# Introduction to Risk and Return

## Over a Century of Capital Market History in One Easy Lesson

Treasury bills are about as safe an investment as you can make. There is no risk of default, and their short maturity means that the prices of Treasury bills are relatively stable.

There is still some uncertainty about inflation.

- Bond prices *fall when interest rates rise* and *rise when interest rates fall*.
- Common stocks' price change with *all the ups and downs of the issuing companies*.

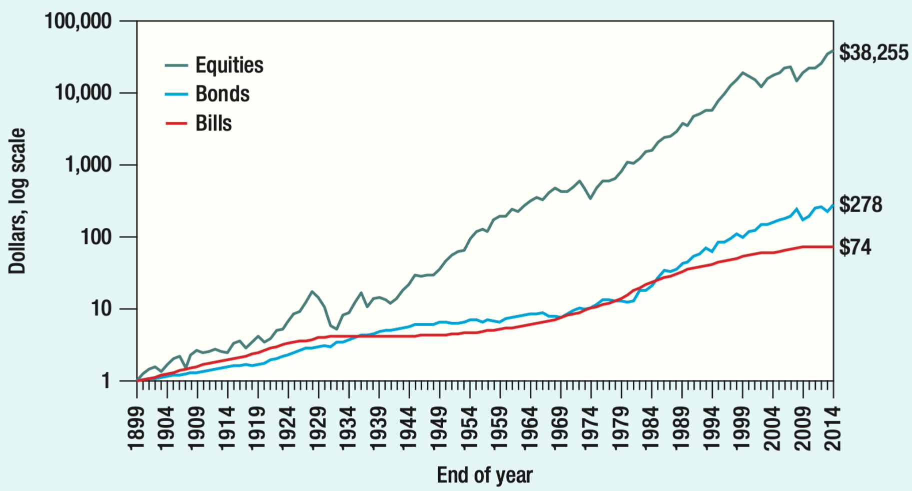


**FIGURE 7.1**

How an investment of $1 at the end of 1899 would have grown by the end of 2014, assuming reinvestment of all dividend and interest payments.

***Source:*** E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

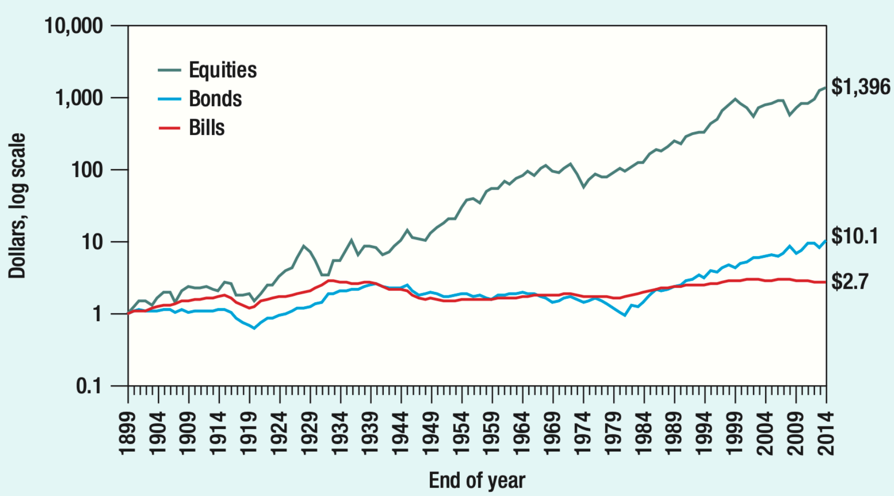


**FIGURE 7.2**

How an investment of $1 at the end of 1899 would have grown by the end of 2014, assuming reinvestment of all dividend and interest payments. Compare this plot with Figure 7.1, and note how inflation has eroded the purchasing power of returns to investors.

***Source:*** E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

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Averages of the 115 annual rates of return for each portfolio:

**TABLE 7.1** Average rates of return on U.S. Treasury bills, government bonds, and common stocks, 1900–2014 (figures in % per year).

