Portfolio Theory and the Capital Asset Pricing Model

The stock market is risky because there is a spread of possible outcomes. The usual measure of this spread is the standard deviation or variance.

The risk of any stock can be broken down into two parts. There is the *specific* or *diversifiable risk* that is peculiar to that stock, and there is the *market risk* that is associated with marketwide variations.

Investors *can* eliminate specific risk by holding a well-diversified portfolio, but they *cannot* eliminate market risk. *All* the risk of a fully diversified portfolio is market risk.

A stock's *contribution* to the risk of a fully diversified portfolio depends on its sensitivity to market changes. This sensitivity is generally known as *beta.*

- A security with a beta of *1.0* has *average market risk*—a well-diversified portfolio of such securities has the same standard deviation as the market index.
- A security with a beta of *0.5* has *below-average market risk*—a well-diversified portfolio of these securities tends to move half as far as the market moves and has half the market's standard deviation.

Harry Markowitz and the Birth of Portfolio Theory

Markowitz's theory

- Combining stocks into portfolios can reduce standard deviation **below** simple weighted-average calculation
- Correlation coefficients make possible
- By changing the proportion of funds invested in stocks, we can change the risk-return characteristics of a portfolio
- Efficient portfolios provide **the highest return for a given level of risk** or **least risk for given level of returns**
- Various weighted combinations of stocks that create specific standard deviation constitute set of efficient portfolios

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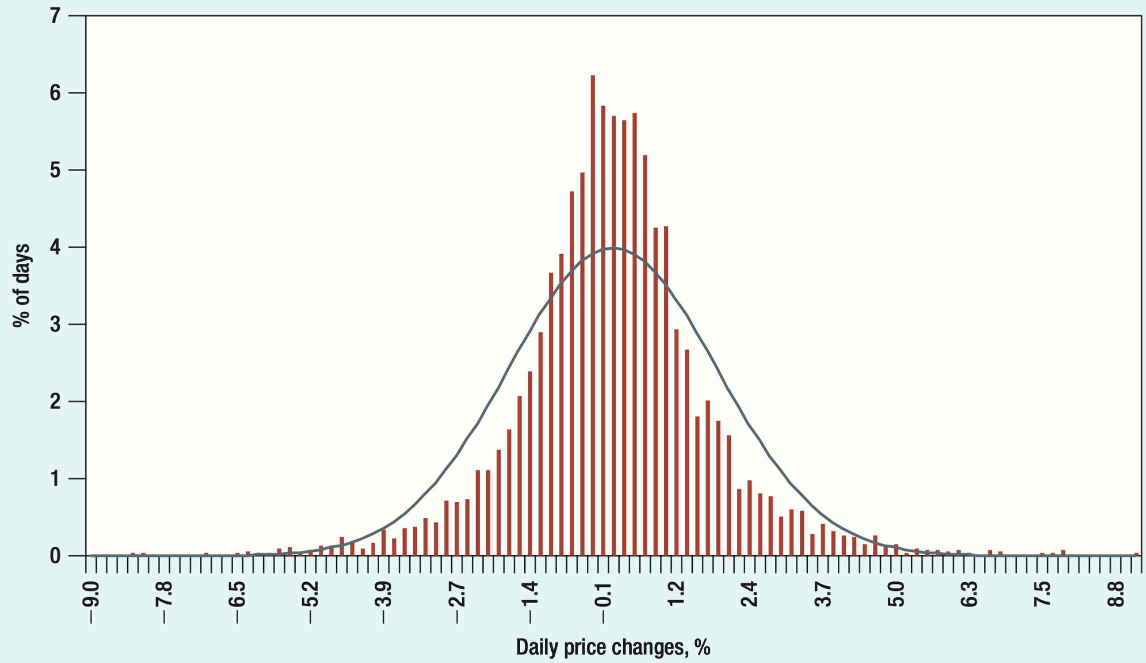


FIGURE 8.1 Daily price changes for IBM are approximately normally distributed. This plot spans 1994 to 2013.

*When measured over a short interval, the past rates of return on any stock conform fairly closely to **a normal distribution**.*

if returns are normally distributed, **expected return and standard deviation** are the *only* two measures that an investor need consider.

Combining Stocks into Portfolios

EXAMPLE

Suppose that you are wondering whether to invest in the shares of Johnson & Johnson (J&J) or Ford.

- J&J offers an expected return of **8.0%** and Ford offers an expected return of **18.8%**.
- The standard deviation of returns is **13.2%** for J&J and **31.0%** for Ford.

→ Ford offers the higher expected return, but it is more risky.

