

Задание 26

1) $u = \varphi(x^2 - y^2)$

$$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} \Rightarrow 2xy + \psi'(x) \Rightarrow \frac{\partial v}{\partial x} = -2y$$

$$\frac{\partial u}{\partial y} = -2y = -\frac{\partial v}{\partial x} \Rightarrow 2xy + \psi'(y)$$

$$\begin{aligned} a &\in \mathbb{C} \\ b &\in \mathbb{R} \end{aligned}$$

$$w = u + iv = x^2 - y^2 + i2xy = z^2 \Rightarrow a + bz^2$$

2) $u = \varphi\left(\frac{y}{x}\right)$

$$\frac{\partial u}{\partial x} = -\frac{y}{x^2} = \frac{\partial v}{\partial y} \Rightarrow v = -\frac{1}{2}\frac{y^2}{x^2} + \psi(x)$$

$$\frac{\partial u}{\partial y} = \frac{1}{x} = -\frac{\partial v}{\partial x} \Rightarrow v = -\ln x + \psi(y)$$

Задание 27

1) $\int_C z dz = \left[z = e^{i\varphi} \rightarrow dz = ie^{i\varphi} d\varphi \right] = \int_0^{2\pi} i e^{i\varphi} \cdot e^{i\varphi} d\varphi = 0$

2) $\int_C \bar{z} dz = \left[\bar{z} = e^{-i\varphi} \quad dz = ie^{i\varphi} d\varphi \right] = \int_0^{2\pi} i e^{i\varphi - i\varphi} d\varphi = i\varphi \Big|_0^{2\pi} = i2\pi$

Задание 28

1) $\int_C \frac{ydx - xdy}{x^2 + y^2}$

Centre: (0, 0)

$$x = \cos \varphi \quad dx = -\sin \varphi d\varphi$$

$$y = \sin \varphi \quad dy = \cos \varphi d\varphi$$

$$\int_0^{2\pi} \frac{\cos^2 \varphi d\varphi + \sin^2 \varphi d\varphi}{\cos^2 \varphi + \sin^2 \varphi} = \left[\int_0^{2\pi} \cos^2 \varphi d\varphi = \pi = \int_0^{2\pi} \sin^2 \varphi d\varphi \right] = -(\pi + \pi) = -2\pi$$