***Source:*** E. Dimson, P. R. Marsh, and M. Staunton, *Triumph of the Optimists: 101 Years of Investment Returns,* (Princeton, NJ: Princeton University Press, 2002), with updates provided by the authors.

| | Average Annual Rate of Return | | |
|---|---|---|---|
| | Nominal | Real | Average Risk Premium (Extra Return versus Treasury Bills) |
| Treasury bills | 3.8 | 1.0 | 0 |
| Government bonds | 5.4 | 2.4 | 1.5 |
| Common stocks | 11.5 | 8.4 | 7.7 |

*The average rate of inflation over this period was about 3% per year.*

*By taking on the risk of common stocks, investors earned a **risk premium** of 11.5 – 3.8 = **7.7%** over the return on Treasury bills.*

Annual rates of return for common stocks fluctuate so much that averages taken over short periods are meaningless, so we look back over such **a long period** to measure average rates of return

## Arithmetic Averages and Compound Annual Returns

If the **cost of capital** is estimated from **historical returns or risk premiums**, use **arithmetic averages**, not compound annual rates of return.

## Using Historical Evidence to Evaluate Today's Cost of Capital

Suppose there is an investment project that has the same risk as the **market portfolio**.

- **Market return** $r_m$
- **Risk-free interest rate** $r_f$

$\rightarrow r_m = r_f + normal\ risk\ premium$

**Crucial assumption:** There is a normal, stable risk premium on the market portfolio, so that the expected *future* risk premium can be measured by the average past risk premium.

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***What the risk premium really is?***

- Many financial managers and economists believe that long-run historical returns are the best measure available and work with a risk premium of about **7.7%**
- Others have a gut instinct that investors don't need such a large risk premium to persuade them to hold common stocks

# Measuring Portfolio Risk

***How to estimate discount rates for assetsthat are neither safe projects nor projects with average risk?***

Need to learn:

- how to measure risk
- the relationship between risks borne and risk premiums demanded

# Variance and Standard Deviation

The standard statistical measures of spread are **variance** and **standard deviation**.

- The **variance** of the market return is the expected squared deviation from the expected return.

$$Variance(\tilde{r}_m) = the\ expected\ value\ of\ (\tilde{r}_m - r_m)^2 = \frac{1}{N-1}\sum_{t=1}^{N}(\tilde{r}_{mt} - r_m)^2$$

→ $\tilde{r}_m$ is the actual return

→ $\tilde{r}_{mt}$ is the market return in period t

→ $r_m$ is the expected return of $\tilde{r}_m$, that is, the mean of the values of $\tilde{r}_{mt}$

- The **standard deviation** is simply the square root of the variance.

$$Standard\ deviation\ of\ \tilde{r}_m = \sqrt{variance(\tilde{r}_m)}$$

*Standard deviation is often denoted by $\sigma$ and variance by $\sigma^2$.*

# Measuring Variability

We use variance or standard deviation to summarize the spread of possible outcomes. They are natural indexes of risk.

To **estimate the variability of any portfolio** of stocks or bonds by:

- Identifying the possible outcomes
- Assigning a probability to each outcome
- Grinding through the calculations

Possibilities come from observing the past variabilities.

The annual standard deviations and variances observed for our three portfolios over the period 1900–2014 were:

| Portfolio | Standard Deviation ($\sigma$) | Variance ($\sigma^2$) |
|---|---|---|
| Treasury bills | 2.9 | 8.2 |
| Government bonds | 9.1 | 83.3 |
| Common stocks | 19.9 | 395.6 |

## How Diversification Reduces Risk

**Diversification:** Strategy designed to reduce risk by spreading the portfolio across many investments.

The individual stocks for the most part are **more variable** than the market indexes.

**Diversification reduces variability.**

→ The market portfolio is made up of individual stocks, but its variability doesn't reflect the average variability of its components.