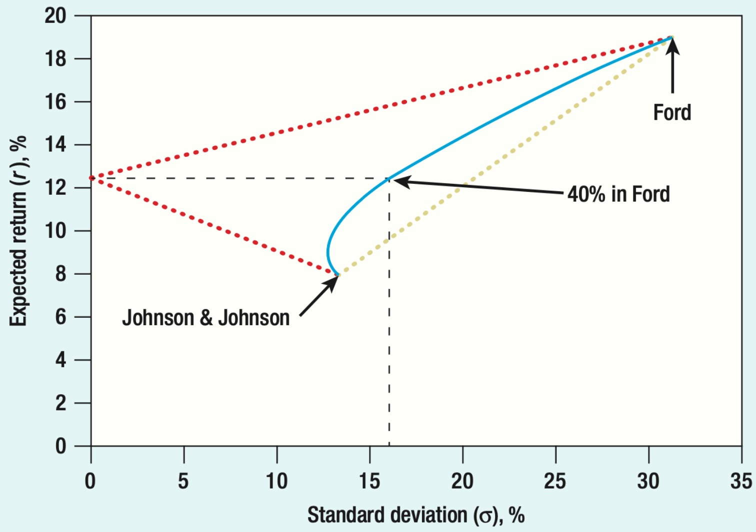


FIGURE 8.3

The curved line illustrates how expected return and standard deviation change as you hold different combinations of two stocks. For example, if you invest 40% of your money in Ford and the remainder in Johnson & Johnson, your expected return is 12.3%, which is 40% of the way between the expected returns on the two stocks. The standard deviation is 15.9%, which is less than 40% of the way between the standard deviations of the two stocks. This is because diversification reduces risk.

The curved blue line: ρ = +.19

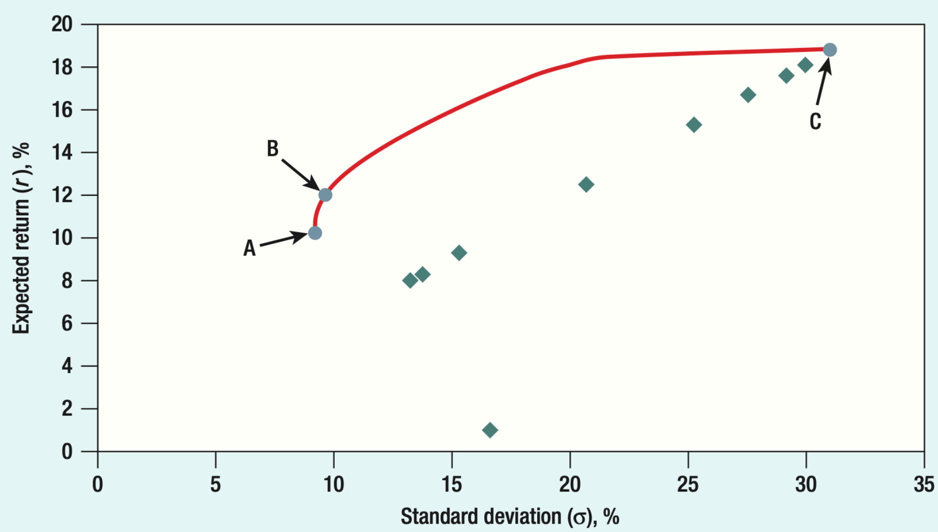
The gold dotted line: ρ = +1

The red dotted line: ρ = –1

- If you want to stake all on getting rich quickly, you should put all your money in Ford *(at the highest point of the blue curve)*.
- If you want a more peaceful life, you should invest most of your money in J&J, but you should keep at least a small investment in Ford *(at the leftmost point on the blue curve where about 10% of the portfolio is Ford)*.

FIGURE 8.4

Each diamond shows the expected return and standard deviation of 1 of the 10 stocks in Table 8.1. There are many possible combinations of expected return and standard deviation from investing in a *mixture* of these stocks. If you like high expected returns and dislike high standard deviations, you will prefer portfolios along the red line. These are *efficient* portfolios. We have marked the three efficient portfolios described in Table 8.1 (A, B, and C).



The figure shows the possible outcomes of choosing from 10 stocks to make a portfolio. Each diamond marks the combination of risk and return offered by a different individual security.

Which combination is best?

- To <u>go up (to increase expected return)</u> and <u>to the left (to reduce risk)</u>.

- Go as far as you can, and you will end up with one of the portfolios that lies along the red line.

→ Portfolios on the red line are **efficient portfolios**, which offer **the highest expected return for any level of risk**.

We want to deploy an investor's funds to **give the highest expected return for a given standard deviation**.

We Introduce Borrowing and Lending

By introducing a risk-free asset r_f that allows investors to lend or borrow at the risk-free rate, we can expand the risk-return opportunities available for investment.

The new portfolio of risky free asset and all risky stocks can be viewed as **a portfolio of risk free asset and a risky portfolio** by all risky stocks.

$$r = (1-\alpha) \times r_f + \alpha \times r_s$$
$$\sigma = \alpha \times \sigma_s$$

Suppose that you can also **lend or borrow money at some risk-free rate** of interest r_f.

If you invest some of your money in Treasury bills (i.e., lend money) and place the remainder in common stock portfolio S, you can obtain any combination of expected return and risk along the straight line joining r_f and S in Figure 8.5.

