P235. 3. 解：(1) $\|f\|_\infty = |f(0)| = |(-1)^3| = 1$

$\|f\|_1 = \int_0^1 |(x-1)^3|\,dx = \frac{1}{4}$

$\|f\|_2 = \left[\int_0^1 [(x-1)^3]^2 dx\right]^{\frac{1}{2}} = \frac{\sqrt{7}}{7}$

(2) $\|f\|_\infty = |f(1)| = \frac{1}{2}$

$\|f\|_1 = \int_0^1 |x-\frac{1}{2}|\,dx = \frac{1}{4}$

$\|f\|_2 = \left[\int_0^1 (x-\frac{1}{2})^2 dx\right]^{\frac{1}{2}} = \frac{\sqrt{3}}{6}$

4. ~~解~~：证明：

(a) 不构成内积，理由：

$\because f(x)=0 \Rightarrow \langle f,f\rangle = \int_a^b [f(x)]^2 dx = 0 \Leftrightarrow f(x)=0$

但反之不成立

(b) 构成内积

6. 解：~~由~~ (1) 由已知，得 $\int_0^1 e^t dt = e-1$

$\int_0^1 te^t dt = 1$

$\therefore \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e-1 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} x_1 = 4e-10 \\ x_2 = 18-6e \end{matrix}$

$\therefore S(t) = (18-6e)t + 4e-10$ 即为所求

(2) 由已知，得

$\int_0^1 \cos(\pi t)\,dt = 0$

$\int_0^1 t\cos(\pi t)\,dt = \frac{-2}{\pi^2}$

$\therefore \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-2}{\pi^2} \end{bmatrix}$ $\therefore S(t) = \frac{12}{\pi^2} - \frac{24}{\pi^2}t$ 即为所求

$\therefore x_1 = \frac{12}{\pi^2} \quad x_2 = -\frac{24}{\pi^2}$

11. 解：由已知，得

$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \quad f = \begin{bmatrix} 4 \\ 4.5 \\ 6 \\ 8 \\ 8.5 \end{bmatrix}$

由 $A^TAx = A^Tf$ 有：

$\begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 31 \\ 105.5 \end{bmatrix}$

$\Rightarrow x_1 = \cancel{8.05}\ 2.45$

$x_2 = \cancel{0.45}\ 1.25$

所以，S(t)=2.45+1.25t 即为所求。