# An adaptive fuzzy dead-zone compensation scheme for nonlinear systems

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**Abstract**

The dead-zone nonlinearity is frequently encountered in many industrial automation equipments and its presence can severely compromise control system performance. Due to the possibility to express human experience in an algorithmic manner, fuzzy logic has been largely employed in the last decades to both control and identification of uncertain dynamical systems. In spite of the simplicity of this heuristic approach, in some situations a more rigorous mathematical treatment of the problem is required. In this work, an adaptive fuzzy controller is proposed for nonlinear systems subject to dead-zone input. The convergence properties of the tracking error will be proven using Lyapunov stability theory and Barbalat's lemma. An application of this adaptive fuzzy scheme to a Van der Pol oscillator is introduced to illustrate the controller design method. Numerical results are also presented in order to demonstrate the control system performance.



## I. INTRODUCTION

Dead-zone is a hard nonlinearity that can be commonly found in many industrial actuators, especially those containing hydraulic valves and electric motors. Dead-zone characteristics are often unknown and, as previously reported in the research literature, its presence can drastically reduce control system performance and lead to limit cycles in the closed-loop system.

The increasing number of works dealing with systems subject to dead-zone input shows the great interest of the engineering community in this particular nonlinearity. The most common approaches are adaptive schemes (Tao and Kokotović, 1994; Wang et al., 2004; Zhou et al., 2006; Ibrir et al., 2007), fuzzy systems (Kim et al., 1994; Oh and Park, 1998; Lewis et al., 1999), neural networks (Šelmić and Lewis, 2000; Tsai and Chuang, 2004; Zhang and Ge, 2007) and variable structure methods (Corradini and Orlando, 2002; Shyu et al., 2005). Many of these works (Tao and Kokotović, 1994; Kim et al., 1994; Oh and Park, 1998; Šelmić and Lewis, 2000; Tsai and Chuang, 2004; Zhou et al., 2006) use an inverse dead-zone to compensate the negative effects of the dead-zone nonlinearity even though this approach leads to a discontinuous control law and requires instantaneous switching, which in practice can not be accomplished with mechanical actuators. An alternative scheme, without using the dead-zone inverse, was originally proposed by Lewis et al. (1999) and also adopted by Wang et al. (2004). In both works, the dead-zone is treated as a combination of a linear and a saturation function. This approach was further extended by Ibrir et al. (2007) and by Zhang and Ge (2007), in order to accommodate non-symmetric dead-zones.

Intelligent control, on the other hand, has proven to be a very attractive approach to cope with uncertain nonlinear systems (Bessa, 2005; Bessa et al., 2005, 2017, 2018, 2019; Dos Santos and Bessa, 2019; Lima et al., 2018, 2020, 2021; Tanaka et al., 2013). By combining nonlinear control techniques, such as feedback linearization or sliding modes, with adaptive intelligent algorithms, for example fuzzy logic or artificial neural networks, the resulting intelligent control strategies can deal with the nonlinear characteristics as well as with modeling imprecisions and external disturbances that can arise.

This paper presents an adaptive fuzzy controller for nonlinear systems subject to dead-zone input. An unknown and non-symmetric dead-band is assumed. The dead-zone nonlinearity is also considered as a combination of linear and saturation functions, but an adaptive fuzzy inference system is introduced, as universal function approximator, to cope with the unknown saturation function. Based on a Lyapunov-like analysis using Barbalat's lemma, the convergence properties of the closed-loop system is analytically proven. To show the applicability of the proposed control scheme, a Van der Pol oscillator is chosen as illustrative example. Simulation results of the adopted mechanical system are also presented to demonstrate the control system efficacy.

## II. PROBLEM STATEMENT AND CONTROL SYSTEM DESIGN

Consider a class of $n^{\text{th}}$-order nonlinear and non-autonomous systems:

$$x^{(n)} = f(\mathbf{x}, t) + b(\mathbf{x}, t)\upsilon \tag{1}$$

where the scalar variable $x$ is the output of interest, $x^{(n)}$ is the $n$-th derivative of $x$ with respect to time $t$, $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]$ is the system state vector, $f, b : \mathbb{R}^n \to \mathbb{R}$ are both nonlinear functions and $\upsilon$ represents the output of a dead-zone function, as shown in Fig. 1.

The dead-zone nonlinearity presented in Fig. 1 can be mathematically described by:

$$\upsilon = \begin{cases} m_l\,(u - \delta_l) & \text{if} \quad u \le \delta_l \\ 0 & \text{if} \quad \delta_l < u < \delta_r \\ m_r\,(u - \delta_r) & \text{if} \quad u \ge \delta_r \end{cases} \tag{2}$$

where $u$ represents the controller output variable.

