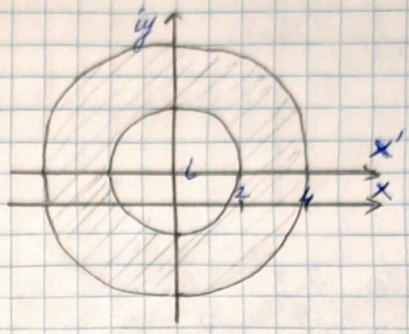
Задачи с курса HSE

Задача 1

1) $1 \le |z - i| \le 4$



$w(z) = |z - i| = w(z - i) = |z|$

$S = \pi r^2 \to S = \pi r_1^2 - \pi r_2^2$

$S = \pi(4^2 - 1^2) = 12\pi$

centre — $z = i$

2) $|z - 4i| + |z + 4i| = 10$

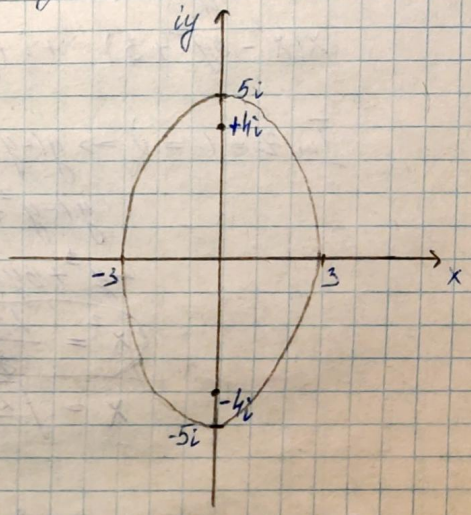
$|x + iy - 4i| + |x + iy + 4i| = 10$

$\sqrt{x^2 + i(y-4)^2} + \sqrt{x^2 + i(y+4)^2} = 10$

$|z| = \sqrt{z z^*} \Rightarrow |z - 4i| = \sqrt{(x + i(y-4))(x - i(y-4))^*}$

$|z + 4i| = \sqrt{(x + i(y+4))(x - i(y+4))^*}$

$\sqrt{x^2 + (y-4)^2} + \sqrt{x^2 + (y+4)^2} = 10$



ellipse — $r_1 + r_2 = 2a$

$|z - 4i| \quad |z + 4i| \quad 10 \Rightarrow a = 5$

$F_1 = c \qquad F_2 = c$

$a^2 = b^2 + c^2 \Rightarrow 5^2 = b^2 + 4^2$

$b = \pm 3$

3) $\operatorname{Im} \frac{1}{z} = 1$

$\frac{1}{z} = \frac{z^*}{z z^*} = \frac{z^*}{|z|^2} = \underbrace{\frac{x - yi}{x^2 + y^2}}_{z'}$

$\operatorname{Im} z' = -\frac{y}{x^2 + y^2} = 1 \Rightarrow -y = x^2 + y^2$

$x^2 + y^2 + y + \frac{1}{4} = \frac{1}{4}$

$x^2 + (y + 0.5)^2 = (0.5)^2$

$x^2 + y^2 + y = 0 \quad | + y + 1$

$x^2 + (y+1)^2 = y + 1$