```
In[194]:= coord = {t, x, y, z};
          n = 4;(*# of spacetime dimensions*)

          (*Minkowski*)

In[196]:= gdd = {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
          gUU = {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};

          (*Christoffel  symbols*)

In[198]:= ΓUdd = Table[1/2 Sum[gUU[[i]][[l]]
              (D[gdd[[l]][[k]], coord[[j]]] + D[gdd[[l]][[j]], coord[[k]]] - D[ gdd[[j]][[k]], coord[[l]]]),
            {l, 1, n}], {i, 1, n}, {j, 1, n}, {k, 1, n}];

          (*Gauge field A with 'a' as internal index and d as spacetime index= Aad*)

In[199]:= m = 3;(*# of internal indices (SU(2))*)

In[213]:= Aad = {{0, U[t], χ1[t, z], ϕ1[t, z]}, {0, χ2[t, z], U[t], ϕ2[t, z]}, {0, 0, 0, U[t]}}
Out[213]= {{0, U[t], χ1[t, z], ϕ1[t, z]}, {0, χ2[t, z], U[t], ϕ2[t, z]}, {0, 0, 0, U[t]}}

In[214]:= MatrixForm[Aad]
Out[214]//MatrixForm=
```

$$\begin{pmatrix} 0 & U[t] & \chi1[t, z] & \phi1[t, z] \\ 0 & \chi2[t, z] & U[t] & \phi2[t, z] \\ 0 & 0 & 0 & U[t] \end{pmatrix}$$

```
In[215]:= Fadd = Table[D[Aad[[a]][[j]], coord[[i]]] - D[Aad[[a]][[i]], coord[[j]]] +
            g Sum[LeviCivitaTensor[3] [[a, b, c]] × Aad[[b]][[i]] × Aad[[c]][[j]], {b, 1, m}, {c, 1, m}],
           {a, 1, m}, {i, 1, n}, {j, 1, n}];

In[216]:= Fadd[[1]] // MatrixForm
Out[216]//MatrixForm=
```

$$\begin{pmatrix} 0 & U'[t] & \chi1^{(1,0)}[t, z] & \phi1^{(1,0)}[t, z] \\ -U'[t] & 0 & 0 & g\,U[t]\,\chi2[t, z] \\ -\chi1^{(1,0)}[t, z] & 0 & 0 & g\,U[t]^2 - \chi1^{(0,1)}[t, z] \\ -\phi1^{(1,0)}[t, z] & -g\,U[t]\,\chi2[t, z] & -g\,U[t]^2 + \chi1^{(0,1)}[t, z] & 0 \end{pmatrix}$$

```
In[217]:= FaUU = Table[Sum[gUU[[i]][[k]] × gUU[[j]][[l]] × Fadd[[a]][[k]][[l]], {k, 1, n}, {l, 1, n}],
            {a, 1, m}, {i, 1, n}, {j, 1, n}];

          (*Lagrangian*)
```

In[218]:= $L = \frac{1}{4}$ Sum[FaUU[[a]][[α]][[β]] × Fadd[[a]][[α]][[β]], {a, 1, m}, {α, 1, n}, {β, 1, n}]

Out[218]=
$$\frac{1}{4}\Big(2 g^2 U[t]^2 \chi1[t,z]^2 + g^2 (U[t]\,\phi1[t,z] - \phi2[t,z]\,\chi1[t,z])^2 + g^2 (-U[t]\,\phi1[t,z] + \phi2[t,z]\,\chi1[t,z])^2 + 2 g^2 U[t]^2 \chi2[t,z]^2 + g^2 (U[t]\,\phi2[t,z] - \phi1[t,z]\,\chi2[t,z])^2 + g^2 (-U[t]\,\phi2[t,z] + \phi1[t,z]\,\chi2[t,z])^2 + g^2 \left(U[t]^2 - \chi1[t,z]\,\chi2[t,z]\right)^2 + g^2 \left(-U[t]^2 + \chi1[t,z]\,\chi2[t,z]\right)^2 - 6 U'[t]^2 + \left(g\, U[t]^2 - \chi1^{(0,1)}[t,z]\right)^2 + \left(-g\, U[t]^2 + \chi1^{(0,1)}[t,z]\right)^2 + \left(-g\, U[t]^2 - \chi2^{(0,1)}[t,z]\right)^2 + \left(g\, U[t]^2 + \chi2^{(0,1)}[t,z]\right)^2 - 2\,\phi1^{(1,0)}[t,z]^2 - 2\,\phi2^{(1,0)}[t,z]^2 - 2\,\chi1^{(1,0)}[t,z]^2 - 2\,\chi2^{(1,0)}[t,z]^2\Big)$$

