University of Toronto Scarborough
Department of Computer & Mathematical Sciences

MAT B41H **2013/2014**

Assignment #2

This assignment is due at the start of your tutorial in the period September 23 – September 27, 2013

A. Suggested reading: Marsden & Tromba, Chapter 2, sections 2.1 and 2.2.

B. Problems:

1. (a) Let $\boldsymbol{u} = (u_1, u_2, u_3) \in \mathbb{R}^3$ and let $\alpha,\ \beta,\ \gamma$ denote the angles between $\boldsymbol{u}$ and the coordinate axes. Show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

   (b) Let $\boldsymbol{v}$ and $\boldsymbol{w}$ be nonzero vectors in $\mathbb{R}^3$ with $\boldsymbol{w} \neq -\boldsymbol{v}$.

   (i) Show that $\boldsymbol{x} = \|\boldsymbol{w}\|\,\boldsymbol{v} + \|\boldsymbol{v}\|\,\boldsymbol{w}$ bisects the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$.

   (ii) Show that the vectors $\|\boldsymbol{w}\|\,\boldsymbol{v} + \|\boldsymbol{v}\|\,\boldsymbol{w}$ and $\|\boldsymbol{w}\|\,\boldsymbol{v} - \|\boldsymbol{v}\|\,\boldsymbol{w}$ are orthogonal.

2. Describe geometrically the region in $\mathbb{R}^3$ which consists of those points $\boldsymbol{x}$ which satisfy

$$\boldsymbol{x} \cdot \boldsymbol{x} - \boldsymbol{x} \cdot (1, -2, 3) \leq 1.$$

3. Solve, for λ, the equation $\det(A - \lambda I) = 0$ where $A \in M_{3,3}(\mathbb{R})$ is given by

$$A = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 0 & 0 \\ 3 & 0 & -4 \end{pmatrix}.$$

   (If you are not able to get exact solutions you may use Newton's method to find approximate solutions.)

4. Determine if $A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 2 & 4 \\ 1 & 3 & -3 \end{pmatrix}$ has an inverse. If it does, use the cofactor/adjoint method to compute it.

5. (a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = 5x^2 + 2y^2 - 3$.
   (i) What is the domain and range of f?
   (ii) Restrict the domain so that f is one-to-one on the new domain.
   (iii) Restrict the codomain so that f is onto the new codomain.

   (b) What is the domain and range of the following functions f. Sketch the domain.
   (i) $f(x, y) = \log_2(x - y)$
   (ii) $f(x, y) = \dfrac{1}{\sqrt{9 - x^2 - y^2}}$.

6. For each of the following expressions sketch a picture showing the regions in $\mathbb{R}^2$ where the expression is positive or negative. Also indicate where the expression is zero or not defined.

   (a) $(2x^2 + 3y^2 - 7)(3xy - 1)$
   (b) $\dfrac{y + x^2 - 5}{y + 2x + 2}$
   (c) $|\, x + y \,| - |\, x - y \,|$
   (d) $\sin(y^2 - x^2)$.

7. Give a rough sketch of the surface in $\mathbb{R}^3$ defined by $0 = 3x^2 + 3y^2 - 6x + 12y - z + 15$.

8. Pictured below are computer generated contour diagrams and graphs. Indicate which contour diagram corresponds to each graph.