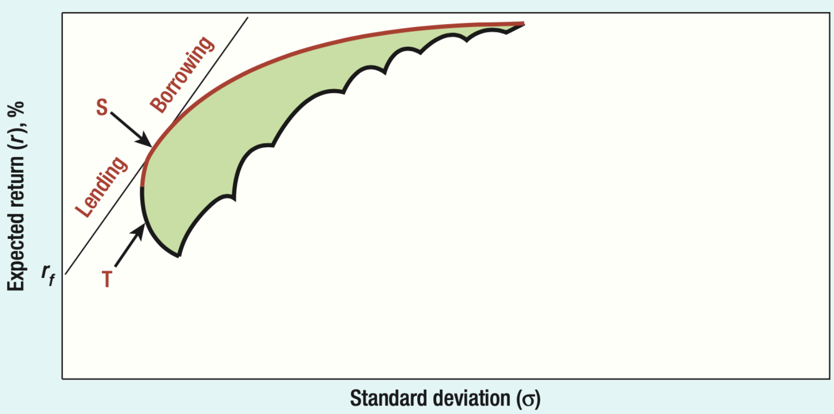
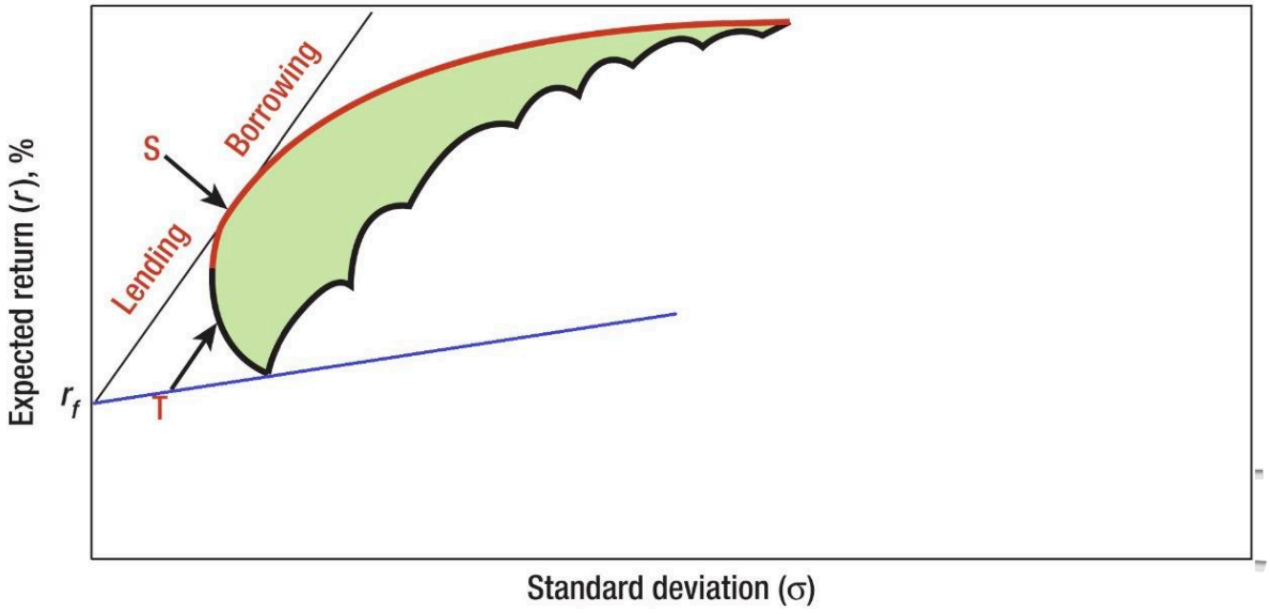


FIGURE 8.5

Lending and borrowing extend the range of investment possibilities. If you invest in portfolio S and lend or borrow at the risk-free interest rate, r_f, you can achieve any point along the straight line from r_f through S. This gives you the highest expected return for any level of risk. There is no point in investing in a portfolio like T.

The new choice set is **the angle area between black line and blue line**



EXAMPLE

Suppose that portfolio S has an **expected return of 15%** and a **standard deviation of 16%**.

Treasury bills offer an **interest rate (r_f) of 5%** and are risk-free (i.e., **their standard deviation is zero**).

1. **Lending**

If you **invest half your money in portfolio S and lend the remainder at 5%**, the expected return on your investment is likewise halfway between the expected return on S and the interest rate on Treasury bills:

$$r = \frac{1}{2} \times expected\ return\ on\ S + \frac{1}{2} \times interest\ rate = 10\%$$

And the standard deviation is halfway between the standard deviation of S and the standard deviation of Treasury bills:

$$\sigma = \frac{1}{2} \times standard\ deviation\ of\ S + \frac{1}{2} \times standard\ deviation\ of\ bills = 8\%$$

→ *This calculation has been simplified since* $\sigma_2 = 0$*.*

2. **Borrowing**

You borrow at the Treasury bill rate **an amount equal to your initial wealth**, and you **invest everything in portfolio S**. You have twice your own money invested in S, but you have to *pay* interest on the loan.

Therefore your expected return is

$$r = 2 \times expected\ return\ on\ S - 1 \times interest\ rate = 25\%$$

And the standard deviation of your investment is

$$\sigma = 2 \times standard\ deviation\ of\ S - 1 \times standard\ deviation\ of\ bills = 32\%$$

Some conclusions:

When you **lend** a portion of your money, you end up **partway between** r_f **and S**; if you can **borrow** money at the risk-free rate, you can extend your possibilities **beyond S**.

<u>You can get the **highest expected return** by a mixture of portfolio S and borrowing or lending.</u>

S is the *best* efficient portfolio. There is no reason ever to hold, say, portfolio T.

How to find the best efficient portfolio?

Start on the vertical axis at r_f and draw the steepest line you can to the curved red line of efficient portfolios. That line will be **tangent to the red line**. **The efficient portfolio at the tangency point** is better than all the others.

→ <u>It offers the highest *ratio* of risk premium to standard deviation.</u>

Sharpe ratio: <u>The ratio of the risk premium to the standard deviation.</u>

$$Sharpe\ ratio = \frac{Risk\ premium}{standard\ deviation} = \frac{r - r_f}{\sigma}$$

Investors track Sharpe ratios to measure the risk-adjusted performance of investment managers.

