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Assignment 2
Due: Wednesday, September 20, 2017, upload before 11:59pm

1) (10 pts.) Do Exercise 34 of Section 1.6 (page 80).
The Logic Problem, taken from WFF'N PROOF, The Game of Logic, has these two assumptions: 1. "Logic is difficult or not many students like logic." 2. "If mathematics is easy, then logic is not difficult." By translating these assumptions into statements involving propositional variables and logical connectives, determine whether each of the following are valid conclusions of these assumptions:

ASSUMPTIONS:
Let L = "Logic is difficult", S = "Many students like logic", M = "Mathematics is easy"
1. $L \vee \neg S$
2. $M \rightarrow \neg L$

a) That mathematics is not easy, if many students like logic.

1. $L \vee \neg S$	
2. $M \rightarrow \neg L$	
3. $\neg M \vee \neg L$	2, contrapositive
4. $\neg S \vee \neg M$	1, 3 resolution
5. $S \rightarrow \neg M$	4, contrapositive

b) That not many students like logic, if mathematics is not easy.
$\neg M \rightarrow \neg S$ is not a valid conclusion.

c) That mathematics is not easy or logic is difficult.
$\neg M \vee L$ is not a valid conclusion.

d) That logic is not difficult or mathematics is not easy.

1. $M \rightarrow \neg L$	
2. $\neg M \vee \neg L$	1, contrapositive

e) That if not many students like logic, then either mathematics is not easy or logic is not difficult.

1. $L \vee \neg S$	
2. $M \rightarrow \neg L$	
3. $\neg M \vee \neg L$	2, contrapositive
4. $S \vee (\neg M \vee \neg L)$	3, Addition
5. $\neg S \rightarrow (\neg M \vee \neg L)$	4, contrapositive

2) (10 pts.) Do Exercise 18 of Section 1.7 (page 91).
Prove that if n is an integer and $3n + 2$ is even, then n is even using

a) a proof by contraposition.
Assume n is odd, and that $n = 2x + 1$ by the definition of odd integers, in order to show that $3n + 2$ is odd.

$$\begin{aligned} 3n + 2 &= 3(2x + 1) + 2 \\ &= 6x + 5 \\ &= 6x + 4 + 1 \\ &= 2(3x + 2) + 1 \end{aligned}$$

It can be seen that $2(3x + 2) + 1$ is the same as $2x + 1$ as they are both two times an integer plus one, so $3n + 2$ is odd.

b) a proof by contradiction.
Assume $3n + 2$ is even and n is odd and $n = 2x + 1$ by the definition of odd integers

$$\begin{aligned} 3n + 2 &= 3(2x + 1) + 2 \\ &= 6x + 5 \\ &= 6x + 4 + 1 \\ &= 2(3x + 2) + 1 \end{aligned}$$

It can be seen that $2(3x + 2) + 1$ is the same as $2x + 1$ as they are both two times an integer plus one, so $3n + 2$ is odd. However since we assumed $3n + 2$ is even the contradiction proves it.

3) (10 pts.) Do Exercise 4 of Section 1.8 (page 108).
Use a proof by cases to show that min(a, min(b, c)) = min(min(a, b), c) whenever a, b, and c are real numbers.

6 cases:

1. $a \geq b \geq c$
   min(a, min(b, c)) = min(min(a, b), c)
   min(a, c) = min(b, c)
   c = c
2. $a \geq c \geq b$
   min(a, min(b, c)) = min(min(a, b), c)
   min(a, b) = min(b, c)
   b = b
3. $b \geq a \geq c$
   min(a, min(b, c)) = min(min(a, b), c)
   min(a, c) = min(a, c)
   c = c
4. $b \geq c \geq a$
   min(a, min(b, c)) = min(min(a, b), c)
   min(a, c) = min(a, c)
   a = a
5. $c \geq a \geq b$
   min(a, min(b, c)) = min(min(a, b), c)
   min(a, b) = min(b, c)
   b = b
6. $c \geq b \geq a$
   min(a, min(b, c)) = min(min(a, b), c)
   min(a, b) = min(a, c)
   a = a

Since all cases are true the statement is true.

4) (10 pts.) Prove that for any sets A and B, $A = (A - B) \cup (A \cap B)$.

1.	$A = (A - B) \cup (A \cap B)$	
2.	$= \{x \mid x \in A \wedge x \notin B\} \cup \{x \mid x \in A \wedge x \in B\}$	1, set difference, intersection
3.	$= \{x \mid (x \in A \wedge x \notin B) \vee (x \mid x \in A \wedge x \in B)\}$	2, union
4.	$= \{x \mid (x \in A \wedge (x \notin B \vee x \in B)\}$	3, distributive property
5.	$= \{x \mid x \in A\}$	
6.	$= A$	

What this proves is that when we combine the part of A that is in B and the part of A that isn’t in B we get A.

5) (10 pts.) Solve for the value of the following :

a) $$\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j)$$

1. 1 + 1 = 2
2. 1 + 2 = 3
3. 1 + 3 = 4
4. 2 + 1 = 3
5. 2 + 2 = 4
6. 2 + 3 = 5
7. = 21

b) $$\sum_{j=0}^{8}(3^j - 2^j)$$

1. 3^0 – 2^0 = 0
2. 3^1 – 2^1 = 1
3. 3^2 – 2^2 = 5
4. 3^3 – 2^3 = 19
5. 3^4 – 2^4 = 65
6. 3^5 – 2^5 = 211
7. 3^6 – 2^6 = 665
8. 3^7 – 2^7 = 2059
9. 3^8 – 2^8 = 6305
10. = 9330

6) (10 pts.) Let x and y be integers. Determine whether the following relations are reflexive, symmetric, antisymmetric, or transitive:

   a) $xy \geq 1$

      Reflexive: No, because we cannot have (0,1)
      Symmetric: Yes, because $xy = yx$.
      Antisymmetric: No, because (3, 5) and (5,3) are both in R.
      Transitive: Yes, because $xy \geq 1$ and $yz \geq 1$, so $xz \geq 1$.
      It is neither an equivalence or partial order relation.

   b) $x \equiv y \pmod 9$

      Reflexive: Yes, because $x \equiv x \pmod 9$
      Symmetric: Yes, because $x \equiv y \pmod 9 \rightarrow y \equiv x \pmod 9$
      Antisymmetric: No, because (0, 9) and (9,0) are both in R.
      Transitive: Yes, because $x \equiv y \pmod 9$ and $y \equiv z \pmod 9$, so $x \equiv z \pmod 9$.
      It is an equivalence relation.
      $[0] = \{\ldots, -9, 0, 9, 18, \ldots\}$
      $[1] = \{\ldots, -8, 1, 10, 19, \ldots\}$
      $[2] = \{\ldots, -7, 2, 11, 20, \ldots\}$
      $[3] = \{\ldots, -6, 3, 12, 21, \ldots\}$
      $[4] = \{\ldots, -5, 4, 13, 22, \ldots\}$
      $[5] = \{\ldots, -4, 5, 14, 23, \ldots\}$
      $[6] = \{\ldots, -3, 6, 15, 24, \ldots\}$
      $[7] = \{\ldots, -2, 7, 16, 25, \ldots\}$
      $[8] = \{\ldots, -1, 8, 17, 26, \ldots\}$

Justify your statements. Finally, determine which of the above relations are equivalence and partial order relations. For equivalence relations, construct the equivalence classes.