Considering that in many engineering components, as for instance hydraulic valves and electric motors, the slopes in both sides of the dead-zone are similar, the following physically motivated assumptions can be made for the dead-zone model presented in Eq. (2):

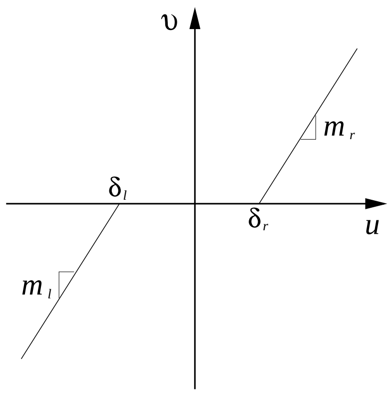



Figure 1: Dead-zone nonlinearity.

**Assumption 1** *The dead-zone output $\upsilon$ is not available to be measured.*

**Assumption 2** *The slopes in both sides of the dead-zone are equal and positive, i.e., $m_l = m_r = m > 0$.*

**Assumption 3** *The dead-band parameters $\delta_l$ and $\delta_r$ are unknown but bounded and with known signs, i.e., $\delta_{l\,\min} \leq \delta_l \leq \delta_{l\,\max} < 0$ and $0 < \delta_{r\,\min} \leq \delta_r \leq \delta_{r\,\max}$.*

In this way, Eq. (2) can be rewritten in a more appropriate form (Lewis et al., 1999; Wang et al., 2004):

$$\upsilon = m[u - d(u)] \tag{3}$$

where $d(u)$ can be obtained from Eq. (2) and Eq. (3):

$$d(u) = \begin{cases} \delta_l & \text{if} \quad u \leq \delta_l \\ u & \text{if} \quad \delta_l < u < \delta_r \\ \delta_r & \text{if} \quad u \geq \delta_r \end{cases} \tag{4}$$

**Remark 1** *Considering Assumption 3 and Eq. (4), it can be easily verified that $d(u)$ is bounded: $|d(u)| \leq \delta$, where $\delta = \max\{-\delta_{l\,\min}, \delta_{r\,\max}\}$.*

The proposed control problem is to ensure that, even in the presence of an unknown dead-zone input, the state vector $\mathbf{x}$ will follow a desired trajectory $\mathbf{x}_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$ in the state space.

Regarding the development of the control law, the following assumptions should also be made:

**Assumption 4** *The state vector $\mathbf{x}$ is available.*

**Assumption 5** *The desired trajectory $\mathbf{x}_d$ is once differentiable in time. Furthermore, every element of vector $\mathbf{x}_d$, as well as $x_d^{(n)}$, is available and with known bounds.*

Let $\tilde{x} = x - x_d$ be defined as the tracking error in the variable $x$, and

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d = [\tilde{x}, \dot{\tilde{x}}, \ldots, \tilde{x}^{(n-1)}]$$

as the tracking error vector.

Now, consider a combined tracking error measure:

$$\varepsilon = \mathbf{c}^{\mathrm{T}}\tilde{\mathbf{x}} \tag{5}$$

where $\mathbf{c} = [c_{n-1}\lambda^{n-1}, \ldots, c_1\lambda, c_0]$ and $c_i$ states for binomial coefficients, i.e.,

$$c_i = \binom{n-1}{i} = \frac{(n-1)!}{(n-i-1)!\,i!}\,, \qquad i = 0, 1, \ldots, n-1 \tag{6}$$

which makes $c_{n-1}\lambda^{n-1} + \cdots + c_1\lambda + c_0$ a Hurwitz polynomial.

From Eq. (6), it can be easily verified that $c_0 = 1$, for $\forall n \geq 1$. Thus, for notational convenience, the time derivative of $\varepsilon$ will be written in the following form:

$$\dot{\varepsilon} = \mathbf{c}^{\mathrm{T}}\dot{\tilde{\mathbf{x}}} = \tilde{x}^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}} \tag{7}$$

where $\bar{\mathbf{c}} = [c_{n-1}\lambda^{n-1}, \ldots, c_1\lambda, 0]$.

