Задача 2

$$1+2\varepsilon+3\varepsilon^2+\ldots n\varepsilon^{n-1}=\frac{n}{\varepsilon-1}$$

$$S_n=\frac{b(-1+q^n)}{(1-q)} \longleftrightarrow S_n^*=-\frac{b(-1+q^n)}{q-1} \quad q\to\varepsilon\ ;\ b=1 \to S_n=-\frac{(-1+\varepsilon^n)}{\varepsilon-1}$$

$$S_n'=-\frac{n\varepsilon^{n-1}(\varepsilon-1)+(\varepsilon^n-1)}{(\varepsilon-1)^2}=-\frac{n\varepsilon^n-n\varepsilon^{n-1}+\varepsilon^n-1}{(\varepsilon-1)^2}=\frac{n\varepsilon^{n-1}-n\varepsilon^n-\varepsilon^n+1}{(\varepsilon-1)^2}=$$

$$=\frac{n\varepsilon^n(\varepsilon-1)-\varepsilon^n+1}{(\varepsilon-1)^2}=\frac{\varepsilon^n(n(\varepsilon-1)-1)+1}{(\varepsilon-1)^2}$$

Задача 3

1) $\operatorname{Im} z=1$ $\qquad z=x+iy$

$$w(z)=z^3+3z-i \longrightarrow (x+iy)^3+3(x+iy)-i=$$

$$=(x+iy)(x^2-y^2+2ixy)+3x+3iy-i=$$

$$=x^3-y^2x+2ix^2y+ix^2y-iy^3-2xy^2+3x+3iy-i=$$

$$=\underbrace{x^3-xy^2-2xy^2+3x}_{u}+i\underbrace{(3x^2y+3y-y^3-1)}_{v}-$$

$$u=x(x^2-y^2-2y^2+3)$$

$$v=y(-y^2+3x^2+3)-1 \qquad \operatorname{Im} z=1 \longrightarrow y=1$$

$$\cancel{v=y(-y^2+3x^2+3)-1=1}$$

$$\cancel{-y^3+3x^2y+3y-2=0}$$

$$\cancel{x^2=\frac{2+y^3-3y}{3y}}$$

$$\cancel{x=\frac{2+y^3-3y}{3y}}$$

$$x^3-x-2x+3x+i(3x^2+3-1-1)$$

$$\underbrace{x^3}_{u}+i\underbrace{(3x^2+1)}_{v}$$

2) $|z-i|=1$

$$w(z)=\frac{1}{z-2i} \longrightarrow w(z)=\frac{1}{x+yi-2i}=\frac{x-i(y-2)}{x^2+(2+y)^2} \qquad \begin{matrix} y=\sin t+1\\ x=\cos t\end{matrix}$$

$$u=\frac{x}{x^2+(y-2)^2} \qquad u=\frac{\cos t}{\cos^2 t+(\sin t-2)^2}=\frac{\cos t}{\cos^2+\sin^2-2\sin t+1}=\frac{\cos t}{2(1-\sin t)}$$

$$v=-\frac{y-2}{x^2+(y-2)^2} \longrightarrow v=\frac{\sin t-1}{\cos^2 t+(\sin t-2)^2}=\ldots=\frac{\sin t-1}{2(1-\sin t)}=\frac{1}{2}$$