# Basics of Image Processing

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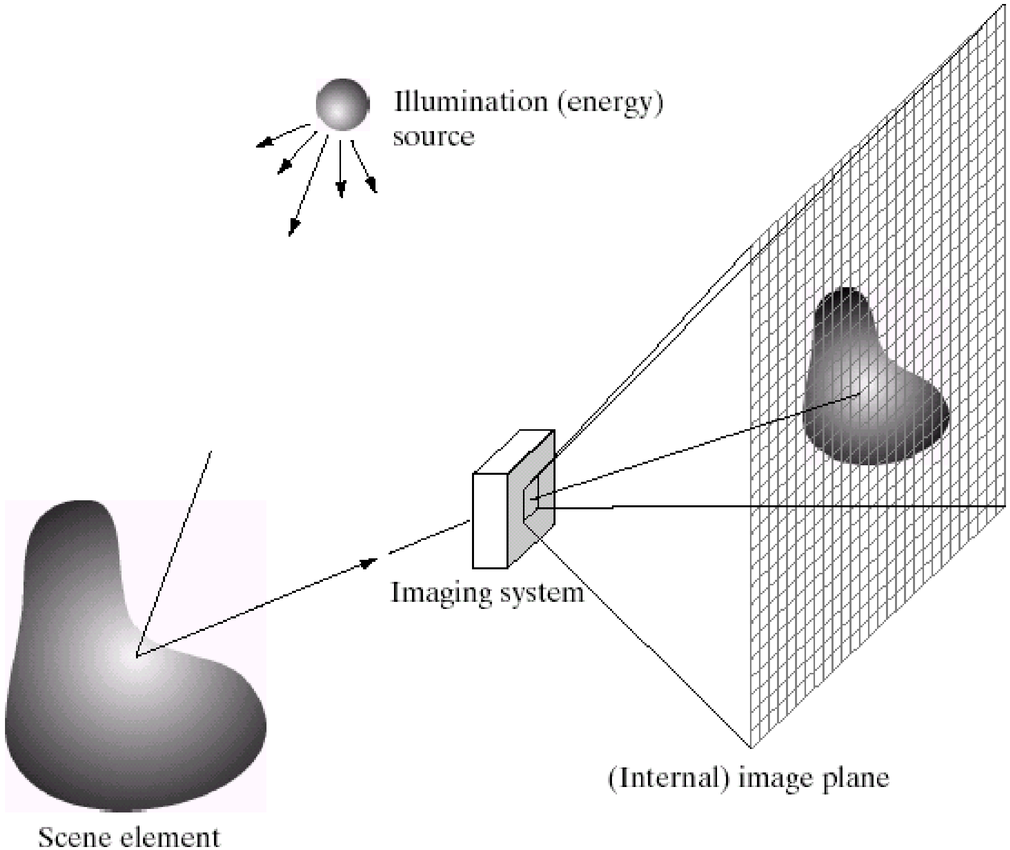
# Digital Camera

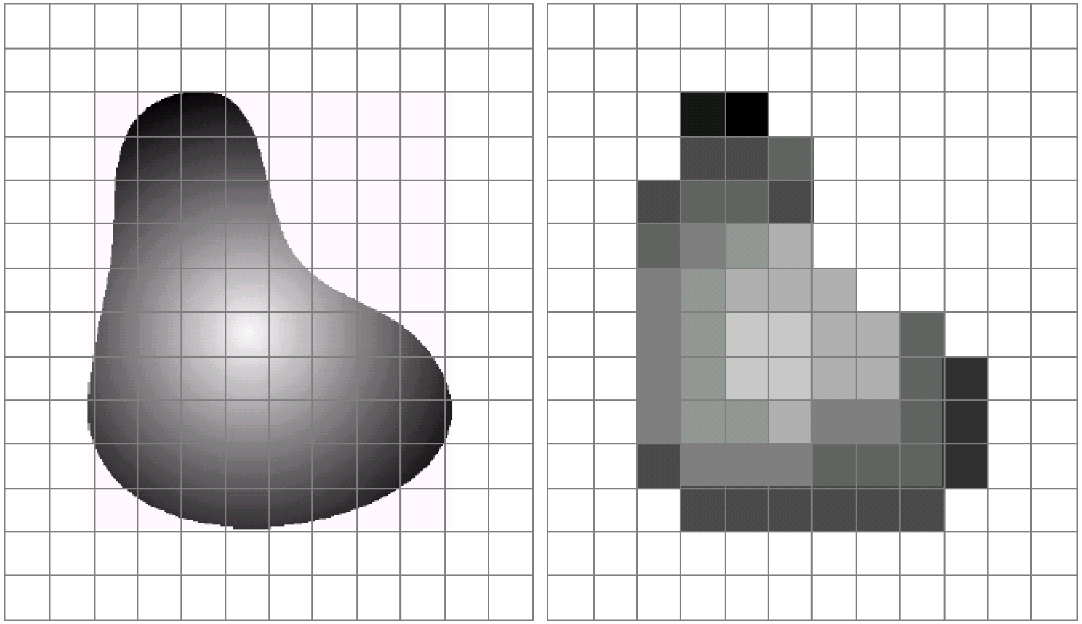

## A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons
- Two common types: Charge Coupled Device (CCD) and CMOS
- http://electronics.howstuffworks.com/digital-camera.htm

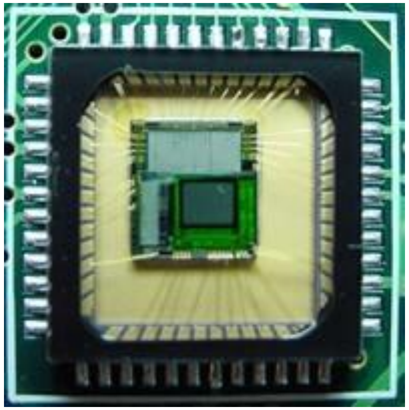
# How Light is Recorded



# Sensor Array



(a) (b)

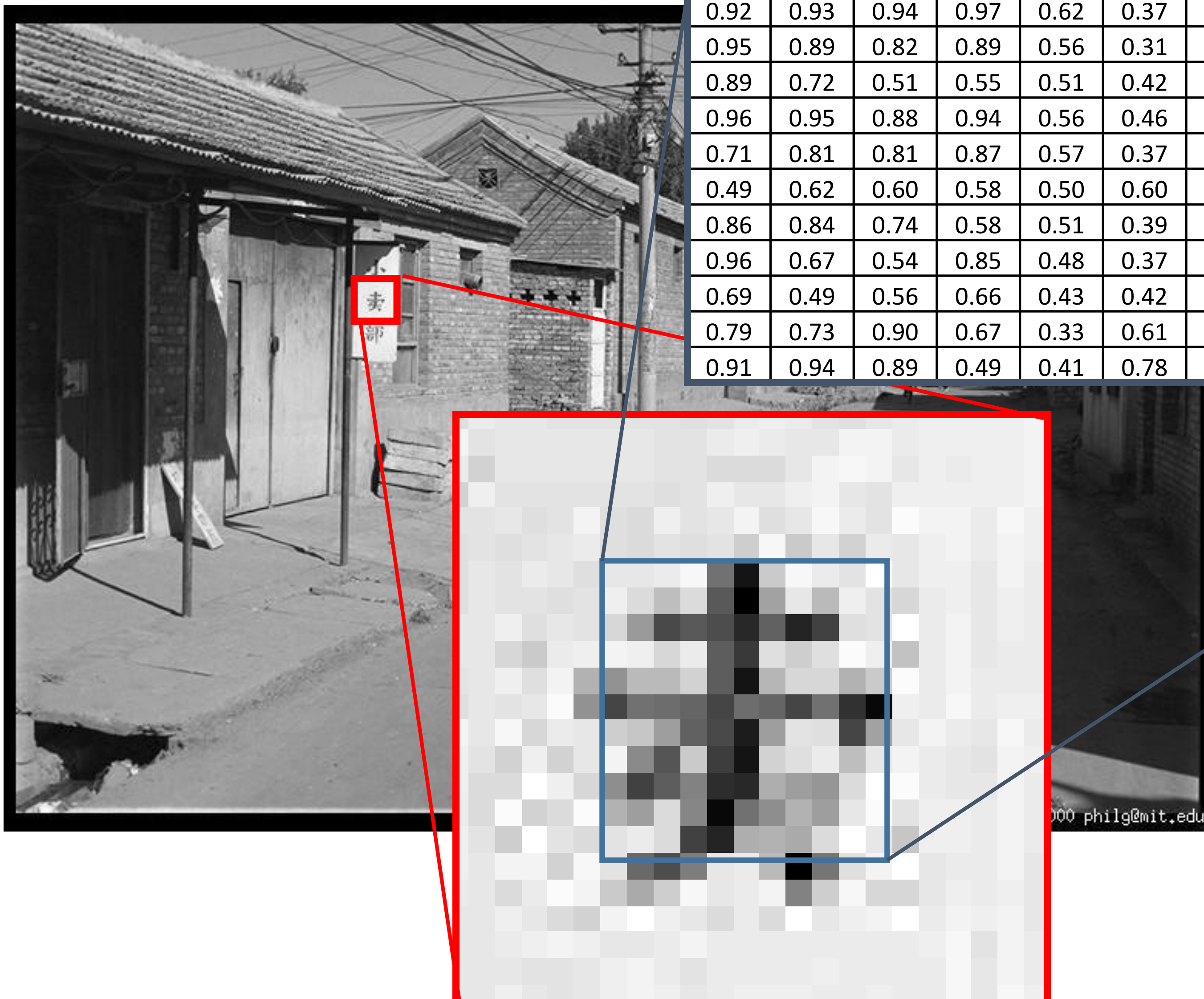


CMOS sensor

(a) Continuous image projected onto a sensor array
(b) Result of image sampling and quantization

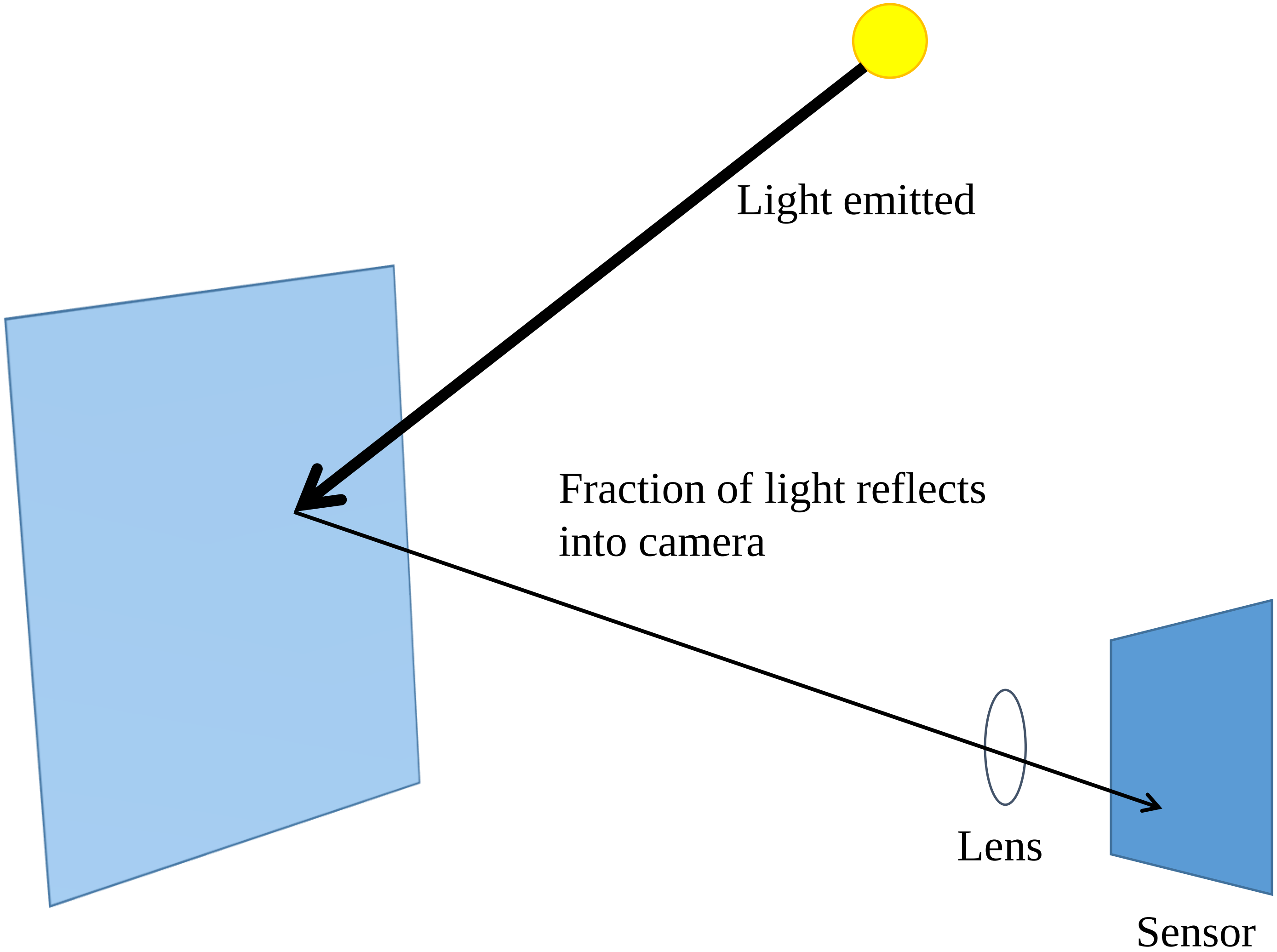
- Each sensor cell records amount of light coming in at a small range of orientations

# Raster Image (Pixel Matrix)




| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.92 | 0.93 | 0.94 | 0.97 | 0.62 | 0.37 | 0.85 | 0.97 | 0.93 | 0.92 | 0.99 |
| 0.95 | 0.89 | 0.82 | 0.89 | 0.56 | 0.31 | 0.75 | 0.92 | 0.81 | 0.95 | 0.91 |
| 0.89 | 0.72 | 0.51 | 0.55 | 0.51 | 0.42 | 0.57 | 0.41 | 0.49 | 0.91 | 0.92 |
| 0.96 | 0.95 | 0.88 | 0.94 | 0.56 | 0.46 | 0.91 | 0.87 | 0.90 | 0.97 | 0.95 |
| 0.71 | 0.81 | 0.81 | 0.87 | 0.57 | 0.37 | 0.80 | 0.88 | 0.89 | 0.79 | 0.85 |
| 0.49 | 0.62 | 0.60 | 0.58 | 0.50 | 0.60 | 0.58 | 0.50 | 0.61 | 0.45 | 0.33 |
| 0.86 | 0.84 | 0.74 | 0.58 | 0.51 | 0.39 | 0.73 | 0.92 | 0.91 | 0.49 | 0.74 |
| 0.96 | 0.67 | 0.54 | 0.85 | 0.48 | 0.37 | 0.88 | 0.90 | 0.94 | 0.82 | 0.93 |
| 0.69 | 0.49 | 0.56 | 0.66 | 0.43 | 0.42 | 0.77 | 0.73 | 0.71 | 0.90 | 0.99 |
| 0.79 | 0.73 | 0.90 | 0.67 | 0.33 | 0.61 | 0.69 | 0.79 | 0.73 | 0.93 | 0.97 |
| 0.91 | 0.94 | 0.89 | 0.49 | 0.41 | 0.78 | 0.78 | 0.77 | 0.89 | 0.99 | 0.93 |

