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Prof. Rego CS 182

1. **Chapter 1.1 Exercise 32**: Construct a truth table for each of these compound propositions.

   a) $p \to \neg p$

p	$\neg p$	$p \to \neg p$
T	F	F
F	T	T

   b) $p \leftrightarrow \neg p$

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

   c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

   d) $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

   e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$q \to \neg p$	$p \leftrightarrow q$	$(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	T	T	T	T

   f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

2. **Chapter 1.3 Exercise 50**: In this exercise we will show that {↓} is a functionally complete collection of logical operators.

   a) Show that p ↓ p is logically equivalent to ¬p.

p	¬p	p ↓ p
T	F	F
F	T	T

   b) Show that (p ↓ q) ↓ (p ↓ q) is logically equivalent to p ∨ q.

p	q	p ↓ q	p ∨ q	(p ↓ q) ↓ (p ↓ q)
T	T	F	T	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	F

   c) Conclude from parts (a) and (b), and Exercise 49, that {↓} is a functionally complete collection of logical operators.

p	q	¬(p ∨ q)	p ↓ q	p ∧ q	p ↓ p	q ↓ q	(p ↓ p) ↓ (q ↓ q)
T	T	F	F	T	F	F	T
T	F	T	T	F	F	T	F
F	T	F	F	F	T	F	F
F	F	F	F	F	T	T	F

The logical NOR is a functionally complete collection of logical operators because it can be used by itself to be AND, OR, NOT.

Chapter 1.3 Exercise 52: Show that {|} is a functionally complete collection of logical operators.

p	¬p	p | p
T	F	F
F	T	T

p	q	p ∧ q	-(p ∧ q)	p | q	(p | q) | (p | q)	p ∨ q	p | p	q | q	(p | p) | (q | q)
T	T	T	F	F	T	T	F	F	T
T	F	F	T	T	F	T	F	T	T
F	T	F	T	T	F	T	T	F	T
F	F	F	T	T	F	F	T	T	F

The logical NAND is functionally complete because it can construct AND, OR, NOT using only itself.

3. **Chapter 1.4 Exercise 42**: Express each of these system specifications using predicates, quantifiers, and logical connectives.

   a) Every user has access to an electronic mailbox.
   Let U(x) = “x is a user”, M(y) = “y is a mailbox”, A(x,y) = “x has access to y”
   $\forall x\ (U(x) \rightarrow (\exists y\ (M(y) \land A(x, y))))$

   b) The system mailbox can be accessed by everyone in the group if the file system is locked.
   Let L = “the file system is locked”, M = “the system mailbox”, A(x,y) = “x has access to y”
   $L \rightarrow \forall x\ A(x, M)$

   c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.
   Let F(x) = “x is the firewall”, D(x) = “x is in a diagnostic state”, P(x) = “x is the proxy server”
   $\forall x\ \forall y\ ((F(x) \land D(x)) \rightarrow (P(y) \rightarrow D(y))$

   d) At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode.
   Let R(x) = “x is a router”, F(x) = “x is functioning normally”, T = “the throughput is between 100kbps and 500 kbps”, P(x) = “x is the proxy server”, D(x) = “x is in diagnostic state”
   $\forall x\ (T \land (P(x) \land \neg D(x))) \rightarrow (\exists y\ R(y) \land F(y))$

4. **Chapter 1.5 Exercise 36**: Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

   a) No one has lost more than one thousand dollars playing the lottery.
      Let L(x, y) = "in the lottery person x has lost y dollars"
      Original - $\neg\exists x\, \exists y(y > 1000 \land L(x, y))$
      Negation - $\exists x\, \exists y(y > 1000 \land L(x, y))$
      English – **Someone has lost more than one thousand dollars playing the lottery.**

   b) There is a student in this class who has chatted with exactly one other student.
      Let C(x, y) = "student x has chatted with student y"
      Original - $\exists x\, \exists y(y \neq x \land \forall z(z \neq x \rightarrow (z = y \leftrightarrow C(x, z))))$
      Negation - $\forall x\, \forall y(y \neq x \rightarrow \exists z(z \neq x \land (z = y \leftrightarrow C(x, z))))$
      English – **Every student in this class has either chatted with no other student or has chatted with more than one student.**

   c) No student in this class has sent e-mail to exactly two other students in this class.
      Let S(x, y) = "student x has sent email to student y"
      Original -$\neg\exists x\, \exists y\, \exists z(x \neq y \land x \neq z \land z \neq y \land \forall r(r \neq x \rightarrow (S(x, r) \leftrightarrow (r = y \lor r = z))))$
      Negation - $\exists x\, \exists y\, \exists z(x \neq y \land x \neq z \land z \neq y \land \forall r(r \neq x \rightarrow (S(x, r) \leftrightarrow (r = y \lor r = z))))$
      English – **Some student in this class has sent e-mail to exactly two other students in this class.**

   d) Some student has solved every exercise in this book.
      Let E(x, y) = "student x has solved exercise y"
      Original - $\exists x\, \forall y\, E(x, y)$
      Negation - $\forall x\, \exists y\, \neg E(x, y)$
      English – **All students have not solved every exercise in this book.**

   e) No student has solved at least one exercise in every section of this book.
      Let E(x, y) = "student x has solved exercise y", S(y, z) = "exercise y is in section z"
      Original - $\neg\exists x\, \forall z\, \exists y(S(y, z) \land E(x, y))$
      Negation - $\exists x\, \forall z\, \exists y(S(y, z) \land E(x, y))$
      English – **Some student has solved at least one exercise in every section of this book.**