# CSCC63 W17 – Assignment 1

Due: Submit to MarkUs by 11.59pm, Jan 28, 2017

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**Turing Machine Conventions.** Use the following conventions when describing a Turing machine.

Turing machine algorithms are described in *stages*, with indentation for *blocks* that represent *loops*. Each stage is written simply (and clearly) enough that it is obvious it can be implemented on Turing machine. Description always starts with input which is always a string. Use the notation $\langle \mathbf{O} \rangle$ to represent the string encoding of object $\mathbf{O}$ (e.g., $\langle \mathbf{G} \rangle$ represents string encoding of graph $\mathbf{G}$). A TM can decode such strings and reject automatically if input does not follow proper encoding. Descriptions must always include clear, explicit conditions for accepting and rejecting. Use Example 3.23 on pp.185-187 as a reference.

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1. (15 marks) For this problem we will consider the notion of computability as it applies to functions.

   For some alphabet $\Sigma$, we say that the function $f : \Sigma^* \to \Sigma^*$ is *computable* if there exists a Turing machine $M$ such that for all strings $s \in \Sigma^*$, $M$ accepts input $w$ with final configuration $q_{accept} f(w)$. In other words, when $M$ is started with input $w$ on its tape, it eventually enters its accepting state with only $f(x)$ on its tape and its head on the first symbol of $f(w)$.

   (a) Let $\Sigma = \{0, 1\}$. Given a string $a \in \Sigma^*$, we denote by $int(a)$ the non-negative integer obtained by interpreting $a$ as a binary number. For example, $int(10011) = 19$.
   Write a *high level* and an *implementation level* description for a Turing machine that computes $f(x)$ where $f(x) = 3x$ and $x \in \{0, 1\}^*$. See page 185 for an explanation of high level and implementation level descriptions.

   (b) Give a state transition diagram for the Turing machine defined in (a).

2. (10 marks) Let $L$ be an infinite, recognizable language. Prove that there exists a decidable, infinite language $L_1$ such that $L_1$ is a subset of $L$. HINT: Consider an enumerator.

3. (10 marks) Let $R_1$ and $R_2$ be regular expressions such that $L(R_1) \subseteq L(R_2)$. Define $A = \{\langle R_1, R_2 \rangle \mid L(R_1) \subseteq L(R_2)\}$. Show that $A$ is decidable.

4. (10 marks) In the following question we will prove that given a language $L$, $L$ is *recognizable* iff a *decidable* language $L_D$ exists such that $L = \{u | \exists v, u\#v \in L_D\}$.

   (a) Show that if a language $L$ is recognizable, then there exists a decidable language $L_D$ such that $L = \{u \mid \exists v, u\#v \in L_D\}$.

   (b) Show that if there exists a decidable language $L_D$ and a language $L$ such that $L = \{u \mid \exists v, u\#v \in L_D\}$ then language $L$ is recognizable.

5. (5 marks) Show that the collection $L_D$ of decidable languages are closed under intersection. Repeat for the collection $L_R$ of recognizable languages.

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Total Marks: 50