*Even a little diversification can provide a substantial reduction in variability.*

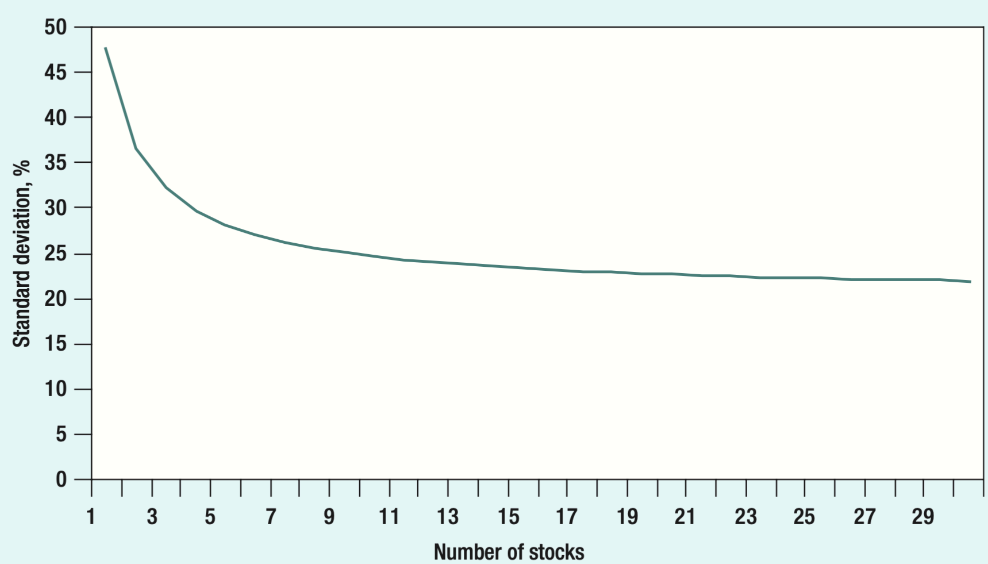



**FIGURE 7.9**

Average risk (standard deviation) of portfolios containing different numbers of stocks. The stocks were selected randomly from stocks traded on the New York Exchange from 2006 through 2010. Notice that diversification reduces risk rapidly at first, then more slowly.

***Note:*** The figure shows the average risk of randomly selected portfolios divided equally among *N* stocks. The group of stocks consists of all those continuously quoted on the NYSE, 2006–2010.

**Diversification works because prices of different stocks do not move exactly together.**

***Specific risk vs. market risk***

- **Specific risk**

  The risk that potentially can be eliminated by diversification is called **specific risk**.

  - Specific risk stems from the fact that many of the perils that surround an individual company are peculiar to that company and perhaps its immediate competitors.
  - Specific risk may be called *unsystematic risk, residual risk, unique risk,* or *diversifiable risk*.
  - Example: Boeing 737 max two deadly crashes

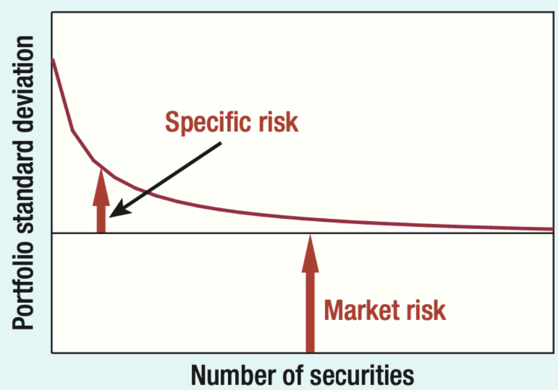
- **Market risk**

  Risk that you can't avoid, regardless of how much you diversify, is generally known as **market risk**.

