P235. 7. 证明：使用正交产生公式得 φ_1

$\varphi_{-1}=0 \quad \varphi_0=1 \quad \varphi_1=t-\frac{\langle t,\varphi_0\rangle}{\langle \varphi_0,\varphi_0\rangle}\varphi_0=t-\frac{\langle t,1\rangle}{\langle 1,1\rangle}$

$\varphi_2=t^2-\frac{\langle t^2,1\rangle}{\langle 1,1\rangle}-\frac{\langle t^2,\varphi_1\rangle}{\langle \varphi_1,\varphi_1\rangle}\varphi_1$

$\langle t\varphi_1-a_1\varphi_1-b_1\varphi_0,\varphi_1\rangle=\langle t\varphi_1,\varphi_1\rangle-a_1\langle\varphi_1,\varphi_1\rangle$

$=\langle t\varphi_1,\varphi_1\rangle-\frac{\langle t\varphi_1,\varphi_1\rangle}{\langle\varphi_1,\varphi_1\rangle}\langle\varphi_1,\varphi_1\rangle=0$

$\langle t\varphi_1-a_1\varphi_1-b_1\varphi_0,\varphi_0\rangle=\langle t\varphi_1-b_1\varphi_0,\varphi_0\rangle=\langle t\varphi_1-b_1,1\rangle=\langle t(t-\frac{\langle t,1\rangle}{\langle 1,1\rangle})-b_1,1\rangle$

$=\langle t^2,1\rangle-\langle t\frac{\langle t,1\rangle}{\langle 1,1\rangle},1\rangle-\langle\frac{\langle t\varphi_1,1\rangle}{\langle 1,1\rangle},1\rangle=\langle t^2,1\rangle-\langle t\frac{\langle t,1\rangle}{\langle 1,1\rangle},1\rangle-\langle t(t-\frac{\langle t,1\rangle}{\langle 1,1\rangle}$

$),1\rangle=0$

$\therefore t\varphi_1-a_1\varphi_1-b_1\varphi_0$ 为正交多项式且最高次数为2，若 t^2 系数为1

$\therefore$ 递推公式 $\varphi_2=t\varphi_1-a_1\varphi_1-b_1\varphi_0$ 满足

令 $\varphi_0,\varphi_1,\varphi_2\cdots\varphi_n$ 满足递推公式，只要证明 $t\varphi_n-a_n\varphi_n-b_{n-1}\varphi_{n-1}$ 具有正交性且最高次为 $n+1$ 其系数为1，即得证递推公式满足 $n+1$，从而递推公式得证。

$\langle t\varphi_n-a_n\varphi_n-b_n\varphi_{n-1},\varphi_n\rangle=\langle t\varphi_n-a_n\varphi_n,\varphi_n\rangle=\langle t\varphi_n,\varphi_n\rangle-a_n\langle\varphi_n,\varphi_n\rangle=\langle t\varphi_n,\varphi_n\rangle$

$-\frac{\langle t\varphi_n,\varphi_n\rangle}{\langle\varphi_n,\varphi_n\rangle}\langle\varphi_n,\varphi_n\rangle=0$

$\therefore$ 递推公式有 $t\varphi_j=\varphi_{j+1}+a_j\varphi_j+b_j\varphi_{j-1}$ $\langle t\varphi_n-a_n\varphi_n-b_n\varphi_{n-1},\varphi_{n-1}\rangle$

$=0$

$\therefore$ 递推公式得证

8. 解：由已知，得

$(f(x),P_0(x))=\int_{-1}^{1}\sin\frac{\pi}{2}x\,dx=0 \qquad (f(x),P_1(x))=\int_{-1}^{1}x\sin\frac{\pi}{2}x\,dx$

$(f(x),P_2(x))=\int_{-1}^{1}(\frac{3}{2}x^2-\frac{1}{2})\sin\frac{\pi}{2}x\,dx$

$(f(x),P_3(x))=\int_{-1}^{1}(\frac{5}{2}x^3-\frac{3}{2}x)\sin\frac{\pi}{2}x\,dx$

$a_0^*=\frac{(f(x),P_0(x))}{2} \qquad a_1^*=\frac{3(f(x),P_1(x))}{2} \qquad a_2^*=\cdots \quad a_3^*=\cdots$

得 $S_3^*(x)=a_0^*P_0(x)+a_1^*P_1(x)+a_2^*P_2(x)+a_3^*P_3(x)$

$\approx 1.5531913x-0.5622285x^3$