In[219]:= % // Expand

Out[219]=
$$\frac{3}{2} g^2 U[t]^4 + \frac{1}{2} g^2 U[t]^2 \phi1[t,z]^2 + \frac{1}{2} g^2 U[t]^2 \phi2[t,z]^2 - g^2 U[t]\,\phi1[t,z]\,\phi2[t,z]\,\chi1[t,z] + \frac{1}{2} g^2 U[t]^2 \chi1[t,z]^2 + \frac{1}{2} g^2 \phi2[t,z]^2 \chi1[t,z]^2 - g^2 U[t]\,\phi1[t,z]\,\phi2[t,z]\,\chi2[t,z] - g^2 U[t]^2 \chi1[t,z]\,\chi2[t,z] + \frac{1}{2} g^2 U[t]^2 \chi2[t,z]^2 + \frac{1}{2} g^2 \phi1[t,z]^2 \chi2[t,z]^2 + \frac{1}{2} g^2 \chi1[t,z]^2 \chi2[t,z]^2 - \frac{3}{2} U'[t]^2 - g\, U[t]^2 \chi1^{(0,1)}[t,z] + \frac{1}{2} \chi1^{(0,1)}[t,z]^2 + g\, U[t]^2 \chi2^{(0,1)}[t,z] + \frac{1}{2} \chi2^{(0,1)}[t,z]^2 - \frac{1}{2} \phi1^{(1,0)}[t,z]^2 - \frac{1}{2} \phi2^{(1,0)}[t,z]^2 - \frac{1}{2} \chi1^{(1,0)}[t,z]^2 - \frac{1}{2} \chi2^{(1,0)}[t,z]^2$$

In[220]:= Collect[%, g]

Out[220]=
$$g^2 \left(\frac{3 U[t]^4}{2} + \frac{1}{2} U[t]^2 \phi1[t,z]^2 + \frac{1}{2} U[t]^2 \phi2[t,z]^2 - U[t]\,\phi1[t,z]\,\phi2[t,z]\,\chi1[t,z] + \frac{1}{2} U[t]^2 \chi1[t,z]^2 + \frac{1}{2} \phi2[t,z]^2 \chi1[t,z]^2 - U[t]\,\phi1[t,z]\,\phi2[t,z]\,\chi2[t,z] - U[t]^2 \chi1[t,z]\,\chi2[t,z] + \frac{1}{2} U[t]^2 \chi2[t,z]^2 + \frac{1}{2} \phi1[t,z]^2 \chi2[t,z]^2 + \frac{1}{2} \chi1[t,z]^2 \chi2[t,z]^2\right) - \frac{3}{2} U'[t]^2 + \frac{1}{2} \chi1^{(0,1)}[t,z]^2 + \frac{1}{2} \chi2^{(0,1)}[t,z]^2 + g \left(-U[t]^2 \chi1^{(0,1)}[t,z] + U[t]^2 \chi2^{(0,1)}[t,z]\right) - \frac{1}{2} \phi1^{(1,0)}[t,z]^2 - \frac{1}{2} \phi2^{(1,0)}[t,z]^2 - \frac{1}{2} \chi1^{(1,0)}[t,z]^2 - \frac{1}{2} \chi2^{(1,0)}[t,z]^2$$

(*EM tensor*)

In[206]:= Tμν =
Table[Sum[gUU[[α]][[β]] × Fadd[[a]][[μ]][[α]] × Fadd[[a]][[ν]][[β]], {a, 1, m}, {α, 1, n}, {β, 1, n}],
{μ, 1, n}, {ν, 1, n}] − $\frac{1}{4}$ Table[gdd[[μ]][[ν]], {μ, 1, n}, {ν, 1, n}] ×
Sum[FaUU[[a]][[α]][[β]] × Fadd[[a]][[α]][[β]], {a, 1, m}, {α, 1, n}, {β, 1, n}];

```
In[ ]:= Tij = Table[Tμν[[i]][[j]], {i, 1, 3}, {j, 1, 3}];

In[ ]:= CovDFaUU = Table[Sum[D[FaUU[[a]][[k]][[j]], coord[[k]]] +
           Sum[ ΓUdd[[k]][[k]][[l]] FaUU[[a]][[l]][[j]] + ΓUdd[[j]][[k]][[l]] FaUU[[a]][[k]][[l]], {l, 1, n}] ,
          {k, 1, n}], {a, 1, m}, {j, 1, n}];

In[ ]:= NonaU = Table[Sum[g LeviCivitaTensor[3][[a, b, c]] × Aad[[b]][[i]] × FaUU[[c]][[i]][[j]],
          {b, 1, m}, {c, 1, m}, {i, 1, n}], {a, 1, m}, {j, 1, n}];

In[ ]:= YMeq = CovDFaUU + NonaU;
```