Basics of Machine Learning

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- Logistic Regression and Ensemble Methods (Decision Tree Adaboost) （3）
  逻辑回归与集成学习算法
- Overfitting, Underfitting and Cross-Validation
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- Scientific Reading and Writing（3）
  科技文阅读与写作

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随堂实验

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  线性回归与梯度下降
- Linear Classification with Stochastic Gradient Descent (2)
  线性分类、支持向量机、随机梯度算法

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- Classification with AdaBoost (4)
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- Recommendation Systems (4)
  推荐系统

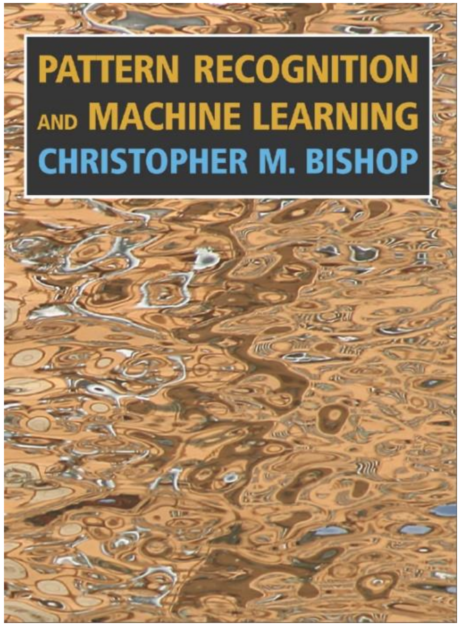
考核标准+参考书

- **考核标准**

考试（50%）+ 平时成绩（25%）+ 技术报告（25%）

- **参考书**

  - Pattern Recognition and Machine Learning **By Bishop**
  - Understanding Machine Learning: From Theory to Algorithms By Shai Shalev-Shwartz and Shai Ben-David
  - 《机器学习》By 周志华