  - Market risk stems from the fact that there are other economywide perils that threaten all businesses.
  - That is why stocks have a tendency to move together. And that is why investors are exposed to market uncertainties, no matter how many stocks they hold.
  - Market risk may be called *systematic risk* or *undiversifiable risk*.
  - Example: Covid-19, great recession

**FIGURE 7.11**

**Diversification eliminates specific risk. But there is some risk that diversification *cannot* eliminate. This is called *market risk.***




- If you have only a single stock, **specific risk is very important**.
- **For a reasonably well-diversified portfolio, *only* market risk matters.**

# Calculating Portfolio Risk

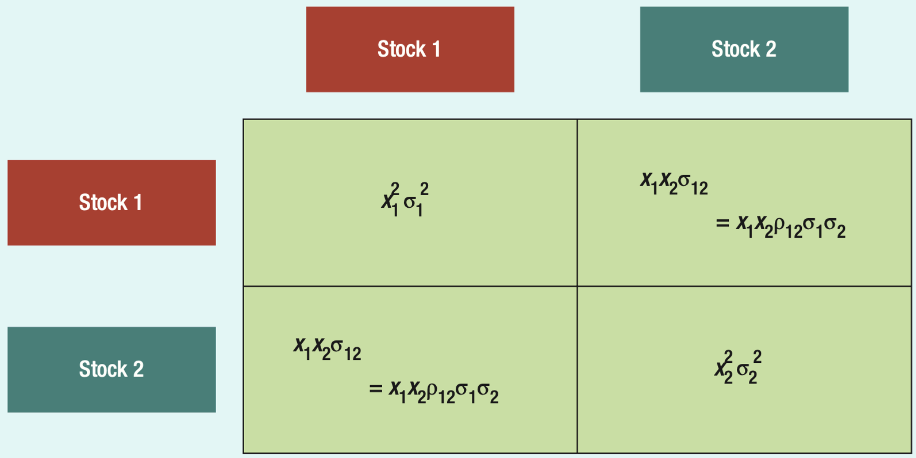



**FIGURE 7.12**

The variance of a two-stock portfolio is the sum of these four boxes. $x_1$, $x_2$ = proportions invested in stocks 1 and 2; $\sigma_1^2$, $\sigma_2^2$ = variance of stock returns; $\sigma_{12}$ = covariance of returns ($\rho_{12}\ \sigma_1\ \sigma_2$); $\rho_{12}$ = correlation between returns on stocks 1 and 2.

$$Portfolio\ variance = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2(x_1x_2\rho_{12}\sigma_1\sigma_2)$$

The **greatest** payoff to diversification comes when the two stocks are **negatively correlated**.

When there is **perfect negative correlation**, there is always a portfolio strategy (represented by a particular set of portfolio weights) that will **completely eliminate risk**. It's too bad perfect negative correlation doesn't really occur between common stocks.

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***What if there are 3 stocks in the portfolio?***

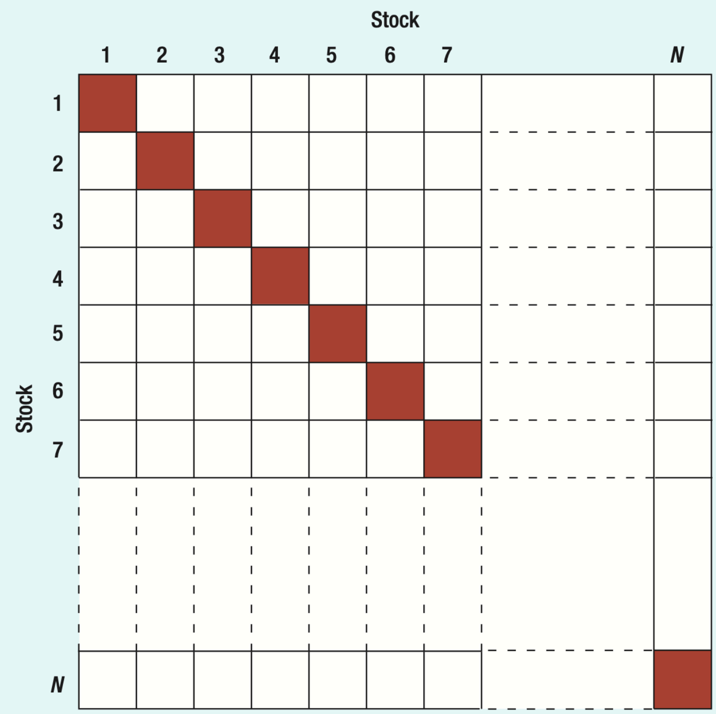
First calculate 2 of them as a portfolio, then add the third one onto the existed portfolio.

