University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #1

This assignment is due at the start of your tutorial in the period September 16 – September 20, 2013.

A. Suggested reading: Marsden & Tromba, Chapter 1, sections 1.1, 1.2, 1.3 and 1.5.

B. Problems:

Most of these problems are a review of prerequisite material.

1. (a) Using only the definition of derivative find $f'(a)$ when $f(x) = 2x^2 + x - 3$.

   (b) Using only the definition of the Riemann integral find $\int_0^2 f(x)\,dx$ when $f(x) = 2x^2 + x - 3$.

   (c) Find $\dfrac{dF(x)}{dx}$ where $F(x) = \displaystyle\int_{\cos x}^{1-x^3} e^{t^2}\,dt$

2. (a) Without using integral tables, evaluate each of the following

   (i) $\displaystyle\int \frac{x^6 + x^3}{1 + x^2}\,dx$ (ii) $\displaystyle\int \frac{(\ln w)^3}{w}\,dw$

   (iii) $\displaystyle\int \sin^4 x \cos^3 x\,dx$ (iv) $\displaystyle\int z^2 \cos z\,dz$

   (v) $\displaystyle\int \sin(\ln x)\ dx$ (vi) $\displaystyle\int \frac{dx}{(x+1)(x-2)}$

   (vii) $\displaystyle\int x^2\sqrt{9 - x^2}\,dx$

   (b) Find the following definite integrals, if they converge.

   (i) $\displaystyle\int_4^9 \frac{e^{\sqrt{y}}}{\sqrt{y}}\,dy$ (ii) $\displaystyle\int_0^6 \frac{dx}{(x-4)^{2/3}}$ (iii) $\displaystyle\int_0^\infty \frac{x}{e^x}\,dx$

3. The police observe that the skid marks of a stopping car are 200 m long. Assuming the car decelerated at a constant rate of 20 m/sec^2, skidding all the way, how fast was the car going when the brakes were applied?

4. Let $\boldsymbol{v} = (1, -1, 1)$ and $\boldsymbol{w} = (0, 1, -2)$ be vectors in $\mathbb{R}^3$.

   (a) Find the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$.

   (b) Verify the Cauchy-Schwarz inequality and the triangle inequality for $\boldsymbol{v}$ and $\boldsymbol{w}$.

   (c) Find all unit vectors in $\mathbb{R}^3$ which are orthogonal to both $\boldsymbol{v}$ and $\boldsymbol{w}$.

   (d) Find the projection of (i) $\boldsymbol{v}$ onto $\boldsymbol{w}$ and (ii) $\boldsymbol{w}$ onto $\boldsymbol{v}$.

5. (a) Suppose $\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{w} \cdot \boldsymbol{v}$ for all $\boldsymbol{v}$. Is it necessarily true that $\boldsymbol{u} = \boldsymbol{w}$? Justify your answer.

   (b) Suppose $\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{w} \cdot \boldsymbol{v}$ for some $\boldsymbol{v} \neq \mathbf{0}$. Is it necessarily true that $\boldsymbol{u} = \boldsymbol{w}$? Justify your answer.

6. Let $\boldsymbol{b}_1 = \left(\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}}\right)$, $\boldsymbol{b}_2 = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $\boldsymbol{b}_3 = \left(\frac{-2}{\sqrt{30}}, \frac{-5}{\sqrt{30}}, \frac{1}{\sqrt{30}}\right)$.

   (a) Determine if $\boldsymbol{b}_1$, $\boldsymbol{b}_2$ and $\boldsymbol{b}_3$ form an orthonormal basis for $\mathbb{R}^3$. Justify your answer.

   (b) If $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, and $\boldsymbol{b}_3$ should form an orthonormal basis, find the coordinates of $\boldsymbol{v} = (1, 0, 1)$ in the $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{b}_3$ system.

7. Let $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & -1 & 4 \\ -3 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} \frac{3}{2} & -1 & -\frac{1}{2} \\ \frac{7}{3} & -\frac{5}{3} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{1}{3} & \frac{1}{6} \end{pmatrix}$, and $C = \begin{pmatrix} -1 & 3 & 2 \\ 4 & -1 & 1 \\ 3 & 2 & 3 \end{pmatrix}$.

   (a) Compute $\det A$, $\det B$, $\det C$, $\det AB$ and $\det(A + B)$.

   (b) Verify that A and B are inverse matrices and use this fact to

      (i) solve the simultaneous equations:

$$\begin{aligned} x \quad\quad + 3z &= 1 \\ 2x - \ y + 4z &= 2 \\ -3x + 2y + \ z &= 3 \end{aligned}$$

   and

      (ii) show that the only $\boldsymbol{v} \in \mathbb{R}^3$ such that $A\boldsymbol{v} = \mathbf{0}$ is the zero vector.

   (c) Can you find a non-zero vector $\boldsymbol{v} \in \mathbb{R}^3$ such that $C\boldsymbol{v} = \mathbf{0}$? Would the argument used in (b)(ii) work here? Explain.