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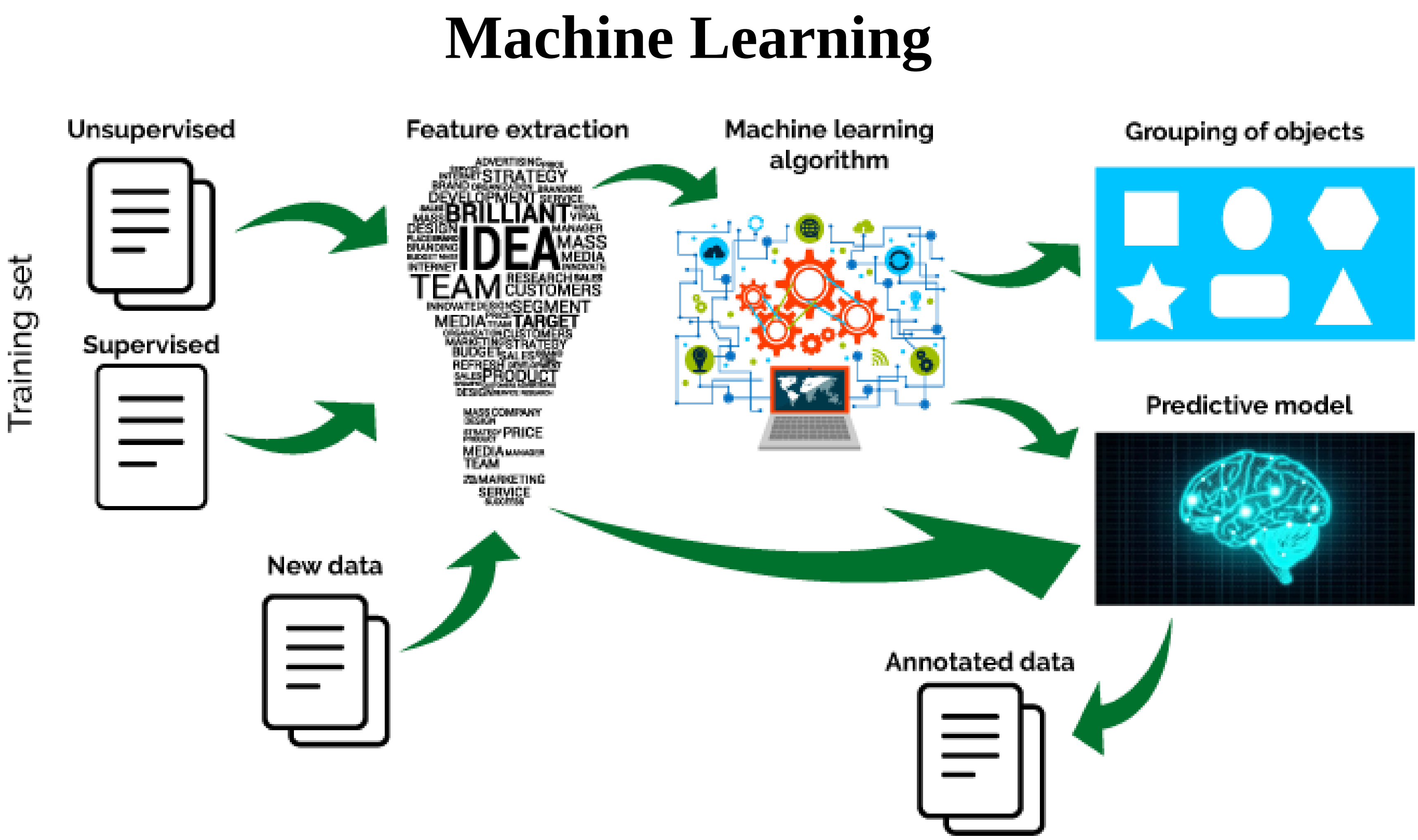
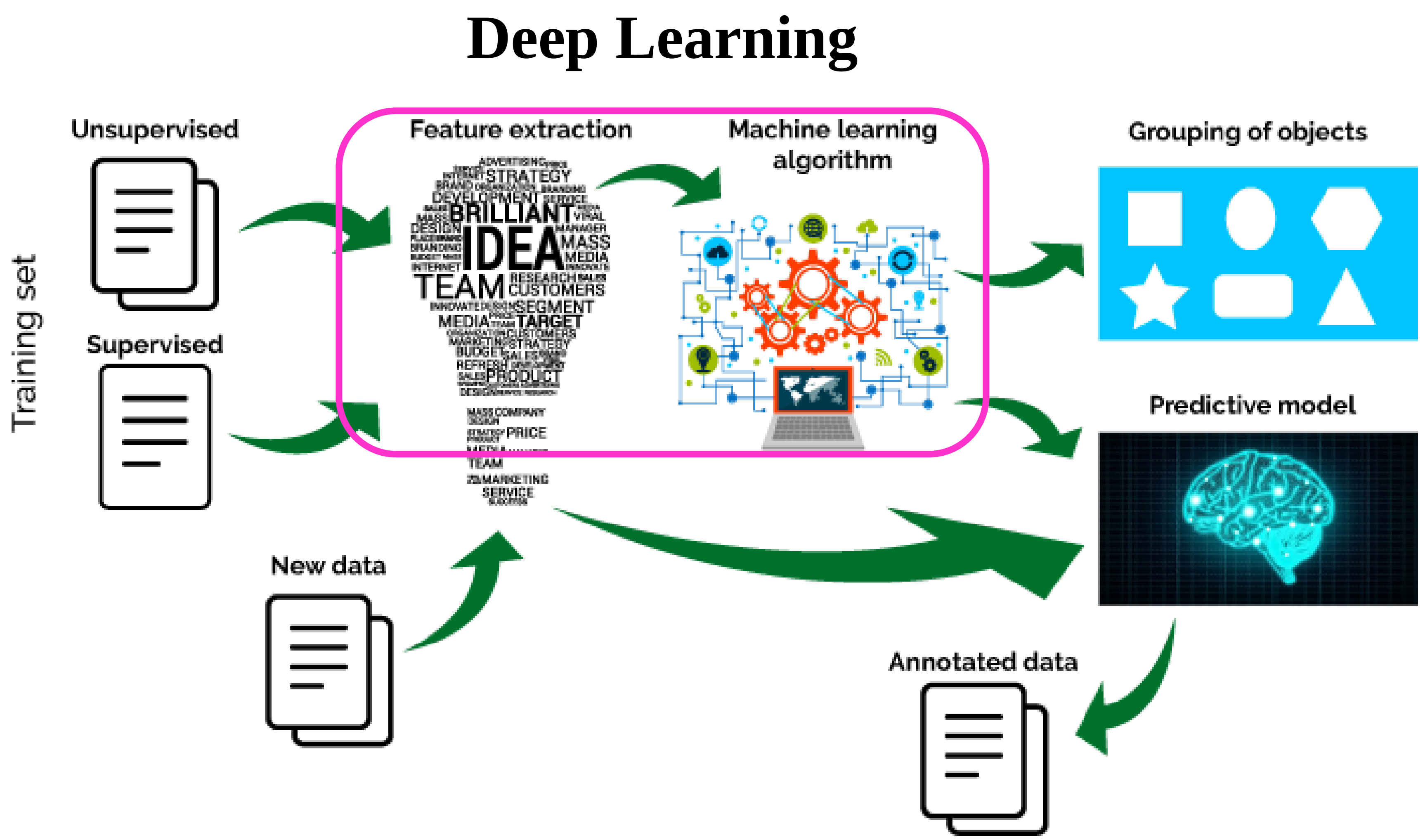
Big Picture: Machine Learning

