How to Calculate Present Values

Future Values and Present Values

Calculating Future Values

Money has a *time value*: **a dollar today is worth more than a dollar tomorrow**

Money grows at a **compound rate** and the interest that you earn is called **compound interest**.

- **Present value (PV):** Value today of a future cash flow.
- **Future value (FV):** Amount to which an investment will grow after earning interest.

$$Future\ value = FV = PV \times (1+r)^t$$

Calculating Present Values

In general, suppose that you will receive a cash flow of C_t dollars at the end of year t.

$$Present\ value = PV = \frac{C_t}{(1+r)^t} = C_t \times discount\ factor$$

The rate, r, in the formula is called **the discount rate**, and the present value is *the discounted value* of the cash flow, C_t.

→ *The cost of capital is the return foregone by not investing in financial markets.*

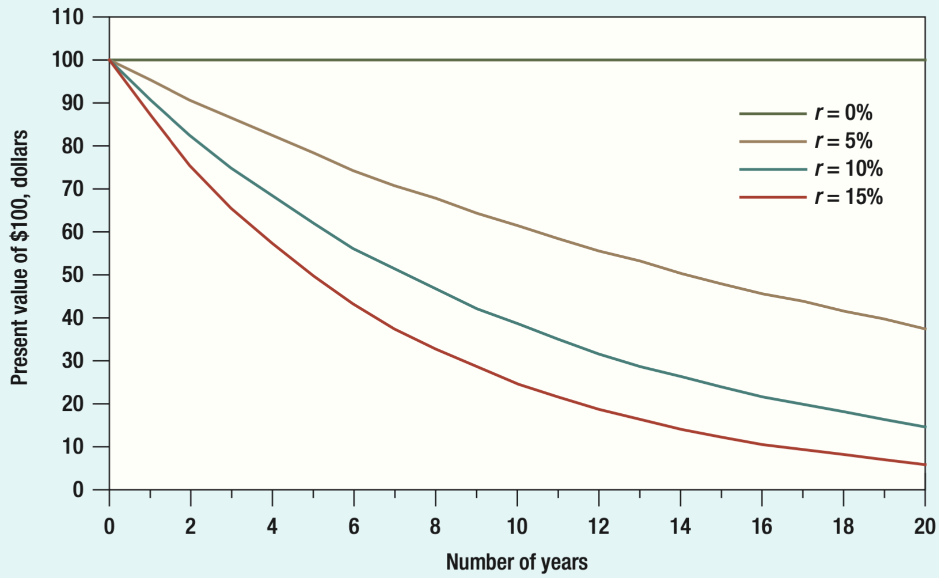
Instead of *dividing* the future payment by $(1+r)^t$, you can equally well *multiply* the payment by $1/(1+r)^t$. The expression $1/(1+r)^t$ is called the **discount factor**.

→ *measuring the present value of one dollar received in year t*

$$Discount\ factor = DF = \frac{1}{(1+r)^t}$$

FIGURE 2.2

Present value of a future cash flow of \$100. Notice that the longer you have to wait for your money, the less it is worth today.



Net Present Value

Net present value (NPV) equals present value minus the required investment:

$$NPV = PV - investment$$

Present value is the value of the investment today; *net present value* is the addition that the investment makes to your wealth.

Risk and Present Value

A safe dollar is worth more than a risky dollar.

→ Most investors dislike risky ventures and won't invest in them unless they see the prospect of a higher return.

It is still proper to discount the payoff by the rate of return offered by a risk-equivalent investment in financial markets, by thinking of *expected* payoffs and the *expected* rates of return on other investments.

- Higher risk projects require a higher rate of return
- Higher required rates of return cause lower PVs

Present Values and Rates of Return

$$Return = \frac{profit}{investment}$$

The investment can be justified by either one of the following two rules:

- **Net present value rule.** Accept investments that have positive net present values.
- **Rate of return rule.** Accept investments that offer rates of return in excess of their opportunity

costs of capital.

→ *Both rules give the same answer. In some cases where the rate of return rule is unreliable, use the net present value rule.*

Calculating Present Values When There Are Multiple Cash Flows

The *total* present value of a stream of cash flows is:

$$PV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots + \frac{C_T}{(1+r)^T} = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

→ the **discounted cash flow** (or **DCF**) formula

The *net* present value (NPV) :

$$NPV = C_0 + PV = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t}$$

The Opportunity Cost of Capital

When you discount the expected cash flows by the opportunity cost of capital, you are asking how much investors in the financial markets are prepared to pay for a security that produces a similar stream of future cash flows.

Some mistakes concerning the bank loan:

1. The interest rate on the loan has nothing to do with the risk of the project: it reflects the good health of your existing business.
2. Whether you take the loan or not, you still face the choice between the office building and an equally risky investment in the stock market.

Looking for Shortcuts—Perpetuities and Annuities

How to Value Perpetuities

Perpetuity: Financial concept in which cash flow is theoretically received forever.

