Computer Science Fundamentals:
Intro to Algorithms, Systems, & Data Structures

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Contents

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Preface

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Please note: These are my personal notes, and while I strive for accuracy, there may be errors. I encourage you to refer to the original slides for precise information.
Comments and suggestions for improvement are always welcome.

Prerequisites

— 1 —
Circuits and Logic

1.1 Representing Information

1.1.1 Electricity & Information: Volts, Amps, & Watts

Figuring out how to represent information is tricky: Nature encodes information in DNA, though it may be hard to store because of decay (This is an active area of research). Punching holes in cards was a common method of storing information, but it's difficult to manipulate [1]. Ideally:

- **Inexpensive:** We want to reproduce at scale with low costs.
- **Stable:** Reliably store information for long periods.
- **Mutable:** The ability to manipulate information easily.

Definition 1.1: Electricity & Information

Electricity is a flow of electrons, which can be used to represent information. We can use the presence or absence of an electric current to represent binary values:

- **1** for presence of current;
- **0** for absence of current.

This is the basis of digital electronics and computing.

This is great for our applications, as electricity is relatively inexpensive given the scale of production.

Theorem 1.1: Noise & Error Accumulation

We ought to keep in mind that electricity is not perfect. Though we design systems to measure information, slight inaccuracies or environmental factors may introduce noise, which over time corrupts information.

It's important that we understand the difference between analog and digital systems:

Definition 1.2: Analog vs. Digital

An **analog** system is one that uses continuous signals to represent information, while a **digital** system uses discrete values (e.g., binary) to represent information.

Example 1.1: Real World Analog vs. Digital

Vinyl records are analog, as the grooves on the record represent sound waves continuously. In contrast, a digital system would be a CD or MP3 file, where sound is represented as discrete samples of the original sound wave. ■

Our main focus will be on digital systems, representing the strength of electricity as binary values. First we will briefly understand the terminology used in electrical systems:

Definition 1.3: Voltage, Amps, & Watts

Definition wise we have the following terms in electrical systems:

- **Voltage (Volts):** The potential difference between two points in an electrical circuit, measured in volts (V).
- **Amperage (Amps):** The flow of electric **current**, measured in amperes (A/I).
- **Resistance (Ohms):** The opposition to the flow of electric current, are ohms (Ω/R).
- **Power (Watts):** The rate at which electrical energy is transferred, are watts (W).

We calculate all such as follows:

- **Voltage:** $V = I \cdot R$ (Voltage = Current × Resistance).
- **Current:** $I = P/V$ (Current = Power / Voltage).
- **Resistance:** $R = V/I$ (Resistance = Voltage / Current).
- **Power:** $P = V \cdot I$ (Power = Voltage × Current).

These ratios between Voltage, Current, and Resistance are part of **Ohm's Law**.

Let's understand this with a common analogy to water flow:

Example 1.2: Water & Electric Flow Analogy

Imagine a water pipe system:

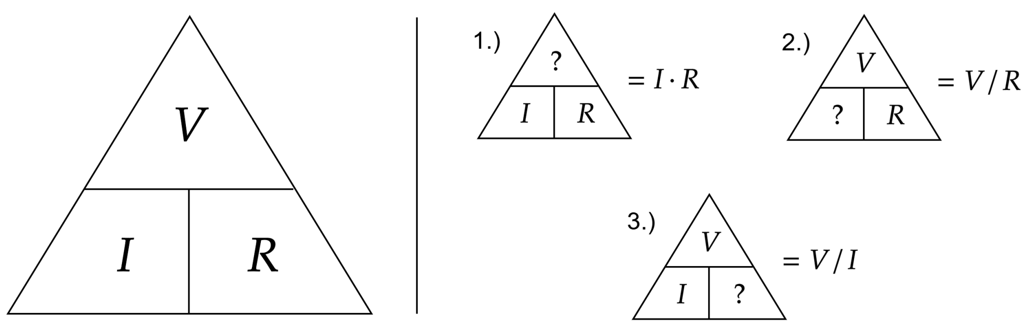
- **Voltage** is the water pressure in the pipes, the force pushing water through the system.
- **Current** is the amount of water flowing through the pipes at any given time.
- **Resistance** is the size of the pipes, which affects how easily water can flow.
- **Power** is the total amount of water that flowed through the system over time.

■

The relationship between Voltage, Current, and Resistance has a handy visualization:

Definition 1.4: Ohm's Triangle

Ohm's Triangle is a visual representation of the relationship between Voltage, Current, and Resistance. If any two values are known, the third can be calculated using the triangle:



Here, Voltage (V) is at the top, with Current (I) and Resistance (R) at the bottom corners:

1. **Voltage** is unknown: $V = I \cdot R$.
2. **Current** is unknown: $I = V/R$.
3. **Resistance** is unknown: $R = V/I$

A common mnemonic to remember is "Viral" for VIR (Voltage, Current, Resistance).