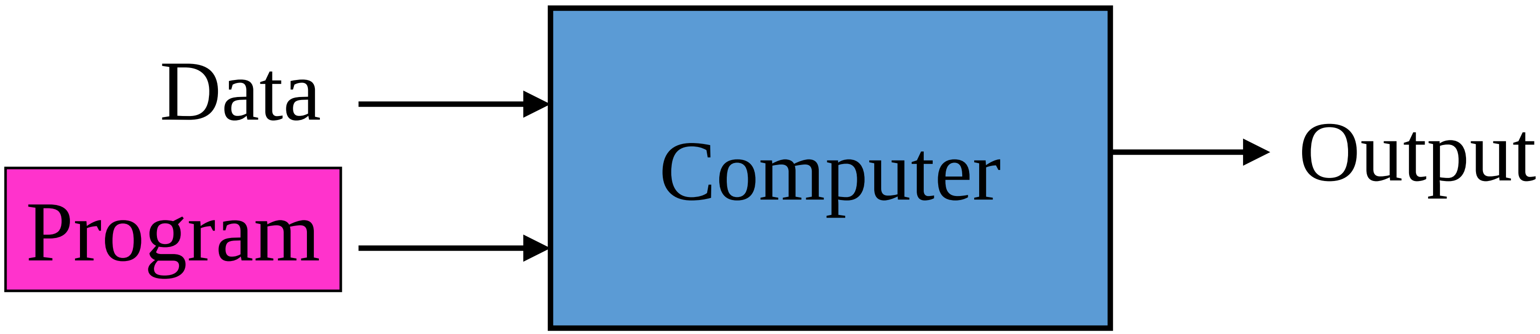


Big Picture: Machine Learning

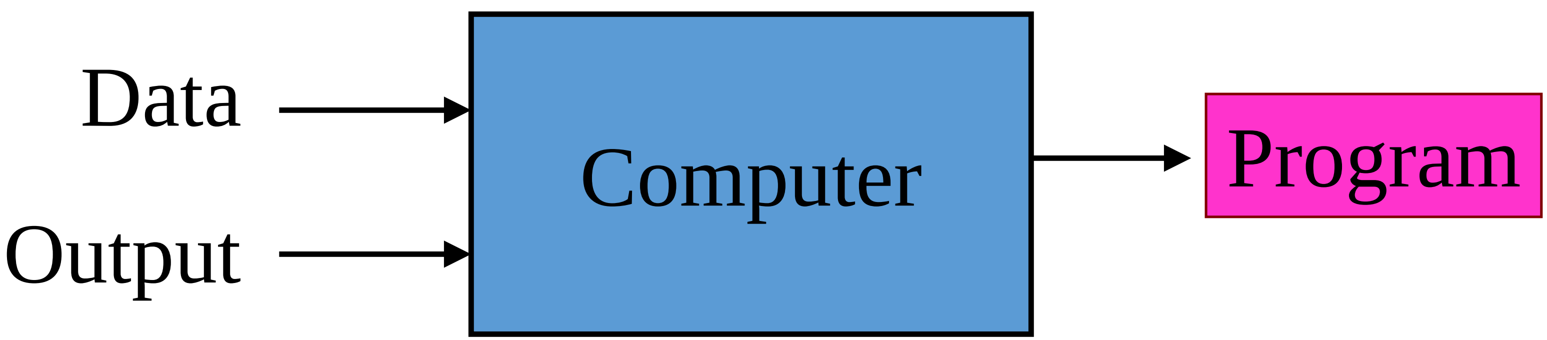


Traditional Programming and Machine Learning

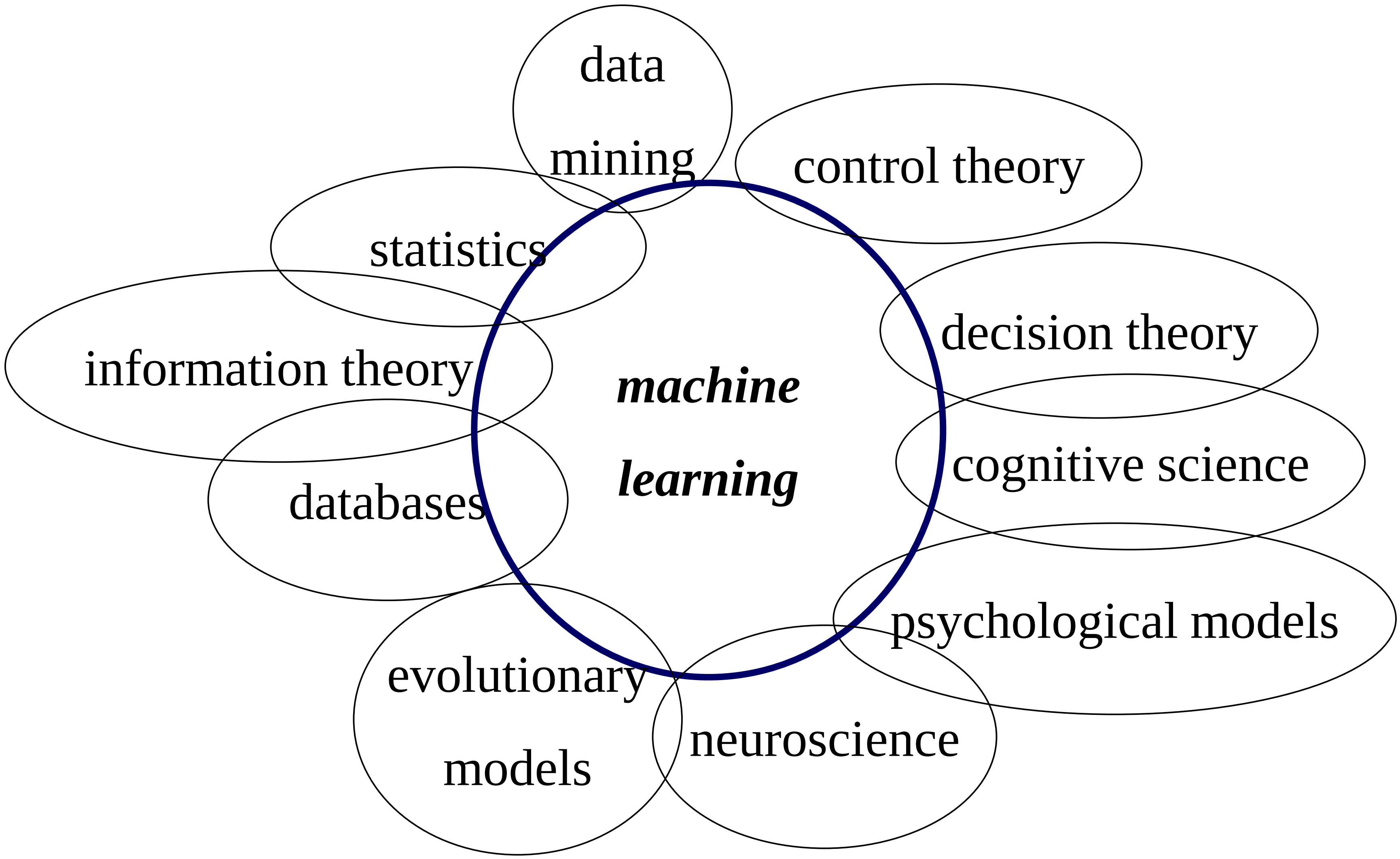
- **Traditional Programming**



- **Machine Learning**



Related Fields



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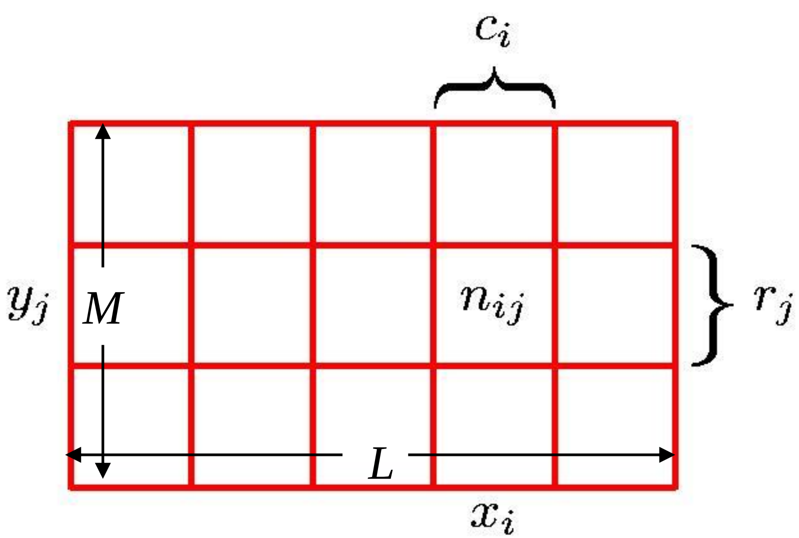
Probability Theory

- **Random Variables**

$$P(A) = \frac{1}{6}, A = 1, 2, \ldots, 6$$

- Random variables describe the outcome of a random experiment in terms of a (real) number
- A random experiment is an experiment that can (in principle) be repeated several times under the same conditions
- Discrete or continuous random variables
- Independent and identically distributed (iid) experiment vs non-iid experiment

Probability Theory



- **Marginal Probability**

$$P(X = x_i) = \frac{c_i}{L}$$

- **Joint Probability**

$$P(X = x_i, Y = y_i) = \frac{n_{ij}}{L \times M} = \frac{c_i \times r_j}{L \times M}$$