The annual rate of return on a perpetuity is equal to the promised annual payment divided by the present value:

$$Return = \frac{cash\ flow}{present\ value}$$

$$r = \frac{C}{PV}$$

The present value of a perpetuity given the discount rate r and the cash payment C:

$$PV = \frac{C}{r}$$

Today's value of a perpetuity starting $t + 1$ years from now (starting in year $t + 1$ while now is year 0):

$$PV = \frac{C}{r} \times \frac{1}{(1+r)^t}$$

How to Value Annuities

An **annuity** is an asset that pays a fixed sum each year for a specified number of years.

If the interest rate is r, then the present value of an annuity that pays $\$C$ a period for each of t periods is:

$$Present\ value\ of\ t\_year\ annuity = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

The *t*-year **annuity factor**:

$$\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

$$\text{Present value of annuity} = \underbrace{\frac{1}{r}}_{\text{perpetuity formula}} \times \left[\underbrace{1}_{\text{\$1 starting next year}} - \underbrace{\frac{1}{(1+r)^t}}_{\text{minus \$1 starting at } t+1}\right]$$

Valuing Annuities Due

A level stream of payments starting immediately is called an **annuity due.** An annuity due is worth **(1 + r)** times the value of an ordinary annuity.

→ *the first cash flow in the annuity comes at time 0*

$$PV\ of\ annuity\ due = C + C \times \left[\frac{1}{r} - \frac{1}{r(1+r)^{t-1}}\right]$$

$$= PV\ of\ t\_year\ annuity \times (1+r) = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right] \times (1+r)$$

More Shortcuts—Growing Perpetuities and Annuities

Growing Perpetuities

Growing Perpetuity: Cash flow is theoretically growing constantly at g (g<r) and received forever.

$$PV = \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \dots$$
$$= \frac{C_1}{1+r} + \frac{C_1(1+g)}{(1+r)^2} + \frac{C_1(1+g)^2}{(1+r)^3} + \dots$$

If we assume that $r > g$, the calculation simplifies to:

$$Present\ value\ of\ growing\ perpetuity = \frac{C_1}{r-g}$$

Why should g be smaller than r?

- If not, the sum of the PVs of each term does not converge to a certain value
- In the real business world, it is implied by the diminishing marginal returns
- If one asset grows **forever** at a rate lager than r, eventually it is the only asset that survives and occupies the whole world

Application in business valuation

- The growing perpetuity is a basic model of mature firms that are growing constantly and slowly
- The dividend payments to shareholders are also growing constantly and slowly

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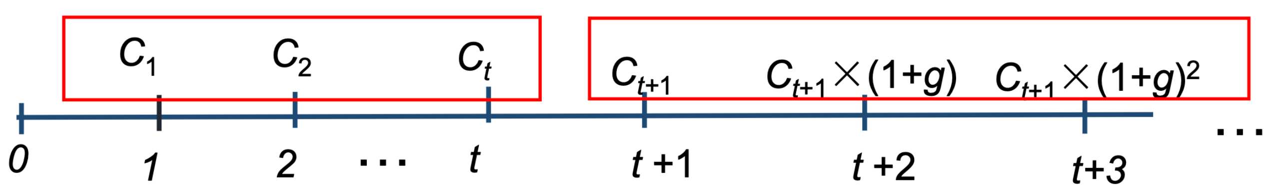
A company can grow very rapidly now, but will eventually come to a stage of low and constant growth.



$$PV_0 = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_t}{(1+r)^t} + \frac{C_{t+1}}{r-g} \times \frac{1}{(1+r)^t}$$

Growing Annuities

$$PV\ of\ growing\ annuity = C \times \frac{1}{r-g}\left[1 - \frac{(1+g)^t}{(1+r)^t}\right]$$

Summary: shortcuts

- **Perpetuity:** a cash flow is theoretically received forever. **(perpetual, not growing)**
- **Growing Perpetuity:** cash flow is theoretically growing constantly at g ($g<r$) and received forever. **(perpetual, growing)**
- **Annuity:** an asset that pays a fixed sum each year for a specified number of years. **(not perpetual, not growing)**
- **Growing Annity:** an asset that pays a growing cash flow each year for a specified number of years. **(not perpetual, growing)**

	Cash Flow ($)						
Year:	**0**	**1**	**2 . . .**	**. . . $t-1$**	**t**	**$t+1$ . . .**	**Present Value**
Perpetuity		1	1 . . .	1	1	1 . . .	$\frac{1}{r}$
t-period annuity		1	1 . . .	1	1		$\frac{1}{r}-\frac{1}{r(1+r)^t}$
t-period annuity due	1	1	1 . . .	1			$(1+r)\left(\frac{1}{r}-\frac{1}{r(1+r)^t}\right)$
Growing perpetuity		1	$1\times(1+g)\ldots$	$1\times(1+g)^{t-2}$	$1\times(1+g)^{t-1}$	$1\times(1+g)^{t}\ldots$	$\frac{1}{r-g}$
t-period growing annuity		1	$1\times(1+g)\ldots$	$1\times(1+g)^{t-2}$	$1\times(1+g)^{t-1}$		$\frac{1}{r-g}\left[1-\frac{(1+g)^t}{(1+r)^t}\right]$

TABLE 2.2 Some useful shortcut formulas. Both the growing perpetuity and growing annuity formula must assume that the discount rate r is greater than the growth rate g. If $r = g$, the formulas blow up and are useless.