Now for completeness sake, we distinguish the following:

Definition 1.5: Energy vs. Power

Energy is the capacity to do work, measured in joules (J). **Power** is the rate at which work/energy is done or used, measured in watts (W). This is given by the formulation:

$$P = E/t$$

where P is power, E is energy, and t is time.

Example 1.3: Energy-Power Water Analogy

Continuing with the water analogy:

- **Energy** is the total amount of water stored in a tank.
- **Power** is how fast water flows out of the tank per second.

If we have a large tank (more energy), and water flows out slowly, we have high energy but low power. Conversely, if we open the tap wide (high power), we use up the water quickly. ■

We will wrap up such with a final analogy that uses numbers:

Example 1.4: Mathematical Water Analogy

- **Water Gun:** Imagine a water gun with very high pressure granted by the resistance of its small nozzle, so only a little water comes out.
  - Pressure (Voltage) = 10 V
  - Water Flow (Current) = 1 A
  - Power = 10 V × 1 A = 10 W
- **Large Hose:** Now, consider a large fire hose with lower pressure but a much wider opening with less resistance, allowing a lot of water to flow.
  - Pressure (Voltage) = 2 V
  - Water Flow (Current) = 5 A
  - Power = 2 V × 5 A = 10 W

Both systems consumed the same amount of power (10 W), despite supporting different voltages, currents, and possibly energy supplies. **Question:** What is the resistance of each system? ■

1.1.2 Combinational Devices

We now focus on the conduits of representing information digitally:

Definition 1.6: Digital Current Encoding Threshold

Given a line of voltage V, which we measure, V_{TH} serves as a threshold:

$$\text{0-bit} < V_{TH} < \text{1-bit}$$

In practice, we have noise ϵ in our measurements, making it hard to discern $V_{TH} + \epsilon$ from $V_{TH} - \epsilon$. To mitigate this, we pad the threshold from both sides called the **forbidden zone**:

$$\text{0-bit} \leq V_L < \text{“Forbidden Zone”} < V_H \leq \text{1-bit}$$

Where V_L (low-level) and V_H (high-level) are the region markers for valid voltage distinction.

Definition 1.7: Combinational Device

A **combinational device** is follows four specifications (spec.) called the, **static discipline**:

- **Input:** A set of input signals (i.e., measuring voltage levels).
- **Output:** A set of output signals (i.e., outputting voltage levels).
- **Functional Spec:** A mapping of all possible input combinations to an output value.
- **Timing Spec:** Detailing an upper bound t_{PD} **(Propagation Delay)**, which is the minimum amount of time needed for the output to stabilize on a new value after an input change.

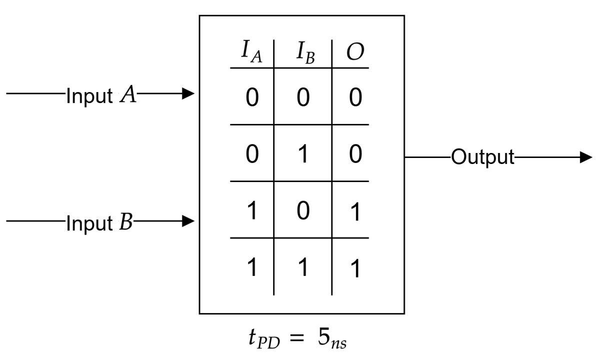


Figure 1.1: A combinational device with inputs A and B, and a truth table detailing mappings towards the output. The $t_{PD} = 5_{ns}$ (nanoseconds).

Definition 1.8: Combinational Digital Systems

A combinational device may also be made up of multiple other combinational devices. It must follow that:

- Each device is indeed a combinational device.
- Every input is connected to a single output.
- Each parent input will at most visit the same child input once (i.e., no cycles).

The t_{PD} of the system is the sum of sub-devices t_{PD}'s along a path such that it is the maximum such t_{PD} path in the system.

