Velocity Constraint

If the pipe flow velocity is also important in optimization, its constraint can be introduced by applying the continuity as:

$$V_{\min} \leq \sum_{J=1}^{NPA} \frac{Q_N}{\frac{\pi}{4} D_{NJ}^2} X_{NJ} \leq V_{\max} \tag{4}$$

where Q_N = flow rate of pipe N, $V_{\min}$ and $V_{\max}$ = minimum and maximum allowable flow velocities in the pipes, respectively.

2.5 Loop Constraints

Two conservative rules of continuity and energy govern the flow distribution in the pipe-networks. Flow continuity should be satisfied at each node i.e.,

$$\sum Q_{in} - \sum Q_{out} = Q_D \tag{5}$$

where Q_{in} and Q_{out} = respectively, the flow discharges to and out of the node and Q_D = the demand or supply flow at the node. The principle of energy conservation should be also satisfied for each loop i.e.,

$$\sum h_f - \sum E_p = 0 \tag{6}$$

in which E_p = the energy added to the system by a pump and h_f = head loss, computed for pipes of the loop using Darcy–Weisbach or Hazen-Williams equation that is:

$$h_f = \frac{\omega L Q^{\alpha}}{C^{\alpha} D^{\beta}} \tag{7}$$

where C = Hazen-Williams coefficient, ω = numerical constant depending on the problem's units and α and β = Hazen-Williams exponents.

Developing Eqs. 6 and 7 for a looped network, results in a nonlinear system of equations. In order to determine the flow distribution in the network, these equations must be numerically solved. In fact, this is the central difficulty in optimizing the looped networks that makes the linear programming methods hard and limited to be applied. As a remedy, a hydraulic simulation program like EPANET is used in which the continuity and energy constraints are automatically satisfied. In this study, the hydraulic simulation is not directly employed in the optimization analysis. To solve the ILP here, the pipe flow discharges in the network should be initially known. For this purpose, an iterative procedure was originally proposed by Samani and Mottaghi (2006). The procedure is more discussed here as follows:

1. Define an arbitrary path for each node to join it with the reference point (the reservoir). For this purpose, one pipe of each loop is put aside in such a manner that the looped network becomes a quasi-benched one. Although, choosing the ignored pipes is quite optional but it would be better to select those, likely including the minimum discharge in their loops. Since these pipes are not involved in the ILP's constraints they are not optimized. However, to minimize the cost objective function, ILP reasonably assigns them to be the smallest commercial size from the commercial list. The probable low-discharge pipes can be initially identified using the critical path method (Kadu et al. 2008) or simply, by assigning a diameter size to all pipes and running the hydraulic simulator, EPANET here, once. This results in a flow distribution in the network only according to the pipes' lengths and friction coefficients.

2. Assume pipe diameters using the commercially available list. This is, in fact, an initial guess to start solving the ILP. Samani and Mottaghi (2006) showed that the proposed iterative procedure is very efficient in terms of convergence. It was concluded that the initial values do not play a significant role in the time of optimization. Indeed, the initial pipe diameters are introduced to EPANET, only to provide an initial flow distribution. As a general assumption here, all pipes are initially given a same diameter size.
3. Substitute the flow discharges determined through step 2 in pressure and velocity constraints, Eqs. 3 and 4.
4. Using MATLAB, the branch and bound method is called to minimize the ILP's objective function (1), subject to the constraints (2), (3) and (4). As a result, integer variables X_{NJ} and the corresponding pipe sizes are determined. Nevertheless, the solution is not done since the pipe discharges may not be still final; so
5. Compare the resultant pipe sizes with the assumed values. If they are identical, the problem has been solved, otherwise, use the resulting pipes as the updated variables and repeat the procedure starting from step 2. This algorithm is followed until the stopping criterion is satisfied.

After a few numbers of iterations, the optimum design is eventually obtained. However, all the ignored pipes necessarily include the smallest commercial size in it. This is the main deficiency of the current ILP that does not lead to the global optimum. In fact, dealing with the non-linearity and discreteness of the looped equations impose some limitations on the ILP. Genetic algorithm is the most common evolutionary approach that has been frequently used in many researches. GA is found to be very easy to implement and capable of solving multi-modal problems (Goldberg 1989; Haupt and Haupt 2004). Because of the random-natural mechanism of evolution, GA converges much slower than mathematical methods. This becomes more serious and time consuming in large scale problems with many decision variables. Regarding the relative merits and deficiencies of the ILP and GA, it seems that these two could cover each other. Therfore, the hybrid schemes are taken into account in order to develop an effiecnt slover that would be fast as well as able in reaching to the global optimum designs.