Separate the investor's job into two stages

1. **The best portfolio of common stocks** must be selected—S in our example.
2. This portfolio must be **blended with borrowing or lending** to **obtain an exposure to risk that suits the particular investor's taste**. Each investor, therefore, should put money into just two benchmark investments—a risky portfolio S and a risk-free loan (borrowing or lending).

- In a market that is **not fully competitiv**e, if you have better information than your rivals, you will want the portfolio to include relatively large investments in **the stocks you think are undervalued**.
- In a **competitive market**, there is no reason to hold a different portfolio of common stocks from anybody else. In other words, you might just as well **hold the market portfolio**. That is why many professional investors invest in a market-index portfolio and why most others hold well-diversified portfolios.

The Relationship Between Risk and Return

Treasury bills have a beta of *0* and ***a risk premium of 0***.

*The market portfolio of common stock*s has average market risk: its beta is *1.0*.

The difference between the return on the market and the interest rate is termed the *market risk premium.* Since 1900 the market risk premium $(r_m - r_f)$ has averaged 7.7% a year.

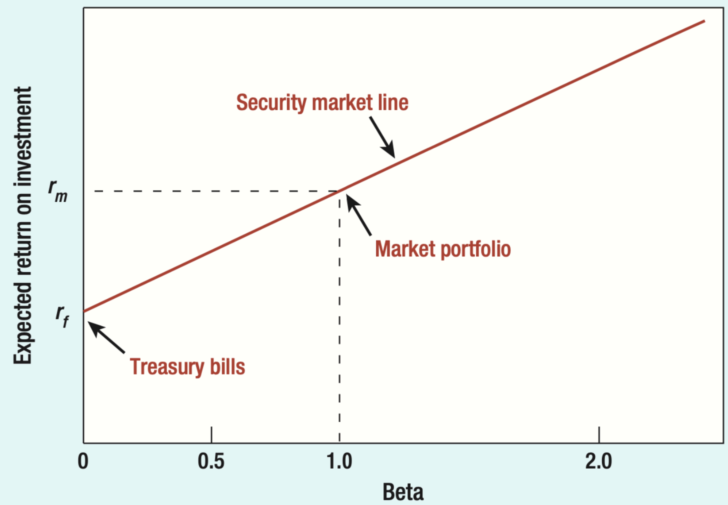


FIGURE 8.6

The capital asset pricing model states that the expected risk premium on each investment is proportional to its beta. This means that each investment should lie on the sloping security market line connecting Treasury bills and the market portfolio.

The **capital asset pricing model,** or **CAPM**: In a competitive market, the expected risk premium varies in **direct proportion** to beta.

→ In Figure 8.6 all investments must plot along the sloping line, known as the **security market line.**

The expected risk premium on an investment with a beta of 0.5 is, therefore, *half* the expected risk premium on the market; the expected risk premium on an investment with a beta of 2 is *twice* the expected risk premium on the market.

$$Expected\ risk\ premium\ on\ stock = beta \times expected\ risk\ premium\ on\ market$$

$$r - r_f = \beta(r_m - r_f)$$

Some Estimates of Expected Returns

The application of CAPM:

- To give an estimate of the expected return
- To find the discount rate for a new capital investment

In practice, choosing a discount rate is seldom so easy.

- You must learn how to adjust the expected return for the extra risk caused by company borrowing
- Also need to consider the difference between short- and long-term interest rates

Review of the Capital Asset Pricing Model

The basic principles of portfolio selection:

1. Investors like **high expected return and low standard deviation**. Common stock portfolios that offer the highest expected return for a given standard deviation are known as *efficient portfolios.*
2. If the investor can lend or borrow at the risk-free rate of interest, one efficient portfolio is better than all the others: **the portfolio that offers the highest ratio of risk premium to standard deviation** (that is, portfolio S in Figure 8.5). **A risk-averse investor** will put part of his money in this efficient portfolio and part in the risk-free asset. **A risk-tolerant investor** may put all her money in this portfolio or she may borrow and put in even more.
3. The composition of this best efficient portfolio depends on the investor's assessments of expected returns, standard deviations, and correlations. But suppose everybody has the same information and the same assessments. If there is **no superior information**, each investor should hold the same portfolio as everybody else; in other words, **everyone should hold the market portfolio**.

The risk of individual stocks:

1. Do not look at the risk of a stock in isolation but at **its contribution to portfolio risk**. This contribution depends on the stock's sensitivity to changes in the value of the portfolio.
2. **A stock's sensitivity to changes in the value of the *market* portfolio is known as *beta.*** Beta, therefore, measures the marginal contribution of a stock to the risk of the market portfolio.