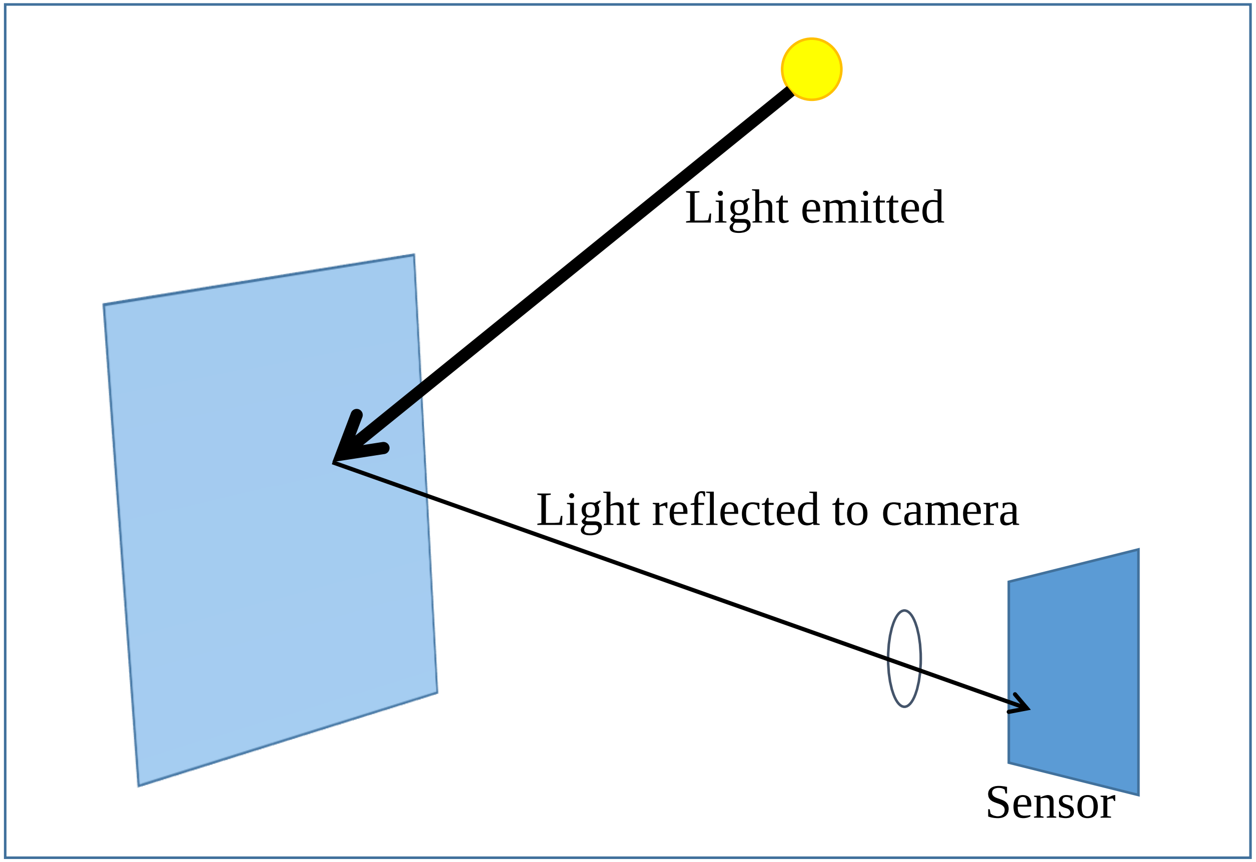
# How does a Pixel Get its Value?



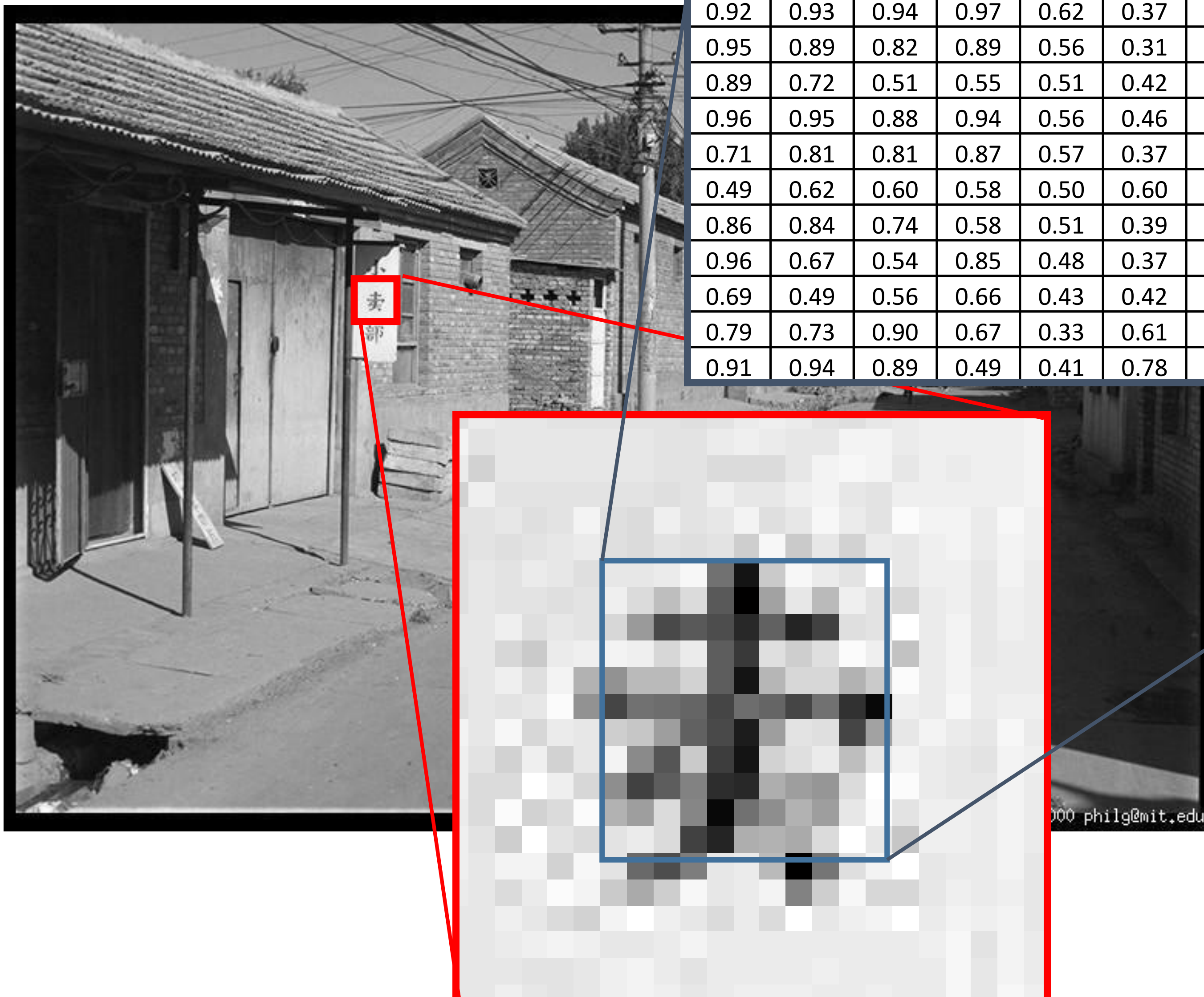
# How does a Pixel Get its Value?

## Major factors

- Illumination strength and direction
- Surface geometry
- Surface material
- Nearby surfaces
- Camera gain/exposure

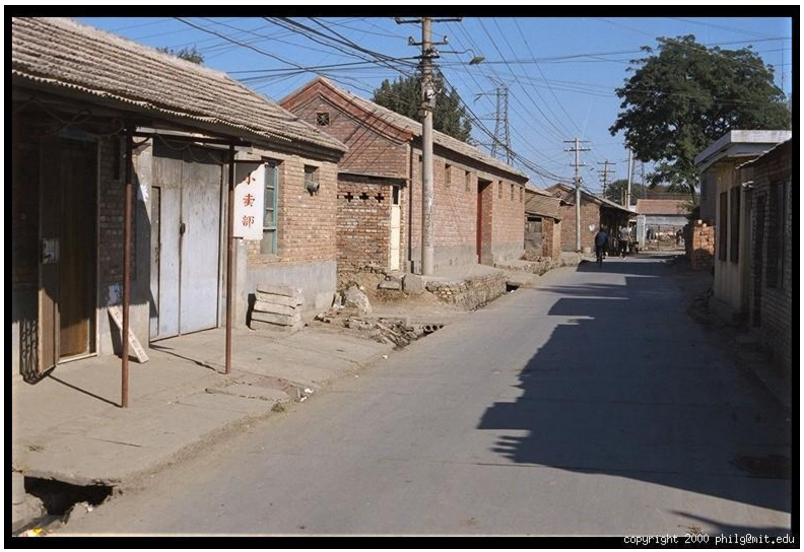


# Gray Image (One Single Matrix)




| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.92 | 0.93 | 0.94 | 0.97 | 0.62 | 0.37 | 0.85 | 0.97 | 0.93 | 0.92 | 0.99 |
| 0.95 | 0.89 | 0.82 | 0.89 | 0.56 | 0.31 | 0.75 | 0.92 | 0.81 | 0.95 | 0.91 |
| 0.89 | 0.72 | 0.51 | 0.55 | 0.51 | 0.42 | 0.57 | 0.41 | 0.49 | 0.91 | 0.92 |
| 0.96 | 0.95 | 0.88 | 0.94 | 0.56 | 0.46 | 0.91 | 0.87 | 0.90 | 0.97 | 0.95 |
| 0.71 | 0.81 | 0.81 | 0.87 | 0.57 | 0.37 | 0.80 | 0.88 | 0.89 | 0.79 | 0.85 |
| 0.49 | 0.62 | 0.60 | 0.58 | 0.50 | 0.60 | 0.58 | 0.50 | 0.61 | 0.45 | 0.33 |
| 0.86 | 0.84 | 0.74 | 0.58 | 0.51 | 0.39 | 0.73 | 0.92 | 0.91 | 0.49 | 0.74 |
| 0.96 | 0.67 | 0.54 | 0.85 | 0.48 | 0.37 | 0.88 | 0.90 | 0.94 | 0.82 | 0.93 |
| 0.69 | 0.49 | 0.56 | 0.66 | 0.43 | 0.42 | 0.77 | 0.73 | 0.71 | 0.90 | 0.99 |
| 0.79 | 0.73 | 0.90 | 0.67 | 0.33 | 0.61 | 0.69 | 0.79 | 0.73 | 0.93 | 0.97 |
| 0.91 | 0.94 | 0.89 | 0.49 | 0.41 | 0.78 | 0.78 | 0.77 | 0.89 | 0.99 | 0.93 |

# Color Image

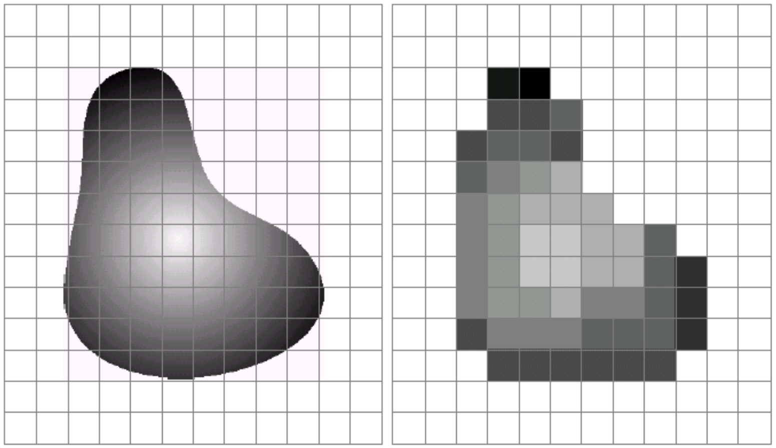



R

G

B

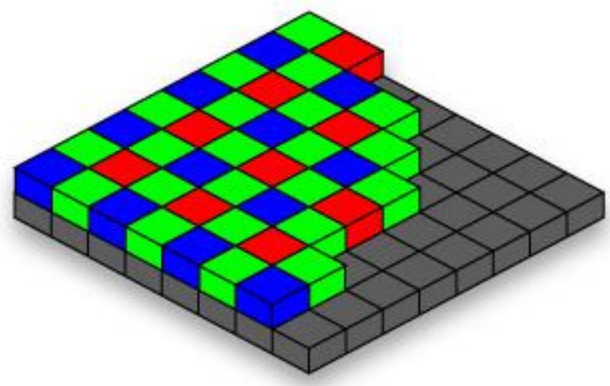
# Digital Color Images

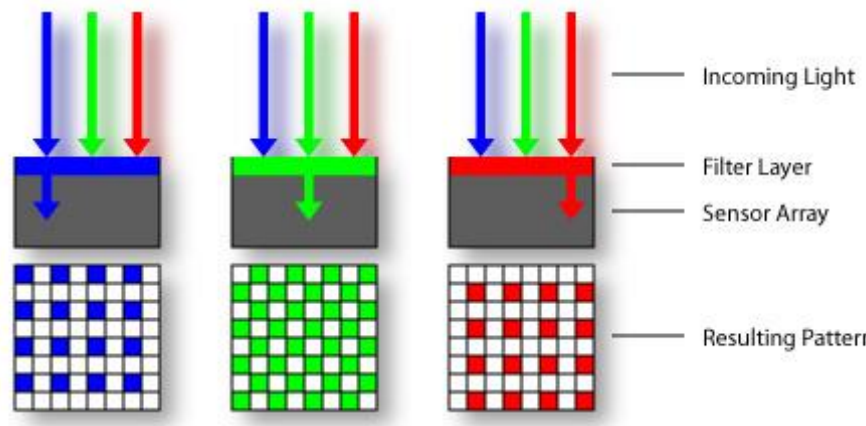


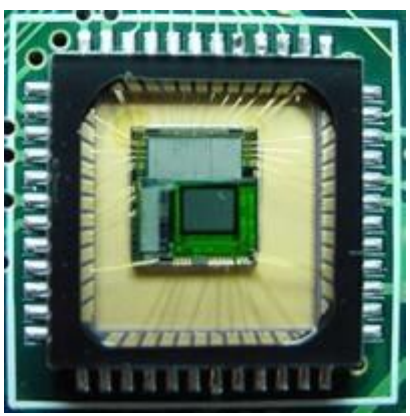
a b

**FIGURE 2.17** (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Bayer Filter





CMOS sensor

# Digital Image Storage

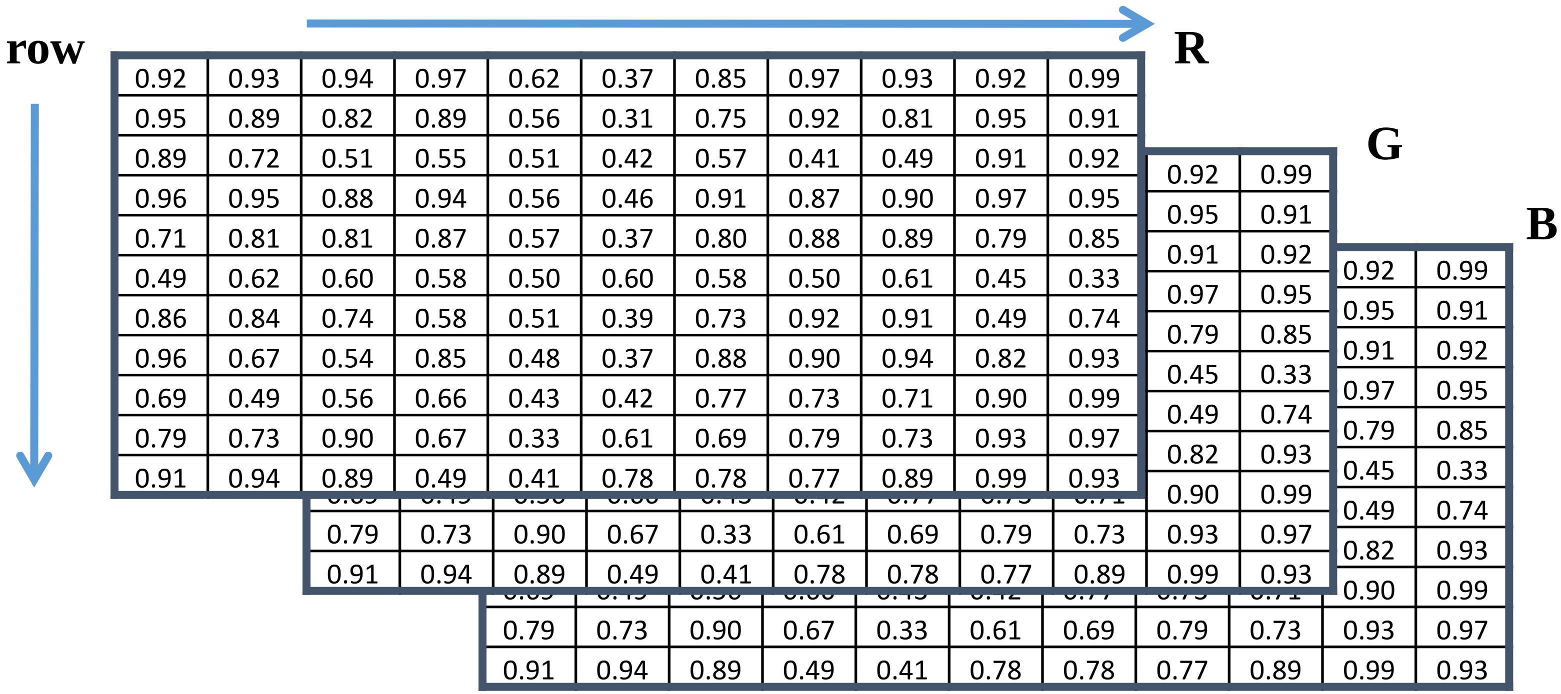
## Stored in two parts

- Header
  - width, height ,… ,cookie
    - Cookie is an indicator of what type of image file
- Data
  - uncompressed, compressed, ASCII, binary.