3 Coupled GA-ILP Optimization Model

In the ILP, described above, the looped network should be transformed to a quasi-branched one considering arbitrary paths in which one pipe is set aside from each loop. While *NL* and *NP* are respectively the number of loops and pipes in the network, there are therefore, *NL* ignored pipes and *NP*-*NL* involved pipes in the ILP process.

As already discussed, althogh *NL* ignored pipes are included in the cost objective function, but they are not in the pressure constraints. Consequently, the ignored pipes will be assigned to be the smallest commercial size by the ILP. As a remedy for this limitation, an external optimization procedure is applied here in which the decision variables are only the *NL* pipes, those are not properly optimized in the ILP. For this purpose, a simple binary genetic algorithm is applied that only optimizes the ignored pipes and dictates them to the ILP which the inner solver. The proposed solution algorithm is:

1. GA is started by setting its parameters such as population size, mutation and crossover ratios, method of selection and paring and also stopping criterion. Each set of decision variables named as chromosome consists of *NL* diameter size of the ignored pipes.
2. An initial population of chromosomes is randomly generated from the list of commercial pipe sizes. For each chromosome, the ILP is then called in which the ignored pipe diameters (from GA) are kept to be unchanged during the ILP iterative procedure. Afterward, the optimum diameters for the involved pipes are also deterimined and returned to the GA along with their corresponding cost.
3. Based on the minimized cost values from the ILP, good chromosomes take part in producing new generations. Using the method of weighted random pairing (Haupt and Haupt 2004), parents are selected and children are then generated by the uniform crossover operator. Afterward, a few genes are mutated. The new generation is finally produced.
4. The desirable convergnce is checked. If it is satisfactory the calculations are stopped otherwise, the procedure with the new generation, goes to step 2.

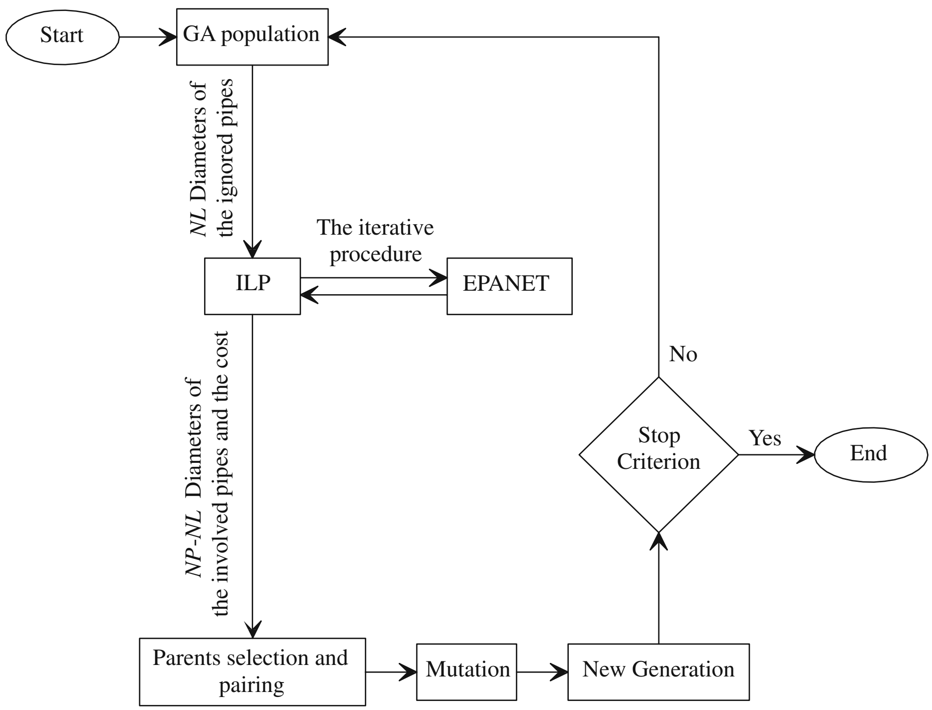

Fig. 1 GA-ILP algorithm

The algorithm is continued until gaining (on) the global optimum design. Figure 1 demonstrates the described procedure in which all the model's components are schematically shown in connection with each other.