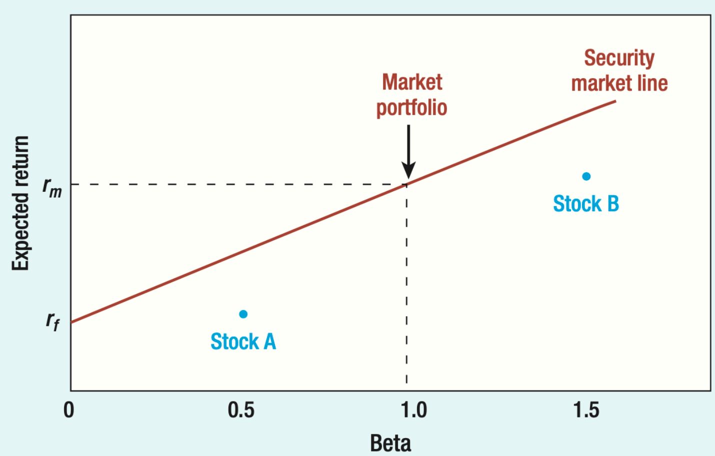
CAPM:

If everyone holds the market portfolio, and if beta measures each security's contribution to the market portfolio risk, then it is no surprise that the **risk premium** demanded by investors is **proportional to beta**.

What If a Stock Did Not Lie on the Security Market Line?

FIGURE 8.7

In equilibrium no stock can lie below the security market line. For example, instead of buying stock A, investors would prefer to lend part of their money and put the balance in the market portfolio. And instead of buying stock B, they would prefer to borrow and invest in the market portfolio.



An investor can **always** obtain an expected risk premium of $\beta(r_m - r_f)$ by holding a mixture of the market portfolio and a risk-free loan. So in well-functioning markets **nobody** will hold a stock that offers an expected risk premium of **less** than $\beta(r_m - r_f)$.

If we take all stocks together, we have the market portfolio. Therefore, we know that stocks *on average* lie on the line. Since none lies *below* the line, then **there also can't be any that lie *above* the line**.

Thus **each and every stock *must*** lie on the security market line and offer an expected risk premium of $r - r_f = \beta(r_m - r_f)$.

Validity and Role of the Capital Asset Pricing Model

Some common agreements:

- Investors require some extra return for taking on risk.
- Investors are concerned with those risks that they cannot eliminate by diversification.

  If this were not so, stock prices will increase whenever two companies merge to spread their risks. And investment companies which invest in the shares of other firms are more highly valued than the shares they hold.

Tests of the Capital Asset Pricing Model

- **Problem 1: Risk premium is not proportional to the beta**

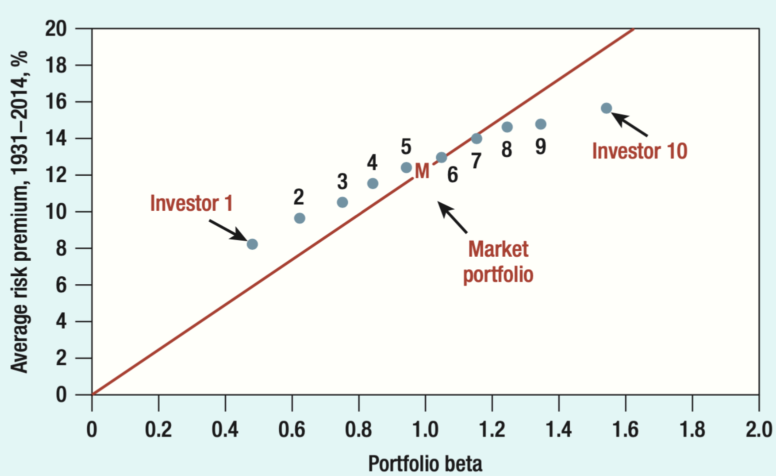


FIGURE 8.8

The capital asset pricing model states that the expected risk premium from any investment should lie on the security market line. The dots show the actual average risk premiums from portfolios with different betas. The high-beta portfolios generated higher average returns, just as predicted by the CAPM. But the high-beta portfolios plotted below the market line, and the low-beta portfolios plotted above. A line fitted to the 10 portfolio returns would be "flatter" than the market line.

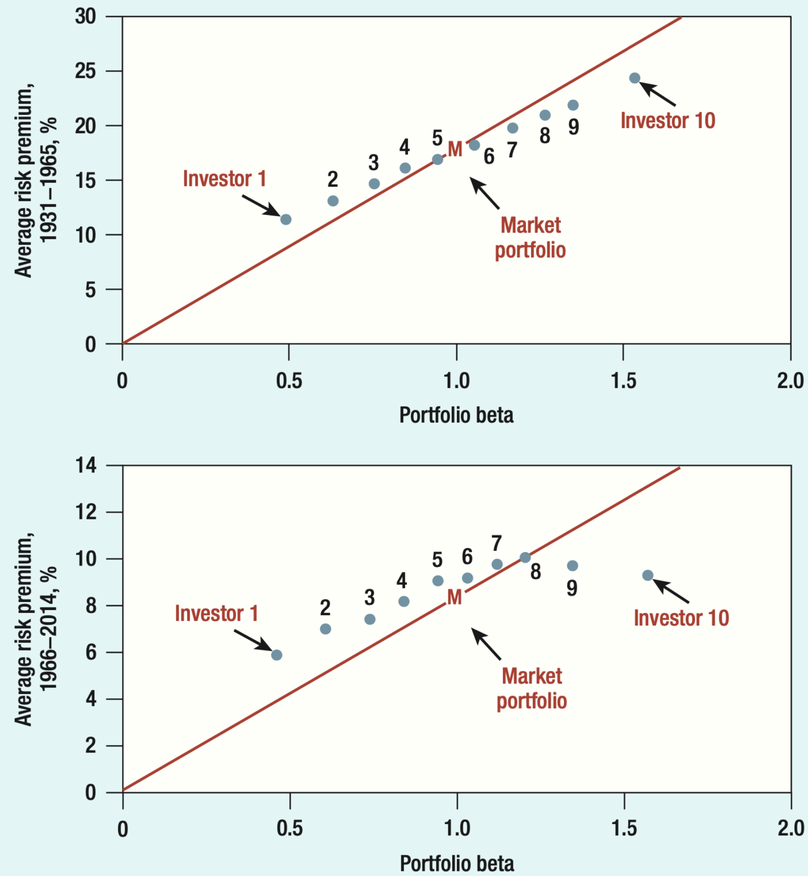
Source: F. Black, "Beta and Return," *Journal of Portfolio Management* 20 (Fall 1993), pp. 8–18. Used with permission of Institutional Investor, Inc., **www.iijournals.com**. All rights reserved. Updates courtesy of Adam Kolasinski.

