

# Valuing Bonds

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The interest rates on government bonds are benchmarks for all interest rates. When government interest rates go up or down, corporate rates follow more or less proportionally.

## Using the Present Value Formula to Value Bonds

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Every year until the bond matures, bondholders collect regular interest payments. At maturity, when you get the final interest payment, they also get back the **face value** of the bond, which is called the bond's **principal**.

A **bond** is a legally binding agreement between a borrower and a lender.

### *Key features of a bond*

1. *Par value:* **Face amount/principal**; paid at maturity. Normally it is \$100, or \$1,000, or \$1 million...
2. *Coupon interest rate:* Stated interest rate. **Multiply by par value to get dollars of interest.** Generally fixed.
3. *Maturity:* Years until bond must be repaid. Declines.
4. *Issue date:* Date when bond was issued.
5. *Default risk:* Risk that issuer will not make interest or principal payments.

## A Short Trip to Paris to Value a Government Bond

**The bond's coupon:** A coupon or coupon payment is **the annual interest rate paid on a bond**, expressed as **a percentage of the face value** and paid from issue date until maturity.

Bond prices are usually expressed as a percentage of face value.

$$PV(\text{bond}) = PV(\text{annuity of coupon payments}) + PV(\text{final payment of principal})$$

→ Thus the bond can be valued as a package of an annuity (the coupon payments) and a single, final payment (the repayment of principal).

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The bond's **yield to maturity (YTM)**: The rate of return earned on a bond held to maturity. Also called "promised yield".

### **YTM = Discount Rate = Opportunity Cost of Capital**

- The YTM of a **pure discount bond** is the spot discount rate of same maturity:  $y = r_t$
- The YTM of a **coupon bond** is:

$$Price = \sum_{t=1}^N \frac{C_t}{(1+y)^t} + \frac{P}{(1+y)^T}$$

→  $C_t$  is the coupon received at  $t$ .  $P$  is the par value of the bond.

### YTM is the IRR from bond investment.

The yield to maturity (YTM) of a bond is single rate that discounts the payments on the bond to the purchase price.

### YTM vs. the spot discount rate

- *Yield to maturity* is the total rate of return that will have been earned by a bond when it makes all interest payments and repays the original principal.
- *The spot rate* is the rate of return earned by a bond when it is bought and sold on the secondary market without collecting interest payments.
- The spot interest rate for a zero-coupon bond is *the same* as the YTM for a zero-coupon bond.

**Current yield** is an investment's annual income (interest or dividends) divided by the current price of the security. This measure examines the current price of a bond, rather than looking at its face value.

$$Current\ yield = \frac{annual\ cash\ in\ flows}{market\ price}$$

- A bond that is priced **above** its face value is said to sell at a *premium*.
  - Investors who buy a bond at a premium face a capital loss over the life of the bond, so the yield to maturity on these bonds is always **less** than the current yield.
  - If **coupon rate > YTM**, price rises above par, and bond sells at a premium.
- A bond that is priced **below** face value sells at a *discount*.
  - Investors in discount bonds look forward to a capital gain over the life of the bond, so the yield to maturity on a discount bond is **greater** than the current yield.
  - When **YTM rises, above the coupon rate**, the bond's value falls below par, so it sells at a discount.

## Back to the United States: Semiannual Coupons and Bond Prices

**Treasury bills:** The short-term securities issued by the Treasury maturing in a year or less.

→ zero-coupon

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Treasury bonds, notes, and bills are traded in the *fixed-income market*.

**Fixed income securities** are financial claims with promised cash flows of fixed amount paid at pre-determined dates.

→ Bond is also called fixed income security.

e.g. Treasury bills, notes, bonds, commercial paper, corporate bonds, mortgage-backed securities...

### ***Valuation factors of fixed income securities***

- Cash Flow Factors: Maturity, principal, coupon
  - Time Value Factors: Interest rate
  - Risk Factors: Credit risk, liquidity risk, inflation risk, timing risk (callability), currency risk
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Not everyone can buy Treasury bonds, notes, or bills on the stock exchange. They are traded by a network of bond dealers, who quote prices at which they are prepared to buy and sell.

YTM is *reported* as an **annual rate**.

## **How Bond Prices Vary with Interest Rates**

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**Bond prices and interest rates *must* move in opposite directions.**

The yield to maturity, **our measure of the interest rate on a bond**, is *defined* as **the discount rate that explains the bond price**.

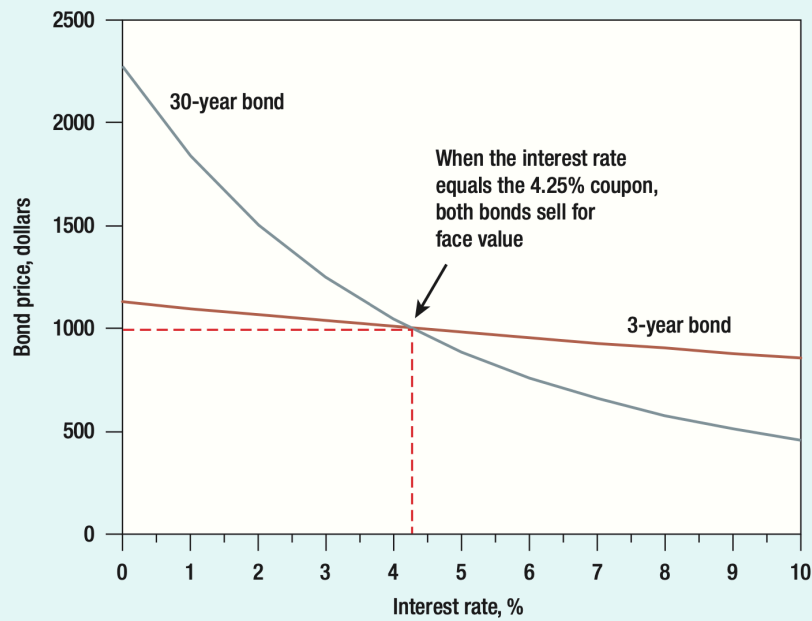
**When bond prices fall, interest rates (that is, yields to maturity) must rise. When interest rates rise, bond prices must fall.**