## File types

- JPEG, BMP, PPM

# Images in Matlab

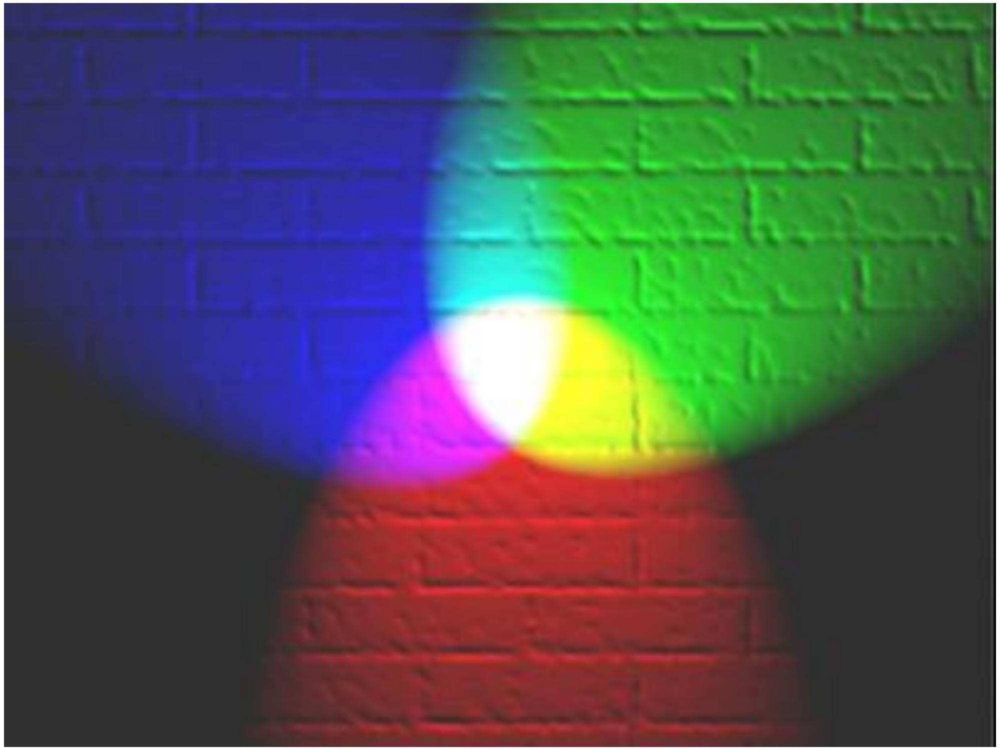



| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.92 | 0.93 | 0.94 | 0.97 | 0.62 | 0.37 | 0.85 | 0.97 | 0.93 | 0.92 | 0.99 |
| 0.95 | 0.89 | 0.82 | 0.89 | 0.56 | 0.31 | 0.75 | 0.92 | 0.81 | 0.95 | 0.91 |
| 0.89 | 0.72 | 0.51 | 0.55 | 0.51 | 0.42 | 0.57 | 0.41 | 0.49 | 0.91 | 0.92 |
| 0.96 | 0.95 | 0.88 | 0.94 | 0.56 | 0.46 | 0.91 | 0.87 | 0.90 | 0.97 | 0.95 |
| 0.71 | 0.81 | 0.81 | 0.87 | 0.57 | 0.37 | 0.80 | 0.88 | 0.89 | 0.79 | 0.85 |
| 0.49 | 0.62 | 0.60 | 0.58 | 0.50 | 0.60 | 0.58 | 0.50 | 0.61 | 0.45 | 0.33 |
| 0.86 | 0.84 | 0.74 | 0.58 | 0.51 | 0.39 | 0.73 | 0.92 | 0.91 | 0.49 | 0.74 |
| 0.96 | 0.67 | 0.54 | 0.85 | 0.48 | 0.37 | 0.88 | 0.90 | 0.94 | 0.82 | 0.93 |
| 0.69 | 0.49 | 0.56 | 0.66 | 0.43 | 0.42 | 0.77 | 0.73 | 0.71 | 0.90 | 0.99 |
| 0.79 | 0.73 | 0.90 | 0.67 | 0.33 | 0.61 | 0.69 | 0.79 | 0.73 | 0.93 | 0.97 |
| 0.91 | 0.94 | 0.89 | 0.49 | 0.41 | 0.78 | 0.78 | 0.77 | 0.89 | 0.99 | 0.93 |

- Image represented as a matrix
- Suppose we have a N×M RGB image called “im”
  - im(1,1,1) = top-left pixel value in R-channel
  - im(y, x, b) = y pixels down, x pixels to right in the $b^{th}$ channel
  - im(N, M, 3) = bottom-right pixel in B-channel
- imread(filename) returns a uint8 image (values 0 to 255)
  - Convert to double format (values 0 to 1) with im2double

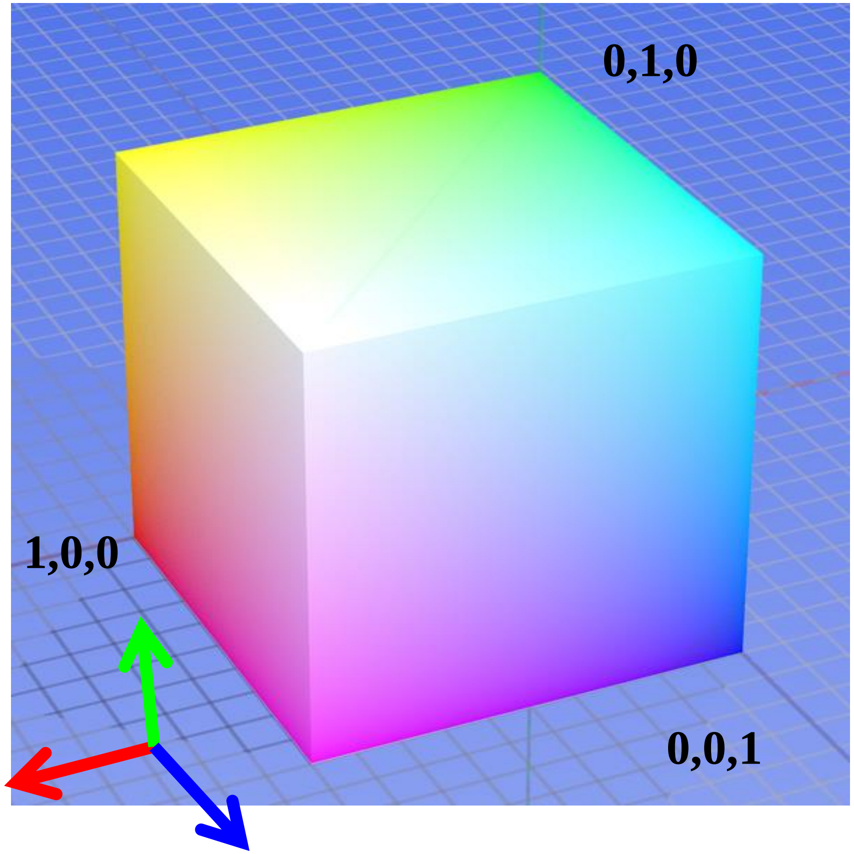
# Color Space

How can we represent color?



# Color Space: RGB

Default color space




**R**
(G=0,B=0)

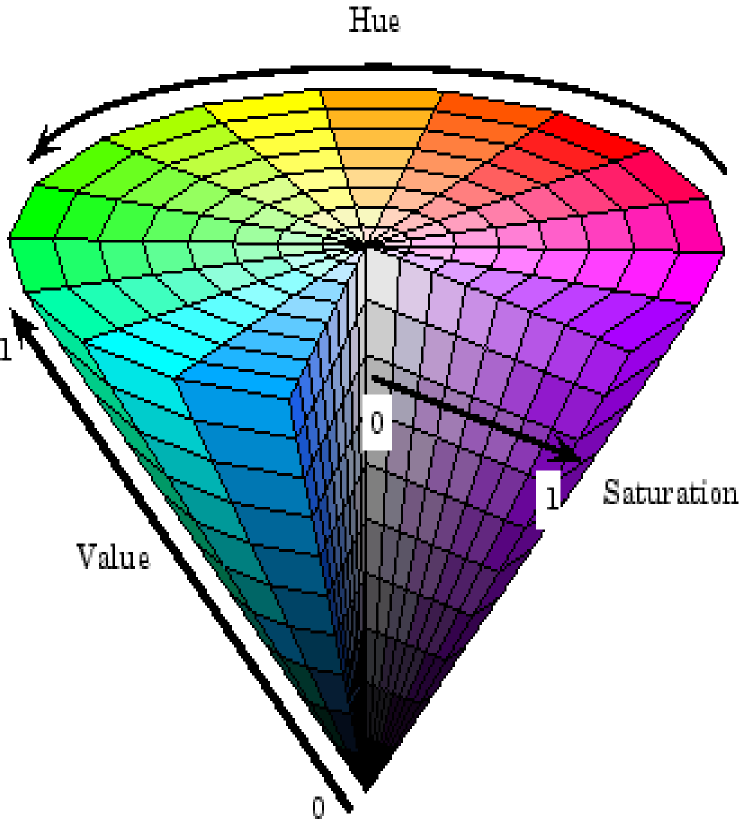
**G**
(R=0,B=0)

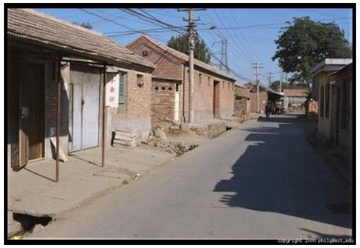
**B**
(R=0,G=0)

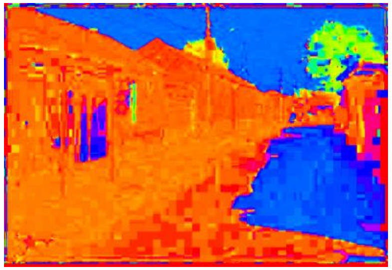
- Some drawbacks
  - Strongly correlated channels
  - Non-perceptual

# Color Space: HSV

Intuitive color space



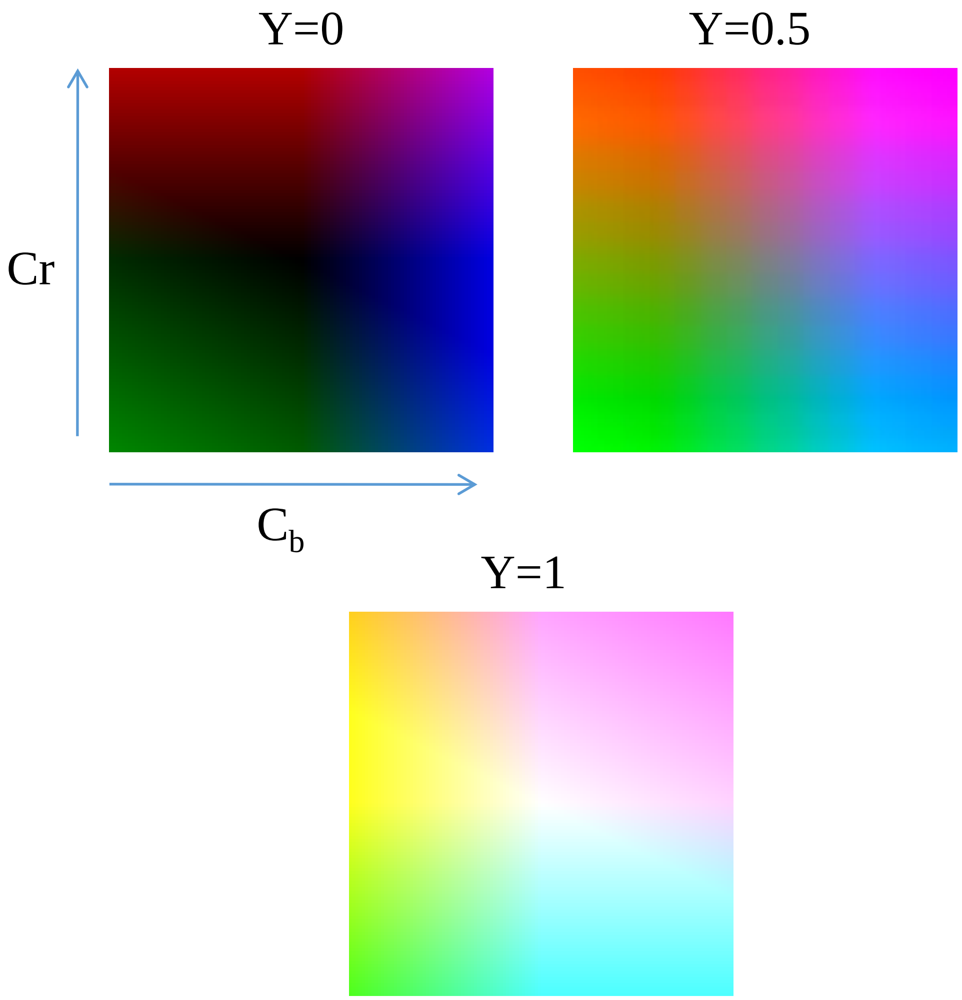


**H**
(S=1,V=1)

**S**
(H=1,V=1)

**V**
(H=1,S=0)

# Color Space: YCbCr




- Fast to compute, good for compression, used by TV

# Color Space: YCbCr

$$Y' = 16 + \frac{65.738 \cdot R'_D}{256} + \frac{144.384 \cdot G'_D}{256} + \frac{25.064 \cdot B'_D}{256}$$

**Y**
(Cb=0.5,Cr=0.5)

$$C_B = 128 - \frac{37.945 \cdot R'_D}{256} - \frac{74.494 \cdot G'_D}{256} + \frac{112.439 \cdot B'_D}{256}$$

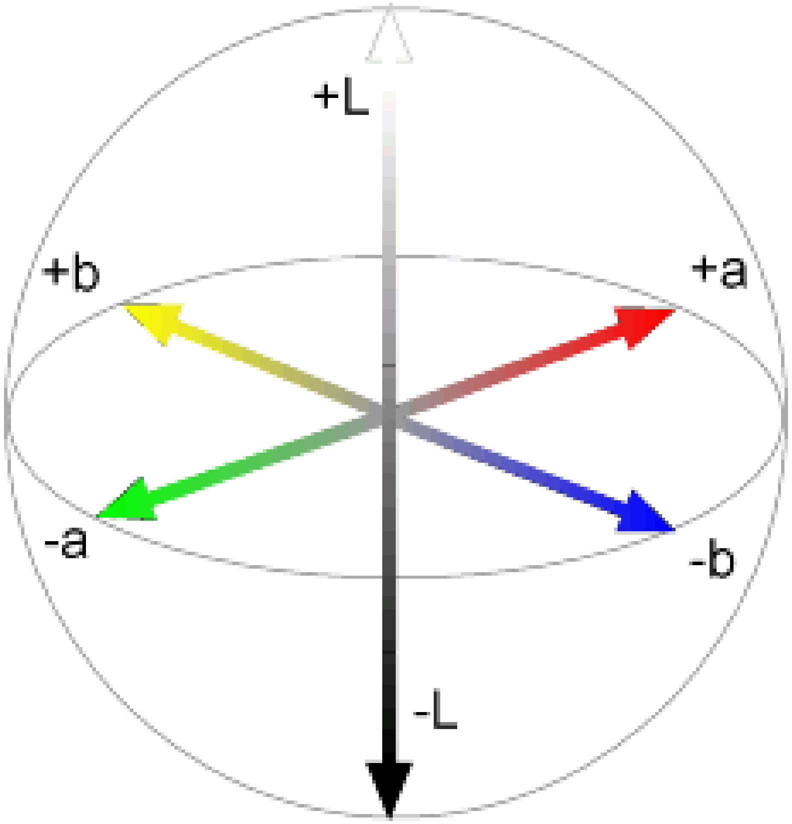
***C_B***
(Y=0.5,Cr=0.5)

$$C_R = 128 + \frac{112.439 \cdot R'_D}{256} - \frac{94.154 \cdot G'_D}{256} - \frac{18.285 \cdot B'_D}{256}$$