- **Conditional Probability**

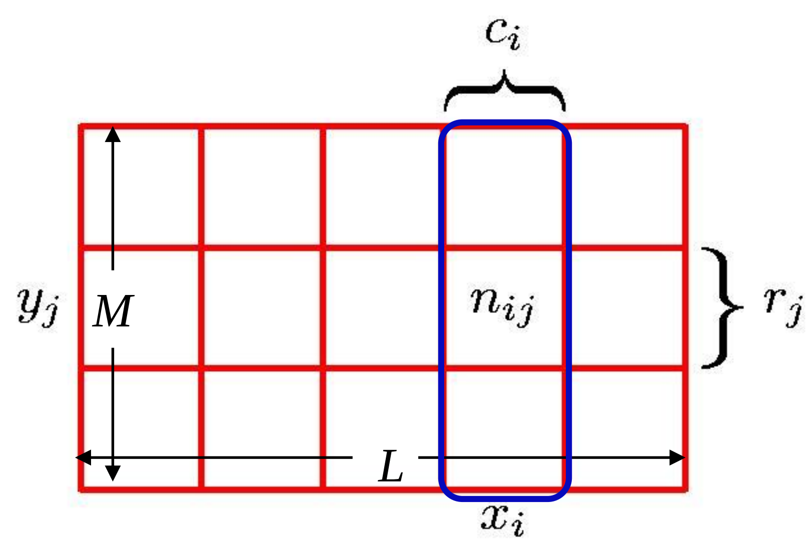
$$P\left(Y = y_j \mid X = x_i\right) = \frac{r_j}{M}$$

Probability Theory

Sum Rule

$$P(X = x_i) = \frac{c_i}{L} = \frac{1}{L \times M}\sum_j n_{ij}$$

$$= \sum_j P(X = x_i, Y = y_j)$$



Product Rule

$$P(X = x_i, Y = y_i) = \frac{n_{ij}}{L \times M} = \frac{r_j}{M} \cdot \frac{c_i}{L}$$

$$= P\left(Y = y_j \mid X = x_i\right) P(X = x_i)$$

Marginalization

Marginal Probability Joint Probability

$$P(X = x_i) = \sum_j P(X = x_i, Y = y_j)$$

$$= \sum_j P\big(X = x_i \big| Y = y_j\big)\, P(Y = y_j)$$

Conditional Probability Marginal Probability

Y \ X	x_1	x_2	x_3	x_4	$p_y(Y)\downarrow$
y_1	$4/32$	$2/32$	$1/32$	$1/32$	$8/32$
y_2	$2/32$	$4/32$	$1/32$	$1/32$	$8/32$
y_3	$2/32$	$2/32$	$2/32$	$2/32$	$8/32$
y_4	$8/32$	0	0	0	$8/32$
$p_x(X) \rightarrow$	$16/32$	$8/32$	$4/32$	$4/32$	$32/32$

Margin

This concept is called "marginal" because it can be found by summing values in a table along rows or columns, and writing the sum in the **margins** of the table

Contents

Bayes' Theorem

The Rules of Probability

$$\text{Sum Rule: } P(X) = \sum_{Y} P(X, Y)$$

$$\text{Product Rule: } P(X, Y) = P(Y|X)P(X)$$

Bayes' Theorem

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)} \qquad P(X) = \sum_{Y} P(X|Y)P(Y)$$

Bayes' Theorem

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

Posterior probability $P(Y|X)$: the likelihood of event Y occurring given that X is true, $P(Y|X)$ is a conditional probability

Posterior probability $P(X|Y)$: the likelihood of event X occurring given that Y is true, $P(X|Y)$ is a conditional probability

Prior probability $P(X)$ and $P(Y)$: the probabilities of observing X and Y independently of each other (the marginal probability)

Bayes' Theorem

$$P(\text{"taking a shower"}|\text{"wet"}) = P(\text{"wet"}|\text{"taking a shower"})\frac{P(\text{"taking a shower"})}{P(\text{"wet"})}$$

$$P(\text{reason}|\text{observation}) = P(\text{observation}|\text{reason})\frac{P(\text{reason})}{P(\text{observation})}$$

- Often useful in diagnosis situations,
  since $P(\text{observation}|\text{reason})$ might be easily determined
- Useful for reasoning
- Often delivers surprising results

