AI (FALL 2018) - ASSIGNMENT 2
CSP and KRR
Due: 11:59pm, Sunday, Oct. 28,2018

1. Provide formulations for each of the following problems as CSPs: specify the variables, domains and constraints.

   (a) Magic square: an *order 3* magic square is a 3×3 square grid. Fill it with distinct positive integers in the range $1, 2, ..., 9$ such at the sum of the integers in each row, column and diagonal is equal.

   (b) Hamiltonian tour: given a graph of n cities connected by roads, choose a particular order to visit all cities without repeating any.

   (c) Crypto-arithmetic puzzle: $\texttt{INT} \times \texttt{L} = \texttt{AAAI}$. For the equation to be correct, find out what distinct digit each letter represents.

2. Consider the following CSP with binary constraints. There are 4 variables: A, B, C, D with their respective domains:

   $D_A = \{1, 2, 3, 4\}$, $D_B = \{3, 4, 5, 8, 9\}$, $D_C = \{2, 3, 5, 6, 7, 9\}$, $D_D = \{3, 5, 7, 8, 9\}$.

   The constraints are:

   - $C_1 : A \geq B$
   - $C_2 : B > C$ or $C - B = 2$
   - $C_3 : C \neq D$

   (a) Find the first solution by using the Forward Checking algorithm with the MRV heuristics, *i.e.*, always choose the variable with smallest remaining number of elements in the domain to instantiate, breaking ties in the alphabetic order. Assign values in the current domain of each variable in increasing order. At each node indicate:

      i. The variable being instantiated and the value being assigned to it.
      ii. The CurDom for each variable.
      iii. Mark any node with an empty CurDom with DWO.

   (b) Enforce GAC on the constraints and give the resultant variable domains. You should show which values of a domain are removed at each step, and which arc is responsible for removing the value. After this first step, use the GAC algorithm to find the first solution.

3. Consider the following formulae asserting that a binary relation is symmetric, transitive, and serial:

   $S_1 : \forall x \forall y (P(x, y) \supset P(y, x))$

   $S_2 : \forall x \forall y \forall z ((P(x, y) \wedge P(y, z))) \supset P(x, z)$

   $S_3 : \forall x \exists y P(x, y)$

   Prove by resolution that

$$S_1 \wedge S_2 \wedge S_3 \supset \forall x P(x, x).$$

   In other words, if P is symmetric, transitive and serial, then P is reflexive.

4. Bob has been murdered, and Alexander, Peter and Michael are the only suspects (meaning exactly one of them is the murderer). Alexander says that Peter was the victim's friend, but that Michael hated the victim. Peter says that he was out of the town the day of the murder, and besides, he didn't even know the guy. Michael says that he saw Alexander and Peter with the victim just before the murder. You may assume that everyone- except possibly for the murderer-is telling the truth.

   - Use resolution to find the murderer. (formalize the facts as a set of clauses, prove that there is a murderer, and answer the question "who is the murderer ?")
   - Suppose we discover that we were wrong - we cannot assume that there was only a single murderer. Show that in this case the facts do not support anyone's guilt. (for each suspect, present a logical interpretation that supports all the facts but where that suspect is innocent and the other two are guilty).