Thermodynamic quantities: Condensate Ansatz

Ideal Gas limits

In[1]:= Nc = 2; Nf = 2;

$$\text{Pideal} = \frac{\pi^2}{45} T^4 \left(\text{Nc}^2 - 1\right) + \frac{7\,\pi^2}{180} T^4\, \text{Nc}\, \text{Nf};$$

$$\text{Eideal} = \frac{\pi^2}{15} T^4 \left(\text{Nc}^2 - 1\right) + \frac{7\,\pi^2}{60} T^4\, \text{Nc}\, \text{Nf};$$

$$\text{Sideal} = 4\, \frac{\pi^2}{45} T^3 \left(\text{Nc}^2 - 1 + \frac{7}{4}\, \text{Nc}\, \text{Nf}\right);$$

In[5]:= Pideal

Out[5]= $\frac{2\,\pi^2\, T^4}{9}$

In[ ]:= Eideal

Out[ ]= $\frac{19\,\pi^2\, T^4}{12}$

In[ ]:= Sideal

Out[ ]= $\frac{19\,\pi^2\, T^3}{9}$

In[ ]:= $\dfrac{91\,200}{\text{Exp}\left[\frac{6\,\pi}{23\,(0.1184)}\right]}$ // N

Out[ ]= 89.9244

Normalised Plots

```
(*Press=pressure,Sden=entropy density, Eden =energy density*)

Λms = 200;
```

In[131]:=
```
Clear[c1, c2, Λ, β0, T, Λms]
Λ = T; Nc = 2; Nf = 2; β0 = 1/(4 π)^2 (11/3 Nc - 2/3 Nf);
α[T_] = 2 Log[Λ/Λms];
αs[T_] = 1/(4 π β0 α[T]) (1); (*For only loop *)
Press[c1_, c2_, T_, Λms_] =
  -(-1/45 (-1 + Nc^2) π^2 (T)^4 - 7/180 Nc Nf π^2 (T)^4 - 1/(48 π^2) c1^3 (-Nc + Nf)
      (2 c1 - ⅈ T JacobiCN[c1 c2, -1] JacobiDN[c1 c2, -1] JacobiSN[c1 c2, -1] + ⅈ T
        JacobiCN[c1 (c2 - ⅈ/T), -1] JacobiDN[c1 (c2 - ⅈ/T), -1] JacobiSN[c1 (c2 - ⅈ/T), -1]) -
     2 c1^3 (c1 - 2 ⅈ T JacobiCN[c1 c2, -1] JacobiDN[c1 c2, -1] JacobiSN[c1 c2, -1] + 2 ⅈ
        T JacobiCN[c1 (c2 - ⅈ/T), -1] JacobiDN[c1 (c2 - ⅈ/T), -1] JacobiSN[c1 (c2 - ⅈ/T), -1])
      (-(1 / (4 (4 π) αs[T])) - (Nf Log[4]) / (48 π^2) + 1/2 β0 Log[Λ/(4 π T)]));
Thdpot[c1_, c2_, T_, Λms_] =
  (-1/45 (-1 + Nc^2) π^2 (T)^4 - 7/180 Nc Nf π^2 (T)^4 - 1/(48 π^2) c1^3 (-Nc + Nf)
      (2 c1 - ⅈ T JacobiCN[c1 c2, -1] JacobiDN[c1 c2, -1] JacobiSN[c1 c2, -1] + ⅈ T
        JacobiCN[c1 (c2 - ⅈ/T), -1] JacobiDN[c1 (c2 - ⅈ/T), -1] JacobiSN[c1 (c2 - ⅈ/T), -1]) -
     2 c1^3 (c1 - 2 ⅈ T JacobiCN[c1 c2, -1] JacobiDN[c1 c2, -1] JacobiSN[c1 c2, -1] + 2 ⅈ
        T JacobiCN[c1 (c2 - ⅈ/T), -1] JacobiDN[c1 (c2 - ⅈ/T), -1] JacobiSN[c1 (c2 - ⅈ/T), -1])
      (-(1 / (4 (4 π) αs[T])) - (Nf Log[4]) / (48 π^2) + 1/2 β0 Log[Λ/(4 π T)]));
```