***C_R***
(Y=0.5,Cb=05)

# Color Space: CIE L*a*b*

"Perceptually uniform" color space




Luminance = brightness
Chrominance = color

**L**
(a=0,b=0)

**a**
(L=65,b=0)

**b**
(L=65,a=0)

# Questions

- **Which contains more information?**
  - intensity (1 channel)
  - chrominance (Chroma 2 channels)

- **If you had to choose, would you rather go without luminance or chrominance?**

# Only Color Shown – Constant Intensity

# Only Intensity Shown – Constant Color

# What is Image Processing?

- Improving the image qualities
- Improvement of pictorial information for human interpretation
- The processing helps in maximizing clarity, sharpness and details of features of interest towards information extraction

# Contents

# Image Processing vs Computer Vision

**Low Level**

**Image Processing**

**Acquisition, representation, compression, transmission**

**image enhancement**

**edge/feature extraction**

**Pattern matching**

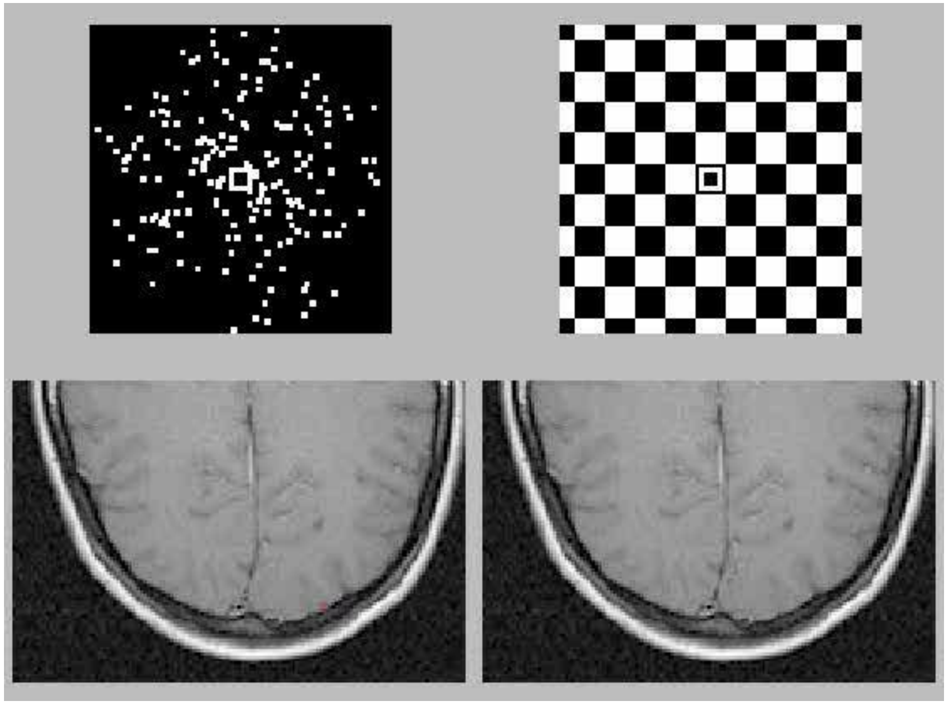
**image "understanding“ (Recognition, 3D)**

**Computer Vision**

**High Level**

# IP and CV are Interdisciplinary Fields

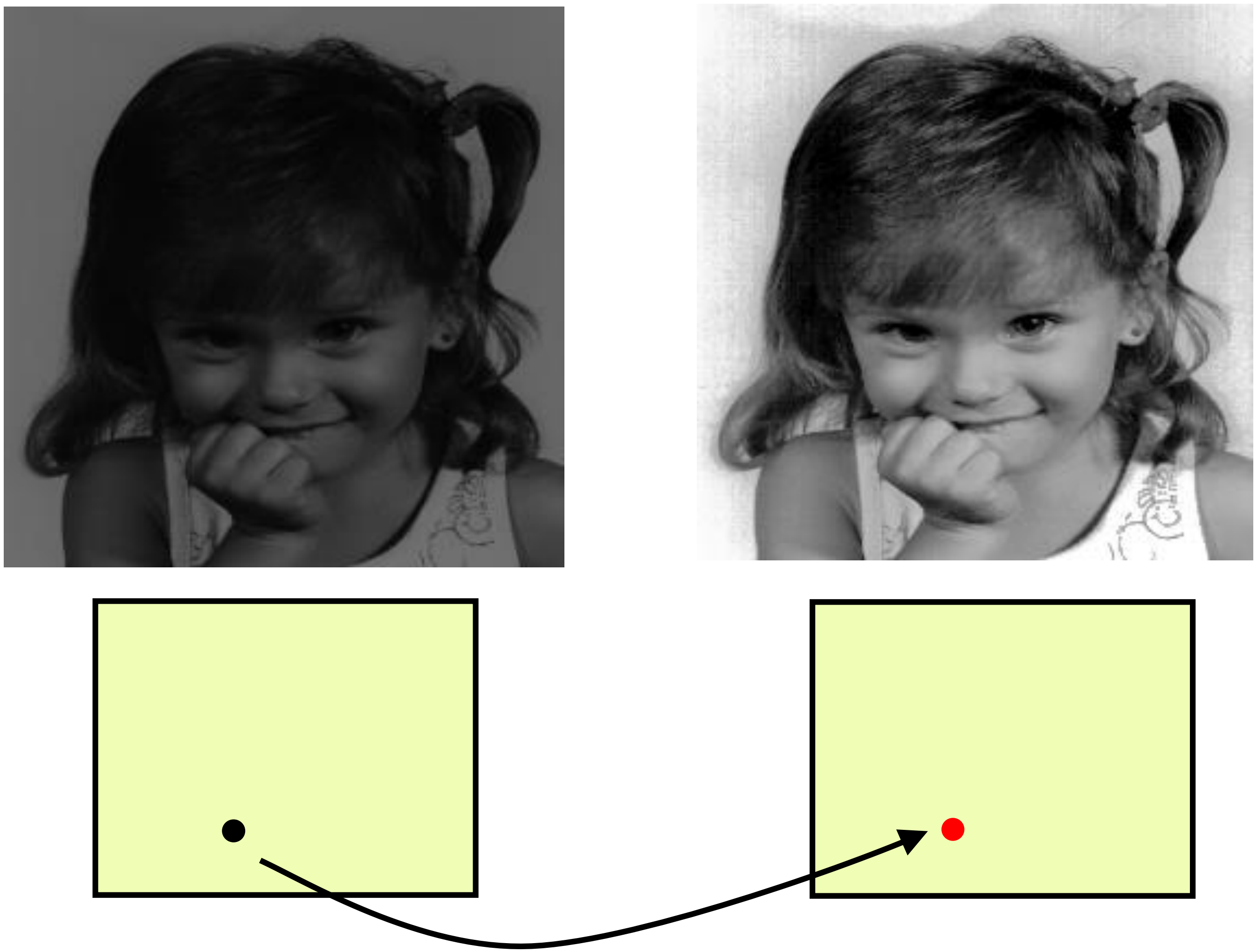
- Mathematical Models (CS, EE, Math)
- Eye Research (Biology)
- Brain Research:
  - Psychophysics (Psychologists)
  - Electro-physiology (Biologists)
  - Functional MRI (Biologists)



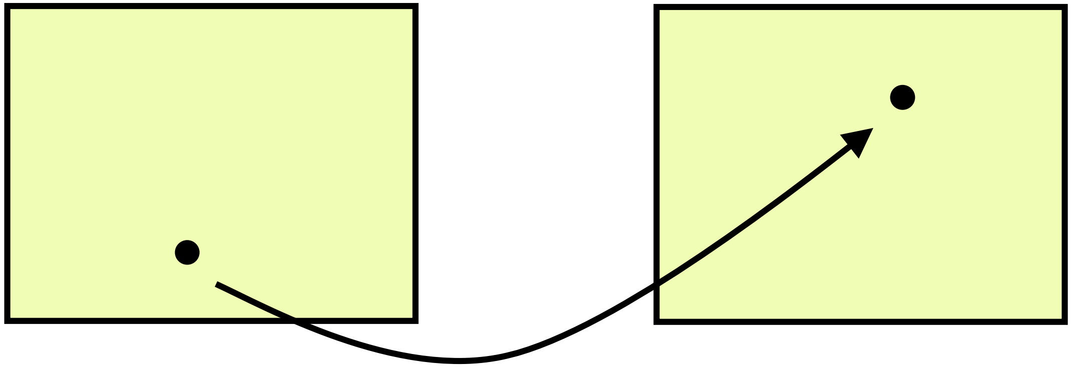
# Image Operations

- Point Operations
- Geometric Operations
- Spatial Operations
- Global Operations (Freq. domain)
- Multi-Resolution Operations
- The Fourier Transform
- TMulti-Resolution

