University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #9

This assignment is due at the start of your tutorial in the period November 25 – November 29, 2013.

A. Suggested reading: Marsden & Tromba, Chapter 5, sections 5.3 – 5.5.

B. Problems:

1. Give a rough sketch of the region and evaluate the following integrals or show divergence. (You may need to change the order of integration.)

   (a) $\displaystyle\int_D \frac{x}{y}\,dA, \quad D = [-2,4] \times [1,3].$

   (b) $\displaystyle\int_D e^x \sin y\,dA, \quad D = [0,2] \times [0,\frac{\pi}{4}].$

   (c) $\displaystyle\int_D x^2 y\,dA,$ D is the region bounded by the lines $x = y$ and $y = 2x + 1$ between $x = 1$ and $x = 3$.

   (d) $\displaystyle\int_{-1}^{1}\int_{y^{2/3}}^{(2-y)^2} \left(\frac{3}{2}\sqrt{x} - 2y\right) dx\,dy.$
   (page 305, #19.)

   (e) $\displaystyle\int_D |\,x + y\,|\,dA,$ where $D = \big[0,1\big] \times \big[-1,1\big]$.

   (f) $\displaystyle\int_0^1\int_{e^y}^{e} \frac{x}{\ln x}dx\,dy.$

   (g) $\displaystyle\int_D \|\,\nabla f\,\|^2\,dA,$ where $f(x,y) = y - x^2 + 1$ and $D = \big\{(x,y) \mid f(x,y) \geq 0,\ y \leq 0\big\}$.

   (h) $\displaystyle\int_D e^x y\,dA,$ where D is the interior of the triangle with vertices $(-1,1)$, $(2,2)$ and $(0,-1)$.

   (i) $\displaystyle\int_0^1\int_x^{\sqrt[3]{x}} e^{x/y}\,dy\,dx.$

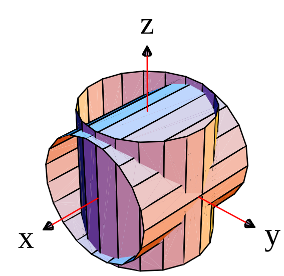
2. Show that
$$4\,\pi \quad \le \quad \iint_D \big(x^2+y^2+1\big)\,dx\,dy \quad \le \quad 20\,\pi\,,$$
where D is the disk of radius 2 centered at the origin.
(page 305, #31)

3. Suppose $f(x,y,z)$ is a continuous function. Write the integral
$$\int_0^1 \int_z^1 \int_0^{x-z} f(x,y,z)\,dy\,dx\,dz$$
in two other orders of integration.
(It may not always be possible to write this integral as a *single* triple integral.)

4. If B is the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$ and $(0,0,1)$, evaluate $\displaystyle\iiint_B y\,dV$.

5. Find the volume of the region B lying below the plane $z = 3-2y$ and above the paraboloid $z = x^2+y^2$.

6. Find the volume of the region which lies inside both $x^2+y^2=r^2$ and $y^2+z^2=r^2$.



7. For the following regions write the triple integral over the region W in the form
$$\int_W f\,dV = \iiint f(x,y,z)\,dz\,dy\,dx\ .$$
   (a) $W = \Big\{(x,y,z) \mid \sqrt{x^2+y^2} \le z \le 1\Big\}$.
   (b) $W = \big\{(x,y,z) \mid \frac{1}{2} \le z \le 1 \text{ and } x^2+y^2+z^2 \le 1\big\}$.

(cf. pages 303–304, #25, 26)