In[86]:=
```
Sden[c1_, c2_, T_, Λms_] = -D[Thdpot[c1, c2, T, Λms], T];
```

In[87]:=
```
Enden[c1_, c2_, T_, Λms_] = -Press[c1, c2, T, Λms] + T Sden[c1, c2, T, Λms];
```

In[88]:=
```
Press1[c1_, c2_, T_, Λms_] = Press[c1, c2, T, Λms] / Pideal;
Thdpot1[c1_, c2_, T_, Λms_] = Thdpot[c1, c2, T, Λms] / Pideal;
Sden1[c1_, c2_, T_, Λms_] = Sden[c1, c2, T, Λms] / Sideal;
Enden1[c1_, c2_, T_, Λms_] = Enden[c1, c2, T, Λms] / Eideal;
```

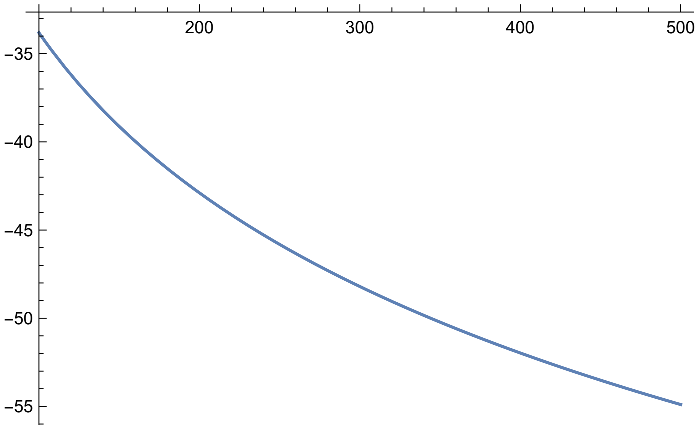
Case A: Condensate is not in equilibrium with quantum fluctuations

Case B: Condensate is in equilibrium with quantum fluctuations

All thermodynamics are only a function of Temperature.

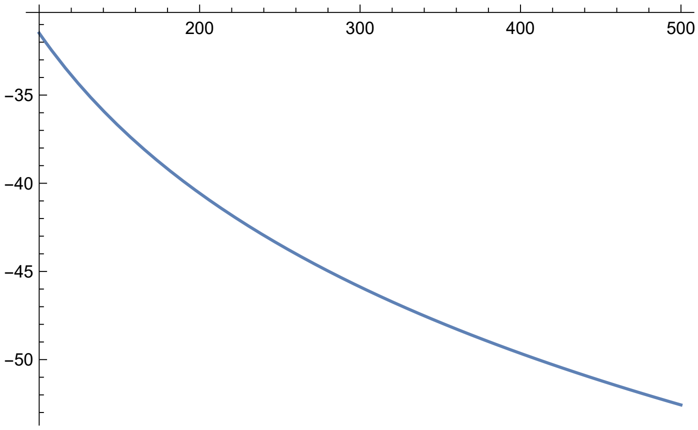
```
(* c1 → 4 I EllipticK[-1] T *)

In[136]:= Sden2[T_, c2_, Λms_] = -D[Thdpot[4 I EllipticK[-1] T , c2, T, Λms], T];

In[137]:= Sden22[T_, c2_, Λms_] = Sden2[T, c2, Λms] / Sideal;

In[147]:= Enden2[T_, c2_, Λms_] = -Press[4 I EllipticK[-1] T, c2, T, Λms] + T Sden2[T, c2, Λms];

In[148]:= Enden22[T_, c2_, Λms_] = Enden2[T, c2, Λms] / Eideal;

(*Normalised pressure*)

In[174]:= Plot[Press1[4 I EllipticK[-1] T, 0, T, 120], {T, 100, 500}]
```

Out[174]=




```
(*Normalised energy density*)

In[151]:= Plot[Enden22[T, 0, 200], {T, 100, 500}]
```