How Interest Is Paid and Quoted

The interest rates are given in <u>annual terms</u> in the financial world, no matter how to get paid actually.

Compound interest

- Interest of this year (period) is accrued to the principal on which interest is calculated next year (period).
- $FV = C \times (1+r)^n$
- Compound interest is used in calculating <u>the true return</u> in any cases.

Simple interest

- Interest of this year is NOT accrued to the principal.
- $FV = C \times (1 + n \times r)$
- Simple interest is applied in either <u>quoting the annual rate in short-term investments</u>, or <u>calculating return in one long period</u>.

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the *quoted* annual interest rate vs. the *effective* annual rate

- $The\ quoted\ annual\ rate = the\ total\ annual\ payment \div the\ number\ of\ payments\ in\ the\ year$
- When interest is paid once a year, the quoted and effective rates are the same.
- When interest is paid more frequently, the effective interest rate is higher than the quoted rate.

APR (Annual Percentage Rate)

- Stated/quoted interest rate in financial institutions
- What investors really earn depends on the frequency of interest payments, i.e. , semi-annually compounded, quarterly compounded, monthly compounded, or even continuously compounded.

EAR (Effective Annual Rate)

- Rate that investors actually earn their interests.
- At the same APR, different interest payment frequency (how the APR is compounded) results in different EARs.

APR and EAR Conversion

$$EAR = \left(1 + \frac{APR}{n}\right)^n - 1$$

$$Interest\ rate\ in\ each\ period = \frac{APR}{n}$$

$$n = \frac{the\ length\ of\ a\ period}{1\ year}$$

Continuous Compounding

$$EAR = \lim_{n \to \infty}\left(1 + \frac{APR}{n}\right)^n - 1 = e^{APR} - 1$$

\$1 invested at a continuously compounded rate of r will grow to $e^r = (2.718)^r$ by the end of the first year. By the end of t years, it will grow to $e^{rt} = (2.718)^{rt}$.

$$Cash\ flow\ by\ the\ end\ of\ year\ t = C \times (1 + EAR)^t = C \times e^{APR \times t}$$

For annuity:

Since r is the continuously compounded rate, $\frac{C}{r}$ received in year t is worth $\frac{C}{r} \times \frac{1}{e^{rt}}$ today. Our annuity formula is therefore

$$PV = \frac{C}{r} - \frac{C}{r} \times \frac{1}{e^{rt}}$$

sometimes written as

$$PV = \frac{C}{r}(1 - e^{-rt})$$

Summary

Firms can best help their shareholders by accepting all projects that are worth more than they cost. In other words, they need to **seek out projects with positive net present values**. To find net present value we first calculate present value. Just discount future cash flows by an appropriate rate *r,* usually called the *discount rate, hurdle rate,* or *opportunity cost of capital:*

$$PV = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + \ldots$$

Net present value is present value plus any immediate cash flow:

$$Net\ present\ value\ (NPV) = C_0 + PV$$

Remember that C_0 is **negative** if the immediate cash flow is an investment, that is, if it is a cash outflow.

The discount rate *r* is determined by **rates of return prevailing in financial markets.** If the future cash flow is absolutely safe, then the discount rate is the interest rate on safe securities such as U.S. government debt. If the future cash flow is uncertain, then the expected cash flow should be discounted at the expected rate of return offered by equivalent-risk securities.

Cash flows are discounted for two simple reasons: because (1) a dollar today is worth more than a dollar tomorrow and (2) a safe dollar is worth more than a risky one. Formulas for PV and NPV are numerical expressions of these ideas.

Financial markets, including the bond and stock markets, are the markets where safe and risky future cash flows are traded and valued. That is why we look to rates of return prevailing in the financial markets to determine how much to discount for time and risk. By calculating the present value of an asset, we are estimating how much people will pay for it if they have the alternative of investing in the capital markets.

You can always work out any present value using the basic formula, but shortcut formulas can reduce the tedium. We showed how to value an investment that makes a level stream of cash flows forever (a *perpetuity*) and one that produces a level stream for a limited period (an *annuity*). We also showed how to value investments that produce growing streams of cash flows.

When someone offers to lend you a dollar at a quoted interest rate, you should always check how frequently the interest is to be paid. For example, suppose that a $100 loan requires six-month payments of $3. The total yearly interest payment is $6 and the interest will be quoted as a rate of 6% compounded semiannually. The equivalent *annually compounded rate* is (1.03)2 – 1 = .061, or 6.1%. Sometimes it is convenient to assume that interest is paid evenly over the year, so that **interest is quoted as a continuously compounded rate**.