Though high-beta stocks performed better than low-beta stocks, the difference was not as great as the CAPM predicts.

FIGURE 8.9

The relationship between beta and actual average return has been weaker since the mid-1960s. Stocks with the highest betas have provided poor returns.

Source: F. Black, "Beta and Return," *Journal of Portfolio Management* 20 (Fall 1993), pp. 8–18. Used with permission of Institutional Investor, Inc., **www.iijournals.com**. All rights reserved. Updates courtesy of Adam Kolasinski.



What is going on here?

→ Actual stock returns reflect expectations, but they also embody lots of *"noise"*—the steady flow of surprises that conceal whether on average investors have received the returns they expected.

→ Perhaps the best that we can do is to focus on **the longest period** for which there is reasonable data.

- **Problem 2: The return has been related to other measures**

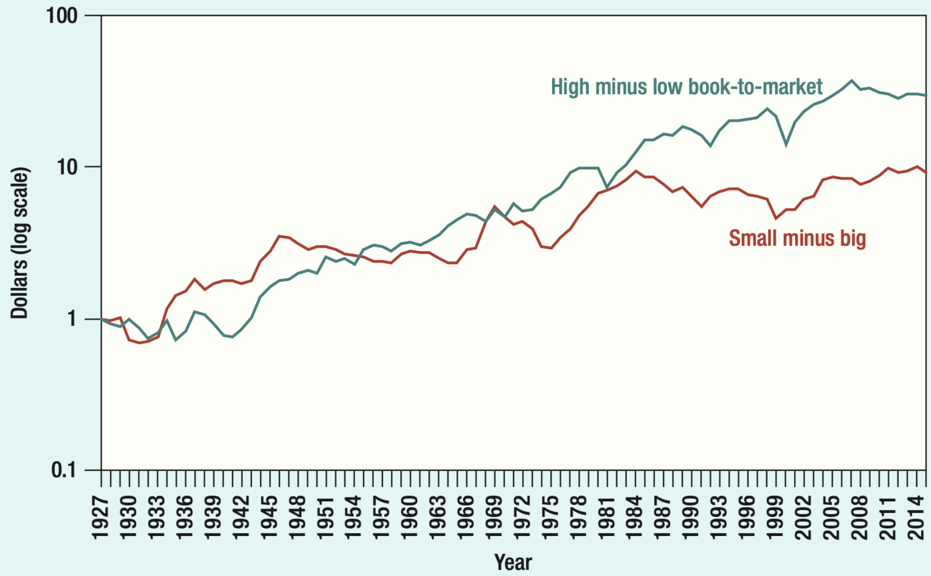


FIGURE 8.10

The red line shows the cumulative difference between the returns on small-firm and large-firm stocks from 1926 to 2014. The green line shows the cumulative difference between the returns on high book-to-market-value stocks (i.e., value stocks) and low book-to-market-value stocks (i.e., growth stocks).

Source: Kenneth French's website, **mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.** Used with permission.

- **The red line:** the cumulative difference between the returns on <u>small-firm stocks and large-firm stocks</u>.
  - Over the long haul **small-cap** stocks' owners have made substantially **higher returns**.
- **The green line:** the cumulative difference between the returns on <u>value stocks and growth stocks</u>.
  - **Value stocks** here are defined as those with **high** ratios of book value to market value. **Growth stocks** are those with **low** ratios of book to market. Notice that **value stocks** have provided a **higher long-run return** than growth stocks. Since 1926 the average annual difference between the returns on value and growth stocks has been 4.8%.

<u>→ Risks in “small-cap” stocks and value stocks that were not captured by beta.</u>

Assumptions behind the Capital Asset Pricing Model

Some "false" assumptions:

- Treasury bills do not guarantee a *real* return. There is still some uncertainty about **inflation**. → not risk-free
- Generally borrowing rates are **higher** than lending rates. → not equal

Some Alternative Theories

Arbitrage Pricing Theory

Arbitrage pricing theory, or **APT,** starts by *assuming* that each stock’s return depends partly on pervasive <u>macroeconomic</u> influences or **“factors”** and partly on **“noise”**—events that are <u>unique to that company</u>.

$$Return = a + b_1(r_{factor\ 1}) + b_2(r_{factor\ 2}) + b_3(r_{factor\ 3}) + \cdots + noise$$

Arbitrage pricing theory states that the expected risk premium on a stock should depend on the expected risk premium **associated with each factor** and **the stock's sensitivity to each of the factors** (b_1, b_2, b_3, etc.).

$$Expected\ risk\ premium = r - r_f = b_1(r_{factor\ 1} - r_f) + b_2(r_{factor\ 2} - r_f) + \cdots$$

The Three-Factor Model

To estimate expected returns, you first need to follow **three steps**:

- **Step 1:** Identify a reasonably short list of macroeconomic factors that could affect stock returns.
- **Step 2:** Estimate the expected risk premium on each of these factors ($r_{factor\ 1} - r_f$, etc.).
- **Step 3:** Measure the sensitivity of each stock to the factors (b_1, b_2, etc.).