# Point Operations

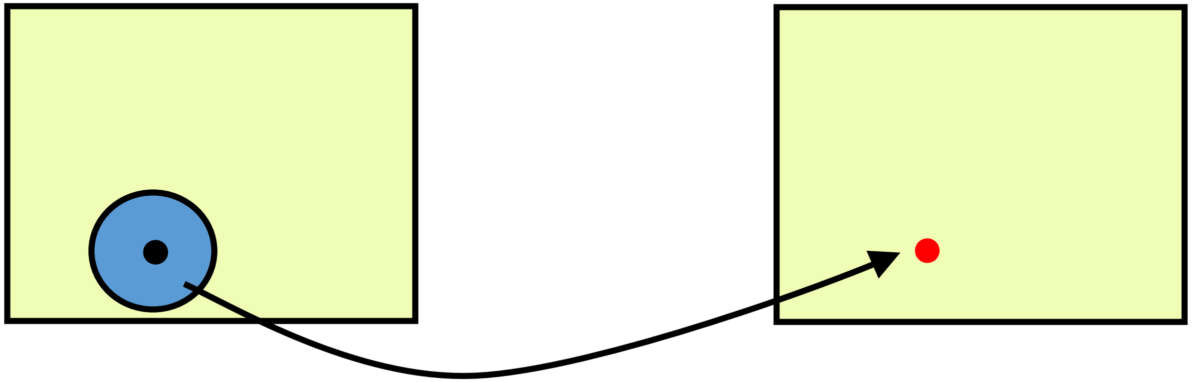


# Geometric Operations



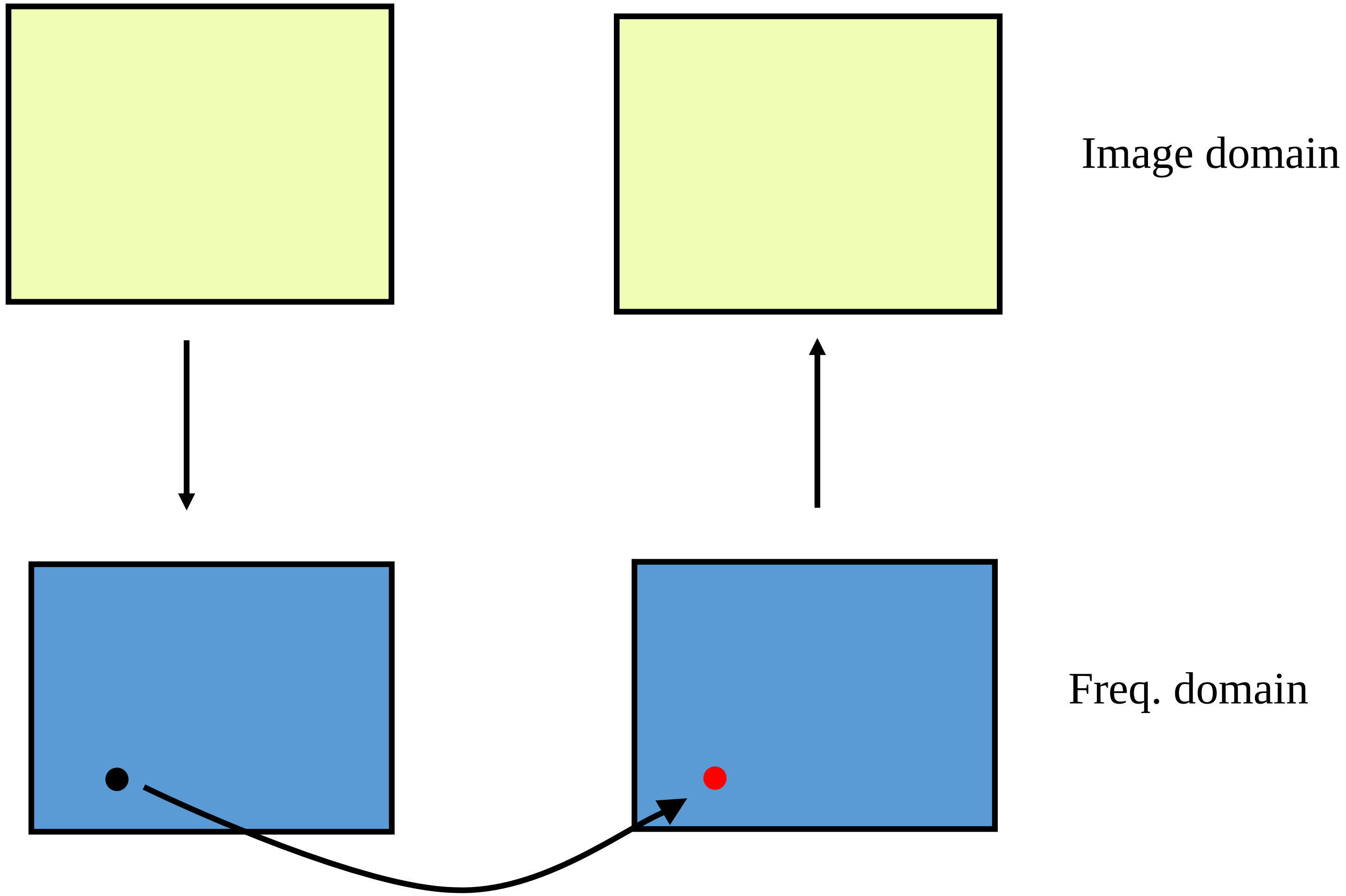
# Geometric and Point Operations

# Spatial Operations

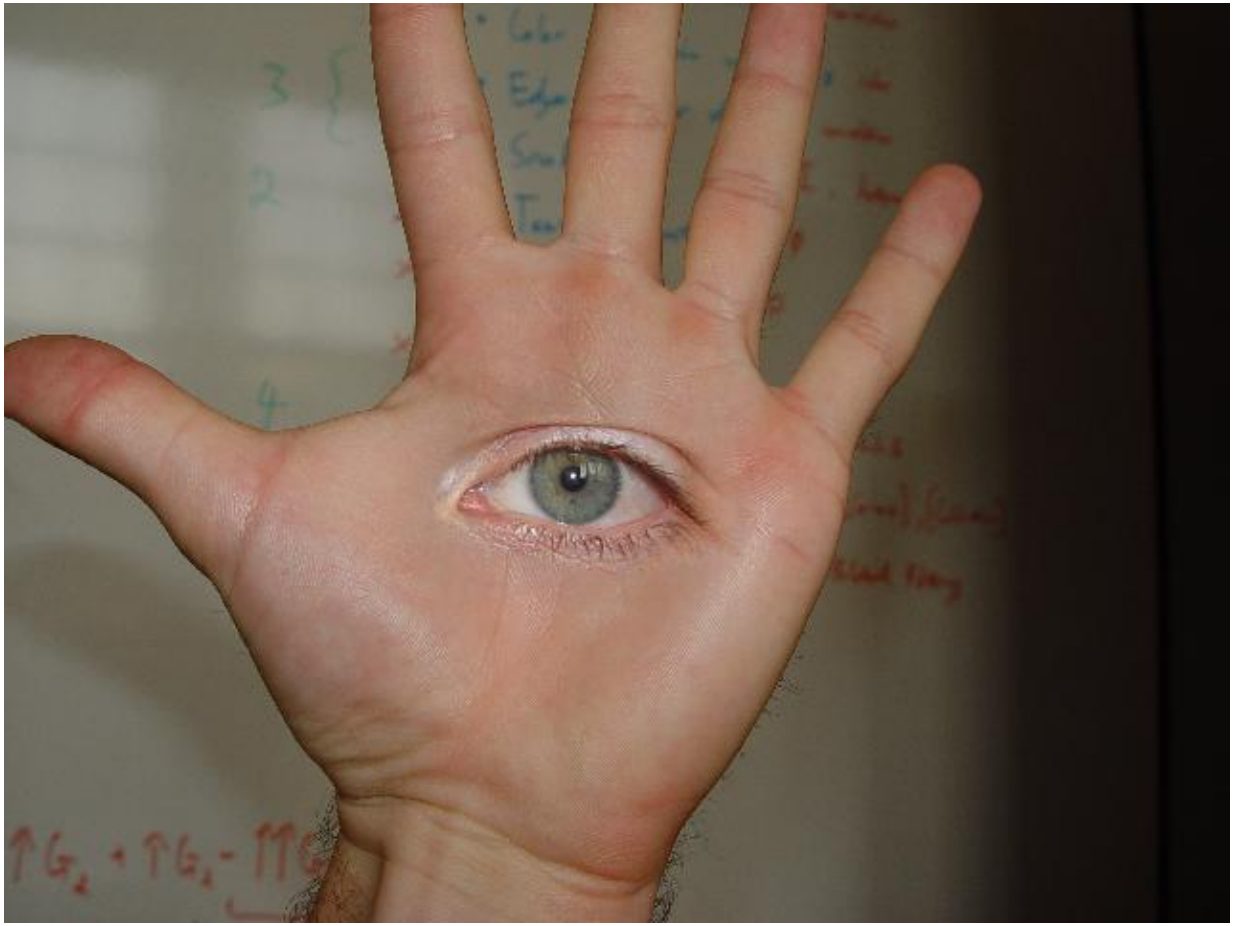


# Global Operations

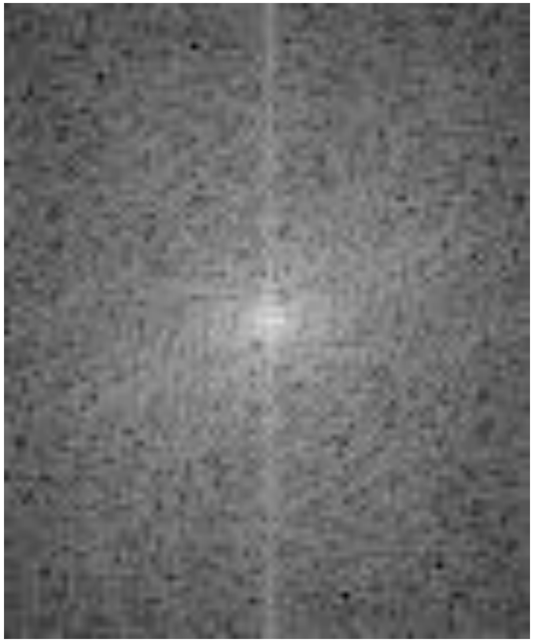
# Global Operations



# Multi-resolution Operations



# Fourier Transform



Jean Baptiste Joseph Fourier  1768-1830

# TMulti-Resolution

Low resolution

High resolution

# Image Denoising

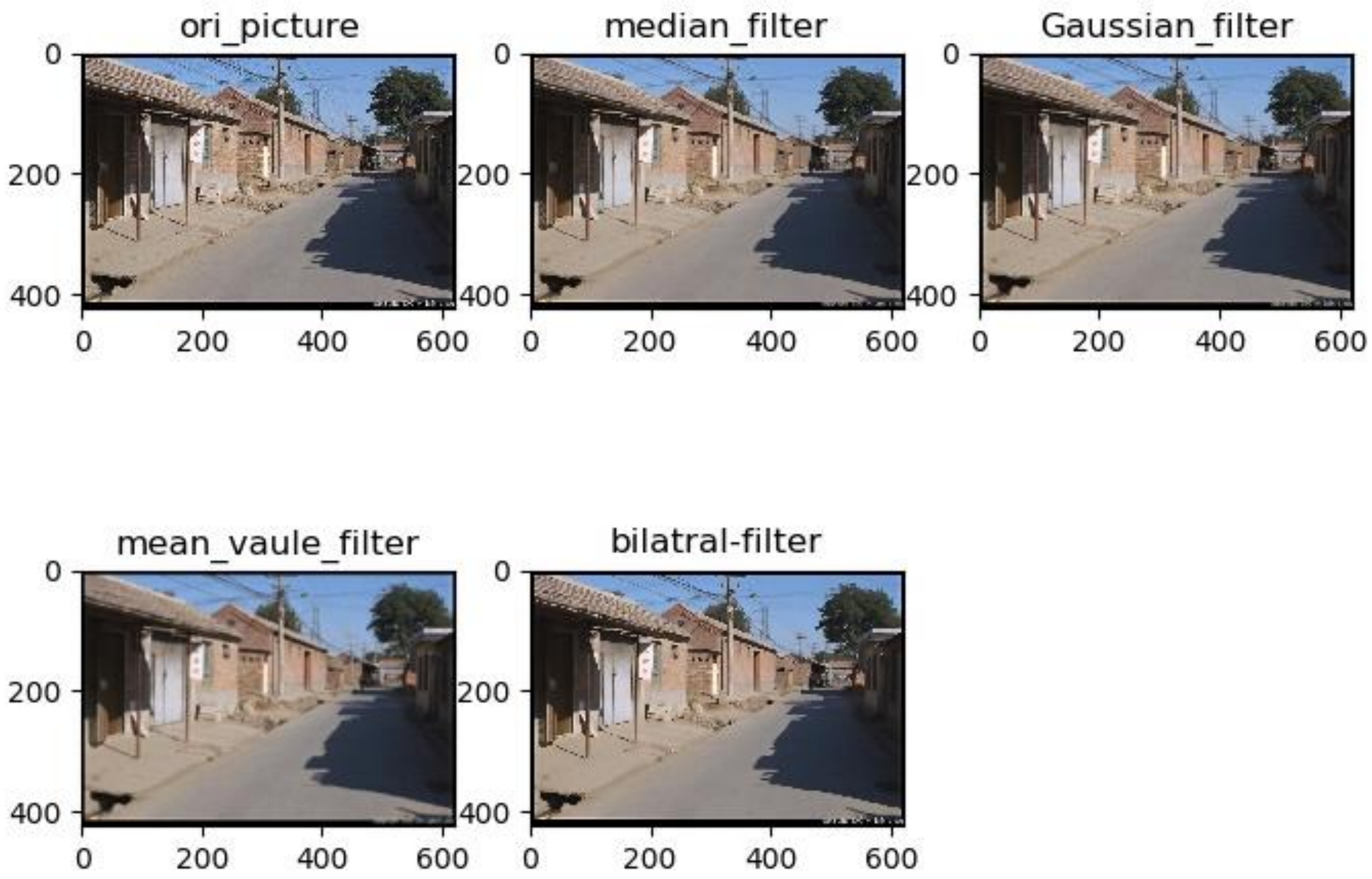
## Definition

- Image Denoising refers to the process of reducing noise in digital images called image denoising

## Denoising method

- Median filtering
- Gaussian filtering
- Mean filtering
- Wiener filtering
- Fourier filtering

# Image Denoising



# Image Enhancement

## Definition

- Enhancing useful information in an image
- It can be a distorted process
- It improve the visual effect of the image, for a given image application

## Image enhancement method

- Airspace based algorithm
- Point algorithm
- Neighborhood enhancement algorithm
- Frequency domain based algorithm

# Image Deblurring

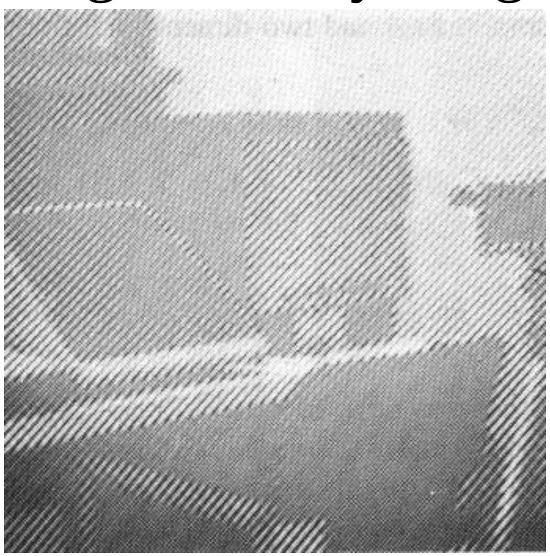
- There are many reasons for image blur, including optical factors, atmospheric factors, artificial factors, technical factors, etc.

- To achieve better processing results, blurring caused by different reasons often requires different processing methods

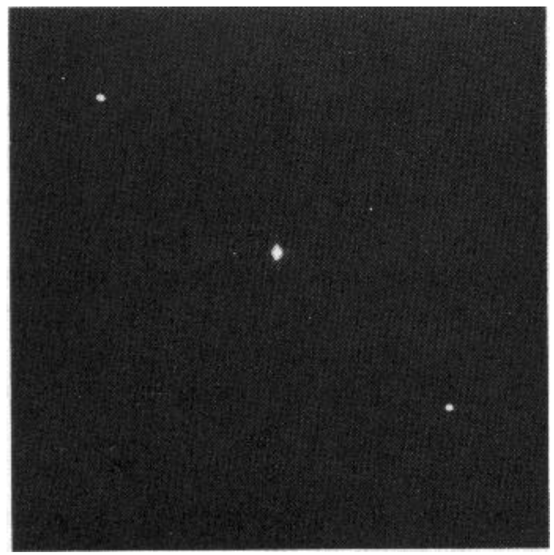
- From a technical point of view, fuzzy image processing methods are mainly divided into three categories:
  - Image enhancement
  - Image restoration
  - Super resolution reconstruction

# Operations in Frequency Domain

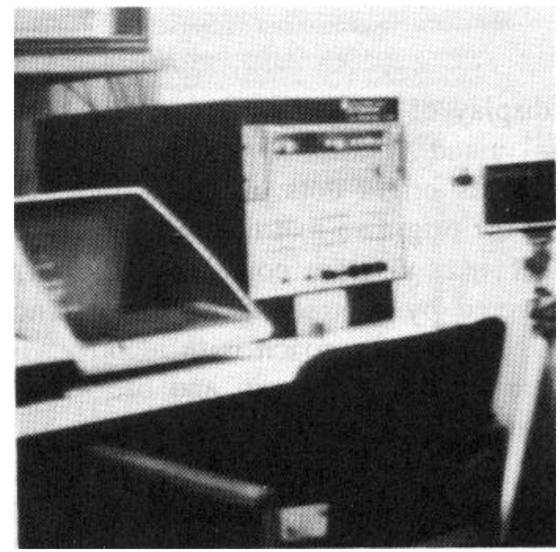
Original Noisy image



Fourier Spectrum



Filtered image



# Image Inpainting

## Definition

- Image restoration refers to the process of reconstructing images and missing or damaged parts of the video
- Image restoration, also known as image interpolation or video interpolation
- Using sophisticated algorithms to replace missing, damaged the image data, mainly to replace some small areas and blemishes

# Image Inpainting

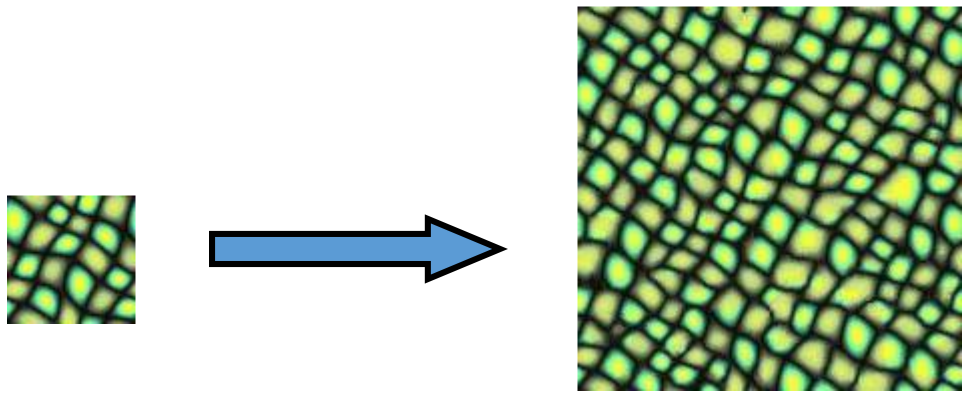
# Video Inpainting

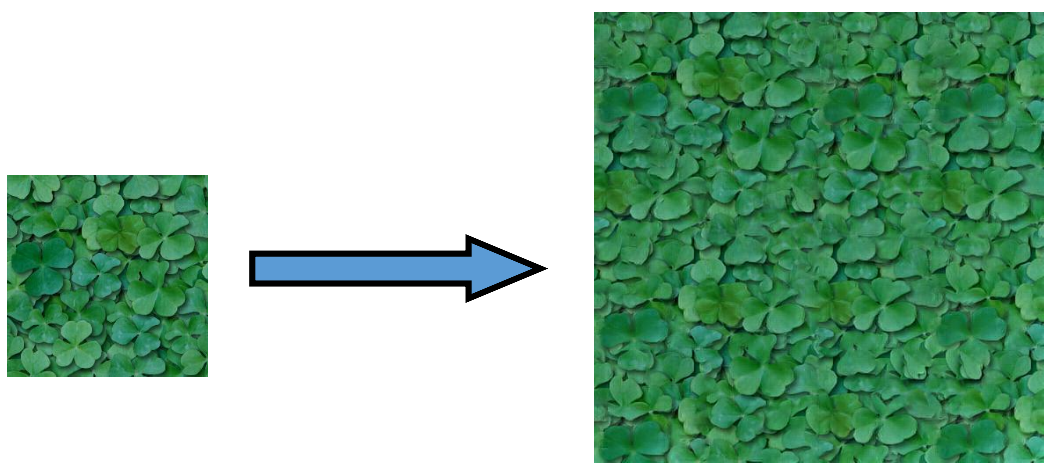

# Texture Synthesis

## Definition

- Texture synthesis is proposed to solve the problem of seam aliasing and other problems in texture mapping
- At present, texture synthesis methods can be divided into two categories:
  - One is Process Texture Synthesis (PTS)
    - It generates texture directly on the surface by simulating the physics generation process
  - the other is Sample Based Texture Synthesis (SBTS)
    - Given a small texture, generate a large similar texture

# Texture Synthesis





# Industry and Applications

- Automobile driver assistant
  - Lane departure warning
  - Adaptive cruise control
  - Obstacle warning
- Digital Photography
  - Image Enhancement
  - Compression
  - Color manipulation
  - Image editing
  - Digital cameras

MobilEye system

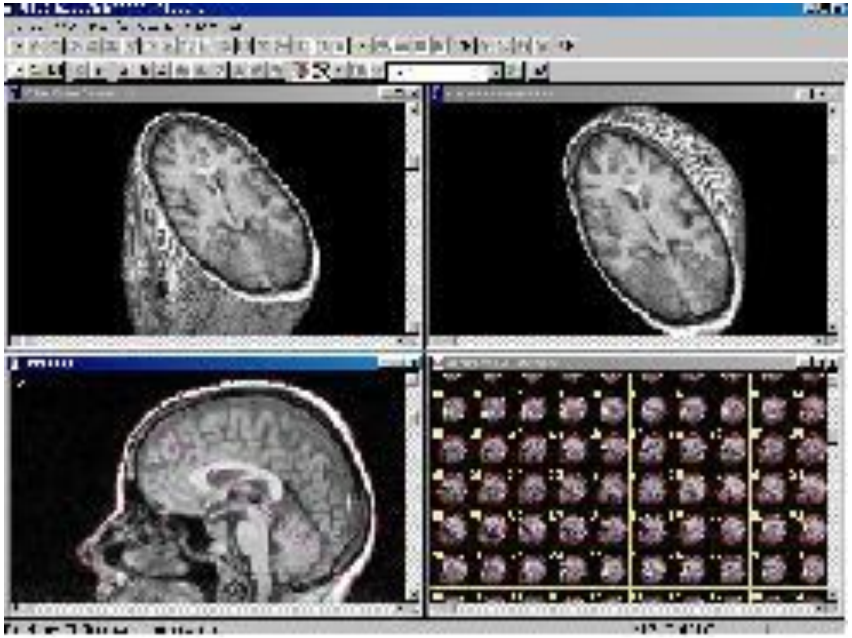
# Industry and Applications

- Film and Video
  - Editing
  - Special effects
- Image Database
  - Content based image retrieval
  - visual search of products
  - Face recognition

