2.解：由已知，得 $y' = \cancel{e^{-\frac{e^{t^2}}{2}}}\ e^{t^2}$

$y_{n+1} = y_n + hf(t,y) = y_n + \frac{e^{t^2}}{2}$

初始条件为 $y_0 = 0.5$ $\quad x = 0.5$ 则

则 $x = \overset{1}{\cancel{0.5}} \Rightarrow y_1 = y_0 + \frac{e^{0.5^2}}{2} = 0.5 + 0.6420127 = 1.140127$

$x = 1.5 \Rightarrow y_2 = y_1 + \cancel{8}\frac{e^{1^2}}{2} = 2.5011536$

$x = 2.0 \Rightarrow y_3 = y_2 + \frac{e^{1.5^2}}{2} = 7.2450215$

5.解：由已知，不妨设 $Re(\lambda) \le 0$，则

$y_{n+1} = y_n + \frac{1}{2}h[\lambda y_n + \lambda y_{n+1}] = (1+\frac{\lambda h}{2})y_n + \frac{\lambda h}{2}y_{n+1}$

假设 y_n 存在扰动 δ_n，则它引起 y_{n+1} 的误差为

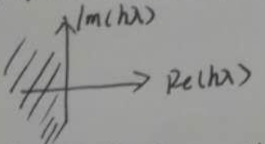
$\delta_{n+1} = (1+\frac{h\lambda}{2})\delta_n + \frac{h\lambda}{2}\delta_{n+1}$

$\Rightarrow \delta_{n+1} = \frac{2+h\lambda}{2-h\lambda}\delta_n$

要保证稳定性，则 $\left|\frac{2+h\lambda}{2-h\lambda}\right| \le 1$ $\quad$ ∵ $h>0$ $Re(\lambda) \le 0$

∴ $\left|\frac{2+h\lambda}{2-h\lambda}\right| \le 1$ 恒成立

∴梯形法无条件稳定，稳定区域为：



阴影部分

6.解：向后欧拉法：$l_{n+1} = y(t_{n+1}) - y_{n+1}$

$= y(t_n) + hy'(t_n) + \frac{h^2}{2}y''(\xi) - y(t_n) - h\lambda y_{n+1}$

$= h\lambda y'(t_{n+1}) + O(h^2) - h\lambda y_{n+1}$

$= hf'_y(t_{n+1},\xi)[y(t_{n+1}) - y_{n+1}] + O(h^2)$

$= hf'_y(t_{n+1},\xi)\cdot l_{n+1} + O(h^2)$

$\Rightarrow l_{n+1} = \frac{1}{1-hf'_y(t_{n+1},\xi)}O(h^2)$ 具有1阶准确度

梯形法：$L_{n+1}=u(t_{n+1})-u(t_n)-\frac{h}{2}[f(t_n,u(t))+f(t_{n+1},u(t_{n+1}))]$

$$=\int_{t_n}^{t_{n+1}}u'(t)dt-\frac{h}{2}[f(t_n,u(t))+f(t_{n+1},u(t_{n+1}))]$$

令 $\tau=\frac{t_{n+1}-t_n}{h}$　$dt=hd\tau$，则有

原式 $=\int_0^1 u'(t_n+\tau h)hd\tau-\frac{h}{2}[f(t_n,u(t_n))+f(t_{n+1},u(t_{n+1}))]$

$$=\int_0^1[(1-\tau)u'(t_n)+\tau u'(t_{n+1})+\frac{\tau(\tau-1)h^2}{2}u'''(t_n+\theta h)]hd\tau$$

$$-\frac{h}{2}[f(t_n,u(t_n))+f(t_{n+1},u(t_{n+1}))]$$

$$=\frac{h}{2}[u'(t_n)+u'(t_{n+1})]-\frac{h}{2}[f(t_n,u(t_n))+f(t_{n+1},u(t_{n+1}))]$$

$$-\frac{h^3}{2}u'''(t_n+\theta h)$$

$$=\frac{h^3}{2}u'''(t_n+\theta h)=O(h^3)$$

∴具有2阶准确度

11.解：(1) 由已知，得

$y'=-100y+100t^2+2t \Rightarrow \lambda=-100$

∴ $h\le\frac{-2}{\lambda}=\frac{-2}{-100}=0.02$

∴当 $h\le0.02$ 时，计算稳定

(2) 同理，有

$h\le\frac{-2.78}{\lambda}=0.0278$

当 $h\le0.0278$ 时，计算稳定

(3) ∵梯形公式无条件稳定

∴ h 无限制