*YTM changes with market interest rate.*

A bond with longer maturity has higher relative (%) price change than one with shorter maturity when interest rate (YTM) changes. All other features are identical.

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The price of long-term bonds is **affected more by changing interest rates** than the price of short-term bonds.



**FIGURE 3.2**

Plot of bond prices as a function of the interest rate. The price of long-term bonds is more sensitive to changes in the interest rate than is the price of short-term bonds.

## Appendix

### Bond And Bond Markets

#### Overview of financial markets

- **Money Markets:** Short-term, liquid, low-risk debt securities
  - Treasury Bills, Bank instruments: CD, CP, BA
- **Capital Markets:** long-term, riskier securities
  - Fixed income securities
    - Treasury notes and bonds, Corporate Bond, Other Bonds
  - Equities
- **Derivative Markets:** derivative Securities
  - Option, Forward and Futures

#### Stock versus Bond

Debt	Equity
Fixed Claim	Residual Claim
High Priority on cash flows	Lowest Priority on cash flows
Tax Deductible	Not Tax Deductible
Fixed Maturity	Infinite life
No Management Control	Management Control

**Hybrids:** Preferred stock, convertible bond, derivative securities

## Valuation of Bonds and Stock

**First Principles:** Value of financial securities = PV of expected future cash flows

To value bonds and stocks, we need to estimate future cash flows' *size (how much)* and *timing (when)*.

Discount future cash flows at an appropriate rate: The rate should be appropriate to the risk presented by the security.

## Bond Valuation

### Pure Discount Bonds

**Pure Discount bonds (or Zero-coupon bonds):** A discount bond with maturity date  $t$  is a bond that pays face value only at  $t$ .

→ the simplest fixed income securities

**Information needed for valuing pure discount bonds:**

- Time to maturity ( $T$ ) = Maturity date - today's date
- Face value ( $F$ )
- Discount rate ( $r$ )

**Present value of a pure discount bond at time 0:**

$$PV = \frac{F}{(1 + r)^T}$$

### Coupon Bonds

**Information needed to value coupon bonds:**

- Coupon payment dates and time to maturity ( $T$ )
- Coupon payment ( $C$ ) per period and Face value ( $F$ )
- Discount rate ( $r$ )

## Value of a Level-coupon bond:

*Value of a level\_coupon bond = PV of coupon payment annuity + PV of face value*

$$PV = \frac{C}{r} \left[ 1 - \frac{1}{(1+r)^T} \right] + \frac{F}{(1+r)^T}$$

## Annual Coupon vs. Semiannual Coupons

- **Annual Coupon Bonds:**

- The bond make coupon payment annually
- Coupon = Coupon Rate × Face Value
- Most European bonds are paying coupon annually

- **Semiannual Coupon Bonds:**

- The bond make coupon payment semiannually
- Coupon = **0.5** × Coupon Rate × Face Value (*Coupon rate are annual rate!*)
- Most US bonds are paying coupon semiannually

## Summary

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Bonds are simply long-term loans. If you own a bond, you are entitled to a regular interest (or coupon) payment and at maturity you get back the bond's face value (or principal). In the U.S., coupons are normally paid every six months, but in other countries they may be paid annually.

**The value of any bond is equal to its cash payments discounted at the spot rates of interest.**

For example, the present value of a 10-year bond with a 5% coupon paid annually equals

$$PV(\% \text{ of face value}) = \frac{5}{(1+r_1)} + \frac{5}{(1+r_2)^2} + \dots + \frac{105}{(1+r_{10})^{10}}$$

This calculation uses a different spot rate of interest for each period.

Spot interest rates are most conveniently calculated from the prices of strips, which are bonds that make a single payment of face value at maturity, with zero coupons along the way. The price of a strip maturing at a future date  $t$  reveals the discount factor and spot rate for cash flows at that date. All other safe cash payments on that date are valued at that same spot rate.

Investors and financial managers use the yield to maturity on a bond to summarize its prospective return. To calculate the yield to maturity on the 10-year 5s, you need to solve for  $y$  in the following equation:

$$PV(\% \text{ of face value}) = \frac{5}{(1+y)} + \frac{5}{(1+y)^2} + \dots + \frac{105}{(1+y)^{10}}$$

**The yield to maturity discounts all cash payments at the same rate, even if spot rates differ.**

Notice that the yield to maturity for a bond can't be calculated until you know the bond's price or present value.

A bond's maturity tells you the date of its final payment, but it is also useful to know the *average* time to each payment. This is called the bond's *duration*. Duration is important because there is a direct relationship between the duration of a bond and the exposure of its price to changes in interest rates. A change in interest rates has a greater effect on the price of long-duration bonds.

