University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #3

This assignment is due at the start of your tutorial in the period September 30 – October 4, 2013.

A. Suggested reading: Marsden & Tromba, Chapter 2, sections 2.2 and 2.3.

B. Problems:

1. A cone-shaped pool near a detergent manufacturing plant has become polluted with by-products from the manufacturing process. The pool has radius 100 m and maximum depth of 30 m and the density of pollutants is given in kg/m^3 by

$$\delta(x, y, z) = \frac{(50 + \sqrt{x^2 + y^2})(30 - z)}{100}$$

where x and y are meters east/west and north/south of the center of the pool and z is meters below the surface. Give contour diagrams for δ at the surface and at a depth of 10 m. Explain how δ varies with depth and distance from the center.

2. For parts (b) and (c) of this question you will need to use symbolic algebra software such as Mathematica or Maple.

   (a) Characterize and sketch several level curves of the following functions

   (i) $f(x, y) = y^2 - xy$

   (ii) $f(x, y) = \dfrac{x^2 - y^2 + 1}{x^2 + y^2}$

   (iii) $f(x, y) = \dfrac{x + y}{x^2}$

   (iv) $f(x, y) = \dfrac{x}{1 + x^2 + y^2}$

   (v) $f(x, y) = \sqrt{\dfrac{1 + 2y - x^2}{y^2 + y}}$.

   (b) Use the contour plotting command on the functions in part (a) and compare with the pictures you produced in part (a).

   (c) Use the 3D plotting command to generate representations of the graphs of the functions in part (a).

3. For each of the following, evaluate the limit or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^{xy}}{3x+5y+2}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{5y^2}{2x^2+y^2}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy}{2x^2-y^2}$

(d) $\displaystyle\lim_{(x,y)\to(1,-2)} \frac{xy+2x-y-2}{(x^2-1)(y+2)}$

(e) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}\ -1}$

(f) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^2+y^2-2}{|x-1|+|y-1|}$.

4. Let $f(x,y) = \begin{cases} \dfrac{(2^x-1)(\sin y)}{xy} & x\,y \neq 0 \\ \ln 2 & x\,y = 0. \end{cases}$

Is f continuous at $(0,0)$? Explain.

5. For each of the following, evaluate $\dfrac{\partial f}{\partial x}$ at the point $\boldsymbol{a}$.

(a) $f(x,y) = \dfrac{\sin x}{x\,y}$; $\boldsymbol{a} = (\frac{\pi}{2},\, 2)$

(b) $f(x,y,z) = xy + y\cos z - x\sin yz$; $\boldsymbol{a} = (2,-1,\pi)$

(c) $f(x,y,z) = \ln\sqrt{2z-xy}$; $\boldsymbol{a} = (2,1,3)$

(d) $f(x,y) = \|(x,y)\|$; $\boldsymbol{a} = (-1,2)$

(e) $f(x,y) = \begin{cases} \dfrac{3xy+5y^3}{x^2+y^2} & ,\,(x,y)\neq(0,0) \\ 0 & ,\,(x,y)=(0,0) \end{cases}$; $\boldsymbol{a} = (0,0)$

6. Compute the derivative of each of the following (if they are differentiable).

(a) $f:\mathbb{R}^2\to\mathbb{R}^3$, $f(x,y) = (x+y\sin x,\ x^2e^{yx},\ 2^{xy})$.

(b) $f:\mathbb{R}^5\to\mathbb{R}^3$, $f(x_1,x_2,x_3,x_4,x_5) = \left(x_1x_2^2x_3^3x_4^4,\ x_5\tan(x_3x_4),\ \dfrac{x_1x_2}{x_3x_5}\right)$.

(c) $f:\mathbb{R}^n\to\mathbb{R}^k$, $f(\boldsymbol{x}) = A\boldsymbol{x}$ where $A\in M_{k,n}(\mathbb{R})$.

(d) $f:\mathbb{R}^n\to\mathbb{R}$, $f(\boldsymbol{x}) = \|\boldsymbol{x}\|$.