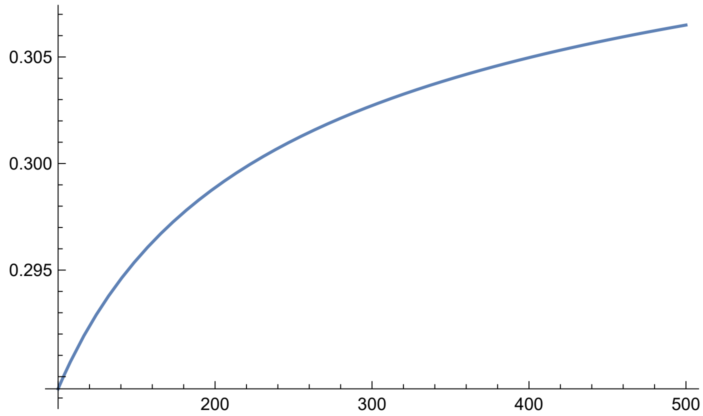
Out[151]=




```
(*w=Pressure/energydensity*)
```

In[126]:=
```
Plot[Press[4 I EllipticK[-1] T, 0, T, 176] / Enden2[T, 0, 176], {T, 100, 500}]
```

Out[126]=




```
(*ideal gas: w=pressure/energydensity*)
```

In[175]:=
```
Pideal / Eideal
```

Out[175]= $\frac{1}{3}$

Thermodynamic quantities: Condensate Ansatz + GW

Case A: Not in equilibrium

Case B: Everything is in equilibrium

In[152]:=
```
Clear[c1, c2, Ap, ωg, t, u]
```

(*c1 = 4iK(-1) T, ω_g = 2πiT *)

In[153]:=
```
u1[t_, T_] = (4 I EllipticK[-1] T) JacobiSN[(4 I EllipticK[-1] T) (-I t + c2), -1]
```

Out[153]= 4 ⅈ T EllipticK[-1] JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]

In[154]:=
```
hp1[t_, T_] = Ap Cos[2 π I T (-I t)]
```

Out[154]= Ap Cos[2 π t T]

(*F^2 term*)

```
In[156]:= 6 D[u1[t, T], t]^2 + 6 u1[t, T]^4 + (1 / 8) hp1[t, T]^4 u[t, T]^4 + hp1[t, T]^2 D[u1[t, T], t]^2 +
  2 hp1[t, T] × u1[t, T] × D[u1[t, T], t] × D[hp1[t, T], t] + u1[t, T]^2 D[hp1[t, T], t]^2

Out[156]= -1536 T^4 EllipticK[-1]^4 JacobiCN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2
   JacobiDN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2 -
  256 Ap^2 T^4 Cos[2 π t T]^2 EllipticK[-1]^4 JacobiCN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2
   JacobiDN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2 +
  1536 T^4 EllipticK[-1]^4 JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^4 +
  256 Ap^2 π T^4 Cos[2 π t T] EllipticK[-1]^3 JacobiCN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]
   JacobiDN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]
   JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1] Sin[2 π t T] -
  64 Ap^2 π^2 T^4 EllipticK[-1]^2 JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2 Sin[2 π t T]^2 +
  1/8 Ap^4 Cos[2 π t T]^4 u[t, T]^4

In[157]:= I11[T_?NumberQ] :=
  NIntegrate[-1536 T^4 EllipticK[-1]^4 JacobiCN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2
     JacobiDN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2 -
    256 Ap^2 T^4 Cos[2 π t T]^2 EllipticK[-1]^4 JacobiCN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2
     JacobiDN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2 +
    1536 T^4 EllipticK[-1]^4 JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^4 +
    32 Ap^4 T^4 Cos[2 π t T]^4 EllipticK[-1]^4 JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^4 +
    256 Ap^2 π T^4 Cos[2 π t T] EllipticK[-1]^3 JacobiCN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]
     JacobiDN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]
     JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1] Sin[2 π t T] - 64 Ap^2 π^2 T^4 EllipticK[-1]^2
     JacobiSN[4 ⅈ (c2 - ⅈ t) T EllipticK[-1], -1]^2 Sin[2 π t T]^2, {t, 0, 1/T}];

(*E^2 term*)

In[∘]:= 3 D[u[t], t]^2 + 1/2 hp[t]^2 u'[t]^2 + hp[t] × u[t] × u'[t] × hp'[t] + 1/2 u[t]^2 hp'[t]^2 // Simplify

Out[∘]= -1/4 c1^2
  (c1^2 (12 + Ap^2 + Ap^2 Cosh[2 t ωg]) JacobiCN[c1 (c2 - ⅈ t), -1]^2 JacobiDN[c1 (c2 - ⅈ t), -1]^2 -
    2 Ap^2 ωg^2 JacobiSN[c1 (c2 - ⅈ t), -1]^2 Sinh[t ωg]^2 + 2 ⅈ Ap^2 c1 ωg JacobiCN[c1 (c2 - ⅈ t), -1]
     JacobiDN[c1 (c2 - ⅈ t), -1] JacobiSN[c1 (c2 - ⅈ t), -1] Sinh[2 t ωg])

In[∘]:= I2[T_?NumberQ] :=
  NIntegrate[-1/4 c1^2 (c1^2 (12 + Ap^2 + Ap^2 Cosh[2 t ωg]) JacobiCN[c1 (c2 - ⅈ t), -1]^2
      JacobiDN[c1 (c2 - ⅈ t), -1]^2 - 2 Ap^2 ωg^2 JacobiSN[c1 (c2 - ⅈ t), -1]^2 Sinh[t ωg]^2 +
     2 ⅈ Ap^2 c1 ωg JacobiCN[c1 (c2 - ⅈ t), -1] JacobiDN[c1 (c2 - ⅈ t), -1]
      JacobiSN[c1 (c2 - ⅈ t), -1] Sinh[2 t ωg]), {t, 0, 1/T}];

(*Choosing c2 and Ap*)

In[158]:= c2 = 0; Ap = 0.001;
```

