University of Toronto at Scarborough
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Midterm Test

MATB41H – Techniques of the Calculus of Several Variables I

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Duration: 110 minutes

1. **[10 points]** Characterize and sketch several level curves of the function

$$f(x,y) = \sqrt{4\,x^2 + y^2}\ .$$

   Carefully indicate where f is zero, positive, negative and not defined.

2. **[10 points]** For each of the following, either calculate $\lim_{(x,y)\to(0,0)} f(x,y)$ (showing your steps) or show that it does not exist.

   (a) $f(x,y) = \dfrac{xy^2}{x^2+y^4}$.

   (b) $f(x,y) = \begin{cases} \dfrac{x\,\sin(xy)}{y} & ,\ \text{if } y \neq 0 \\ 0 & ,\ \text{if } y = 0\ . \end{cases}$

3. **[6 points]** Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x,y) = \begin{cases} \dfrac{x^4 - y^4}{x^2+y^2} & ,\ \text{if } (x,y) \neq (0,0) \\ 1 & ,\ \text{if } (x,y) = (0,0)\ . \end{cases}$$

   Is f continuous at $(0,0)$? (Explain your answer.)

4. **[8 points]** Define the following terms

   (a) interior point of A, where $A \subset \mathbb{R}^n$.

   (b) closed set B, where $B \subset \mathbb{R}^n$.

   (c) bounded set C, where $C \subset \mathbb{R}^n$.

   (d) compact set D, where $D \subset \mathbb{R}^n$.

5. **[15 points]** A researcher looking at the paranormal found that the energy in a certain room was given by $E(x, y, z) = 5x^2 - 3xy + xyz$.

   (a) Find the rate of change in the energy E at the point $\boldsymbol{p} = (3, 4, 5)$ in the direction $\boldsymbol{v} = (1, 1, -1)$.

   (b) In what direction does E change most rapidly at $\boldsymbol{p}$?

   (c) What is the maximum rate of change at $\boldsymbol{p}$?

6. **[15 points]** Let π be the plane in $\mathbb{R}^3$ passing through the points $(1, 0, 4)$, $(2, -1, 0)$ and $(3, 1, 2)$.

   (a) Find an equation (rectangular description) for π.

   (b) Give a parametric description for the line ℓ through $(1, 1, 1)$ and orthogonal to π.

   (c) Find those points on the ellipsoid $4x^2 + 8y^2 + 4z^2 = 7$ where the tangent plane is parallel to π.

7. **[10 points]**

   (a) Find an equation of the tangent plane to the graph of $f(x, y) = 1 - (x^2 + 2y^2)$ at the point $\big(1, 1, f(1, 1)\big)$.

   (b) Find an equation of the tangent plane to the hyperboloid $z^2 - 2x^2 - 2y^2 = 12$ at the point $(1, -1, 4)$.

8. **[15 points]**

   (a) Carefully state the Chain Rule for functions of more than one variable.

   (b) Let $f : \mathbb{R}^4 \to \mathbb{R}^3$ be given by $f(x, y, z, w) = (xzw,\ y^2w^3,\ x^2z)$ and let $g : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $g(x, y, z) = (ye^x,\ yz^2,\ x + yz)$.
   `USE THE CHAIN RULE` to compute $D(g \circ f)(x, y, z, w)$.
   (**NOTE:** You must use the Chain Rule and show all your steps.)

9. **[5 points]** Let $z = f(x, y)$ where $x = v\,e^w$ and $y = w\,e^v$. Compute $\dfrac{\partial f}{\partial v}$ and $\dfrac{\partial f}{\partial w}$.

10. **[6 points]** Give the 4^{th} degree Taylor polynomial about the origin of $f(x, y) = \cos(xy)\,\ln(1 + x^2)$.