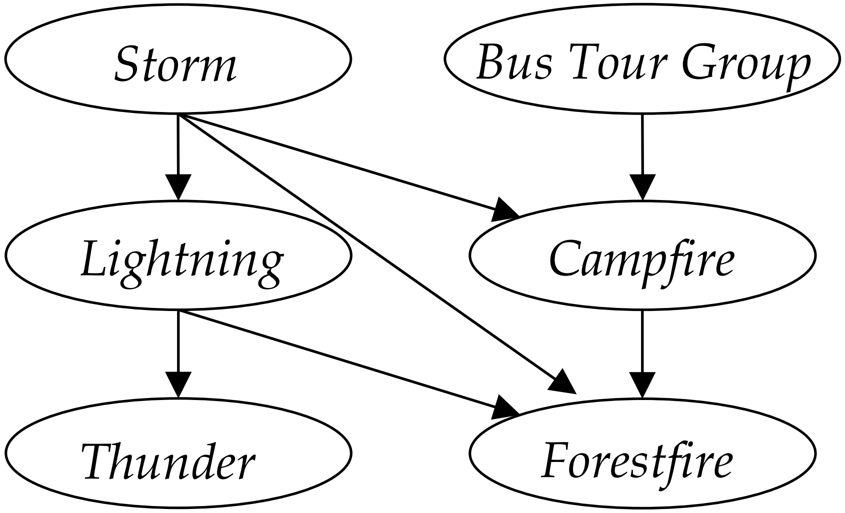
Bayes' Theorem in Bayesian Learning

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- $P(h)$: prior probability of hypothesis h
- $P(D)$: prior probability of training data D
- $P(h|D)$: posterior probability of h given D
- $P(D|h)$: posterior probability of D given h

Bayesian Net

- Network represents conditional independence assertions
- Each node conditionally independent of its non-descendants, given its immediate predecessors (e.g. Campfire and Lightning are independence conditioned on Storm)



conditional probability tables (CPT)

	$S \wedge B$	$S \wedge \neg B$	$\neg S \wedge B$	$\neg S \wedge \neg B$
C	0.4	0.1	0.8	0.2
$\neg C$	0.6	0.9	0.2	0.8

C: Campfire

S: Storm

B: Bus Tour Group

Example

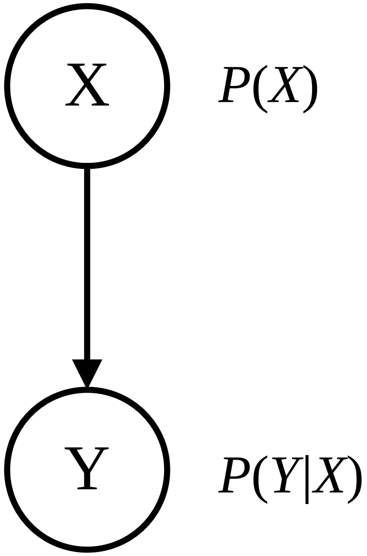
- **Random variables *X* and *Y***

  X: It is raining
  Y: The grass is wet

- ***X* affects *Y***

  Or, *Y* is a symptom of *X*

- **Draw two nodes and link them**
  - Define the CPT(conditional probability tables) for each node
    - $P(\mathrm{X})$ and $P(Y|X)$
  - Typical use: we observe Y and we want to query $P(X|Y)$
  - - Y is an evidence variable
    - X is a query variable



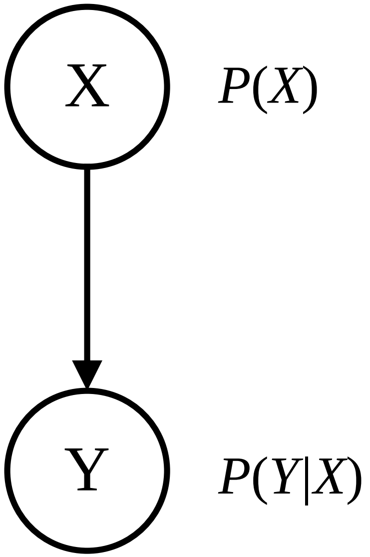
Example

- **What is $P(X|Y)$?**
  - Given that we know the CPTs of each node in the graph

$$P(X \mid Y) = \frac{P(Y \mid X)P(X)}{P(Y)}$$

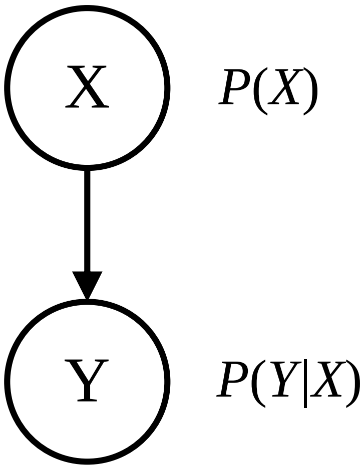
$$= \frac{P(Y \mid X)P(X)}{\sum_X P(X, Y)}$$

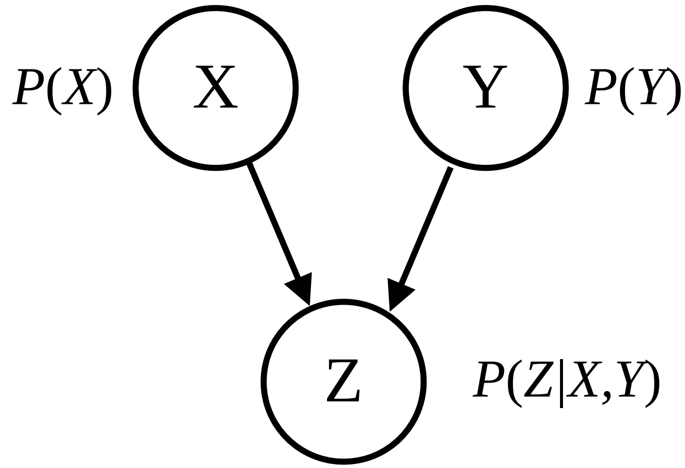
$$= \frac{P(Y \mid X)P(X)}{\sum_X P(Y \mid X)P(X)}$$



