University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #8

This assignment is due at the start of your tutorial in the period November 18 – November 22, 2013.

A. Suggested reading:

1. Marsden & Tromba, Chapter 3, section 3.4.
2. Marsden & Tromba, Chapter 5, sections 5.1–5.4.

B. Problems:

1. Use Lagrange multipliers to find the constrained critical points of f subject to the given constraints.

   (a) $f(x, y) = x^2 + 2y^2, \qquad 4x - 6y = 25$

   (b) $f(x, y) = 2x + 3y, \qquad x^2 + y^2 = 4$

   (c) $f(x, y) = 4x^2 + 9y^2, \qquad xy = 4$

   (d) $f(x, y) = x\,y, \qquad 4x^2 + 9y^2 = 32$

   (e) $f(x, y) = x^2\,y^4, \qquad x^2 + 2y^2 = 6$

   (f) $f(x, y, z) = xy + xz + yz - xyz, \qquad x + y + z = 1, \quad x, y, z \geq 0$

   (g) $f(x, y, z) = x, \qquad x + \dfrac{y}{2} + \dfrac{z}{3} = 0, \quad x^2 + y^2 + z^2 = 1.$

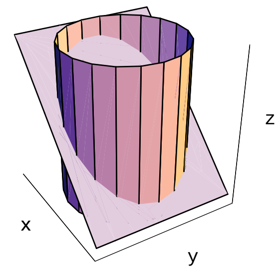
2. A deep space probe in the shape of the sphere $x^2 + y^2 + z^2 = 1$ is located in a part of space where the temperature is given, in degrees Kelvin, by $4x^2 + yz + 15$. Find the warmest point(s) on the surface of the probe. (Justify your answer.)

3. If a rocket is launched with a constant thrust corresponding to an acceleration of s ft/s^2, its height after t seconds is $f(s, t) = \dfrac{1}{2}\big(s - 32\big)\, t^2$ feet. Fuel usage is proportional to $s^2 t$ and fuel capacity is limited by $s^2 t = 10000$. Find the thrust that maximizes height.

4. Find the points on the solid sphere $x^2 + y^2 + z^2 \leq 18$ where $f(x, y, z) = x^2 + 3x + xy + y^2 + z^2$ attains its global extrema. (Justify that global extrema do exist.)

5. A cubical oven can be described by $0 \le x \le 1$, $0 \le y \le 1$ and $0 \le z \le 1$. A metal plate is placed in the oven in such a way that it occupies that portion of the plane $x + y + z = 1$ that lies inside the oven. If the oven is heated so that the temperature at each point (x, y, z) is given by $T\,(x, y, z) = 4 - 2x^2 - y^2 - z^2$ in hundreds of degrees Celsius, what are the hottest and coldest points on the plate?

6. The cylinder $x^2 + y^2 = 1$ intersects the plane $x + z = 1$ in an ellipse. Find the point on that ellipse that is the furthest from the origin.
(Justify your answer including an explanation of why global extrema do exist.)



7. To fill an order for 100 units of its product, a company will distribute production between two plants, plant 1 and plant 2. The total cost is given by $C = f(q_1, q_2) = 0.1\,q_1^2 + 7\,q_1 + 15\,q_2 + 1000$ where q_1 = number of units at plant 1 and q_2 = number of units at plant 2. To minimize costs, how many should be produced at each plant.