University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #7

This assignment is due at the start of your tutorial in the period November 11 – November 15, 2013.

A. Suggested reading: Marsden & Tromba, Chapter 3, sections 3.3 and 3.4.

B. Problems:

1. Let $f : \mathbb{R}^k \to \mathbb{R}$ be a function of class C^3 and let $\boldsymbol{a}$ be a critical point of f.

   (a) Suppose $k = 2$. Explain why the Hessian matrix for f at $\boldsymbol{a}$ cannot be $\begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$.

   (b) Suppose $k = 4$ and that the Hessian matrix for f at $\boldsymbol{a}$ is $\begin{pmatrix} 3 & 2 & 0 & 1 \\ 2 & 2 & -1 & 0 \\ 0 & -1 & -2 & 0 \\ 1 & 0 & 0 & -3 \end{pmatrix}$.

   Is $\boldsymbol{a}$ a local maximum, a local minimum, or a saddle?

2. (a) Find the global maximum and minimum values attained by the function $f(x, y) = x^2 - xy + y^2 + 1$ on the closed square $[-1, 2] \times [-1, 2]$.

   (b) Justify the existence of the global extrema in part (a).

3. The surface area of a rectangular box without a top is to be 108 cm^2. Regard one of the variables as an implicit function of the other two and find the greatest possible volume.

   (You may assume that the optimal dimensions exist to produce a maximal volume.)

4. An electronics retailer determines that the profit P (in dollars) from selling x units of a 16 GB MP3 player and y units of a 32 GB MP3 player is given by $P(x, y) = 8x + 10y - (0.001)(x^2 + xy + y^2) - 10,000$. Find her maximum profit.

5. Find the maximum volume of the largest rectangular box with one corner at the origin and the opposite corner at the point $\boldsymbol{p} = (x, y, z)$ on the paraboloid $z = 1 - \dfrac{x^2}{4} - \dfrac{y^2}{9}$. You may assume that $x,\ y,\ z \geq 0$.

Before attempting the next question you should read the material on the "method of least squares" on pages 214–215 of your textbook.

6. A physics experiment has produced the following table:

Time (sec)	0	2	3	4	5	8
Distance (cm)	10	8	7	6	3	1

   (a) Use the method of least squares to find the straight line that best fits the data. Note: You must use the tools of this course.

   (b) Plot the data and line together.

   (c) Can you estimate how long it will take until the distance is 0 (zero)?

7. Marsden & Tromba, page 201, # 3, 4, 5.