

Задание 24

$$\frac{du}{dx} = \frac{dv}{dy}$$

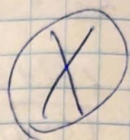
$$\frac{dv}{dy} = -\frac{du}{dx}$$

1) $w = x^2 + y^2$

$$u = x^2 + y^2 \quad v = 0$$

$$\cdot 2x \neq 0$$

$$\cdot 2y \neq 0$$



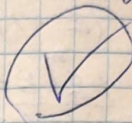
2) $w = x^2 - y^2 + 2ixy$

$$u = x^2 - y^2$$

$$v = 2xy$$

$$\cdot 2x = 2x$$

$$\cdot -2y = 2y$$



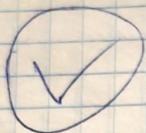
3) $w = \frac{1}{x+iy} = \frac{x-iy}{x^2-y^2}$

$$u = \frac{x}{x^2-y^2}$$

$$\cdot \frac{x^2-y^2-2x^2}{(x^2-y^2)^2} = -\frac{x^2-y^2-2x^2}{(x^2-y^2)^2}$$

$$v = -\frac{y}{x^2-y^2}$$

$$\cdot \frac{2xy}{(x^2-y^2)^2} = (-x) \frac{2xy}{x^2-y^2}$$



Задание 25

1) $|f| = e^{r^2 \cos 2\varphi}$

$$z = re^{i\varphi}$$

$$w = \ln f = \ln |f| e^{i \operatorname{Arg} f} = \underbrace{\ln |f|}_{\operatorname{Re} w} + i \underbrace{\operatorname{Arg} f}_{\operatorname{Im} w}$$

$$u = \ln |f| = r^2 \cos 2\varphi =$$

$$v = \operatorname{arg} f = \dots = r^2 (\cos^2 \varphi - \sin^2 \varphi) = x^2 - y^2$$

$$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} \Rightarrow \partial_x 2xy + \psi(x)$$

$$\frac{\partial v}{\partial x} = 2x = -\frac{\partial u}{\partial y} = 2y$$

$$\rightarrow w = x^2 - y^2 + i 2xy + iC = z^2 + iC$$

$$f(z) = e^{z^2} e^{iC} \quad (C=0)$$

2) $\operatorname{Arg} f = xy$

$$v = xy$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = x \rightarrow u = \frac{1}{2} x^2 + \psi(y)$$

$$\psi'(y) = \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = -y$$

$$\psi(y) = -\frac{1}{2} y^2$$

$$u = \frac{1}{2} (x^2 - y^2)$$

$$w = \frac{1}{2} (x^2 - y^2 + xyi) = \frac{1}{2} z^2$$

$$f = e^{\frac{1}{2} z^2}$$