- Industrial Automation and Inspection
  - vision-guided robotics
  - Inspection systems

# Industry and Applications

- Sports analysis
  - sports refereeing and commentary
  - 3D visualization and tracking sports actions
- Medical and Biomedical
  - Surgical assistance
  - Sensor fusion
  - Vision based diagnosis
- Astronomy
  - Astronomical Image Enhancement
  - Chemical/Spectral Analysis



# Industry and Applications

- Arial Photography
  - Image Enhancement
  - Missile Guidance
  - Geological Mapping
- Robotics
  - Autonomous Vehicles
- Security and Safety
  - Biometry verification (face, iris)
  - Surveillance  (fences, swimming pools)

# Industry and Applications

- Military
  - Tracking and localizing
  - Detection
  - Missile guidance
- Traffic and Road Monitoring
  - Traffic monitoring
  - Adaptive traffic lights

Cruise Missiles

# Question

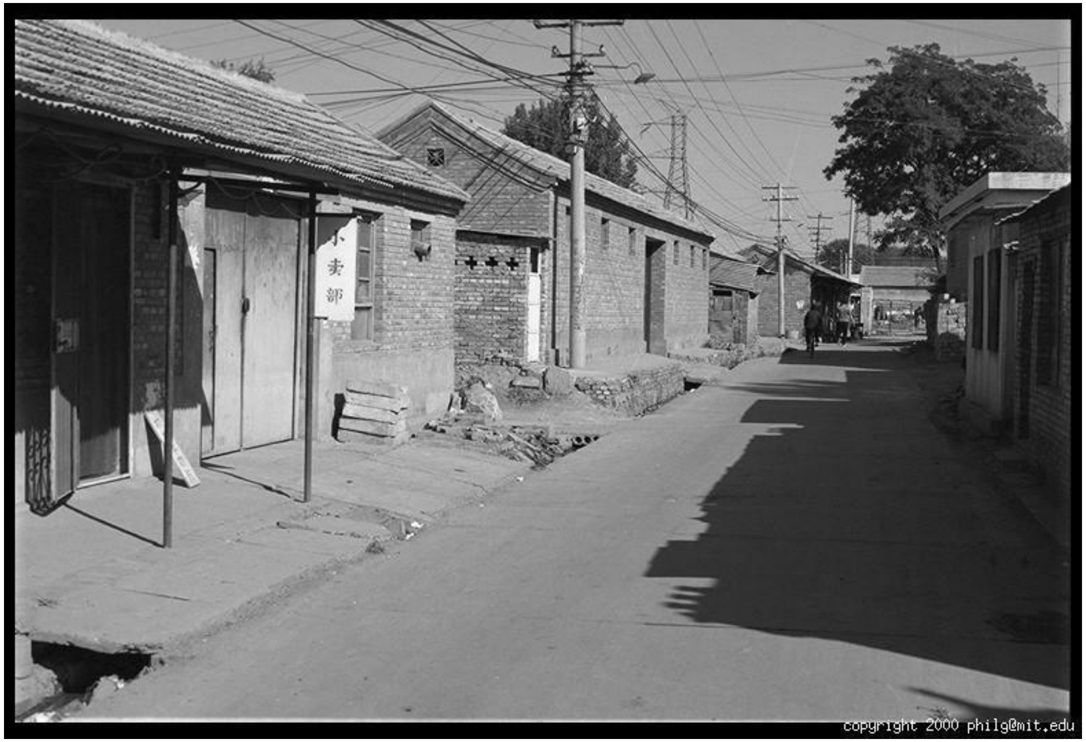
Why Computer Vision is Hard?

# Why Computer Vision is Hard?

- Inverse problems
- Priori-knowledge is required
- Complexity extensive
  - Top-Down vs Bottom-Up paradigm
  - Parallelism
- Non-local operations
  - Propagation of Information

# Contents

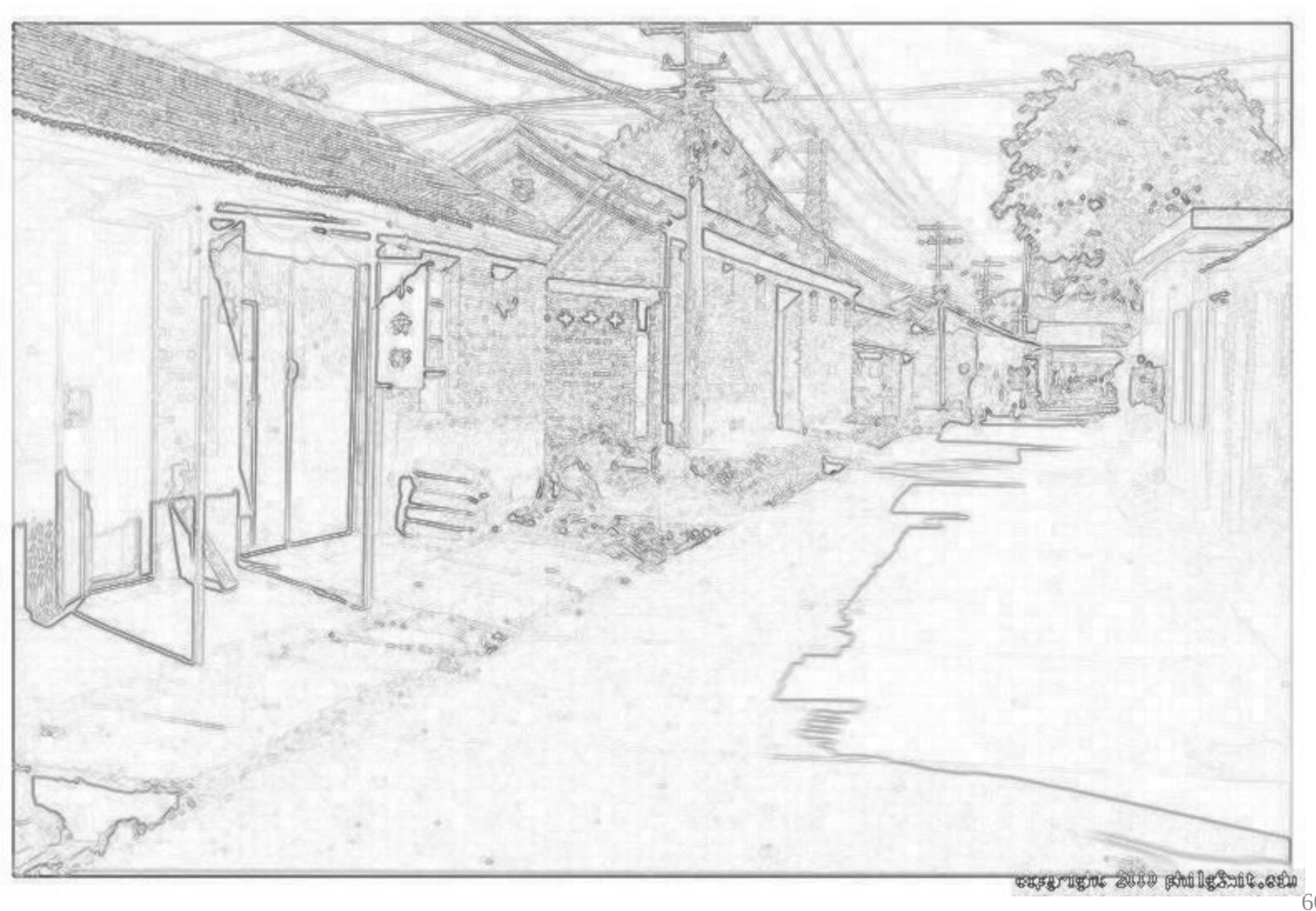
# Basis for Interpreting Intensity Images




- Key idea: for nearby scene points, most factors do not change much
- The information is mainly contained **in local differences** of **brightness**

# Basis for Interpreting Intensity Images

- Darkness = Large Difference in Neighboring Pixels



# Three Views of Filtering

## Image filters in spatial domain

- Filter is a mathematical operation on values of each patch
- Smoothing, sharpening, measuring texture

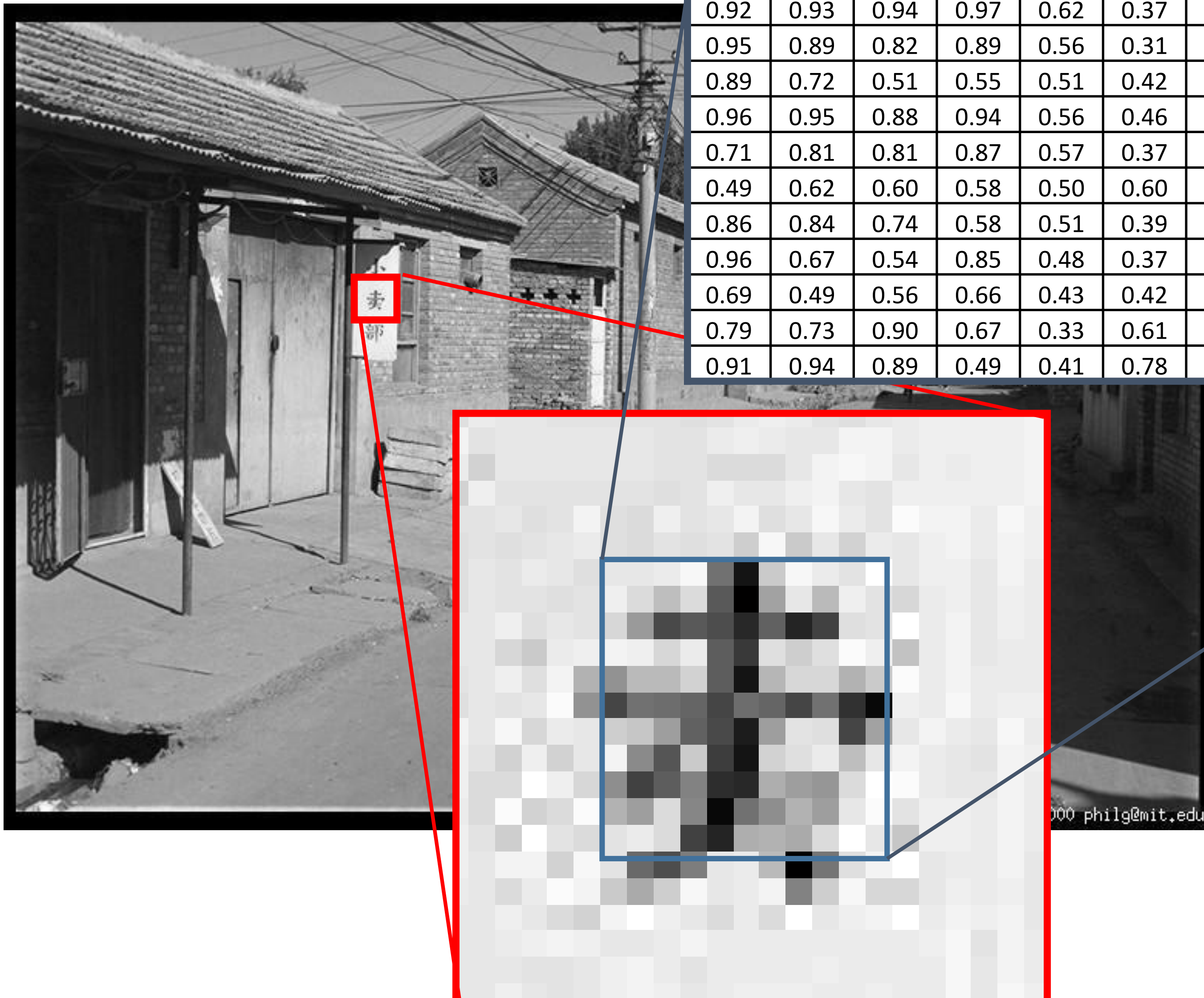
## Image filters in the frequency domain

- Filtering is a way to modify the frequencies of images
- Denoising, sampling, image compression

## Templates and Image Pyramids

- Filtering is a way to match a template to the image
- Detection, coarse-to-fine registration

# Gray Image (Pixel Matrix)




| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.92 | 0.93 | 0.94 | 0.97 | 0.62 | 0.37 | 0.85 | 0.97 | 0.93 | 0.92 | 0.99 |
| 0.95 | 0.89 | 0.82 | 0.89 | 0.56 | 0.31 | 0.75 | 0.92 | 0.81 | 0.95 | 0.91 |
| 0.89 | 0.72 | 0.51 | 0.55 | 0.51 | 0.42 | 0.57 | 0.41 | 0.49 | 0.91 | 0.92 |
| 0.96 | 0.95 | 0.88 | 0.94 | 0.56 | 0.46 | 0.91 | 0.87 | 0.90 | 0.97 | 0.95 |
| 0.71 | 0.81 | 0.81 | 0.87 | 0.57 | 0.37 | 0.80 | 0.88 | 0.89 | 0.79 | 0.85 |
| 0.49 | 0.62 | 0.60 | 0.58 | 0.50 | 0.60 | 0.58 | 0.50 | 0.61 | 0.45 | 0.33 |
| 0.86 | 0.84 | 0.74 | 0.58 | 0.51 | 0.39 | 0.73 | 0.92 | 0.91 | 0.49 | 0.74 |
| 0.96 | 0.67 | 0.54 | 0.85 | 0.48 | 0.37 | 0.88 | 0.90 | 0.94 | 0.82 | 0.93 |
| 0.69 | 0.49 | 0.56 | 0.66 | 0.43 | 0.42 | 0.77 | 0.73 | 0.71 | 0.90 | 0.99 |
| 0.79 | 0.73 | 0.90 | 0.67 | 0.33 | 0.61 | 0.69 | 0.79 | 0.73 | 0.93 | 0.97 |
| 0.91 | 0.94 | 0.89 | 0.49 | 0.41 | 0.78 | 0.78 | 0.77 | 0.89 | 0.99 | 0.93 |

# Image Filtering

- Image filtering: for each pixel, compute function of local neighborhood and output a new value

- Same function applied at each position

- Output and input image are typically the same size

# Image Filtering or **Convolution**

- Linear filtering: a weighted sum/difference of pixel values
- Really important!
  - Enhance images
    - Denoising, smooth, increase contrast, blurring, noise reduction, image sharpening etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching
    - Detect patterns
  - **Deep Learning**
    - Convolutional Neural Networks

# Concept of Convolution


**Black box**

# Concept of Convolution


- $g(x, y) = h(x, y) * f(x, y)$ ; mask convolved with an image
- $g(x, y) = f(x, y) * h(x, y)$ ; image convolved with mask
- The convolution operator (*) is commutative, $h(x, y)$ is the mask or filter.

# Convolution Mask

- Mask is a matrix, usually of size 1x1, 3x3, 5x5 or 7x7 (odd number).