Thermodynamic Potential

In[160]:=
```
Λms = 176; Λ = 2 π T; β0 = (1/ (4 π)^2) ((11 / 3) Nc - (2 / 3) Nf);
α[T_] = 2 Log[Λ / Λms];
αs[T_] = (1 / (4 π β0 α[T])) (1);(*For only loop *)
```

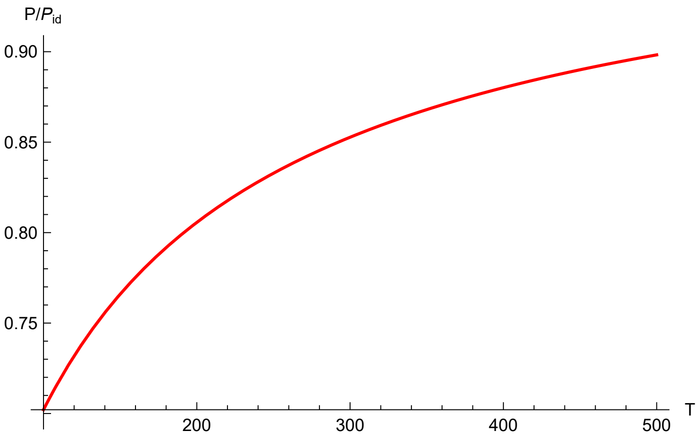
In[163]:=
```
Thdpotgw1[T_] := -((π^2/45) T^4 (Nc^2 - 1) + 7 (π^2/180) T^4 Nc Nf -
    ((-1) / (4 (4 π) αs[T]) + (1 / 2) β0 Log[Λ / (4 π T)] - (1/ (4 π)^2) (Nf / 3) Log[4])
     I11[T] - (1 / 3) (1/ (4 π)^2) (Nf - Nc) θ);
```

In[164]:=
```
Pgw1[T_] = -Thdpotgw1[T];
Sdengw1[T_] = -D[Thdpotgw1[T], T];
Endengw1[T_] = -Pgw1[T] + T Sdengw1[T];
```

In[167]:=
```
Pgw1n[T_] = Pgw1[T] / Pideal;
Thdpotgw1n[T_] = Thdpotgw1[T] / Pideal;
Sdengw1n[T_] = Sdengw1[T] / Sideal;
Endengw1n[T_] = Endengw1[T] / Eideal;

(*Normalised pressure*)
```

In[171]:=
```
Plot[Re[Pgw1n[T]], {T, 100, 500}, AxesLabel → {"T", "P/P_id"}, PlotStyle → Red]
```

Out[171]=




```
(*normalised Energy density*)
```

In[173]:=
```
Plot[Re[Endengw1n[T]], {T, 100, 500}, AxesLabel → {"T", "ϵ/ϵ_id"}, PlotStyle → Red]
```

Out[173]=