# Not so Discrete Math

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## 0.1 Introduction to Sets

In discrete math we work with some group of 'things,' a thing or something we fancily call an **object**. A group or categorization of objects is called a set.

**Theorem 0.1: Set**

Is a collection of objects.

**For Example:**

- $S =$ The set of all students in a classroom.
- $A =$ The set of all vowels in the English alphabet.
- $\mathbb{Z} =$ The set of all integers.

Objects in a **set** are called **elements**.

**Theorem 0.2: Element**

An object that is a member of a given set.

To expand on the previous example:

- $S = \{s_1, s_2, s_3\}$, where $s_1, s_2, s_3$ are students, elements of the set.
- $A = \{a, e, i, o, u\}$, where $a, e, i, o, u$ are elements.
- $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$, elements of integer set.

Curly braces denote a set, commas to separate elements, The '...' or 'ellipse' indicate that the set continues indefinitely in that direction. When the set's pattern is clear to the reader, we use the dotted notation.

Symbols used to denote members of a set:

- '$\in$' = in the set.
- '$\notin$' = not in the set.

**Theorem 0.3: Membership**

If $x$ is an element of set $A$, $x \in A$. If $x$ is not an element of set $A$, $x \notin A$.

**For Example:** Given $A = \{a, e, i, o, u\}$,
$a \in A$, "$a$ is an element of $A$," and $b \notin A$, "$b$ is not an element of $A$."

**Order nor repetition matter:**

- $A = \{1, 2, 3\} = \{3, 2, 1\} = \{1, 2, 3, 3, 3, 3, 3\}$.
- $B = \{a, b, c\} = \{a, b, c, a, b, c\}$.

**Theorem 0.4: Properties of a Set**

- The order of elements does not matter.
- Duplicate elements are not counted.

A subset is a set contained within another set. If the set $B$ is a subset of set $A$, then every element in $B$ is also in $A$ as shown in Figure 1:

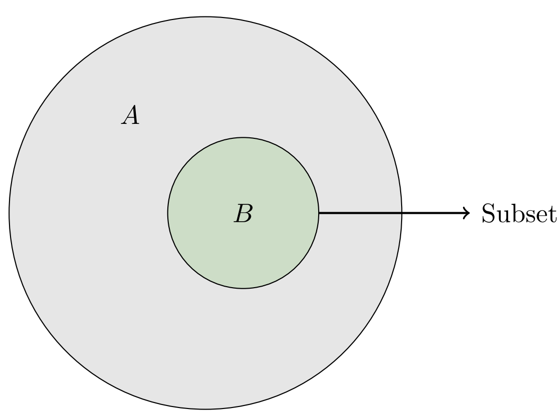


Figure 1:

Written $B \subseteq A$ or $A \supseteq B$, similar to the less than or equal to signs '$\leq$' and '$\geq$'.

**Theorem 0.5: Subset**

If every element in set $B$ is also in set $A$, then $B$ is a subset of $A$.
Denoted: $B \subseteq A$ or $A \supseteq B$.

**For Example:**

- $\{-1, 0\} \subseteq \{-1, 0, 1, 2, 3\}$
- $\{-1, 1, 3\} \subseteq \{-1, 0, 1, 2, 3\}$
- $\{-1, 7\} \nsubseteq \{-1, 0, 1, 2, 3\}$

$\nsubseteq$ denotes 'not a subset of.'

Explicitly defining a set, say $\{1, 2, 3, ...\}$, is called **set-roster notation**.
**Set-builder notation** enables us to define the set's properties.

**Theorem 0.6: Set-Builder Notation**

Follows the general form: $\{x \mid P(x)\}$,

- $x$ = defines some variable.
- " $\mid$ " = is short hand for "such that."
- $P(x)$ = describes the properties $x$ must satisfy.

**For Example:** Lets define the set of even integers

- $\{x \mid x \text{ is an even integer}\}$: "$x$, such that, $x$ is an even integer."
- $\{x \in \mathbb{Z} \mid x \text{ is even}\}$: "$x$ in Integers, such that, $x$ is an even."
- $\{x \in \mathbb{Z} \mid x \text{ is not odd}\}$: "$x$ in Integers, such that, $x$ is not odd."

**It's important to define exactly what variables are.** In the above, $x$ was stated directly as an integer. If not, x could be **water-balloons** or **puppies**.