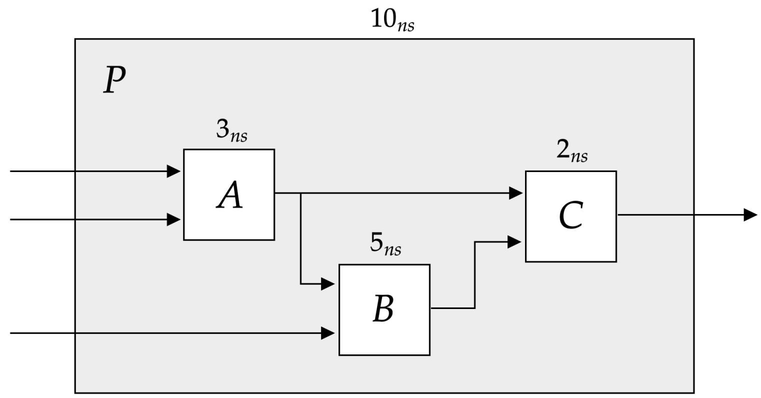


Figure 1.2: A combinational digital system, with a parent device P and children devices A, B and C. We abstract away the mappings focusing on the components and their connections. We see that there are no cycles and all sub components are also combinational devices; Hence, the parent system is a combinational device. The t_{PD} of the system is 10_{ns}, as the longest path takes $A \to B \to C = 3_{ns} + 5_{ns} + 2_{ns} = 10_{ns}$, the effective bottleneck of the system.

Though this introduces a new problem:

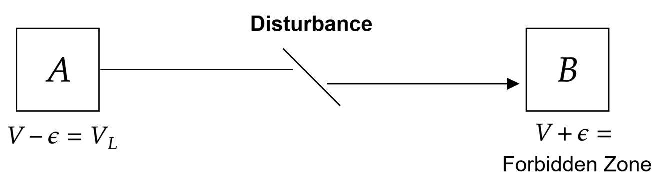
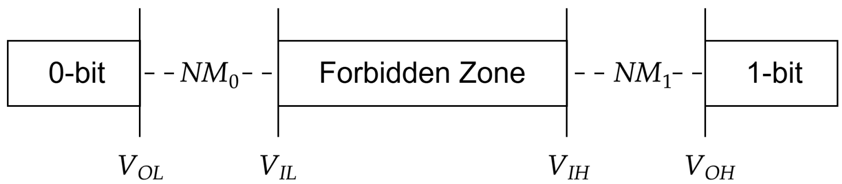


Figure 1.3: Combinational devices A and B communicate; However, A's output (V) is dangerously close to V_L, over the wire there is a disturbance, causing the input of B to enter the forbidden zone.

We offer a simple fix to this problem, by loosening up the thresholds during certain phases:

Definition 1.9: Noise Margins

To mitigate noise from outputs of a combinational device, we decrease the *forbidden zone* (FZ) for the receiving device. The overlap between the output's FZ and the input's FZ is called the **noise margin**. Concretely, we define the following:



Where, V_{OL} and V_{OH} are the output bounds, while V_{IL} and V_{IH} are the new input bounds. Then NM_0 is the noise margin for the 0-bit, and NM_1 is the noise margin for the 1-bit. The smallest of the two is called the **noise immunity** of the device (i.e., the worst case that must be supported).

Now when building our systems or combinational devices we must standardize how a particular device behaves on inputs and outputs to account for the worst case noise.

Definition 1.10: Voltage Transfer Characteristics (VTC)

The **Voltage Transfer Characteristics** (VTC) is a graphical representation which shows how a device's inputs affect its outputs after stabilization. The horizontal axis measures the input voltage, while the vertical axis measures the output voltage.

- **Horizontal Axis (V_{in}):** Contains V_{IL} and V_{IH}:

  $$V_{in} \leq V_{IL} \text{ (0-bit)} \quad \text{and} \quad V_{in} \geq V_{IH} \text{ (1-bit)}$$

  Otherwise, the input is in the forbidden zone.

- **Vertical Axis (V_{out}):** Contains V_{OL} and V_{OH}:

  $$V_{OL} < \text{Invalid Outputs} < V_{OH} \quad \text{such that} \quad V_{in} < V_{OL}, V_{in} > V_{OH}$$

  I.e., if the input is already in the forbidden zone, the output is irrelevant.

It's given that the device must perform properly such that a $V_{in} > V_{IH}$ will always yield a $V_{out} > V_{OH}$, and a $V_{in} < V_{IL}$ will always yield a $V_{out} < V_{OL}$. Each device has its own VTC, plotting the input-output relationship. The resulting curve is the **VTC** of the device.

Diagram next page.

