

**Exercise 1.2.** Let  $n$  be a composite integer. Show that there exists a prime  $p$  dividing  $n$ , with  $p \leq n^{1/2}$ .

*Proof:* Let  $n$  be a composite integer  $\therefore n = ab$  for some integers  $a, b \mid 1 < a, b < n$ .  
 $a, b \leq n^{1/2}$  must hold, or else  $ab > n$ .

$n > 0 \therefore n$  can factor to powers of primes  $p$  (Fundamental Theorem of Arithmetic). If  $p$  is a composite, factor again until a single prime  $p'$  is found.  $p' \mid p$  then  $p' \mid n$  and  $p' \leq n^{1/2}$ . ■.