The Fama–French three-factor model:

$$r - r_f = b_{market}(r_{market\ factor}) + b_{size}(r_{size\ factor}) + b_{book-to-market}(r_{book-to-market\ factor})$$

Step 1: Identify the Factors

Factor	Measured by
Market factor	Return on market index *minus* risk-free interest rate
Size factor	Return on small-firm stocks *less* return on large-firm stocks
Book-to-market factor	Return on high book-to-market-ratio stocks *less* return on low book-to-market-ratio stocks

Step 2: Estimate the Risk Premium for Each Factor

- market risk premium is 7%
- the difference between the annual returns on small and large capitalization stocks averaged 3.5%
- the difference between the returns on stocks with high and low book-to-market ratios averaged 4.8%

Step 3: Estimate the Factor Sensitivities

	Three-Factor Model				CAPM
	Factor Sensitivities				
	b_{market}	b_{size}	$b_{book\text{-}to\text{-}market}$	Expected Return[a]	Expected Return[b]
Autos	1.37	0.62	–0.07	13.4%	12.7%
Banks	1.12	0.02	0.74	13.5	10.6
Chemicals	1.35	0.05	–0.19	10.7	11.3
Computers	1.17	–0.10	–0.33	8.3	9.7
Construction	1.13	0.82	0.57	15.5	12.1
Food	0.52	–0.15	0.00	5.1	5.4
Oil and gas	1.21	–0.20	0.02	9.9	10.1
Pharmaceuticals	0.77	–0.27	–0.31	5.0	4.9
Telecoms	0.87	–0.08	0.04	8.0	8.0
Utilities	0.48	–0.16	0.08	5.2	5.2

TABLE 8.3 Estimates of expected equity returns for selected industries using the Fama–French three-factor model and the CAPM.

[a] The expected return equals the risk-free interest rate plus the factor sensitivities multiplied by the factor risk premiums, that is, $r_f + (b_{market} \times 7) + (b_{size} \times 3.5) + (b_{book\text{-}to\text{-}market} \times 4.8)$.
[b] Estimated as $r_f + \beta(r_m - r_f)$, that is, $r_f + \beta \times 7$. Note that we used *simple* regression to estimate β in the CAPM formula. This beta may, therefore, be different from b_{market} that we estimated from a *multiple* regression of stock returns on the three factors.

Source: The industry indexes are value-weighted indexes from Kenneth French's website, **mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html**. Used with permission.

e.g. Computer stocks

Assuming that the risk-free interest rate is 2%.

$$r - r_f = (1.17 \times 7) - (0.10 \times 3.5) - (0.33 \times 4.8) = 6.2\%$$
$$r = 2\% + 6.2\% = 8.2\%$$

The Three-Factor model is not widely used in estimating the cost of equity, but it is widely used in measuring the performance of mutual funds, pension funds and other professionally managed portfolios.