Belief Nets Represent Joint Probability

- **The joint probability function can be calculated directly from the network**
- **It is the product of the CPTs of all the nodes**
- $\boldsymbol{P(var_1, \ldots, var_n) = \prod_i P(var_i | \text{Parents}(var_i))}$





$\boldsymbol{P(X,Y) = P(X)P(Y|X)}$

$\boldsymbol{P(X,Y,Z) = P(X)\, P(Y)\, P(Z|X,Y)}$

Probability Densities

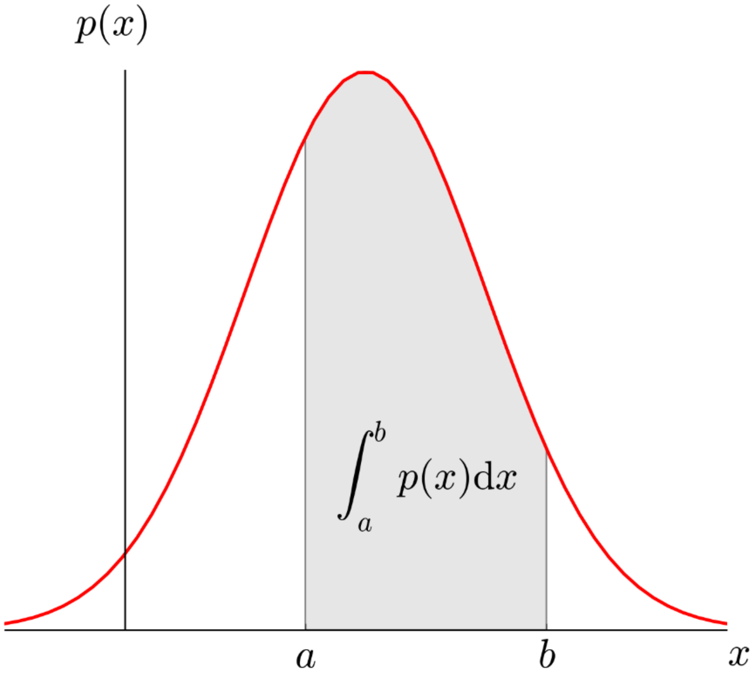
- **The probability density function p(x) has the following properties**

$$p(x) \geqslant 0$$

$$p(x \in (a,b)) = \int_a^b p(x)\,\mathrm{d}x$$

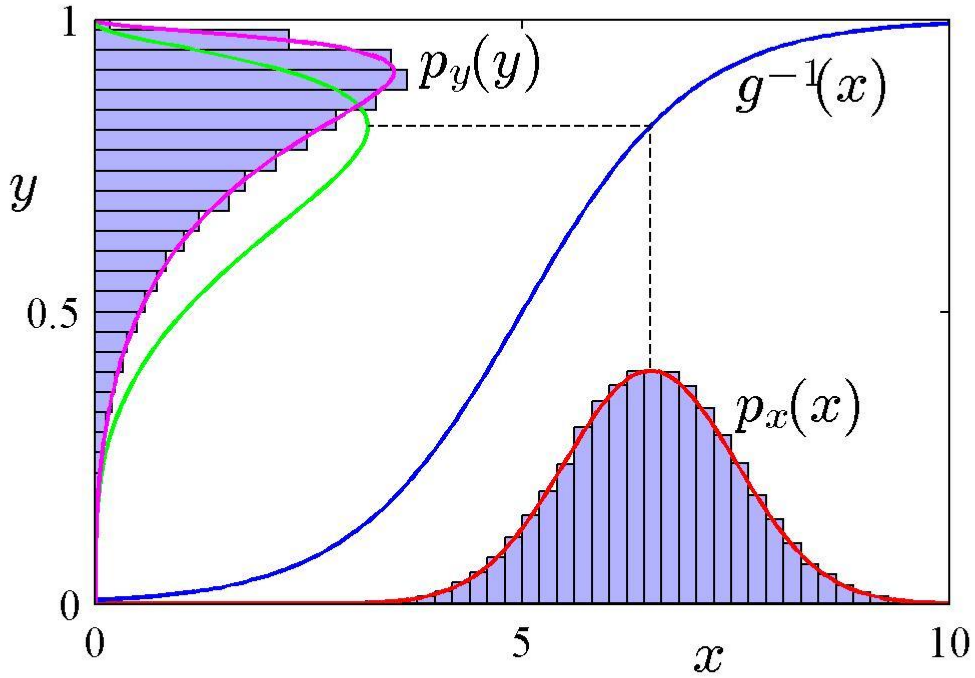
$$P(z) = \int_{-\infty}^{z} p(x)\,\mathrm{d}x$$

$$\int_{-\infty}^{\infty} p(x)\,\mathrm{d}x = 1$$



Transformed Densities

- x has a probability density $p_x(x)$
- $y = h(x)$ is some strictly monotonic continuous function
- Probability density $p_y(y)$ can be transformed from $p_x(x)$



