Задача 4

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

1) $w = x^2 + y^2$ $\quad u = x^2 + y^2$ $\quad v = 0$

$\cdot 2x \neq 0$

$\cdot 2y \neq 0$

X

2) $w = x^2 - y^2 + 2ixy$ $\quad u = x^2 - y^2$ $\quad v = 2xy$

$\cdot 2x = 2x$

$\cdot -2y = 2y$

✓

3) $w = \frac{1}{x+iy} = \frac{x-iy}{x^2-y^2}$ $\quad u = \frac{x}{x^2-y^2}$ $\quad v = -\frac{y}{x^2-y^2}$

$\cdot \frac{x^2-y^2-2x^2}{(x^2-y^2)^2} = -\frac{x^2+y^2}{(x^2-y^2)^2}$

$\cdot \frac{2xy}{(x^2-y^2)^2} = (-)(-)\frac{2xy}{x^2-y^2}$

✓

Задача 5

1) $|f| = e^{r^2 \cos 2\varphi}$ $\quad z = re^{i\varphi}$

$w = \ln f = \ln|f| e^{i \operatorname{Arg} f} = \underbrace{\ln|f|}_{\operatorname{Re} w} + \underbrace{i \operatorname{Arg} f}_{\operatorname{Im} w}$

$u = \ln|f| = r^2 \cos 2\varphi =$

$v = \arg f = \dots \quad = r^2(\cos^2\varphi - \sin^2\varphi) = x^2 - y^2$

$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} \Rightarrow v = 2xy + \psi(x)$

$\frac{\partial v}{\partial x} = 2x = -\frac{\partial u}{\partial y} = 2y$

$\to w = x^2 - y^2 + i2xy + ic = z^2 + ic$

$f(z) = e^{z^2} e^{ic} \quad (c = 0)$

2) $\operatorname{Arg} f = xy$

$v = xy$

$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = x \to u = \frac{1}{2}x^2 + \psi(y)$

$\psi'(y) = \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = -y$

$u = \frac{1}{2}(x^2 - y^2) \quad \Longleftarrow \quad \psi(y) = -\frac{1}{2}y^2$

$w = \frac{1}{2}(x^2 - y^2 + xyi) = \frac{1}{2}z^2$

$f = e^{\frac{1}{2}z^2}$