University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #4

The Term Test will take place on **Monday, October 28, 5:00 pm – 7:00 pm**.

This assignment is due at the start of your tutorial in the period October 7 – October 11, 2013.

A. Suggested reading:

1. Marsden & Tromba, Chapter 1, section 1.3.
2. Marsden & Tromba, Chapter 2, sections 2.3, 2.5 and 2.6.

B. Problems:

1. Let $f(x,y) = \det \begin{pmatrix} e^x & 1 & -1 & 0 \\ e^x y^2 & y^2 & -y^2 & 1 \\ 0 & x+y & 1 & 1 \\ 0 & 1 & x+y & 1 \end{pmatrix}$.

   (a) Calculate $\dfrac{\partial f}{\partial x}$. Decide where $\dfrac{\partial f}{\partial x}$ is zero, positive and negative and indicate this information in a sketch.

   (b) Calculate $\dfrac{\partial f}{\partial y}$ and draw the level curve for $\dfrac{\partial f}{\partial y}$ corresponding to value $c = 1$ as carefully as possible.

2. (a) Give both the parametric and rectangular descriptions of the line (in $\mathbb{R}^3$) joining the points $(-1,1,2)$ and $(2,0,-3)$.

   (b) Give both the parametric and rectangular descriptions of the plane π that passes through the points $(-1,1,2)$, $(2,0,-3)$ and $(2,-1,2)$.

   (c) Give a parametric description of the line through $(0,1,0)$ and orthogonal to π. Where does this line meet in π.

3. Marsden & Tromba, page 70, # 22.

4. Compute $\boldsymbol{u} \times \boldsymbol{w}$ for $\boldsymbol{u} = (2,-1,1)$ and $\boldsymbol{w} = (3,-4,-2)$.

5. Find an equation of the tangent plane to $z = f(x, y)$ at the point $(2, 3, f(2, 3))$ for each of the following:

   (a) $f(x, y) = y^2 - xy$

   (b) $f(x, y) = \dfrac{x^2 - y^2 + 1}{x^2 + y^2}$

   (c) $f(x, y) = \dfrac{x + y}{x^2}$

   (d) $f(x, y) = \dfrac{x}{1 + x^2 + y^2}$

   (e) $f(x, y) = \sqrt{\dfrac{1 + 2y - x^2}{y^2 + y}}$.

6. (a) Compute an equation for the tangent planes of the following surfaces at the indicated points.

   (i) $x^2 + y^2 + z = 7$, $(1, -2, 2)$

   (ii) $(\cos x)(\sin y)e^z = 0$, $(\frac{\pi}{2}, 1, 0)$

   (b) Find an equation for the tangent plane at the point $(1, -5, 0)$ to the graph of the function $z = f(x, y)$ defined implicitly by

$$x^2y + yz^2 + x\,e^{x\,z} = -4$$

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7. (a) Compute the directional derivative of $f(x, y, z) = xz + y^2z^2$ at the point $(3, -1, 2)$ in the direction of the vector $\boldsymbol{v} = (0, -3, 4)$.

   (b) Compute the directional derivative of $f(x, y, z) = xy^2z$ at the point $(3, 4, 5)$ in the direction of the outward normal to the surface $2x^2 + 2y^2 - z^2 = 25$ at this point.

8. Let $f : \mathbb{R}^4 \to \mathbb{R}^3$ be given by $f(x, y, z, w) = (xzw,\ y^2w^3,\ x^2z)$ and let $g : \mathbb{R}^3 \to \mathbb{R}^3$ be given by $g(x, y, z) = (ye^x,\ yz^2,\ x + yz)$.

   (a) Find Df and Dg. Use the chain rule to compute $D(g \circ f)$.

   (b) Compute $g \circ f$ and $D(g \circ f)$ directly.