$y = h(x) = g^{-1}(x)$

$$p_y(y) = p_x(x)\left|\frac{\mathrm{d}x}{\mathrm{d}y}\right| = p_x(g(y))\,|g'(y)|$$

Maximum Likelihood Estimation

- **A density $\boldsymbol{f}$ usually contains parameters $\boldsymbol{\theta} \in \Omega$: $\boldsymbol{f(x|\theta)}$**
  Parameters $\boldsymbol{\theta}$: a scalar or a vector

$$f(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- **Question: How to estimate $\boldsymbol{\theta}$ given data $\boldsymbol{\mathcal{D}} = \{\boldsymbol{x_i}\}$?**
- **Likelihood function of $\boldsymbol{\theta}$ given $\boldsymbol{x}$:**

$$L(\theta|x) = P(X = x|\theta)$$

- **Likelihood function of $\boldsymbol{\theta}$ given $\boldsymbol{\mathcal{D}} = \{\boldsymbol{x_i}\}$:**

$$L_{\mathcal{D}}(\theta) = P(\mathcal{D}|\theta) = \prod_{m} P(x_i|\theta)$$

Maximum Likelihood Estimation

- **Likelihood function of $\boldsymbol{\theta}$ given $\mathcal{D} = \{\boldsymbol{x_i}\}$ (iid $\boldsymbol{x_i}$)**

$$L_{\mathcal{D}}(\theta) = P(\mathcal{D}|\theta) = \prod_i P(x_i|\theta)$$

- **Estimate $\boldsymbol{\theta}$ by**

$$\theta_* = \underset{\theta}{\operatorname{argmax}} \left(\prod_i P(x_i|\theta)\right)$$

- **In practice, often use log likelihood function**

$$\theta_* = \underset{\theta}{\operatorname{argmax}} \log\left(\prod_i P(x_i|\theta)\right)$$

- **Then, we have**

$$\theta_* = \underset{\theta}{\operatorname{argmax}} \left(\sum \log\big(P(x_i|\theta)\big)\right)$$

Maximum a Posteriori Estimation

- **Replace the likelihood in the MLE formula with the posterior, and we get:**

$$\theta_{MAP} = \underset{\theta}{\operatorname{argmax}}\, P(X|\theta)P(\theta)$$

$$= \underset{\theta}{\operatorname{argmax}}\, \log P(X|\theta) + \log P(\theta)$$

$$= \underset{\theta}{\operatorname{argmax}}\, \log \prod_i P(x_i|\theta) + \log P(\theta)$$

$$= \underset{\theta}{\operatorname{argmax}} \sum_i \log P(x_i|\theta) + \log P(\theta)$$

MLE vs MAP

- **If we use uniform prior in MAP estimation, *P*(*θ*) is a const, so we have:**

$$\theta_{MAP} = \operatorname{argmax} \sum_i \log P(x_i|\theta) + \log P(\theta)$$
$$= \operatorname{argmax} \sum_i \log P(x_i|\theta) + const$$
$$= \operatorname{argmax} \sum_i \log P(x_i|\theta) = \theta_{MLE}$$

- **MLE is a special case of MAP, where the prior is uniform**

Contents

Probability and Information Theory

- **Information measure of an event *A***

$$I(A) = -\log_b P(A)$$

***I*(*A*):** self-information or information content, random variable

***P*(*A*):** probability of the event happening

***b*:** base, usually b=2

base 2 = bits   base 3 = trits
base 10 = Hartleys   base e = nats

Information and Probability

Examples

The Chinese football team lost：

$P(A)$=1 $$I(A) = -\log_2 P(A) = 0$$

The Chinese table tennis team lost：

$P(A)$=0 $$I(A) = -\log_2 P(A) = +\infty$$

Probability $P(A)$: The degree of uncertainty of an event

Self-information $I(A)$: The elimination of uncertainty

Entropy

- **Entropy is simply the average (expected) amount of the information from the event**

$$H(A) = -E[\log_2 P(A)] = -\sum_{A} P(A)\log_2 P(A)$$

H*(*A*) is maximized when $\boldsymbol{P(A) = \frac{1}{n}}$ for all *A

- **Joint Entropy**

$$H(A,B) = -E[\log_2 P(A,B)] = -\sum_{A,B} P(A,B)\log_2 P(A,B)$$

- **Conditional entropy of *A* given *B***

$$H(A|B) = -E[\log_2 P(A|B)] = -\sum_{A,B} P(A,B)\log_2 P(A|B)$$

Thank You