Based on Assumptions 4 and 5, the following control law can be proposed:

$$u = \frac{1}{bm}(-f + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}} - \kappa\varepsilon) + \hat{d}(\hat{u}) \tag{8}$$

where $\kappa$ is a strictly positive constant and $\hat{d}(\hat{u})$ an estimate of $d(u)$, that will be computed in terms of the equivalent control $\hat{u} = (bm)^{-1}(-f + x_d^{(n)} - \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})$ by an adaptive fuzzy algorithm.

The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in a linguistic manner as follows:

$$\textit{If } \hat{u} \textit{ is } \hat{U}_r \textit{ then } \hat{d} = \hat{D}_r \;\;;\;\; r = 1, 2, \cdots, N$$

where $\hat{U}_r$ are fuzzy sets, whose membership functions could be properly chosen, and $\hat{D}_r$ is the output value of each one of the $N$ fuzzy rules.

Considering that each rule defines a numerical value as output $\hat{D}_r$, the final output $\hat{d}$ can be computed by a weighted average:

$$\hat{d}(\hat{u}) = \frac{\sum_{r=1}^{N} w_r \cdot \hat{d}_r}{\sum_{r=1}^{N} w_r} \tag{9}$$

or, similarly,

$$\hat{d}(\hat{u}) = \hat{\mathbf{D}}^{\mathrm{T}}\mathbf{\Psi}(\hat{u}) \tag{10}$$

where, $\hat{\mathbf{D}} = [\hat{D}_1, \hat{D}_2, \ldots, \hat{D}_N]$ is the vector containing the attributed values $\hat{D}_r$ to each rule $r$, $\mathbf{\Psi}(\hat{u}) = [\psi_1(\hat{u}), \psi_2(\hat{u}), \ldots, \psi_N(\hat{u})]$ is a vector with components $\psi_r(\hat{u}) = w_r / \sum_{r=1}^{N} w_r$ and $w_r$ is the firing strength of each rule.

To ensure the best possible estimate $\hat{d}(\hat{u})$, the vector of adjustable parameters can be automatically updated by the following adaptation law:

$$\dot{\hat{\mathbf{D}}} = -\varphi\varepsilon\mathbf{\Psi}(\hat{u}) \tag{11}$$

where $\varphi$ is a strictly positive constant related to the adaptation rate.

The boundedness and convergence properties of the closed-loop system are established in the following theorem.

**Theorem 1** *Consider the nonlinear system (1) subject to the dead-zone (2) and Assumptions 1–5. Then, the controller defined by (8), (10) and (11) ensures the boundedness of all closed-loop signals and the exponential convergence of the tracking error, i.e.,* $\tilde{\mathbf{x}} \to \mathbf{0}$ *as* $t \to \infty$.

**Proof:** Let a positive definite Lyapunov function candidate $V$ be defined as

$$V(t) = \frac{1}{2}\varepsilon^2 + \frac{bm}{2\varphi}\mathbf{\Delta}^{\mathrm{T}}\mathbf{\Delta} \tag{12}$$

where $\mathbf{\Delta} = \hat{\mathbf{D}} - \hat{\mathbf{D}}^*$ and $\hat{\mathbf{D}}^*$ is the optimal parameter vector, associated to the optimal estimate $\hat{d}^*(\hat{u}) = d(u)$. Thus, the time derivative of $V$ is

$$\begin{aligned}
\dot{V}(t) &= \varepsilon\dot{\varepsilon} + bm\varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}} \\
&= (\tilde{x}^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})\varepsilon + bm\varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}} \\
&= (x^{(n)} - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}})\varepsilon + bm\varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}} \\
&= \big[f + bm\,u - bm\,d(u) - x_d^{(n)} + \bar{\mathbf{c}}^{\mathrm{T}}\tilde{\mathbf{x}}\big]\varepsilon + bm\varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\mathbf{\Delta}}
\end{aligned}$$

Applying the proposed control law (8) and noting that $\dot{\mathbf{\Delta}} = \dot{\hat{\mathbf{D}}}$, then

$$\begin{aligned}
\dot{V}(t) &= \big[bm(\hat{d} - d) - \kappa\varepsilon\big]\varepsilon + bm\varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}} \\
&= \big[bm\mathbf{\Delta}^{\mathrm{T}}\mathbf{\Psi}(\hat{u}) - \kappa\varepsilon\big]\varepsilon + bm\varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\dot{\hat{\mathbf{D}}} \\
&= -\kappa\varepsilon^2 + bm\varphi^{-1}\mathbf{\Delta}^{\mathrm{T}}\big[\dot{\hat{\mathbf{D}}} + \varphi\varepsilon\mathbf{\Psi}(\hat{u})\big]
\end{aligned}$$