## General Formula for Computing Portfolio Risk

The method for calculating portfolio risk can easily be extended to portfolios of three or more securities.

**FIGURE 7.13**

To find the variance of an *N*-stock portfolio, we must add the entries in a matrix like this. The diagonal cells contain variance terms ($x^2\sigma^2$) and the off-diagonal cells contain covariance terms ($x_i x_j \sigma_{ij}$).




## Limits to Diversification

When there are many securities, the number of covariances is much larger than the number of variances. Thus the variability of a well-diversified portfolio reflects **mainly the covariances**.

Suppose we are dealing with portfolios in which equal investments are made in each of *N* stocks. The proportion invested in each stock is, therefore, 1/*N.*

There are $N$ variance boxes and $N^2 - N$ covariance boxes. Therefore,

$$\begin{aligned} Portfolio\ variance &= N\left(\frac{1}{N}\right)^2 \times average\ variance + (N^2 - N)\left(\frac{1}{N}\right)^2 \times average\ covariance \\ &= \frac{1}{N} \times average\ variance + \left(1 - \frac{1}{N}\right) \times average\ covariance \end{aligned}$$

Notice that as *N* increases, the portfolio variance steadily approaches the average covariance.

If the average covariance were zero, it would be possible to eliminate *all* risk by holding a sufficient number of securities. Unfortunately common stocks move together, not independently. Thus most of the stocks that the investor can actually buy are tied together in a web of positive covariances that set the limit to the benefits of diversification.

**Market risk is the average covariance** that constitutes the bedrock of risk remaining after diversification has done its work.

# Do I Really Have to Add up 25 Million Boxes?

You can "buy the market" by purchasing shares in an **index fund:** a mutual fund or exchange traded fund (ETF) that invests in the market index that you want to track.

*Smart and serious investors hold widely diversified portfolios;* their starting portfolio is often the market itself.

1. start with a widely diversified portfolio, for example a market index fund
2. concentrate on a few stocks as possible additions

# How Individual Securities Affect Portfolio Risk

**Market portfolio is a value-weighted portfolio of all the assets in the economy.**

**The risk of a well-diversified portfolio depends on the market risk of the securities included in the portfolio.**

## Market Risk Is Measured by Beta

**Beta** is a relative measure of risk. It measures the **sensitivity** of stock's return to return on market portfolio.

- Stocks with betas **greater than 1.0** tend to **amplify** the overall movements of the market.
- Stocks with betas **between 0 and 1.0** tend to move in the same direction as the market, but **not as far**.

- The market is the portfolio of all stocks, so the "average" stock has a beta of **1.0**.

*e.g.*

Over the five years from November 2009 to October 2014, Ford had a beta of 1.44. If the future resembles the past, this means that *on average*:

- When the market rises an extra 1%, Ford's stock price will rise by an extra 1.44%.
- When the market falls an extra 2%, Ford's stock prices will fall an extra 2 × 1.44 = 2.88%

## Why Security Betas Determine Portfolio Risk

***Two crucial points about security risk and portfolio risk:***

- Market risk accounts for most of the risk of a well-diversified portfolio.

- The beta of an individual security measures its sensitivity to market movements.

### Where's Bedrock?

With more securities, and therefore better diversification, portfolio risk declines until all specific risk is eliminated and only the bedrock of market risk remains.

Where's bedrock? It depends on **the average beta of the securities selected**.