4 Demonstrative Examples

Now, the proposed model is more investigated by running it against two benchmark examples. Hanoi pipe-network, first presented by Fujiwara and Khang (1990), is selected as Example 1 and a relatively large network, defined by Kadu et al. (2008), is considered as Example 2. The merits of the GA-ILP scheme are then discussed in comparison with the methods previously applied to the examples.

4.1 Example 1

A well-known piping system, Hanoi network (Vietnam) depicted in Fig. 2 is considered to be optimized using the proposed model. This network has been already solved by many researchers and is therefore utilized herein as a comparative benchmark. The network consists of 32 nodes, 34 pipes and 3 loops. A reservoir at node 1 with 100 m Hydraulic Grade Line (HGL) feeds the system. Nodes 2–32 are demand nodes with minimum HGL requirement of 30 m. The nodal demands are shown in the figure and also in Table 1 as well as the pipes' specifications. For all the pipes, Hazen–Williams coefficient is 130. Furthermore, the parameters of Hazen–Williams equation, α, β and ω are 1.85, 4.87 and 10.9031, respectively.

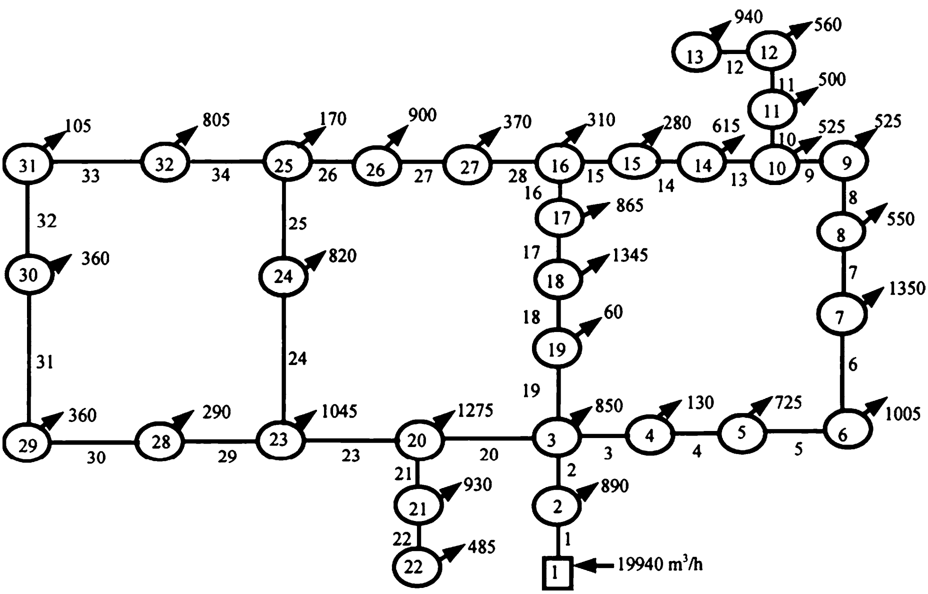

Fig. 2 Hanoi water distribution network

Table 1 Pipe and node details of Hanoi network

Pipe details		Node details	
Pipe	Length (m)	Node	Demand (m^3/h)
1	100	1	−19,940
2	1,350	2	890
3	900	3	850
4	1,150	4	130
5	1,450	5	725
6	450	6	1,005
7	850	7	1,350
8	850	8	550
9	800	9	525
10	950	10	525
11	1,200	11	500
12	3,500	12	560
13	800	13	940
14	500	14	615
15	550	15	280
16	2,730	16	310
17	1,750	17	865
18	800	18	1,345
19	400	19	60
20	2,200	20	1,275
21	1,500	21	930
22	500	22	485
23	2,650	23	1,045
24	1,230	24	820
25	1,300	25	170
26	850	26	900
27	300	27	370
28	750	28	290
29	1,500	29	360
30	2,000	30	360
31	1,600	31	105
32	150	32	805
33	860		
34	950		

In this problem, there are six available pipe diameter sizes in the commercial list those are given in Table 2 with their costs per unit length. There are therefore, 6^{34} different designs required to be evaluated if optimization methods are not utilized.

