Assignment 6

Due: Sunday, December 3rd, 2017, upload before 11:59pm

1) (10 pts.) You roll a four-sided die 3 times. For this problem well use the sample space with 64 equally likely outcomes.

(a) Write down this sample space in set notation.

S = {111, 112, 113, 114, 121, …. , 344, 441, 442, 443, 444}

(b) List all the outcomes in each of the following events.

(i) A = Exactly 2 of the 3 rolls are fours'.

A = {144, 244, 344, 414, 424, 434, 441, 442, 443}

(ii) B = At least 2 of the 3 rolls are fours'.

B = {144, 244, 344, 414, 424, 434, 441, 442, 443, 444}

(iii) C = Exactly 1 of the second and third rolls is a 4.

C = {114, 124, 134, 141, 142, 143, 214, 224, 234, 241, 242, 243, 314, 324, 334, 341, 342, 343, 414, 424, 434, 441, 442, 443}

(iv) $A \cap C$

$A \cap C$ = {414, 424, 434, 441, 442, 443}

2) (15 pts.) Suppose you pick two cards from a deck of 52 playing cards. What is the probability that they are both queens?

a. Calculate the probability when order matters.

P = 4C2/ 52C2

= [4!/((2!)(2!))]/[52!/((2!)(50!))]

= (4*3/2)/(52*51/2)

= 12/(52*51)

= 12/2652

= 1/221

b. Calculate the probability when order doesn't matter.

P = (4/52)*(3/51)

= (1/13)*(1/17)

= 1/221

3) (10 pts.) You roll a twenty-sided die. Determine whether the following pairs of events are independent.

a. You roll an even number and You roll a number less than or equal to 10

P(even number) = 10/20 = ½

P(number less than or equal to 10) = 10/20 = ½

P(even number ∩ number less than or equal to 10) = 5/20 = ¼

P(even number) P(number less than or equal to 10) = ½ * ½ = ¼

Because the probability of the intersection is the same as the probability of the two events multiplied then the two events are independent.

b. You roll an even number and You roll a prime number.

P(even number) = 10/20 = ½

P(prime number) = 8/20 = 2/5

P(even number) P(prime number) = 2/10 = 1/5

P(even number) P(prime number)/ P(prime number) = (1/5)/(2/5) = 1/2

Because the probability of and even number conditioned on a prime number is equal to the probability of an even number then these are independent events because they have no effect on each other.

4) (15 pts.) Two dice are rolled.

A = sum of two dice equals 3

B = sum of two dice equals 7

C = at least one of the dice shows a 1

P(A) = 2/36 = 1/18

P(B) = 6/36 = 1/6

P(C) = 11/36

(a) What is P(A|C)?

P(AC) = 1/18 * 11/36 = 11/648

P(AC)/P(C) = (11/648)/(11/36) = 1/18

P(A|C) = 1/18

(b) What is P(B|C)?

P(BC) = 1/6 * 11/36 = 11/216

P(BC)/P(C) = (11/216)/(11/36) = 1/6

P(B|C) = 1/6

(c) Are A and C independent? What about B and C?

A and C are independent because P(A|C) = P(A). B and C are also independent because P(B|C) = P(B).

5) (20 pts.) Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on this test, the probability that you know the answer is p. If you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly?

Let C = "the event that the student answered the question correctly"

Let K = "the event that the student actually knows the answer"

$$P(K|C) = P(KC)/(PC)$$

$$= [P(C|K)P(K)]/[P(C|K)P(K) + P(C|K^C)P(K^C)$$

$$= p/[p + (1/c)(1-p)]$$

6) (10 pts.) Which is more likely: rolling a total of 8 when 2 dice are thrown or rolling a total of 8 when 3 dice are thrown?

A = total 8 with 2 dice

B = total 8 with 3 dice

P(A) = 5/36 = 30/216

P(B) = 21/216

It is more likely to roll a total of 8 when 2 dice are thrown than when rolling 3 dice because the probability of getting a total of 8 with 2 dice is higher.

7) (15 pts.) Suppose you have an urn containing 7 red and 3 blue balls. You draw three balls at random. On each draw, if the ball is red you set it aside and if the ball is blue you put it back in the urn. What is the probability that the third draw is blue? (If you get a blue ball it counts as a draw even though you put it back in the urn.)

P(RRB) = (7/10)(6/9)(3/8) = 7/40

P(RBB) = (7/10)(3/9)(3/9) = 7/90

P(BRB) = (3/10)(7/10)(3/9) = 7/100

P(BBB) = (3/10)(3/10)(3/10) = 27/1000

P(third draw blue) = 7/40 + 7/90 + 7/100 + 27/1000 = 787/2250

8) (20 pts.) Corrupted by their power, the judges running the popular game show Americas Next Top Scientist have been taking bribes from many of the contestants. Each episode, a given contestant is either allowed to stay on the show or is kicked off.

If the contestant has been bribing the judges she will be allowed to stay with probability 1. If the contestant has not been bribing the judges, she will be allowed to stay with probability 1/3.

Suppose that 1/4 of the contestants have been bribing the judges. The same contestants bribe the judges in both rounds, i.e., if a contestant bribes them in the first round, she bribes them in the second round too (and vice versa).

(a) If you pick a random contestant who was allowed to stay during the first episode, what is the probability that she was bribing the judges?

$$P(ep1) = (1/4 * 1) + (3/4 * 1/3)$$

$$= 1/2$$

(b) If you pick a random contestant, what is the probability that she is allowed to stay during both of the first two episodes?

$$P(ep2) = (1/4 * 1 * 1) + (3/4 * 1/3 * 1/3)$$

$$= 1/3$$

(c) If you pick random contestant who was allowed to stay during the first episode, what is the probability that she gets kicked off during the second episode?

$$P(ep2|ep1) = P(ep1 \cap ep2)/P(ep1)$$

$$= ((1/4 * 1 * 0) + (3/4 * 1/3 * 2/3))/(1/2)$$

$$= (1/6)/(1/2)$$

$$= 1/3$$

Bonus Problem: (20 points) Some games, like tennis or ping pong, reach a state called deuce. This means that the score is tied and a player wins the game when they get two points ahead of the other player. Suppose the probability that you win a point is p and this is true independently for all points. If the game is at deuce what is the probability you win the game?

Note: Provide justifications for all your solutions.

Note: Late submissions will not be accepted. You are allowed a maximum of 3 attempts to submit your assignment. Link for Submission is on Blackboard under ”Homework6” and save the file as ”FirstName. LastName.Assignment6”.