Suppose we constructed a portfolio containing a large number of stocks drawn randomly from the whole market. What would we get?

- The market itself, or a portfolio *very* close to it.
- The portfolio **beta** would be **1.0**, and **the correlation with the market** would be **1.0**.

**The standard deviation of the market were 20% (roughly its average for 1900–2014).**

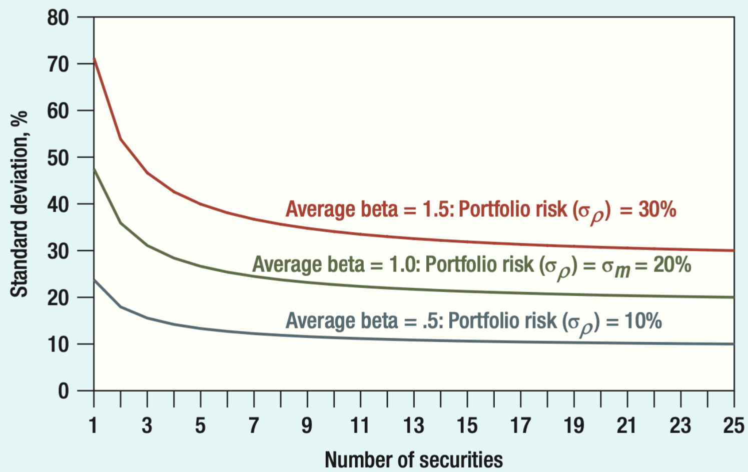



**FIGURE 7.15**

The green line shows that a well diversified portfolio of randomly selected stocks ends up with $\beta = 1$ and a standard deviation equal to the market's—in this case 20%. The upper red line shows that a well diversified portfolio with $\beta = 1.5$ has a standard deviation of about 30%—1.5 times that of the market. The lower blue line shows that a well-diversified portfolio with $\beta = .5$ has a standard deviation of about 10%—half that of the market.

***Note:*** In this figure we assume for simplicity that the total risks of individual stocks are proportional to their market risks.

**The risk of a well-diversified portfolio is proportional to the portfolio beta, which equals the average beta of the securities included in the portfolio.**

The **contribution** of an individual stock to the risk of a well-diversified portfolio depends on its market risk, how sensitive it is to market movement.

## Calculating Beta

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

$\sigma_{im}$ **is the *covariance* between the stock returns and the market returns**

$\sigma_m^2$ **is the variance of the returns on the market**

It turns out that this ratio of covariance to variance measures a stock's contribution to portfolio risk.

**BEYOND THE PAGE**

Try It! Table 7.7: Calculating Anchovy Queen's beta

mhhe.com/brealey12e

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
| 2 | | | | | | | Product of |
| 3 | | | | Deviation | Deviation | Squared | deviations |
| 4 | | | | from | from average | deviation | from average |
| 5 | | Market | Anchovy Q | average | Anchovy Q | from average | returns |
| 6 | Month | return | return | market return | return | market return | (cols 4 × 5) |
| 7 | 1 | −8% | −11% | −10 | −13 | 100 | 130 |
| 8 | 2 | 4 | 8 | 2 | 6 | 4 | 12 |
| 9 | 3 | 12 | 19 | 10 | 17 | 100 | 170 |
| 10 | 4 | −6 | −13 | −8 | −15 | 64 | 120 |
| 11 | 5 | 2 | 3 | 0 | 1 | 0 | 0 |
| 12 | 6 | 8 | 6 | 6 | 4 | 36 | 24 |
| 13 | Average | 2 | 2 | | Total | 304 | 456 |
| 14 | | | | Variance $= \sigma_m^2 = 304/6 = 50.67$ | | | |
| 15 | | | | Covariance $= \sigma_{im} = 456/6 = 76$ | | | |
| 16 | | | | Beta $(\beta) = \sigma_{im}/\sigma_m^2 = 76/50.67 = 1.5$ | | | |

**TABLE 7.7** Calculating the variance of the market returns and the covariance between the returns on the market and those of Anchovy Queen. Beta is the ratio of the variance to the covariance (i.e., $\beta = \sigma_{im}/\sigma_m^2$).