Table 2 Available pipe diameters and unit costs for Hanoi network

Diameter (in)	Unit cost (Dollars)
12	45.726
16	70.400
20	98.387
24	129.333
30	180.748
40	278.280

In order to set up the pressure constraint equations for the ILP, the paths are so selected that only pipes 15, 27 and 33 are not involved (ignored pipes). In fact, one pipe of each loop is not included in the ILP constraints. As earlier discussed, choosing the ignored pipes and consequently the paths is totally arbitrary. However, considering less important pipes (with low discharges) is in more conformity with the ILP's concept and makes it more efficient in the coupled GA-ILP model. Then, only three pipes of 15, 27 and 33 are to be optimized by GA and dictated to the ILP which optimizes other 31 pipes.

GA starts with 20 initial chromosomes, 12 population size, uniform crossover operator, and 0.01 to 0.05 linearly variable mutation ratio. Trend of cost function minimization through the GA-ILP model is shown in Fig. 3. Table 3 also shows the recently optimized pipe diameters, in comparison with the previous works. The nodal pressures in the optimum design are also reported in Table 4.

The minimum cost value is finally optimized to \$6.19 millions with only 1,320 hydraulic evaluations (calling the hydraulic solver) after 20 generations. As seen in Fig. 3, the optimum solution has been earlier achieved, around generation 10.

With the same Hazen–Williams parameters (α, β and ω), Savic and Walters (1997) obtained the minimum cost value of \$6.195 millions with 1,000,000 evaluations using a common GA. With the same α, β but $\omega = 10.5088$ Cunha and Sousa (1999) using Simulated Annealing (SA) and Geem et al. (2002) using Harmony Search (HS) obtained the minimum cost value of \$6.056 millions with 53,000 and 200,000 evaluations, respectively. With $\omega = 10.667$ Liong and Atiquzzaman (2004) obtained \$6.22 millions with 25,402 evaluations using Shuffled Complex Evolution (SCE). An interesting work was also guided by Kadu et al. (2008) in which the search space for GA is efficiently reduced by the concept of the critical path method. In that research, the operation of the modified GA was significantly improved, thereby Hanoi network was optimized to the minimum cost of \$6.19 millions with 18,000 evaluations for $\omega = 10.9031$.

It should be also mentioned that, since the applied computers in the cited works are not identical, the number of evaluations was considered here instead of the run times.

