1.解:由已知,得

$$e_r(\hat{R}) = \frac{\hat{R}-R}{R}$$

$$|e_r(\hat{V})| = \left|\frac{\frac{4}{3}\pi\hat{R}^3 - \frac{4}{3}\pi R^3}{\frac{4}{3}\pi R^3}\right| \le 1\%$$

∴ $0.99 \le (\frac{\hat{R}}{R})^3 \le 1.01$

解得 $0.9966555 \le \frac{\hat{R}}{R} \le 1.0033222$

∴ $|e_r(\hat{R})| = \left|\frac{\hat{R}-R}{R}\right| = 0.0033$

2.解:(1)由已知,得

$$e(\sin\hat{x}) = \sin\hat{x} - \sin x = \sin(x+h) - \sin x$$

(2)由已知,得

$$e_r(\sin\hat{x}) = \frac{\sin\hat{x} - \sin x}{\sin x} = \frac{\sin(x+h) - \sin x}{\sin x}$$

(3)由已知,得

$$(\sin x)' = \cos x$$

条件数 $cond \approx \left|\frac{x\cos x}{\sin x}\right|$

(4)由已知,得

当 $x = n\pi$,其中 $n \in Z$.且 $n \ne 0$ 时,这个问题高度敏感

4.解:由已知,得 $y_1 = y_0 - \frac{1}{100}\sqrt{783}$

$$y_2 = y_1 - \frac{1}{100}\sqrt{783} = y_0 - \frac{2}{100}\sqrt{783}$$

∴ $y_{100} = y_0 - \sqrt{783}$ 　　又∵ $\sqrt{783} \approx 27.982$

∴ 相对误差限为:

$$|e_r(\hat{x})| \le \frac{1}{2d_0} \times 10^{1-p} = \frac{1}{2\times2} \times 10^{-4}$$

∴ 误差限为 $|e_r(\hat{x})| \times \sqrt{783} \approx 0.0007$

5.解:由已知,得 $|\hat{S} - S| = |\hat{L}^2 - L^2| \le 1$

∴ $|(\hat{L}+L)(\hat{L}-L)| \le 1$

∵ $L \approx 100cm$ 　∴ $\hat{L} + L \approx 200cm$

∴ $|(\hat{L}-L)| \le \frac{1}{200} = 0.005$

∴误差不能超过0.005cm

7、解：由已知，可设

$f_1(x) = \frac{1}{(x+1)^6}$ $f_2(x) = (3-2x)^3$ $f_3(x) = \frac{1}{(3+2x)^3}$ $f_4(x) = 99-70x$

则有：$cond_1 \approx \left|\frac{-6x}{x+1}\right| = \left|\frac{6x}{x+1}\right|$

$cond_2 \approx \left|\frac{6x}{3-2x}\right|$

$cond_3 \approx \left|\frac{6x}{3+2x}\right|$

$cond_4 \approx \left|\frac{70x}{99-70x}\right|$

当 $x=\sqrt{2}$ 时，代入有：

$cond_1 \approx 3.5147$ $cond_2 \approx 49.4558$ $cond_3 \approx 1.4558$

$cond_4 \approx 19600 1.0000$

$\because cond_3$ 最小 $\therefore \frac{1}{(3+2\sqrt{2})^3}$ 得到的结果最好