Furthermore, defining $\dot{\hat{\mathbf{D}}}$ according to (11), $\dot{V}(t)$ becomes

$$\dot{V}(t) = -\kappa\varepsilon^2 \tag{13}$$

which implies that $V(t) \le V(0)$ and that $\varepsilon$ and $\mathbf{\Delta}$ are bounded. From the definition of $\varepsilon$ and considering Assumption 5, it can be easily verified that $\dot{\varepsilon}$ is also bounded.
To establish the convergence of the combined tracking error measure, the time derivative of $\dot{V}$ must be also analyzed:

$$\ddot{V}(t) = -2\kappa\varepsilon\dot{\varepsilon} \tag{14}$$

which implies that $\dot{V}(t)$ is also bounded and, from Barbalat's lemma, that $\varepsilon \to 0$ as $t \to \infty$. From the definition of limit, it means that for every $\xi > 0$ there is a corresponding number $\tau$ such that $|\varepsilon| < \xi$ whenever $t > \tau$. According to Eq. (5) and considering that $|\varepsilon| < \xi$ may be rewritten as $-\xi < \varepsilon < \xi$, one has

$$-\xi < c_0\tilde{x}^{(n-1)} + c_1\lambda\tilde{x}^{(n-2)} + \cdots + c_{n-2}\lambda^{n-2}\dot{\tilde{x}} + c_{n-1}\lambda^{n-1}\tilde{x} < \xi \tag{15}$$

Multiplying (15) by $e^{\lambda t}$ yields

$$-\xi e^{\lambda t} < \frac{d^{n-1}}{dt^{n-1}}(\tilde{x}e^{\lambda t}) < \xi e^{\lambda t} \tag{16}$$

Thus, integrating (16) $n-1$ times between 0 and $t$ gives

$$-\frac{\xi}{\lambda^{n-1}}e^{\lambda t} + \left(\frac{d^{n-2}}{dt^{n-2}}(\tilde{x}e^{\lambda t})\Big|_{t=0} + \frac{\xi}{\lambda}\right)\frac{t^{n-2}}{(n-2)!} + \cdots + \left(\tilde{x}(0) + \frac{\xi}{\lambda^{n-1}}\right) \leq \tilde{x}e^{\lambda t} \leq \frac{\xi}{\lambda^{n-1}}e^{\lambda t} + \\ + \left(\frac{d^{n-2}}{dt^{n-2}}(\tilde{x}e^{\lambda t})\Big|_{t=0} - \frac{\xi}{\lambda}\right)\frac{t^{n-2}}{(n-2)!} + \cdots + \left(\tilde{x}(0) - \frac{\xi}{\lambda^{n-1}}\right) \tag{17}$$

Furthermore, dividing (17) by $e^{\lambda t}$, it can be easily verified that the values of $\tilde{x}$ can be made arbitrarily close to 0 (within a distance $\xi$) by taking $t$ sufficiently large (larger than $\tau$), i.e., $\tilde{x} \to 0$ as $t \to \infty$. Considering the $(n-2)^{\text{th}}$ integral of (16), dividing again by $e^{\lambda t}$ and noting that $\tilde{x}$ converges to zero, it follows that $\dot{\tilde{x}} \to 0$ as $t \to \infty$. The same procedure can be successively repeated until the convergence of each component of the tracking error vector is achieved: $\tilde{\mathbf{x}} \to \mathbf{0}$ as $t \to \infty$. □

## III. Illustrative example

In order to illustrate the controller design method and to demonstrate its performance, consider a forced Van der Pol oscillator

$$\ddot{x} - \mu(1 - x^2)\dot{x} + x = bv \tag{18}$$

Without control, i.e., by considering $v = 0$, the Van der Pol oscillator exhibits a limit cycle. The control objective is to let the state vector $\mathbf{x} = [x, \dot{x}]$ track a desired trajectory $\mathbf{x}_d = [\sin t, \cos t]$ situated inside the limit cycle. Figure 2 shows the phase portrait of the unforced Van der Pol oscillator with the limit cycle, two convergent orbits and the desired trajectory.