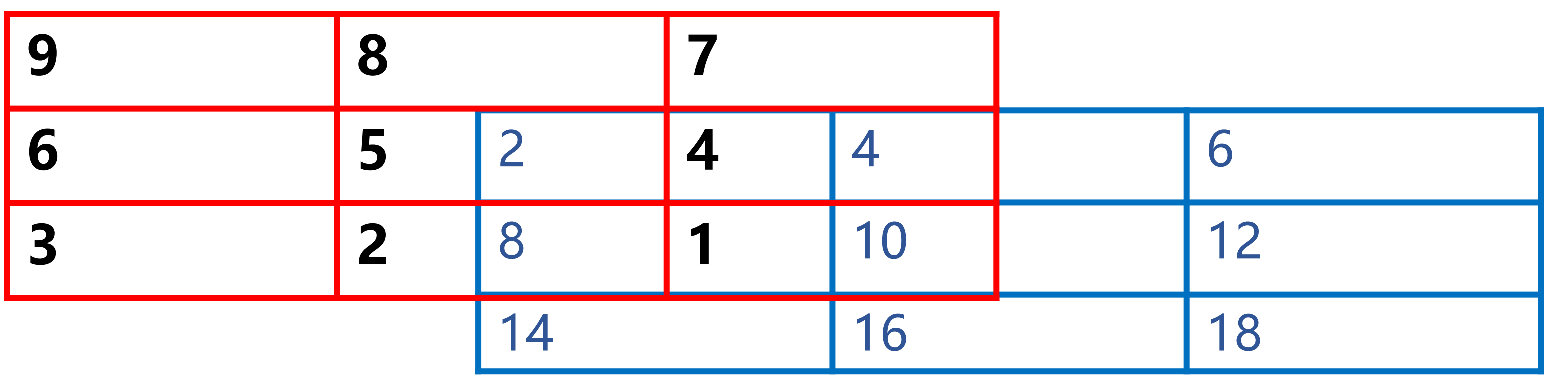
- Convolution steps:
  - Flip the mask horizontally and vertically
  - Slide the mask onto the image
  - Multiply the corresponding elements and add them
  - Repeat this until all image values are calculated

# Convolution Example

- **Image matrix**

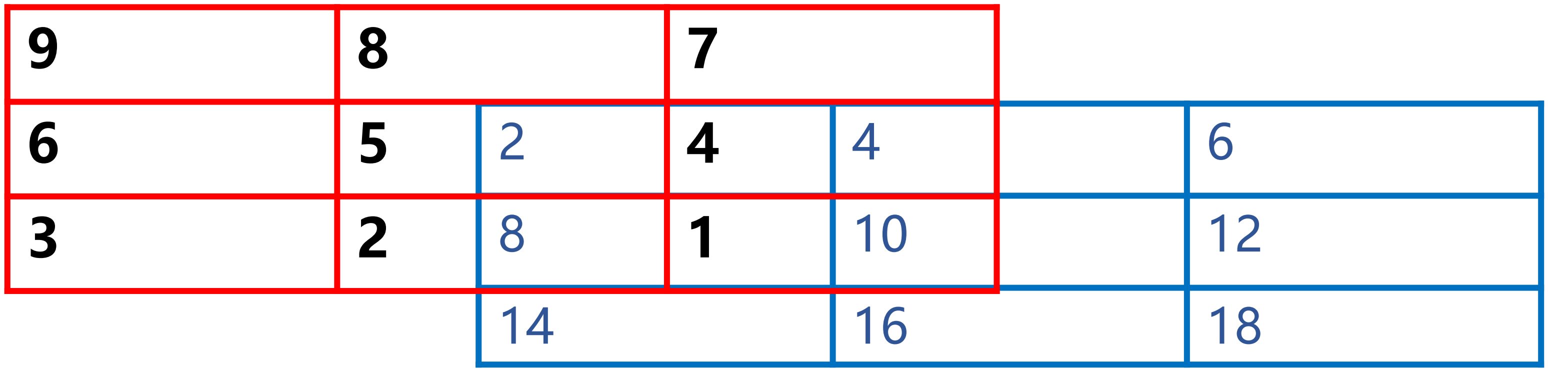
| | | |
|---|---|---|
| 2 | 4 | 6 |
| 8 | 10 | 12 |
| 14 | 16 | 18 |

- **Place the mask over each image element. Multiply the corresponding elements and add them**



# Convolution Example

- **Red – mask**, **blue – image**




- **First pixel: = (5*2) + (4*4) + (2*8) + (1*10)**

  **= 10 + 16 + 16 + 10**

  **= 52**

- Convolution can achieve blurring, edge detection, sharpening, noise reduction, etc.

# Convolution Example

- **Example: box filter**

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

# Image Filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |
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| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

# Image Filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

# Image Filtering

$$g[\cdot,\cdot] \quad \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$$f[.,.]$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$$h[.,.]$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

# Image Filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

# Image Filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

# Image Filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | ? | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

# Image Filtering

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | ? | | | |
| | | | | | | | | | |
| | | | | 50 | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

# Image Filtering

$g[\cdot,\cdot]\ \frac{1}{9}$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$f[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$h[.,.]$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 60 | 90 | 90 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 10 | 20 | 30 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$$h[m,n]=\sum_{k,l} g[k,l]\ f[m+k,n+l]$$

# Box Filter

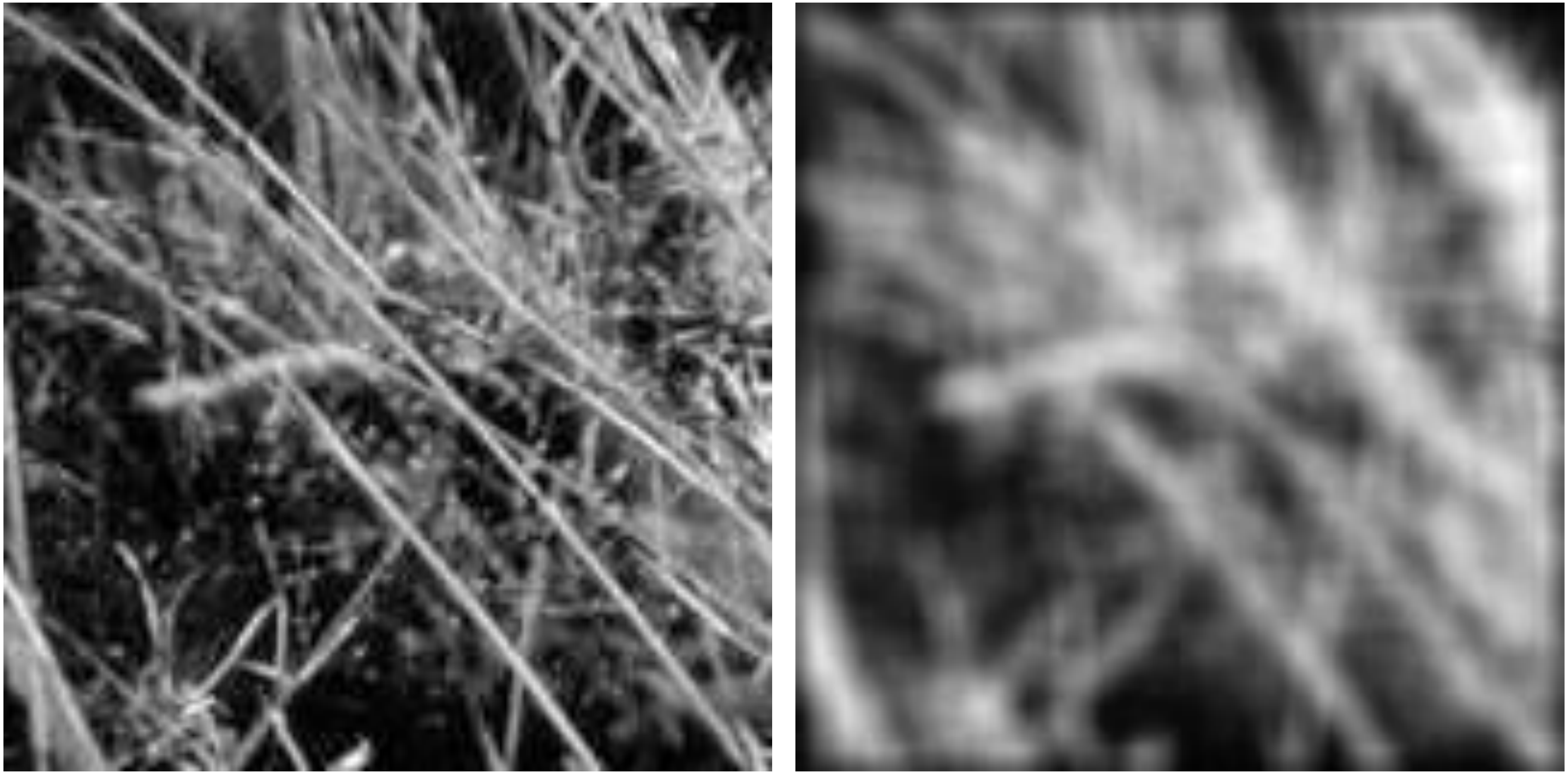
- What does it do?
  - Replaces each pixel with an average of its neighborhood
  - Achieve smoothing effect (remove sharp features)

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

# Box Filter

- Smoothing with box filter



# Practice with Linear Filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

?

# Practice with Linear Filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

Filtered
(**no change**)

# Practice with Linear Filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

?

# Practice with Linear Filters

Original

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

Shifted left
By 1 pixel

# Practice with Linear Filters

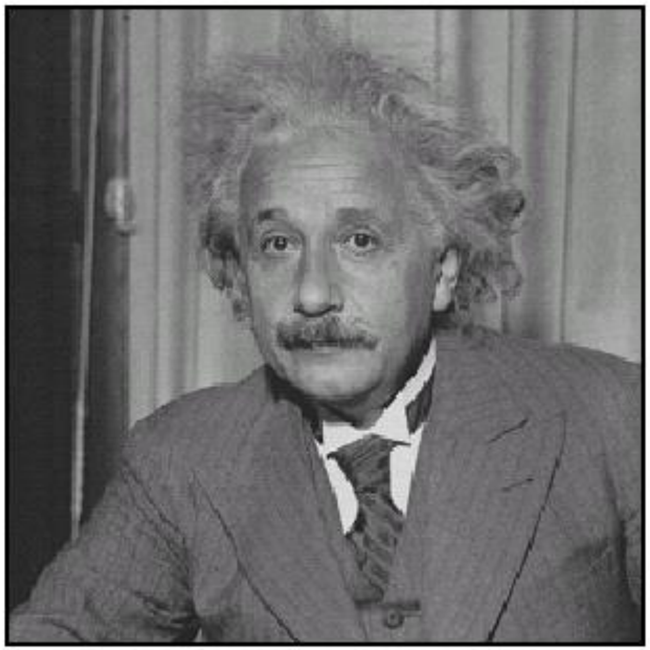
Original

$$\begin{bmatrix}0 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

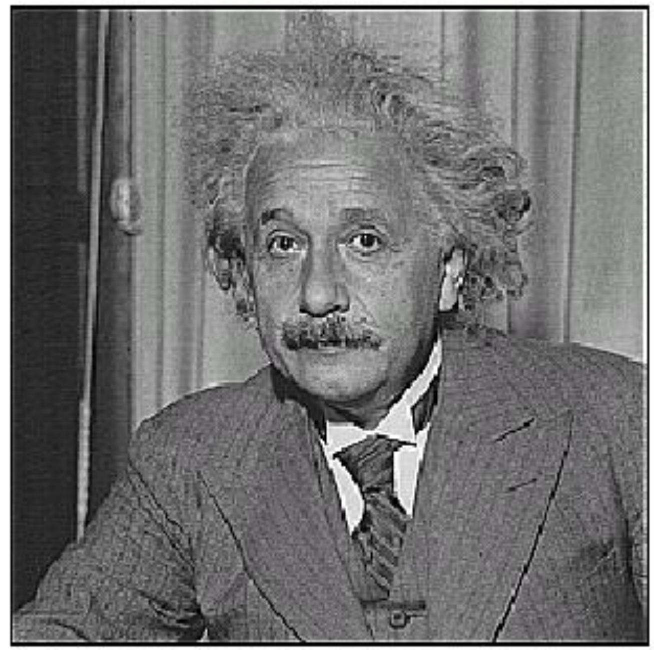
**Sharpening filter**

- Accentuates differences with local average

# Sharpening

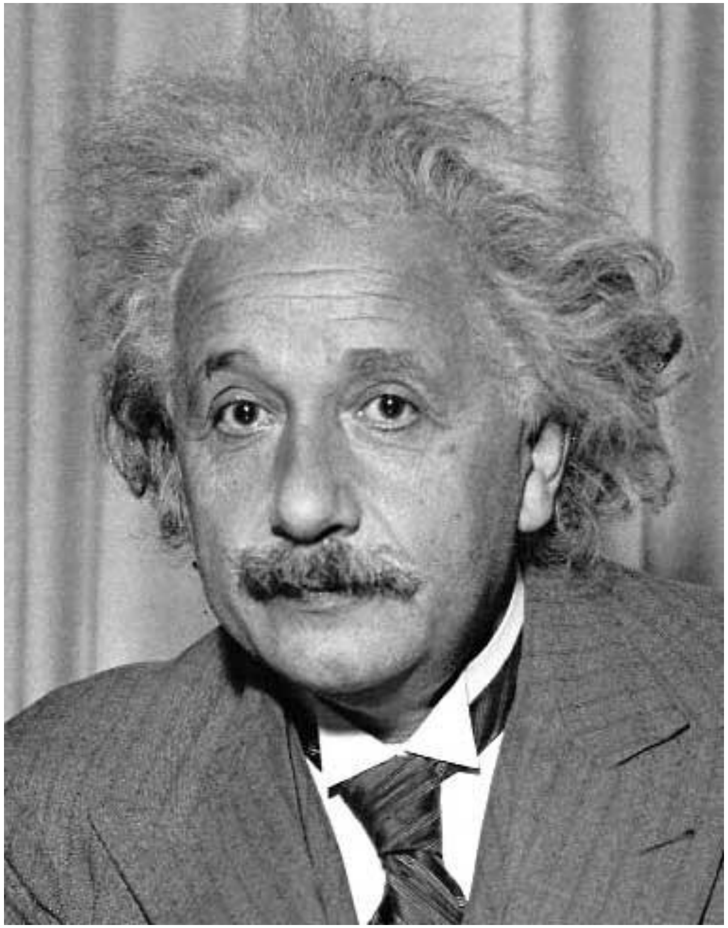


before



after

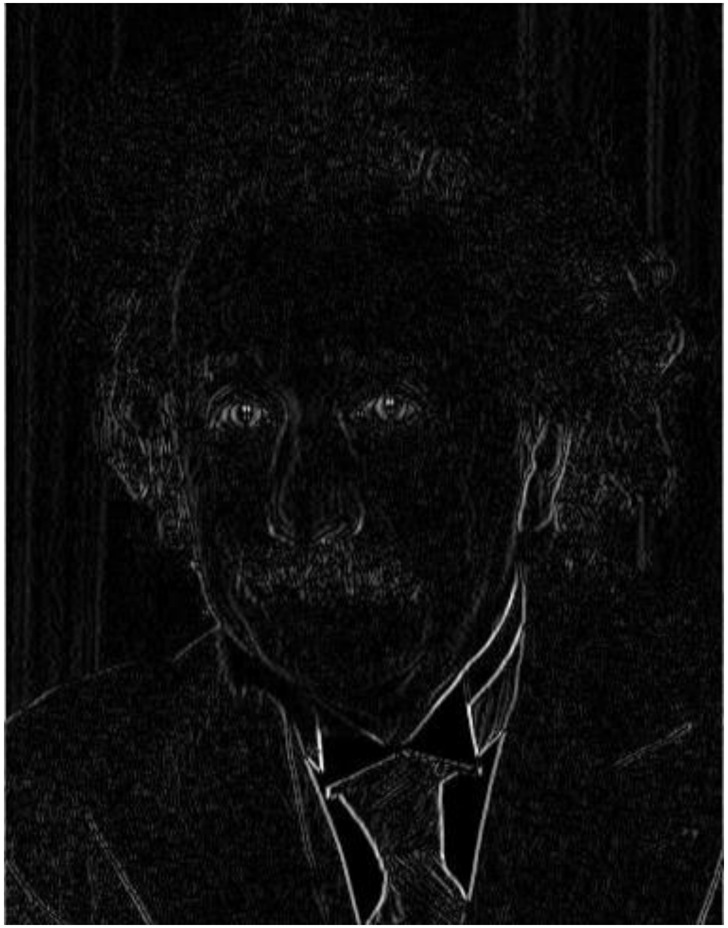
# Other Filters



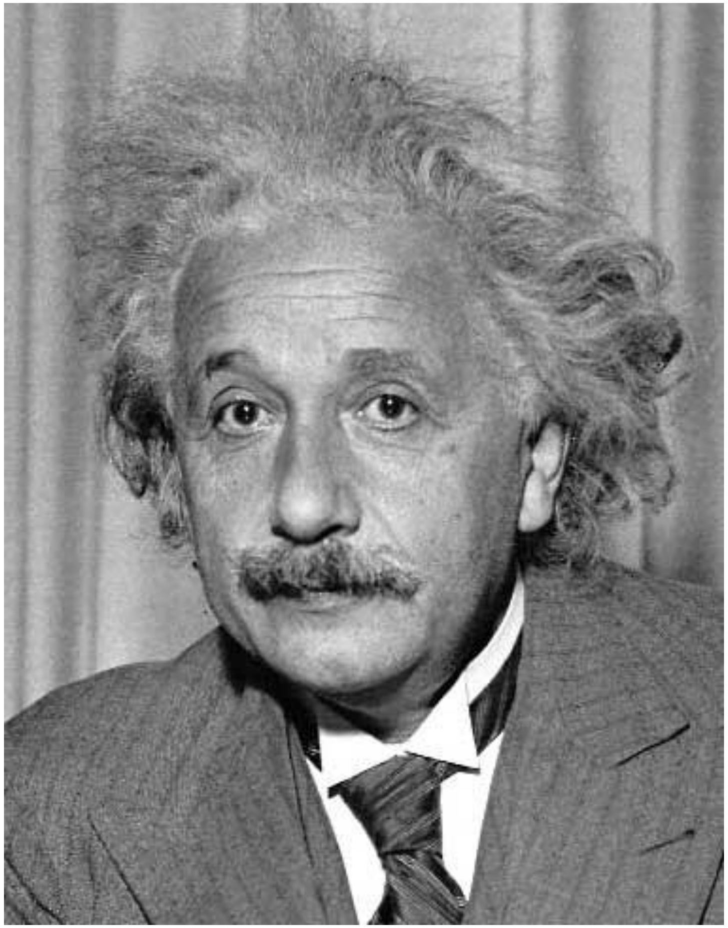
| | | |
|---|---|---|
| 1 | 0 | -1 |
| 2 | 0 | -2 |
| 1 | 0 | -1 |

