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15. 由已知，得

线性多步法需满足：

$$\begin{cases} \alpha_1+\alpha_2+\alpha_3=1 & ① \\ \alpha_1+2\alpha_2+3\alpha_3=\beta_0+\beta_1+\beta_2+\beta_3 & ② \\ \frac{1}{2}(\alpha_1+4\alpha_2+9\alpha_3)=\beta_1+2\beta_2+3\beta_3 & ③ \end{cases}$$

在这里，$\alpha_1=-\alpha \quad \alpha_2=\alpha \quad \alpha_3=1 \quad \beta_0=0 \quad \beta_1=\beta_2=\frac{1}{2}(3+\alpha)$

$\beta_3=0$

$\therefore \alpha_1+\alpha_2+\alpha_3=-\alpha+\alpha+1=1$ 满足①

$\alpha_1+2\alpha_2+3\alpha_3=-\alpha+2\alpha+3=3+\alpha$

$\beta_0+\beta_1+\beta_2+\beta_3=3+\alpha=\alpha_1+2\alpha_2+3\alpha_3$ 满足②

$\frac{1}{2}(\alpha_1+4\alpha_2+9\alpha_3)=\frac{1}{2}(-\alpha+4\alpha+9)=\frac{9+3\alpha}{2}$

$\beta_1+2\beta_2+3\beta_3=\frac{3}{2}(3+\alpha)=\frac{9+3\alpha}{2}=\frac{1}{2}(\alpha_1+4\alpha_2+9\alpha_3)$ 满足③

$\therefore L_{n+1}=O(h^{3+1})=O(h^4)$

故存在一个α值，使原线性多步法是4阶的。