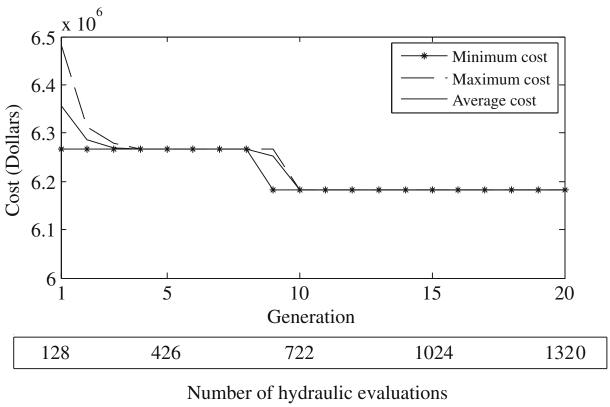

Fig. 3 Cost optimization of Hanoi network by GA-ILP model

Table 3 Optimum solutions by various methods for example 1

Pipe	Savic and Walters (1997) GA	Cunha and Sousa (1999) SA	Geem et al. (2002) HS	Liong and Atiquzzaman (2004) SCE	Kadu et al. (2008) Modified GA	Proposed Model GA-ILP
1	40	40	40	40	40	40
2	40	40	40	40	40	40
3	40	40	40	40	40	40
4	40	40	40	40	40	40
5	40	40	40	40	40	40
6	40	40	40	40	40	40
7	40	40	40	40	40	40
8	40	40	40	30	40	40
9	40	40	40	30	30	30
10	30	30	30	30	30	30
11	24	24	24	30	30	30
12	24	24	24	24	24	24
13	20	20	20	16	16	16
14	16	16	16	12	12	12
15	12	12	12	12	12	12
16	12	12	12	24	16	16
17	16	16	16	30	20	20
18	20	20	20	30	24	24
19	20	20	20	30	24	24
20	40	40	40	40	40	40
21	20	20	20	20	20	20
22	12	12	12	12	12	12
23	40	40	40	30	40	40
24	30	30	30	30	30	30
25	30	30	30	24	30	30
26	20	20	20	12	20	20
27	12	12	12	20	12	12
28	12	12	12	24	12	12
29	16	16	16	16	16	16
30	16	12	12	16	12	12
31	12	12	12	12	12	12
32	12	16	16	16	16	16
33	16	16	16	20	20	20
34	20	24	24	24	24	24
Cost ($ millions)	6.195	6.056	6.056	6.220	6.190	6.190
No. of evaluations	1,000,000	53,000	200,000	25,402	18,000	1,320
ω	10.9031	10.5088	10.5088	10.667	10.9031	10.9031

As concluded from the above discussion the GA-ILP model, requires much less evaluations than the previous evolutionary methods. For instance, the total number of evaluations for Hanoi network here is only 7.3% of the best pervious work done by Kadu et al. (2008).

Table 4 Nodal pressures of Hanoi network

Node	Pressure (m)	Node	Pressure (m)
1	100.00	17	39.28
2	97.08	18	50.04
3	60.82	19	57.13
4	56.38	20	49.59
5	50.88	21	40.04
6	45.13	22	34.76
7	43.81	23	43.42
8	42.28	24	37.73
9	41.09	25	34.07
10	37.61	26	30.51
11	36.01	27	30.32
12	34.83	28	38.05
13	30.53	29	30.08
14	32.06	30	30.58
15	30.96	31	30.90
16	31.13	32	31.81

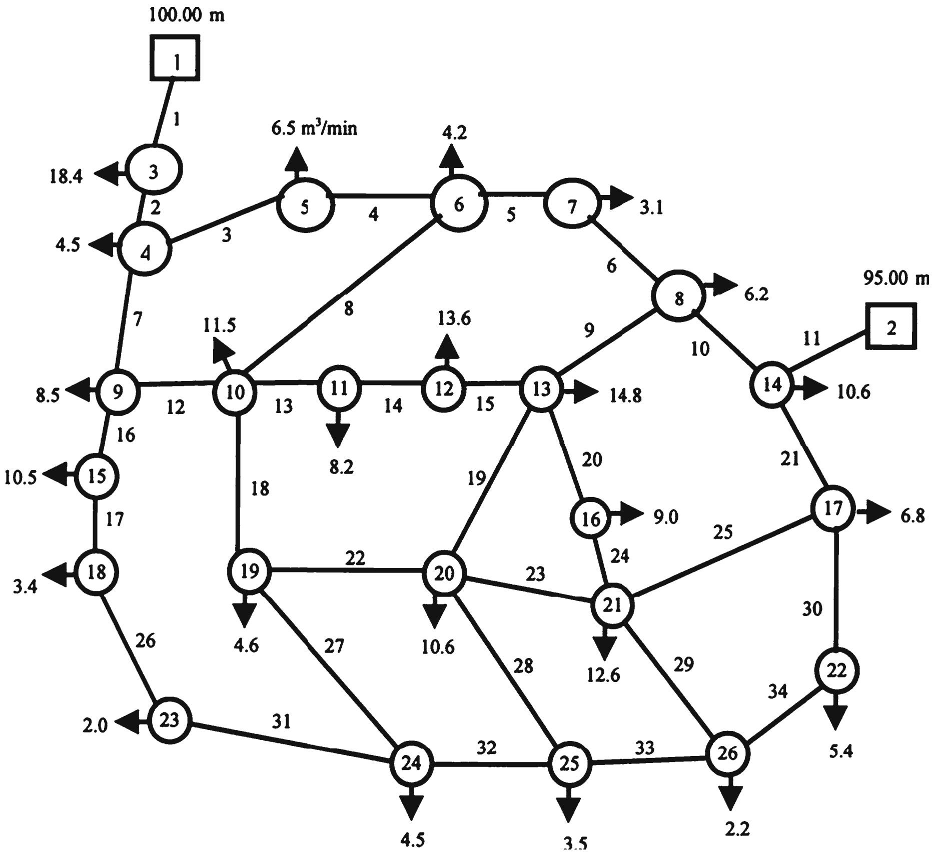

Fig. 4 Water distribution network of example 2 (Kadu et al. 2008)