# Diversification and Value Additivity

***Is a diversified firm more attractive to investors than an undiversified one?***

**No.**

- If investors were *not* able to hold a large number of securities, then they might want firms to diversify for them. But investors *can* diversify. In many ways they can do so **more easily than firms**.
- If investors can diversify **on their own** account, they **will not pay any *extra* for firms that diversify**. And if they have a **sufficiently wide choice** of securities, they **will not pay any *less*** because they are unable to invest separately in each factory.

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**Diversification does not add to a firm's value or subtract from it. The total value is the sum of its parts.**

**Value additivity:** If the capital market establishes a value PV(A) for asset A and PV(B) for B, the market value of a firm that holds only these two assets is *PV(AB) = PV(A) + PV(B)*. A three-asset firm combining assets A, B, and C would be worth *PV(ABC) = PV(A) + PV(B) + PV(C)*, and so on for any number of assets.

Because of value additivity, the Net Present Value rule for capital budgeting works **even under uncertainty**.

# Appendix: International Diversification

Investors can reduce their risk by diversifying within a country as well as across countries.

Since the returns are not perfectly correlated, risk could have been reduced by diversifying internationally *rather than* just domestically.

# Summary

## Chapter 7

Our review of capital market history showed that **the returns to investors have varied according to the risks they have borne**. At one extreme, very safe securities like U.S. Treasury bills have provided an average return over 115 years of only 3.8% a year. The riskiest securities that we looked at were common stocks. The stock market provided an average return of 11.5%, a premium of 7.7% over the safe rate of interest.

This gives us **two benchmarks** for the opportunity cost of capital. If we are evaluating **a safe project**, we discount at the current risk-free rate of interest. If we are evaluating **a project of average risk**, we discount at the expected return on the average common stock. Historical evidence suggests that this return is 7.7% above the risk-free rate, but many financial managers and economists opt for a lower figure. That still leaves us with a lot of assets that don't fit these simple cases. Before we can deal with them, we need to learn how to measure risk.

Risk is best judged in a portfolio context. Most investors do not put all their eggs into one basket: They diversify. Thus the effective risk of any security cannot be judged by an examination of that security alone. Part of the uncertainty about the security's return is diversified away when the security is grouped with others in a portfolio.

**Risk in investment means that future returns are unpredictable.** This spread of possible outcomes is usually measured by **standard deviation**. The standard deviation of the *market portfolio,* as represented by the Standard and Poor's Composite Index, has averaged around **20%** a year.

Most individual stocks have higher standard deviations than this, but **much of their variability represents *specific* risk** that can be **eliminated by diversification**. **Diversification cannot eliminate *market* risk.** Diversified portfolios are exposed to variation in the general level of the market.

A security's **contribution to the risk of a well-diversified portfolio** depends on how the security is liable to be affected by a general market decline. This sensitivity to market movements is known as ***beta* (β).** **Beta measures the amount that investors expect the stock price to change for each additional 1% change in the market.** The average beta of all stocks is **1.0**. A stock with a beta **greater than 1** is unusually **sensitive** to market movements; a stock with a beta **below 1** is unusually **insensitive** to market movements. **The standard deviation of a well-diversified portfolio is proportional to its beta.** Thus a diversified portfolio invested in stocks with a beta of 2.0 will have twice the risk of a diversified portfolio with a beta of 1.0.

One theme of this chapter is that diversification is a good thing *for the **investor**.* **This does not imply that *firms* should diversify.** Corporate diversification is **redundant** if investors can diversify on their own account. Since diversification does not affect the value of the firm, present values add even when risk is explicitly considered. Thanks to *value additivity,* the net present value rule for capital budgeting works even under uncertainty.