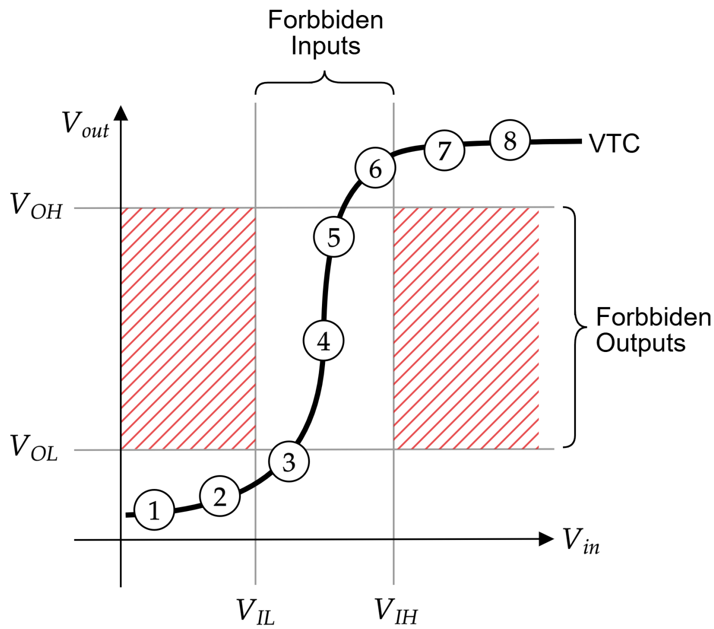


Figure 1.4: A Voltage Transfer Characteristics (VTC) diagram, showing the input-output relationship of a device. The horizontal axis represents the input voltage, while the vertical axis represents the output voltage. The invalid output regions are shaded in red. The VTC is the bold line that crosses each point. Possible points: (1-2) received a low input and output reading, (3-6) undefined, and (7-8) high input and output reading. E.g., an inverter device (inverts logic) would be a vertical flip of the above VTC curve.

Notice how in Figure (1.4) the center white region is taller than it is wide:

Theorem 1.2: Properties of VTC – Gain & Nonlinearity

Since more leeway is allowed for input voltages, the following suffices $V_{OH} - V_{OL} > V_{IH} - V_{IL}$. We can compactly write this as:

- **Width of the transition (x-axis):** $\Delta V_{in} = V_{IH} - V_{IL}$.
- **Height of the swing (y-axis):** $\Delta V_{out} = V_{OH} - V_{OL}$.

Since $\Delta V_{out} > \Delta V_{in}$, the **gain** (average slope) satisfies: (avg.) gain $= \dfrac{\Delta V_{out}}{\Delta V_{in}} > 1$.

Because of this ratio (gain > 1) small deviations (wiggles) in the input are amplified (exaggerated) in the output, which **regenerates** the signal (i.e., the output is a reinforced version of the input). The slope of the VTC must be **nonlinear** to ensure flat stable regions around 0 and 1 bits, and steep transitions between the forbidden zones (as seen in Figure 1.4).

1.1.3 Building Electrical Channels: Transistors

To even begin to manage currents and voltages, we will need a way to control the flow of electricity:

Definition 1.11: Transistor

A **transistor** is a small electronic semiconductor device. A **semiconductor** (e.g., silicon) is a material with electrical conductivity between that of a **conductor** (well at conducting electricity) and an **insulator** (not conducting electricity). It has two main functions:

- **Switching:** Transistors can act as electronic switches, turning current on or off.
- **Amplification:** Transistors can amplify weak electrical signals, making them stronger.

These devices can easily heat up, so for lower current applications, transistors are encased in resin/plastic, while in higher current applications, they have a side made with metal to dissipate heat. These resistors are often attached to a **heat sink**, which is a piece of metal that draws heat away from the transistor.

A transistor has three terminals (pins) made of silicon or germanium:

- **Emitter (E):** The terminal through which current flows out of the transistor.
- **Base (B):** The terminal that controls the transistor's operation.
- **Collector (C):** The terminal through which current flows into the transistor.

These may be in different order depending on the make and model. A **part number** is often printed on the front, which may be used to look up the manufacturer's **datasheet** (specifications of the transistor).

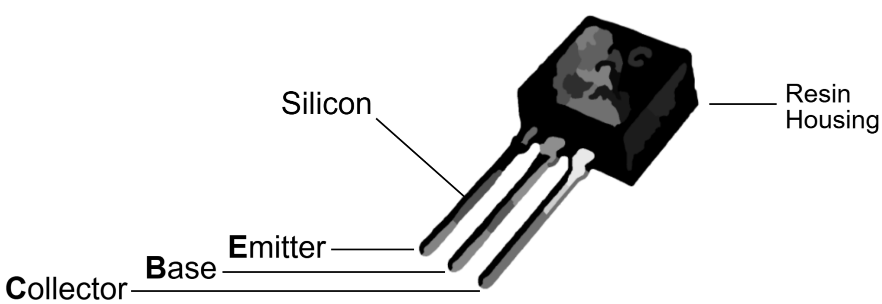


Figure 1.5: A transistor with its three terminals: Emitter (E), Base (B), and Collector (C), all of which are composed of silicon, while the inner workings are encased in resin.

Bibliography

[1] Chris Terman. 6.004 computation structures, 2017. Undergraduate course, Spring 2017.