4.2 Example 2

A water distribution network with two reservoirs, 26 nodes, 34 pipes and 9 loops (Fig. 4) is considered here which was first defined and optimized by Kadu et al. (2008). Two reservoirs with 100 and 95 m HGL supply the network from nodes 1 and 2 respectively. Nodes and pipes numbers are labeled in the figure as well as nodal demands in m^3/min. Other information of the network such as pipe lengths, and minimum HGL required for each node are also given in Table 5. The Hazen–Williams coefficient for all pipes is 130 and α, β and ω are 1.85, 4.87 and 10.68, respectively. 14 commercial pipe sizes are available to be used in this problem those are listed in Table 6 with their costs per unit length. On this basis, there are therefore

Table 5 Pipe and node details of example 2

Pipe details		Node details		
Pipe	Length (m)	Node	Minimum HGL (m)	Demand (m^3/min)
1	300	1	100	–
2	820	2	95	–
3	940	3	85	18.4
4	730	4	85	4.5
5	1,620	5	85	6.5
6	600	6	85	4.2
7	800	7	82	3.1
8	1,400	8	82	6.2
9	1,175	9	85	8.5
10	750	10	85	11.5
11	210	11	85	8.2
12	700	12	85	13.6
13	310	13	82	14.8
14	500	14	82	10.6
15	1,960	15	85	10.5
16	900	16	82	9.0
17	850	17	82	6.8
18	650	18	85	3.4
19	760	19	82	4.6
20	1,100	20	82	10.6
21	660	21	82	12.6
22	1,170	22	80	5.4
23	980	23	82	2.0
24	670	24	80	4.5
25	1,080	25	80	3.5
26	750	26	80	2.2
27	900			
28	650			
29	1,540			
30	730			
31	1,170			
32	1,650			
33	1,320			
34	3,250			

Table 6 Available pipe diameters and unit costs for example 2

Diameter (in)	Unit cost (rupees)
150	1,115
200	1,600
250	2,154
300	2,780
350	3,475
400	4,255
450	5,172
500	6,092
600	8,189
700	10,670
750	11,874
800	13,261
900	16,151
1,000	19,395

14^{34} different possible designs for this example that should be evaluated in absence of optimization methods.

In order to optimize the network using the proposed scheme, firstly the ILP is set up defining the flow paths. For this purpose 9 pipes of 8, 9, 20, 34, 33, 32, 31, 22 and 23 are ignored from the 9 loops those are optimized by the GA. Consequently, remaining 25 pipes are then optimized by the ILP.

The optimization is started with 40 initial chromosomes, 20 population size, uniform crossover operator, and constant 0.02 mutation ratio. Trend of cost function minimization through the model is shown in Fig. 5. As seen there, after only 35 generations corresponding to 4,440 evaluations the minimum cost of 131,312,815 (rupees) is obtained. The optimum pipe diameters and nodal pressures are also shown in Tables 7 and 8, respectively. This example was before optimized by Kadu et al. (2008) using a conventional GA that resulted in the minimum cost function of 131,678,935 (366,120 units costlier). This result was achieved after 2,000 generations, corresponding to about 120,000 evaluations. Adding to this, those authors applied their modification method to improve the GA performance and could reduce the