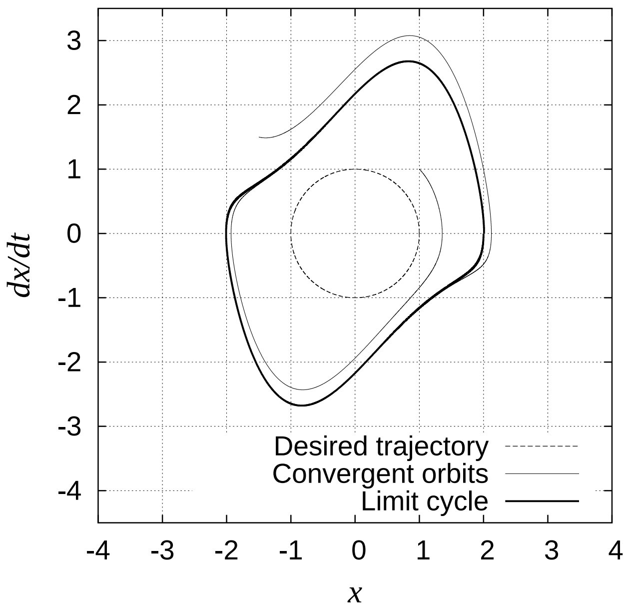



Figure 2: Phase portrait of the unforced Van der Pol oscillator

According to the previously described scheme and considering $\varepsilon = \dot{\tilde{x}} + \lambda\tilde{x}$, the control law can be chosen as follows

$$u = \frac{1}{bm}[-\mu(1 - x^2)\dot{x} + x + \ddot{x}_d - \lambda\dot{\tilde{x}} - \kappa\varepsilon] + \hat{d}(\hat{u})$$

The simulation studies were performed with an implementation in C, with sampling rates of 500 Hz for control system and 1 kHz for the Van der Pol oscillator, and the differential equations were numerically solved using the fourth order Runge-Kutta method. The chosen parameters were $b = 1$, $m = 1$, $\mu = 1$, $\delta_l = -0.4$, $\delta_r = 0.3$, $\lambda = 0.6$, $\kappa = 10$ and $\varphi = 3$. Concerning the fuzzy inference system, triangular and trapezoidal membership functions, respectively $\mu_{\text{tri}}$ and $\mu_{\text{tri}}$, were adopted for $\hat{U}_r$:

$$\mu_{\text{tri}} = \max\left\{\min\left(\frac{\hat{u} - a}{b - a}, \frac{c - \hat{u}}{c - b}\right), 0\right\} \tag{19}$$

where $a$, $b$ and $c$, with $a < b < c$, represent the abscissae of the three corners of the underlying triangle membership function,

$$\mu_{\text{trap}} = \max\left\{\min\left(\frac{\hat{u}-a}{b-a}, 1, \frac{d-\hat{u}}{d-c}\right), 0\right\} \tag{20}$$

where $a$, $b$, $c$ and $d$, with $a < b < c < d$, represent the abscissae of the four corners of the underlying trapezoidal membership function.

The central values of the adopted membership functions were $C = \{-5.0; -1.0; -0.5; 0.0; 0.5; 1.0; 5.0\} \times 10^{-1}$ (see Fig. 3).

