University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #6

Term Test Room Assignments	
Monday, 28 October, 5:00–7:00 pm	
Surname	go to room
A to S	IC 130
T to X	IC 230
Y to Z	IC 200

This assignment is due at the start of your tutorial in the period November 4 – November 8, 2013.

A. Suggested reading: Marsden & Tromba, Chapter 3, sections 3.1, 3.2 and 3.3.

B. Problems:

1. (a) Let $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^n$ be inverse functions. Suppose f and g are differentiable. Show that the determinants of Df and Dg are not 0, and that $Dg = (Df)^{-1}$.

   (b) If $f, g : \mathbb{R}^3 \to \mathbb{R}^3$ are inverse functions and Df is given by $Df = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 3 & 1 & 3 \end{pmatrix}$, compute Dg.

2. Let $A = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 2 & 0 \\ -3 & 0 & 9 \end{pmatrix}$.

   (a) Find the eigenvalues of A.

   (b) For each eigenvalue, find an eigenvector having length one.

   (c) Let B be the matrix whose rows are the eigenvectors found in part (b). Show that B is an orthogonal matrix.

3. Let A be the 3×3 matrix whose eigenvalues are -1, 1 and 2. What is the determinant of the matrix $A^3 + 2A^2 - A - 5\,I$.

4. The length of an open top rectangular box is increasing at a rate of 1 m/s while the width is increasing at the rate of 0.5 m/s and the height is decreasing at a rate of 1 m/s.

   (a) What is the rate of change of the volume of the box when the length is 5 m, the width is 4 m, and the height is 4 m ?

   (b) What is the rate of change in the total surface area of the box at that same instant?

5. Find and classify all critical points of the following functions:

   (a) $f(x, y) = x^3 - xy + y^3$

   (b) $f(x, y) = x^3 y + 12x^2 - 8y$

   (c) $f(x, y) = 4x - 3x^3 - 2xy^2$

   (d) $f(x, y) = e^x - xe^y$

   (e) $f(x, y) = x\,\ln(x + y)$

   (f) $f(x, y) = \displaystyle\int_x^y (e^{t^2} - e^t)dt$

   (g) $f(x, y, z) = x^3 + x\,z^2 - 3\,x^2 + y^2 + 2\,z^2$

   (h) $f(x, y, z) = x^2\,y + y^2\,z + z^2 - 2\,x.$

6. Find and indentify the local extrema of the following functions.

   (a) $f(x, y) = (x - 1)^2(y - 1)^2$

   (b) $f(x, y, z) = (x - 1)^2(y - 1)^2(z - 1)^2.$