Fig. 5 Cost optimization of example 2 by GA-ILP model

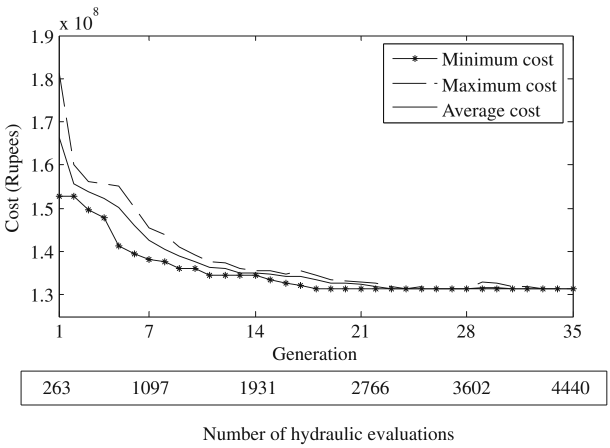

Table 7 Optimum solutions by various methods for example 2

Pipe	Kadu et al. (2008) GA	Present work GA-ILP
1	1,000	1,000
2	900	900
3	400	400
4	350	350
5	150	150
6	250	250
7	800	800
8	150	150
9	400	400
10	500	500
11	1,000	1,000
12	700	700
13	800	800
14	400	400
15	150	150
16	500	500
17	350	350
18	350	350
19	150	150
20	200	150
21	700	700
22	150	150
23	400	450
24	400	400
25	700	700
26	250	250
27	250	250
28	200	200
29	300	300
30	300	300
31	200	200
32	150	150
33	250	200
34	150	150
Cost (rupees)	131,678,935	131,312,815

number of generations to 600 with 36,000 evaluations. However, it is seen that the GA-ILP model has been able to solve this relatively large network with about 3.7% and 12.33% of evaluations of the conventional and modified GA.

5 Summary and Conclusions

Optimal design of water distribution networks is a remarkable goal in hydraulic engineering, which has been focused by many researchers for many years.

Being nonlinear, nonconvex and discrete in nature make this problem difficult to solve. In general, two major aims are followed in optimizing water pipe-networks, (1) obtaining the global solution by (2) developing a computationally efficient

Table 8 Nodal pressures of example 2

Node	Pressure (m)
1	100.00
2	95.00
3	98.97
4	95.70
5	90.96
6	89.54
7	87.87
8	90.06
9	91.86
10	89.18
11	88.99
12	85.17
13	82.21
14	94.50
15	88.58
16	84.68
17	90.94
18	85.65
19	85.29
20	82.29
21	87.49
22	86.58
23	82.32
24	80.21
25	80.20
26	83.06

procedure. Mathematical methods are pioneer in this field but generally result in local solutions for looped networks. These methods like Integer Linear Programming (ILP) are very fast to be implemented but include some limitations to handle the problem's constraints.

Besides, the evolutionary methods inspired by nature have been recently more developed. Genetic Algorithm (GA) as the most popular evolutionary method has been frequently applied to optimization of water distribution networks. It has exhibited such a great success in achieving the global optimum designs. However, GA often needs too many evaluations to solve a problem and is therefore computationally expensive, particularly for large problems.

Regarding the merits and deficits of the mentioned categories, a hybrid optimization model was developed here for optimization of looped network with single demand loading. Through this model, the mathematical method of ILP is linked with a simple GA. To implement the method, particular flow paths are arbitrary considered from a known energy node (e.g., reservoir) to all nodes in the system. As the major part of the system, pipes in the paths are called involved pipes. Developing the energy equation through the paths, results in the ILP's pressure constraints in which one pipe of each loop is not inevitably included. These pipes are called ignored pipes. Indeed, a looped network in the ILP formulation becomes a quasi-branched one. Although, the ignored pipes are in the cost objective function but are not in the ILP constraints. Consequently, while the involved pipes are optimized by the

ILP, the ignored pipes are logically assigned to be the smallest commercial size. This issue makes the ILP dependent on the chosen paths and very likely leads the computations to a local optimum. To solve this problem, the method of GA-ILP was introduced here in which GA and ILP respectively, act as the external and internal solvers. GA determines the diameter of the ignored pipes and dictates them to the ILP. All the other pipes are then deterministically optimized in the ILP using the method of branch and bound.

ILP then returns each chromosome's near-optimum design and its cost to the GA. The evolution process in the GA is continued based on the results obtained from the ILP until the desirable convergence is attained.

As a result, the ILP which optimizes the major part of the network (1) prevents blind and time consuming searches in the GA and (2) promotes each chromosome to a near-optimum design. These resulted in a very fast and efficient optimization as also concluded from the solved examples. Hanoi network with 34 pipes and 3 loops was optimized in this work with only 1,320 evaluations corresponding to 7.3% of the best previous investigation. A relatively large network with 34 pipes and 9 loops was also successfully optimized herein with 4,440 evaluations corresponding to 3.7% of the conventional GA (Kadu et al. 2008) and 366,120 units cheaper.

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