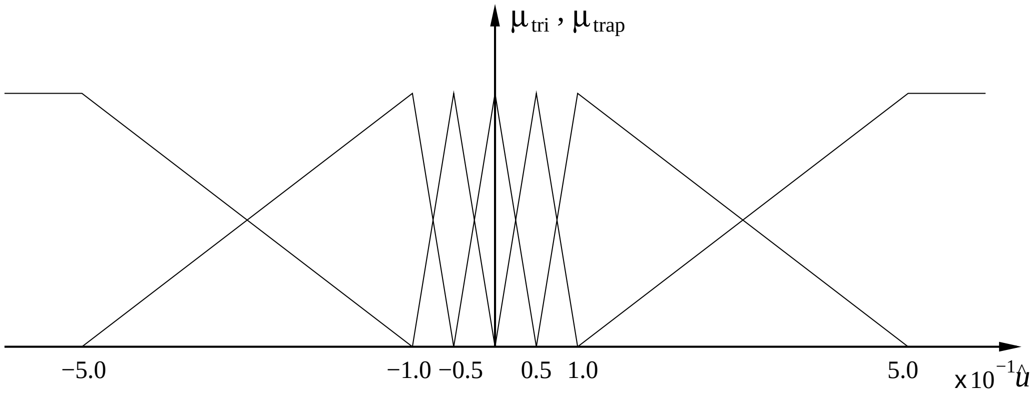
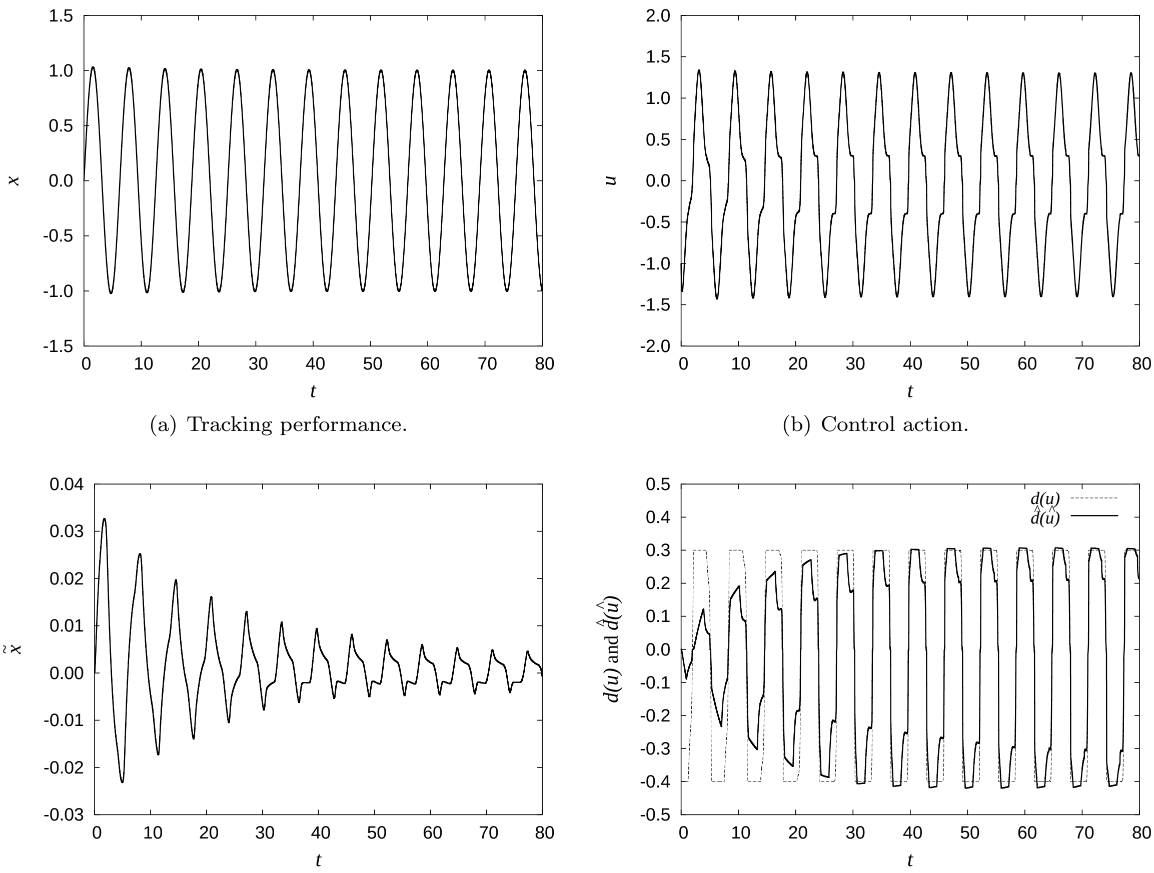



Figure 3: Adopted fuzzy membership functions.

It is also important to emphasize, that the vector of adjustable parameters was initialized with zero values, $\hat{\mathbf{D}} = \mathbf{0}$, and updated at each iteration step according to the adaptation law presented in Eq. (11). Figure 4 gives the corresponding results for the tracking of $x_d = \sin t$.




(a) Tracking performance.

(b) Control action.

(c) Tracking error.

(d) Convergence of $\hat{d}(\hat{u})$ to $d(u)$.

Figure 4: Tracking performance with $x_d = \sin t$.

As observed in Fig. 4, the proposed control law is able to provide trajectory tracking, Fig. 4(a), with a small associated error, Fig. 4(c). Figure 4(d) shows the ability of the adaptive fuzzy scheme to recognize and previously compensate the dead-band characteristics.

## IV. Concluding Remarks

The present work addressed the problem of controlling nonlinear systems subject to dead-zone input. An adaptive fuzzy controller was proposed to deal with the trajectory tracking problem. The boundedness and convergence properties of the closed-loop signals were analytically proven using Lyapunov stability theory and Barbalat's lemma. The control system performance was also confirmed by means of numerical simulations with an application to the forced Van der Pol equation. The adaptive algorithm could automatically recognize the dead-zone nonlinearity and previously compensate its undesirable effects.

## V. Acknowledgements

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