Sobel

Vertical Edge (absolute value)



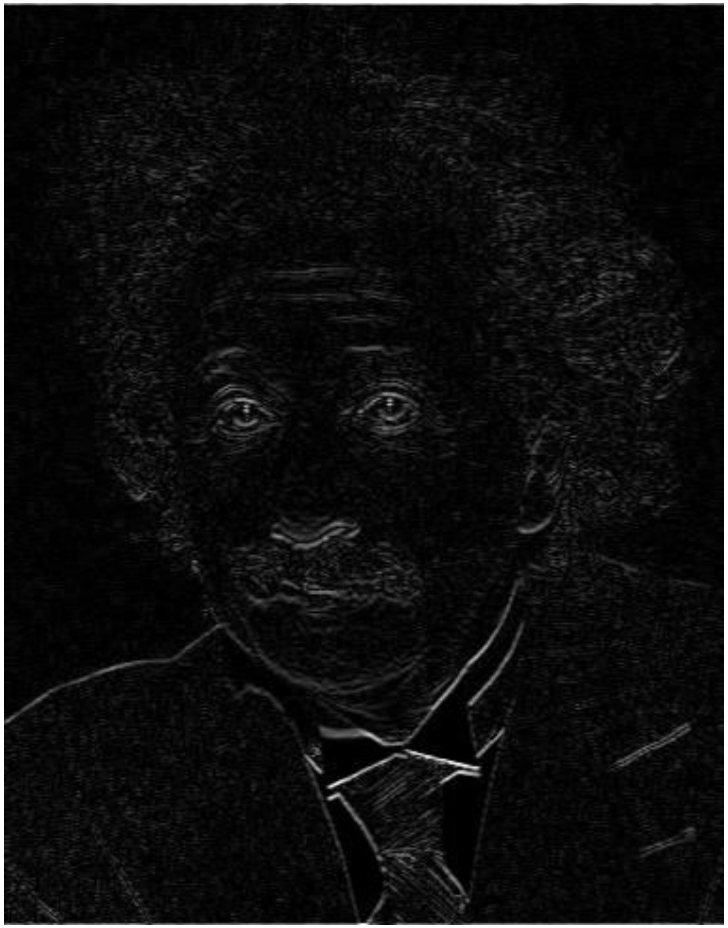
# Other Filters



| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

Sobel

Horizontal Edge
(absolute value)



# Basic Gradient Filters

**Horizontal Gradient**

| | | |
|---|---|---|
| 0 | 0 | 0 |
| -1 | 0 | 1 |
| 0 | 0 | 0 |

or

| | | |
|---|---|---|
| -1 | 0 | 1 |

**Vertical Gradient**

| | | |
|---|---|---|
| 0 | -1 | 0 |
| 0 | 0 | 0 |
| 0 | 1 | 0 |

or

| |
|---|
| -1 |
| 0 |
| 1 |

# Filtering vs Convolution

- 2d filtering
  g=filter f=image
  - h=filter2(g, f); or h= imfilter(f, g);

$$h[m,n] = \sum_{k,l} g[k,l] f[m+k, n+l]$$

- 2d convolution
  - h=conv2(g, f);

$$h[m,n] = \sum_{k,l} g[k,l] f[m-k, n-l]$$

# Key Properties of Linear Filters

- **Linearity:** filter($f_1$ + $f_2$) = filter($f_1$) + filter($f_2$)

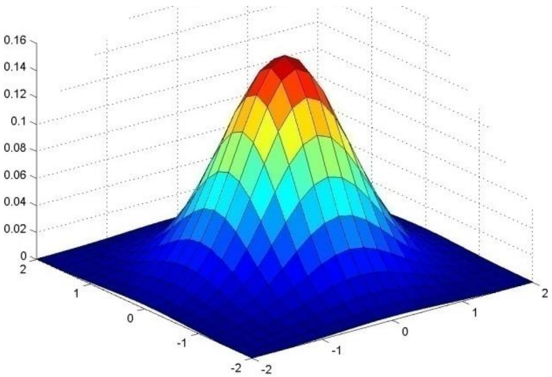
- **Shift invariance:** same behavior regardless of pixel location

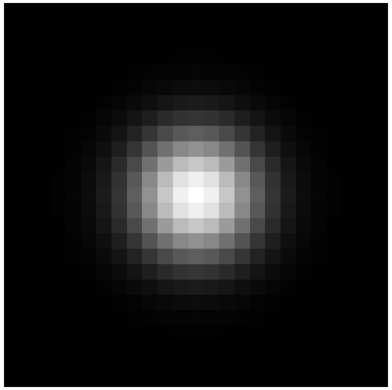
filter(shift(f)) = shift(filter(f))

- Any linear, shift-invariant operator can be represented as a convolution

# Important Filter: Gaussian

- Spatially-weighted average



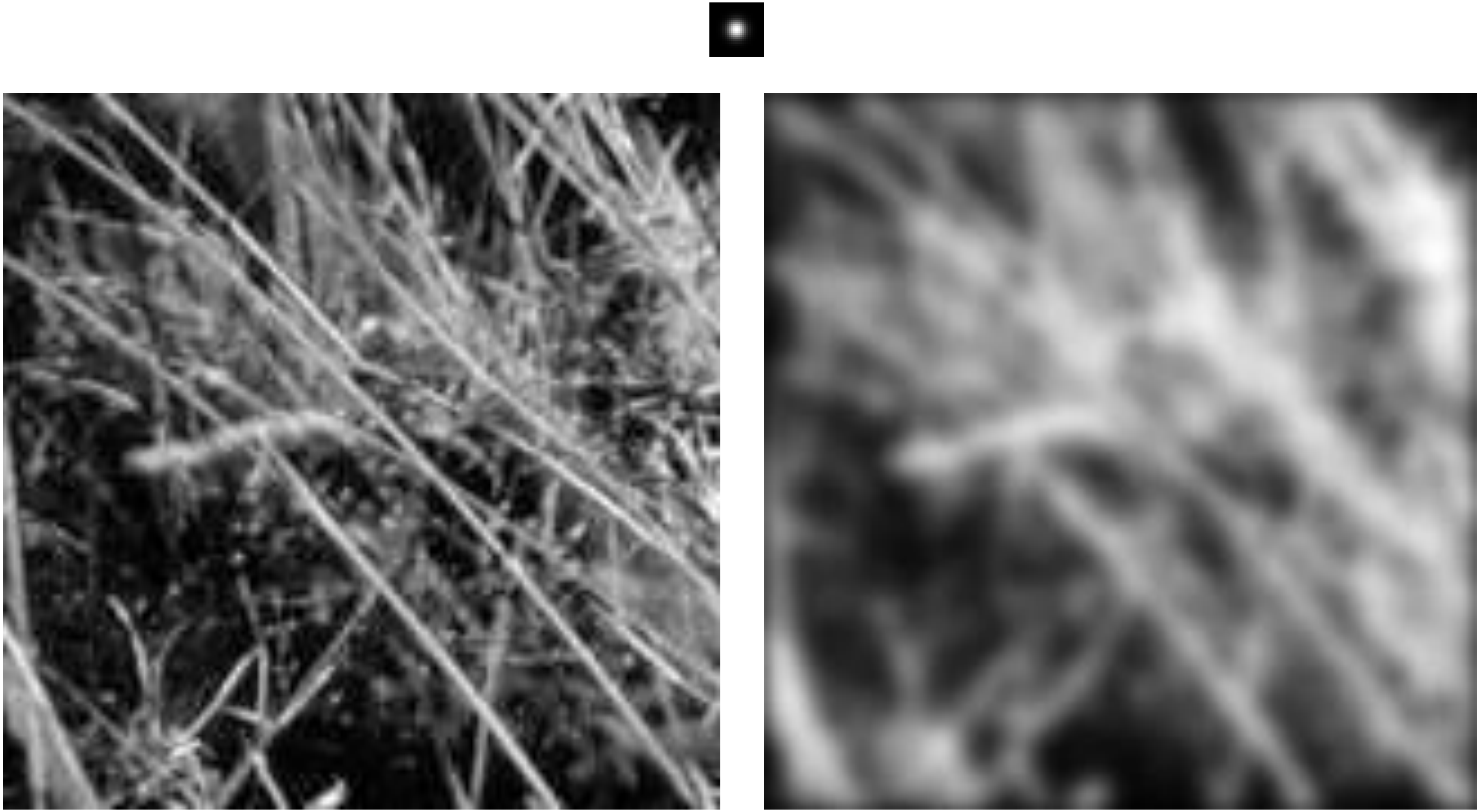



| | | | | |
|---|---|---|---|---|
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.022 | 0.097 | 0.159 | 0.097 | 0.022 |
| 0.013 | 0.059 | 0.097 | 0.059 | 0.013 |
| 0.003 | 0.013 | 0.022 | 0.013 | 0.003 |

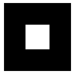
5 x 5, $\sigma$ = 1

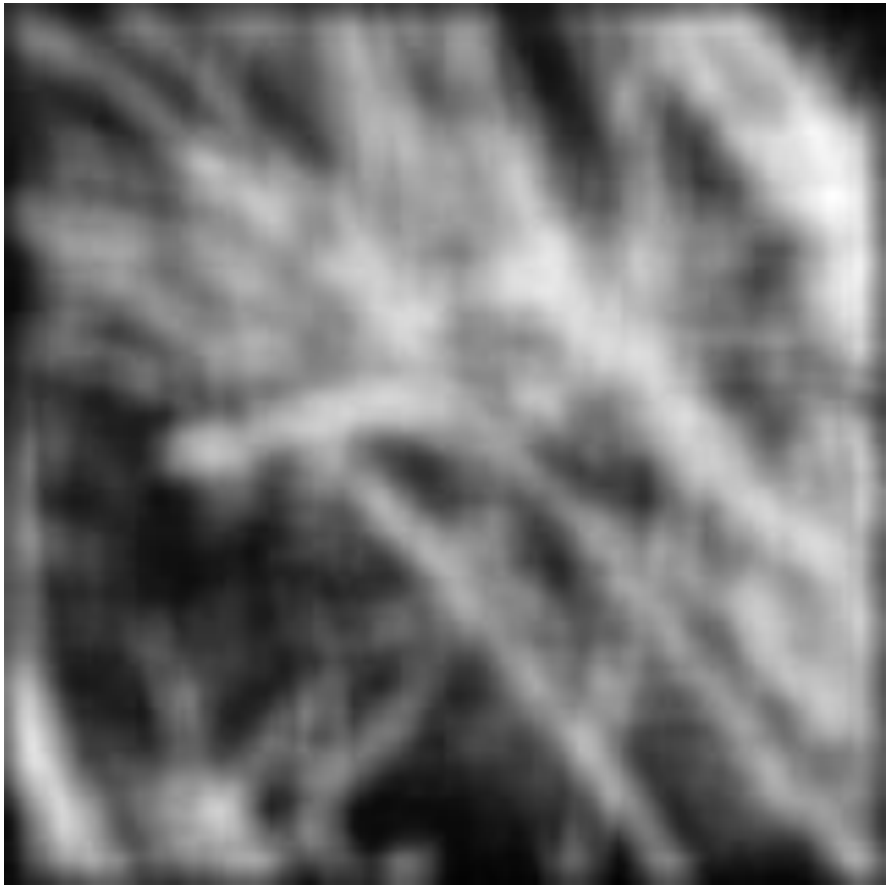
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

# Smoothing with Gaussian Filter



# Smoothing with Box Filter





# Gaussian Filter

- Remove "high-frequency" components from the image (low-pass filter)
  - Images become more smooth
- Convolution with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width $\sigma$ is same as convolving once with kernel of width $\sigma\sqrt{2}$ ?
- Separable kernel
  - Factors into product of two 1D Gaussians

# Separability of Gaussian Filter

$$
\begin{aligned}
G_\sigma(x, y) &= \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}} \\
&= \left( \frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left( \frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right)
\end{aligned}
$$

The 2D Gaussian can be expressed as the product of two functions, one a function of $x$ and the other a function of $y$

In this case, the two functions are the (identical) 1D Gaussian

# Separability Example

2D filtering
(center location only)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix}$$

The filter factors into a product of 1D filters:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Perform filtering along rows:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 & 3 \\ 3 & 5 & 5 \\ 4 & 4 & 6 \end{bmatrix} = \begin{bmatrix} & 11 & \\ & 18 & \\ & 18 & \end{bmatrix}$$

Followed by filtering along the remaining column:

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} & 11 & \\ & 18 & \\ & 18 & \end{bmatrix} = \begin{bmatrix} & & \\ & 65 & \\ & & \end{bmatrix}$$

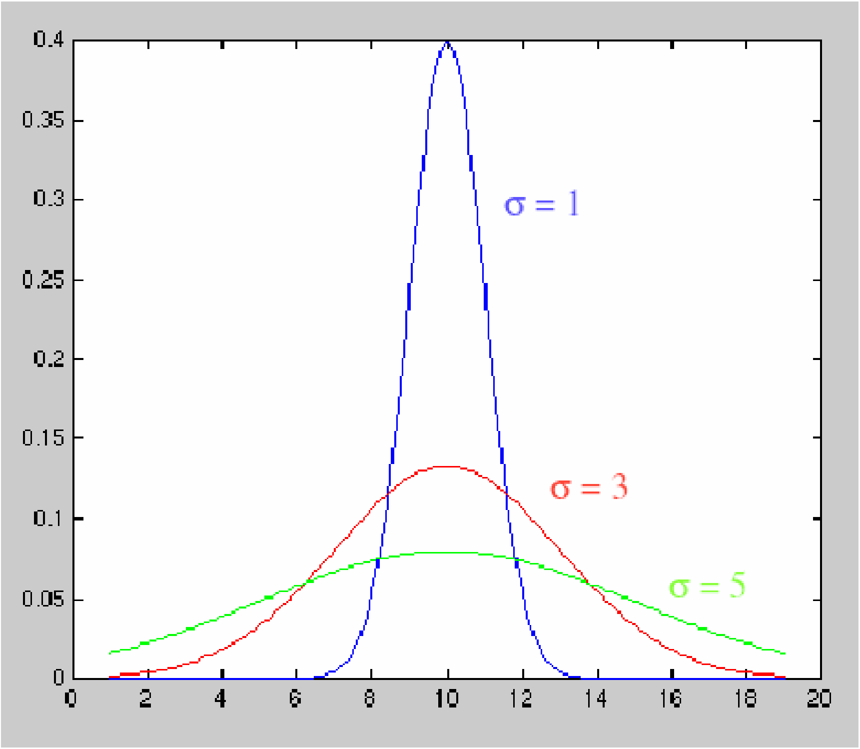
# Separability

Why is separability useful in practice?

# Practical atters

- How big should the filter be?
- Values at edges should be near zero ← important!
- Rule of thumb for Gaussian: set filter half-width to about 3 σ

Effect of σ



# Practical Matters

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

- methods (MATLAB):
  - clip filter (black): imfilter(f, g, 0)
  - wrap around： imfilter(f, g, ‘circular’)
  - copy edge:

    imfilter(f, g, ‘replicate’)
  - reflect across edge:

    imfilter(f, g, ‘symmetric’)

# Thank You