Appendix: Risk and Return

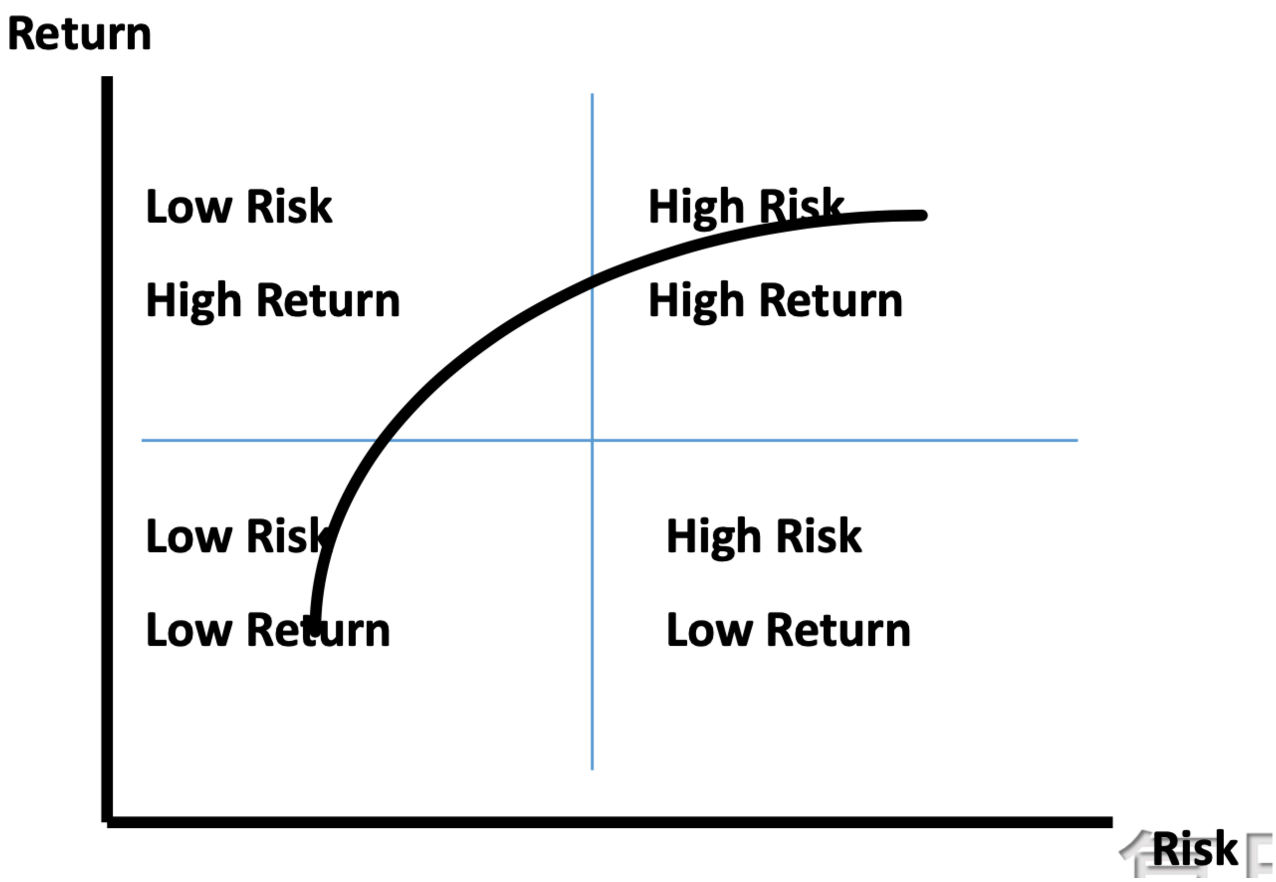
The efficient frontier

The efficient frontier is the set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return.

- Portfolios that lie below the efficient frontier are sub-optimal because they do not provide enough return for the level of risk.
- Portfolios that cluster to the right of the efficient frontier are sub-optimal because they have a higher level of risk for the defined rate of return.
- By adding more stocks to the portfolio one can move towards the efficient portfolio.

The risk-return space is divided into four quadrants:

- The efficient frontier spans the three quadrants
- Rational investors will not invest in high-risk, low-return portfolios



Summary

Chapter 8

The basic principles of portfolio selection boil down to a commonsense statement that investors try to <u>increase the expected return on their portfolios and to reduce the standard deviation of that return</u>. **A portfolio that gives the highest expected return for a given standard deviation, or the lowest standard deviation for a given expected return, is known as an *efficient portfolio.*** To work out which portfolios are efficient, an investor must be able to state the expected return and standard deviation of each stock and the degree of **correlation** between each pair of stocks.

Investors who are r**estricted to holding common stocks** should choose **efficient portfolios that suit their attitudes to risk**. But investors who can also **borrow and lend at the risk-free rate of interest** should choose **the *best* common stock portfolio** *regardless* of their attitudes to risk. Having done that, they can then set the risk of their overall portfolio by deciding what proportion of their money they are willing to invest in stocks. **The best efficient portfolio offers the highest ratio of forecasted risk premium to portfolio standard deviation.**

For an investor who has only the same opportunities and information as everybody else, **the best stock portfolio is the same as the best stock portfolio for other investors**. In other words, he or she should invest in **a mixture of the market portfolio and a risk-free loan** (i.e., borrowing or lending).

A stock's marginal contribution to portfolio risk is measured by its sensitivity to changes in the value of the portfolio. **The marginal contribution of a stock to the risk of the *market portfolio* is measured by *beta.*** That is the fundamental idea behind **the capital asset pricing model (CAPM)**, which concludes that **each security's expected risk premium should increase in proportion to its beta**:

$$Expected\ risk\ premium\ on\ stock = beta \times expected\ risk\ premium\ on\ market$$
$$r - r_f = \beta(r_m - r_f)$$

The capital asset pricing theory is the best-known model of risk and return. It is plausible and widely used but far from perfect. Actual returns are related to beta over the long run, but the relationship is not as strong as the CAPM predicts, and other factors seem to explain returns bet ter since the mid-1960s. **Stocks of small companies, and stocks with high book values relative to market prices, appear to have risks not captured by the CAPM.**

The arbitrage pricing theory offers an alternative theory of risk and return. It states that the expected risk premium on a stock should depend on the stock's exposure to several pervasive mac-roeconomic factors that affect stock returns:

$$Expected\ risk\ premium = r - r_f = b_1(r_{factor\ 1} - r_f) + b_2(r_{factor\ 2} - r_f) + \cdots$$

Here *b*'s represent the individual security's sensitivities to the factors, and $r_{factor} - r_f$ is the risk premium demanded by investors who are exposed to this factor.

Arbitrage pricing theory does not say what these factors are. It asks for economists to hunt for unknown game with their statistical toolkits. Fama and French have suggested **three factors**:

- <u>The return on the market portfolio less the risk-free rate of interest.</u>
- <u>The difference between the return on small- and large-firm stocks.</u>
- <u>The difference between the return on stocks with high book-to-market ratios and stocks with low book-to-market ratios.</u>

In the Fama–French three-factor model, the expected return on each stock depends on its exposure to these three factors.

Each of these different models of risk and return has its fan club. However, all financial economists agree on **two basic ideas**: (1) I<u>nvestors require extra expected return for taking on risk</u>, and (2) they appear to <u>be concerned predominantly with the risk that they cannot eliminate by diversification</u>.